

# STAT 210

## Applied Statistics and Data Analysis:

### Homework 1

Due on Sept. 11/2022

#### Question 1

Consider the following system of equations:

$$\begin{aligned}3x + 2y + 2z + 4w &= 28 \\2x + y + z &= 14 \\2x + 5z + 5w &= 28 \\6x + 2y + 2z + w &= 37\end{aligned}$$

1. Create a matrix in **R** with the coefficients of the system, and a vector with the constants on the right-hand side of the equations. Call them **mat1** and **vec1**, respectively.
2. Find the inverse of **mat1** and call it **mat2**.
3. Create a list named **list1** having as components **mat1**, **vec1**, and **mat2**. Call these components **item1**, **item2**, and **item3**, respectively.
4. Remove **mat1**, **vec1**, and **mat2** from the working directory.
5. Solve the system of equations and call the solution **vec2**.
6. Verify the solution.
7. Verify that if you multiply the inverse matrix **mat2** by **vec1** you get the solution.
8. Find the eigenvectors of **mat1** and **mat2** and verify that the eigenvectors of **mat2** are the reciprocals of the eigenvectors of **mat1**. 倒数

#### Question 2

Consider the function  $f(x) = e^{-|x|}$ , for  $x \in \mathbb{R}$ . We want to use the MonteCarlo method to estimate the value of the integral

$$\int_{-2}^2 f(x) dx$$

1. Plot a graph of this function in the region where you want to calculate the integral.
2. Generate  $N = 1000$  random numbers with uniform distribution in the rectangle  $[-2, 2] \times [0, 1]$ . Count how many points fall below the curve  $f(x) = e^{-|x|}$  and estimate the integral using the fraction of these points with respect to the total number of points and the area of the rectangle. Call the estimator  $I_{1000}$ .
3. Compute analytically the value of the integral and compare with the approximation you obtained in 3. Call  $I$  the value of the integral and calculate  $|I - I_{1000}|$ .
4. Repeat for  $N = 10^k$  for  $k = 4, 5, \dots, 8$  and compute the deviation  $|I - \bar{I}_N|$  from the exact result.

5. Do a log-log plot of the deviation as a function of  $N$ . The points should follow approximately a straight line.