# STAT 210
# Applied Statistics and Data Analysis:
# Homework 7
# Solution

Due on Nov. 06/2022

## Question 1

For this question use the dataset `data1`.

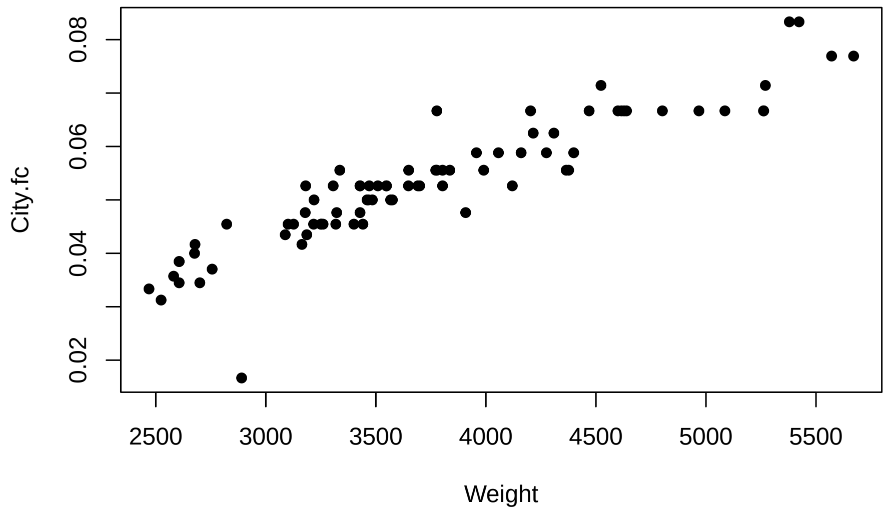
This dataset has information on fuel efficiency, measured in miles per gallon, and seven other variables for 80 different car models. There are two variables related to fuel efficiency, `City.Mpg` and `Highway.Mpg`. We will only consider `City.Mpg`, and we will work with the reciprocal of this variable, 1/`City.Mpg`, which we will call `City.fc` for fuel consumption. We want to explore the relation between this variable and the car's weight (`Weight`).

(i) Read the data and define a new variable called `City.fc` in the data frame equal to the reciprocal of `City.Mpg`. Draw a scatterplot of `City.fc` as a function of `Weight`. Fit a simple linear regression for `City.fc` as a function of `Weight` and add the line to the plot. Comment. Obtain a summary of the regression and comment.

```
data1 <- read.table('data1.txt', header = T)
str(data1)
```

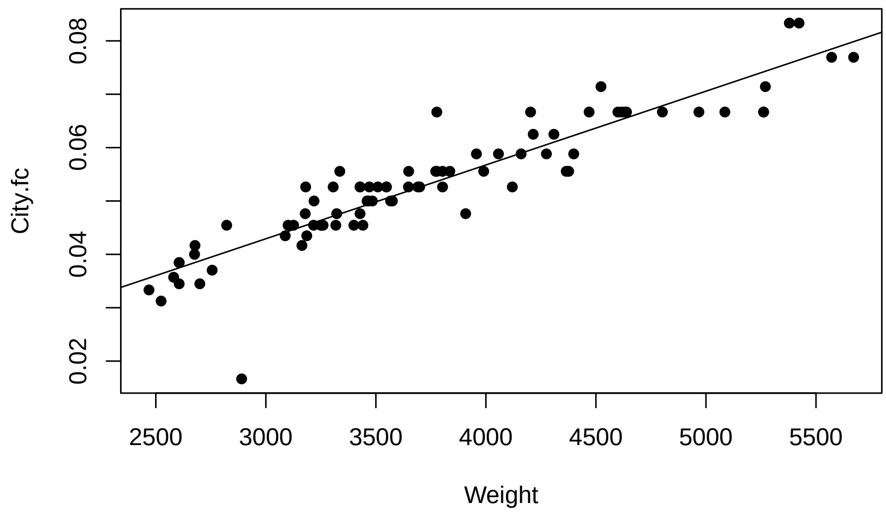
```
## 'data.frame':    81 obs. of  9 variables:
##  $ Model      : chr  "A4" "3_Series" "G35" "X-Type" ...
##  $ Eng.Size   : num  1.8 2.5 3.5 2.5 1.8 2.7 2.5 3 3 3.2 ...
##  $ Cylinders  : int  4 6 6 6 4 6 6 6 6 6 ...
##  $ MSRP       : int  25550 28100 28150 29330 29250 42650 39800 43730 38875 50575 ...
##  $ City.Mpg   : int  22 20 18 19 22 18 19 18 18 19 ...
##  $ Highway.Mpg: int  31 29 26 28 30 25 28 26 25 27 ...
##  $ Weight     : int  3252 3219 3336 3428 3250 3836 3428 3777 3649 3691 ...
##  $ Type       : chr  "Sedan" "Sedan" "Sedan" "Sedan" ...
##  $ Country    : chr  "Germany" "Germany" "Japan" "England" ...
```

```
data1$City.fc <- 1 / data1$City.Mpg
plot(City.fc ~ Weight, data = data1, pch = 16)
```




