

b) We run MH Sampler a long time and collect M samples $(c_1, [m]), \dots, (c_M, [m])$ for $m=1, \dots, M$ after mixing

$$P(c_i | v_1, \dots, v_K) = P(c_i = k | v_1, \dots, v_K) \text{ for } k=1, \dots, K$$

$$\text{Let } g(c_i) = 1[c_i = k]$$

$$\hat{g}(c_1, [1], \dots, c_M, [M]) = \frac{1}{M} \sum_{m=1}^M g(c_m, [m])$$

c) For Gibbs Sampling to work, it must be true that

$$A(c_i | c) = 1 = \min(1, 1)$$

$$= \frac{P(v_i | c_{i \neq j}) \cdot P(v_i | c_{i=j})}{P(v_i | c_{i \neq j}) \cdot P(v_i | c_{i=j})}$$

This will not work, because there is no guarantee this quantity = 1