Problem 1 c) Prove that under any affine transformation, the ratio of parallel line segments is invariant, but the ratio of non-parallel line segments is not invariant.

Given some vector p, an affine transformation is defined as:

$A(p) = M \cdot p + b$

For parallel line segments:

As in part a), we know $f \cdot (k_1 - k_2) = (l_1 - l_2)$, $f \in \mathbb{R}$

Then: $\frac{\|(l_1 - l_2)\|}{\|(k_1 - k_2)\|} = \frac{\|f \cdot (k_1 - k_2)\|}{\|(k_1 - k_2)\|} = f \cdot \frac{\|(k_1 - k_2)\|}{\|(k_1 - k_2)\|} = f$

Using affine transformation:

$$\frac{\|(M \cdot l_1 + b) - (M \cdot l_2 + b)\|}{\|(M \cdot k_1 + b) - (M \cdot k_2 + b)\|} = \frac{\|M \cdot (l_1 - l_2)\|}{\|M \cdot (k_1 - k_2)\|}$$

$$= \frac{\|M \cdot f \cdot (k_1 - k_2)\|}{\|M \cdot (k_1 - k_2)\|} = \frac{f \cdot \|M \cdot (k_1 - k_2)\|}{\|M \cdot (k_1 - k_2)\|} = f$$

Q.E.D. ratio of parallel lines is preserved

For non-parallel line segments:

By counterexample:

Given $k_1 = (0,0,0)$, $k_2 = (0,1,0)$, $l_1 = (0,0,0)$, $l_2 = (0,0,1)$
Forming lines k and l, each with length $= 1$

Now suppose matrix M in A applies a multiplication across axis 2, then:
$k_1 = (0,0,0)$, $k_2 = (0,2,0)$, $l_1 = (0,0,0)$, $l_2 = (0,0,1)$
Now line k has length 2, but line l still has length 1.
Thus, the ratio of non-parallel line segments has not been preserved. Q.E.D.