We fit a model with the function `lm` and add the regression line to the scatterplot:

```
model1 <- lm(City.fc ~ Weight, data = data1)
plot(City.fc ~ Weight, data = data1, pch = 16)
abline(model1)
```




The line seems to fit the data quite well. For the summary we write

```
summary(model1)
```

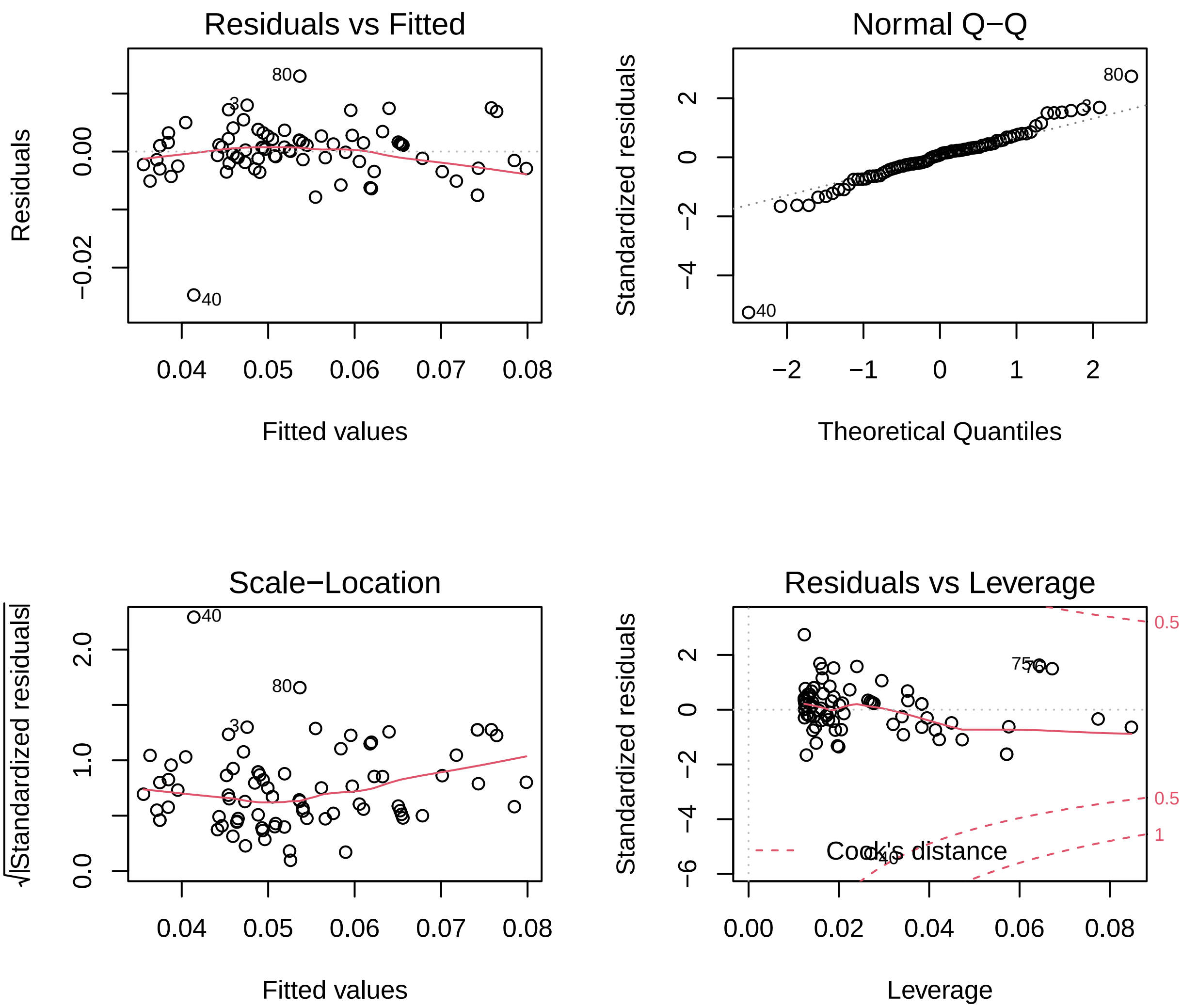
```
##
## Call:
## lm(formula = City.fc ~ Weight, data = data1)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.024734 -0.002015  0.000386  0.002133  0.013001
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.439e-03  2.571e-03    0.56    0.577
## Weight      1.383e-05  6.700e-07   20.64   <2e-16 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.004771 on 79 degrees of freedom
## Multiple R-squared:  0.8435, Adjusted R-squared:  0.8416
## F-statistic: 425.9 on 1 and 79 DF,  p-value: < 2.2e-16
```

