Problem 1 a) Prove that parallel lines in the world reference System are still parallel in the Camera reference System.

We will use k and l, as in the notes, to refer to two parallel lines, with points k_1, k_2 on k and l_1, l_2 on l.

Since k and l are parallel, by definition: $f \cdot (k_1 - k_2) = \cdot (l_1 - l_2)$, $f \in \mathbb{R}$

Let R be a 3x3 rotation matrix, and T be a 3x1 translation vector.

By definition of parallel lines: $(k_1 - k_2) \times (l_1 - l_2) = 0$

We want to show that the rotated and translated lines still have cross product = 0:

$((R \cdot k_1 + T) - (R \cdot k_2 + T)) \times ((R \cdot l_1 + T) - (R \cdot l_2 + T))$

$= (R \cdot k_1 - R \cdot k_2) \times (R \cdot l_1 - R \cdot l_2)$

$= R \cdot (k_1 - k_2) \times R \cdot (l_1 - l_2)$ → distributive property

$= R \cdot (k_1 - k_2) \times f \cdot R \cdot (k_1 - k_2)$ → $f \cdot (k_1 - k_2) = (l_1 - l_2)$, $f \in \mathbb{R}$

$= f \cdot (R \cdot (k_1 - k_2) \times R \cdot (k_1 - k_2))$ → compatible w/ scalar multiplication

$= f \cdot 0$ → self cross product of vector is 0 vector

$= 0$ Q.E.D. parallelism is preserved