

Problem 4 b) Given a 4×4 matrix H representing a projective transformation in 3D space. Prove that the fundamental matrices corresponding to the two pairs of camera matrices (M, M') and (MH, MH') are the same.

Given a 3D point P in the real world, we can write the value of P in each camera space as:

$$p = M \cdot P, \quad p' = M' \cdot P$$

When the camera space is transformed to (MH, MH') we can observe the transformation of the point P as:

$$P_H = H^{-1} \cdot P, \quad P'_H = H^{-1} \cdot P'$$

Finally,

$$\begin{aligned} P_H &= MH P_H \\ &= MH H^{-1} P \\ &= MP \\ &= p \end{aligned}$$

and

$$\begin{aligned} P'_H &= MH' P'_H \\ &= MH' H^{-1} P' \\ &= MP' \\ &= p' \end{aligned}$$

Therefore, the fundamental matrices corresponding to the two pairs of camera matrices are the same.