The estimated values for the intercept and slope are $1.44 \times 10^{-3}$ and $1.38 \times 10^{-5}$, respectively. `Weight` has a very small $p$-value while the $p$-value for the intercept is big, which says that the intercept is not significantly different from zero..

(ii) Draw the diagnostic plots. Do you identify any point as an outlier? If you do, which point is this? Can you identify this point in the initial scatterplot? Can you find a reason why this point is different from the rest?

```r
par(mfrow = c(2,2))
plot(model1)
```




```r
par(mfrow = c(1,1))
```

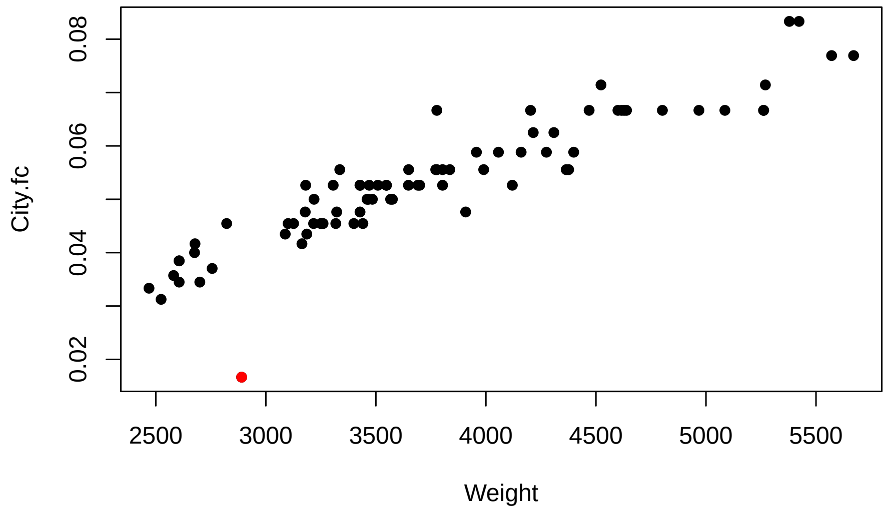
Point 40 is flagged in all the diagnostic plots. It is the point with biggest residual and in the quantile plot is very far from the rest of the points and the reference line. In the Scale-Location plot, the value corresponding to this point is bigger that 2. To identify the point in position 40 we write

```
data1[40,]
```

```
##    Model Eng.Size Cylinders  MSRP City.Mpg Highway.Mpg Weight  Type Country
## 40 Prius      1.5         4 20295       60          51   2890 Sedan   Japan
##       City.fc
## 40 0.01666667
```

