

Problem 1 b) Consider a unit square  $p, q, r, s$  in the world reference system where  $p, q, r$ , and  $s$  are points. Will the same square in the camera reference system always have unit area? Prove or provide a counterexample.

Given a square  $p, q, r, s$ ,  $\text{Area} = \|(q-p) \times (s-p)\|$ , in this case  $\text{Area} = 1$

We can describe the transformation matrix  $\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$  as a single matrix:  $M$

Since the transform from matrix  $M$  is isometric,  $\det(M) = 1$

We can write the transformed formula for area as:

$$\begin{aligned} & \| (M \cdot q - M \cdot p) \times (M \cdot s - M \cdot p) \| \\ &= \det(M) \cdot \|(q-p) \times (s-p)\| \\ &= \|(q-p) \times (s-p)\| \\ &= 1 \quad \text{Q.E.D.} \quad \text{area is preserved} \end{aligned}$$