Problem 4 a) Show that two 3x4 camera matrices $M$ and $M'$ can always be reduced to the following canonical forms by an appropriate projective transformation in 3D space, which is represented by a 4x4 matrix $H$.

$$\hat{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \hat{M}' = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First, find $H_0$ such that $\hat{M} = M \cdot H_0$:

The most obvious value for $H_0 = \begin{bmatrix} A^{-1} & 0 \\ 0 & 0 \end{bmatrix}$, but this creates problems later with $M'$.

We can find a more effective answer in $H_0 = \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix}$, which allows us to cancel terms and produce same effect.

Next, we need to find $H_1$ such that $\hat{M} = M \cdot H_0 \cdot H_1$ and $\hat{M}' = M' \cdot H_0 \cdot H_1$

First, $M' \cdot H_0 = [A' \; b'] \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix} = [A'A^{-1} \;\; -A'A^{-1}b + b'] = \underset{3\times 4}{C}$

Let $C$ be written in row vector form: $C = \begin{bmatrix} — & c_1 & — \\ — & c_2 & — \\ — & c_3 & — \end{bmatrix}$

Now, find $H_1$ such that: $C \cdot H_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$