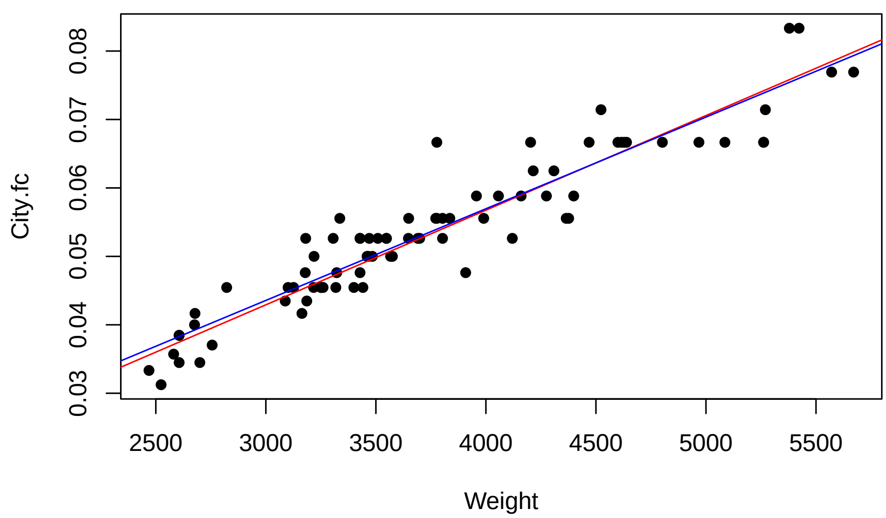
We see that the car model is Prius, a hybrid car, which uses a different system than the rest of the cars in this dataset. We repeat the scatterplot and plot this point in red

```
plot(City.fc ~ Weight, data = data1, pch = 16)
points(City.fc ~ Weight, data = data1[40,], pch = 16, col = 'red')
```




(iii) Fit a new regression model excluding the outlier(s) you identified in the previous section. Draw a scatterplot with both regression lines. Compare the summary tables. Draw the diagnostic plots and comment.

```
data1N <- data1[-40,]
model2 <- lm(City.fc ~ Weight, data = data1N)
plot(City.fc ~ Weight, data = data1N, pch = 16)
abline(model1, col = 'red')
abline(model2, col = 'blue')
```



The two lines are very close, almost indistinguishable.

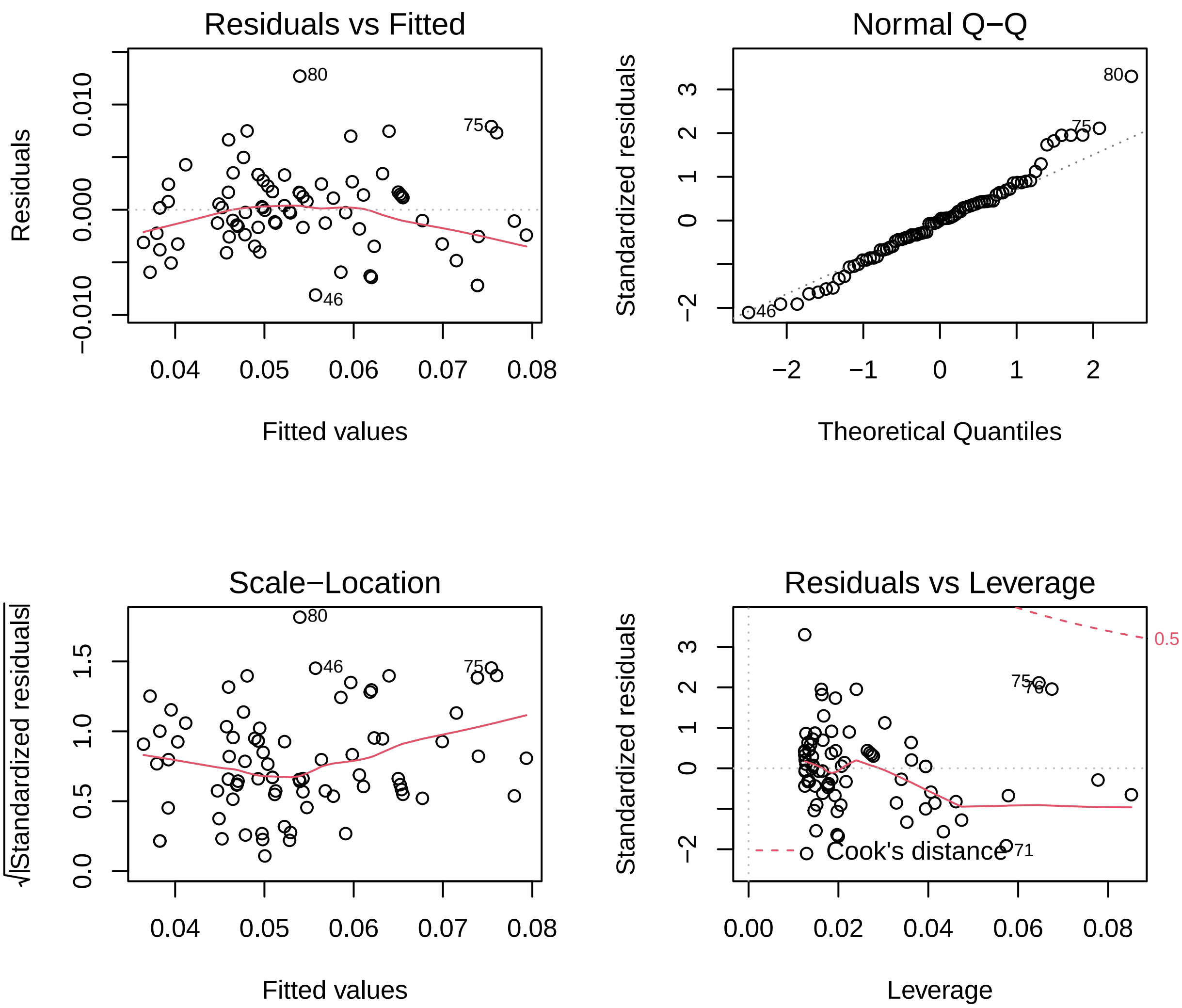
```
summary(model2)
```

```
##
## Call:
## lm(formula = City.fc ~ Weight, data = data1N)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0081052 -0.0023791  0.0000656  0.0017292  0.0126971
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.380e-03  2.108e-03   1.604    0.113
## Weight      1.339e-05  5.479e-07  24.448   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003872 on 78 degrees of freedom
## Multiple R-squared:  0.8846, Adjusted R-squared:  0.8831
## F-statistic: 597.7 on 1 and 78 DF,  p-value: < 2.2e-16
```

The summary tables show some differences but they are small. For instance, the intercept changes from $1.44 \times 10^{-3}$ to $3.38 \times 10^{-3}$ and the slope from $1.383 \times 10^{-5}$ to $1.339 \times 10^{-5}$, which are small changes. The residual standard errors are 0.00477 and 0.00387, and the $R^2$ are 0.844 and 0.885.

We now plot the diagnostic graphs

```
par(mfrow = c(2,2))
plot(model2)
```




```r
par(mfrow = c(1,1))
```

We see that the main difference with the previous model occurs in the quantile plot. The plot excluding point 40 looks better now.

(iv) Run the Shapiro-Wilk test on the residuals for both models and compare the results.

```r
shapiro.test(residuals(model1))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(model1)
## W = 0.88433, p-value = 2.496e-06
```

```r
shapiro.test(residuals(model2))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  residuals(model2)
## W = 0.97636, p-value = 0.1444
```

We see that when point 40 is included, the $p$-value is small and the null hypothesis of normality of the residuals is rejected. On the other hand, when this point is excluded, the $p$ value is large and the null hypothesis is not rejected.

Summing up, point 40 is an outlier but not an influential point in the regression, since the regression equation is not substantially changed when the point is excluded. However, when the point is included, the assumption of normality for the residuals is not verified.

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## Question 2

For this question use the data set `data2`.

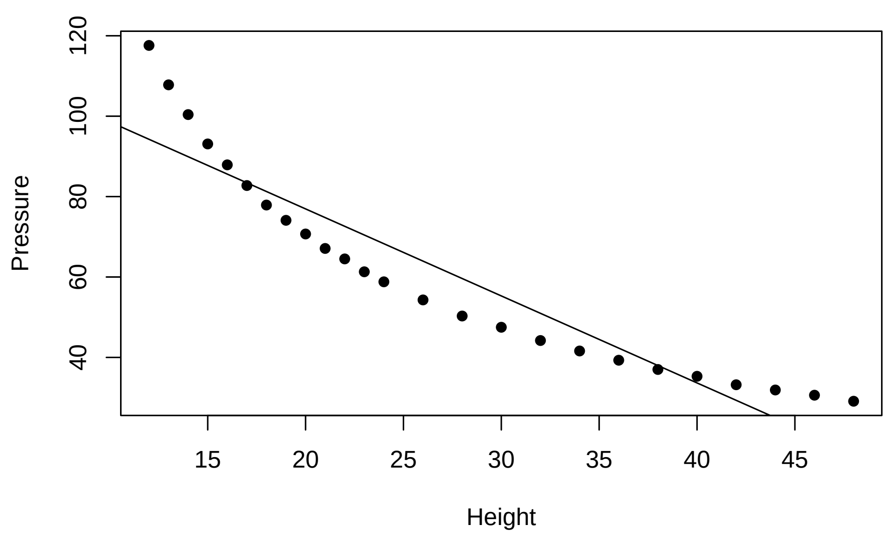
The data for this question come from an experiment to determine the relation between the volume of a gas and the pressure. The file has two variables, `Height` and `Pressure`. Height corresponds to the height of a cylindrical container with a fixed circular base with a movable top that allowed changing the volume of the container. `Height` was measure in inches. `Pressure` is measured in inches of mercury as in a barometer. We want to study the relation between these two variables.

(i) Read `data2` and plot `Pressure` as a function of `Height`. Fit a simple linear regression for `Pressure` as a function of `Height` and add the regression line to the plot. Comment. Obtain a summary for the regression and draw the diagnostic plots. Comment on the results

```
data2 <- read.table('data2.txt', header = T)
str(data2)
```

```
## 'data.frame':    25 obs. of  2 variables:
##  $ Height  : int  48 46 44 42 40 38 36 34 32 30 ...
##  $ Pressure: num  29.1 30.6 31.9 33.2 35.3 37 39.3 41.6 44.2 47.5 ...
```

```
model3 <- lm(Pressure ~ Height, data = data2)
plot(Pressure ~ Height, data = data2, pch = 16)
abline(model3)
```




This is a clearly inadequate model, so we seek a transformation that leads to a better result. That is the purpose of the next section. The summary is

```
summary(model3)
```

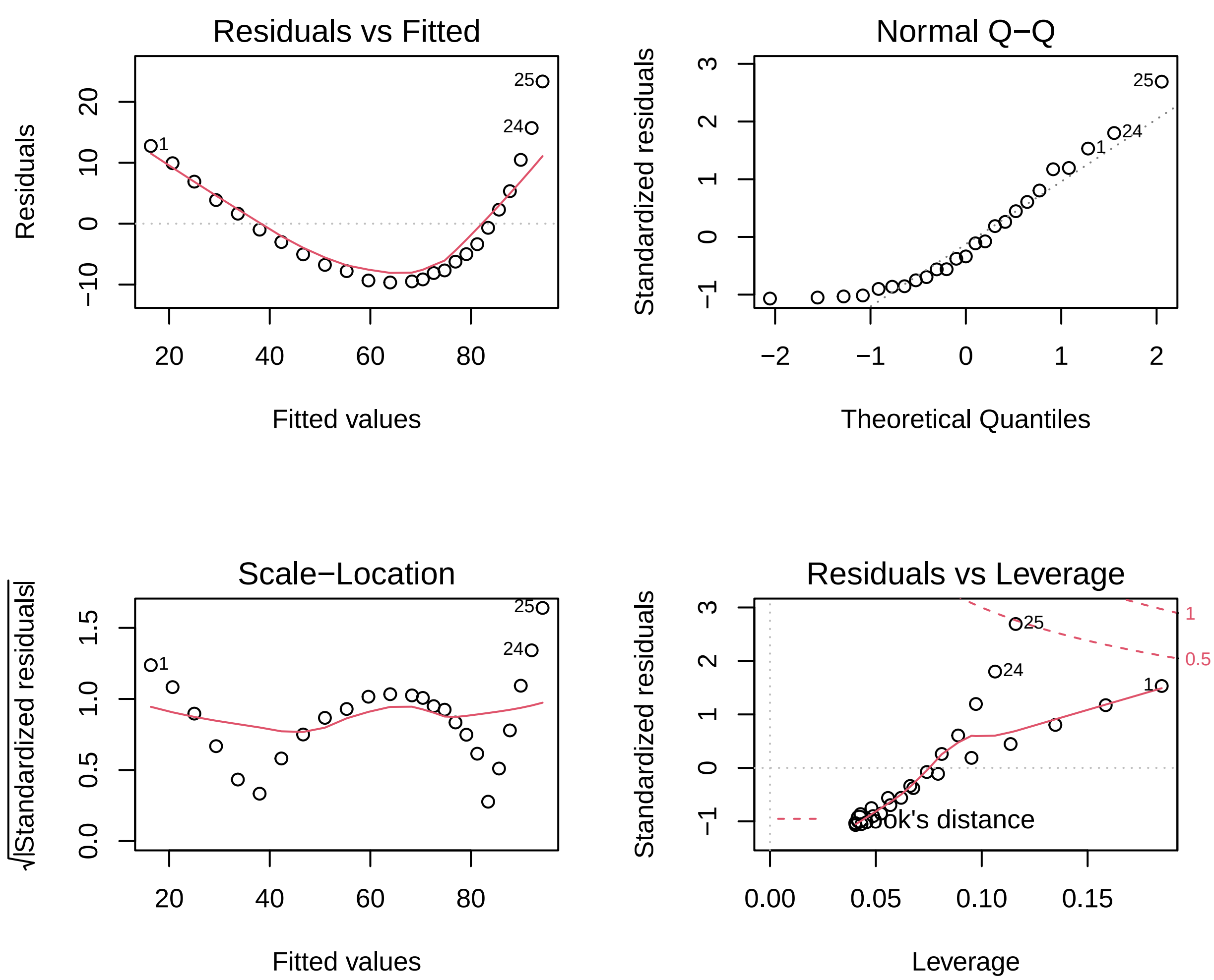
```
##
## Call:
## lm(formula = Pressure ~ Height, data = data2)
```

```
##
## Residuals:
##    Min      1Q Median     3Q    Max
## -9.654 -7.675 -3.012  5.340 23.347
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 120.2233     4.9230   24.42  < 2e-16 ***
## Height       -2.1642     0.1683  -12.86 5.48e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.228 on 23 degrees of freedom
## Multiple R-squared:  0.8779, Adjusted R-squared:  0.8726
## F-statistic: 165.4 on 1 and 23 DF,  p-value: 5.485e-12
```

Observe that the results in the summary do not reflect the fact that the model is not adequate. The $p$-values for the coefficients are both small, and the $R^2$ is almost 88%.

Let us look at the diagnostic plots

```
par(mfrow = c(2,2))
plot(model3)
```

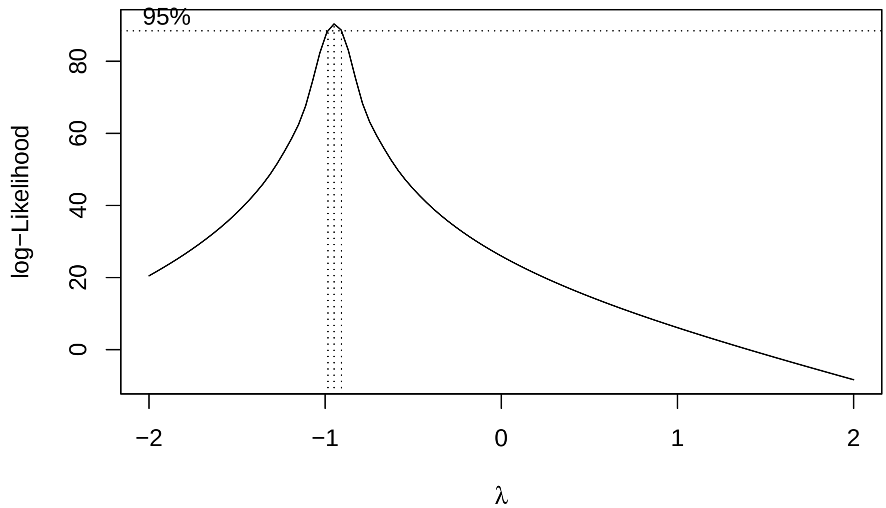


```r
par(mfrow = c(1,1))
```

These plots show clearly that the model is not adequate. All the assumptions made to fit the model are violated in this case.

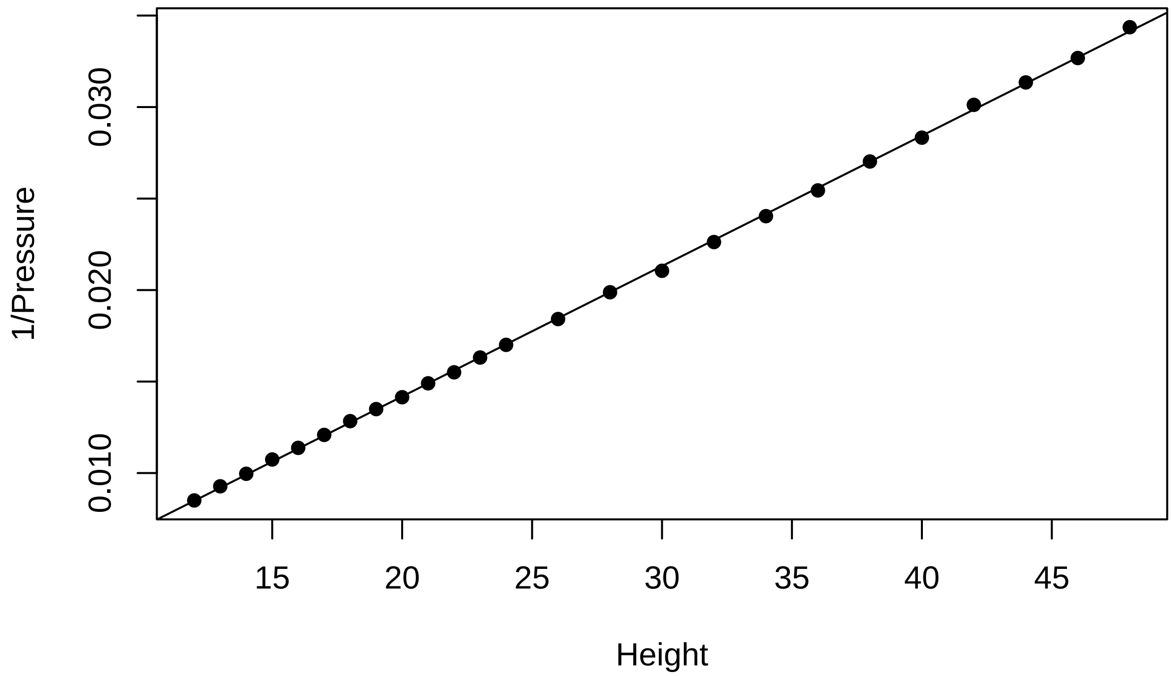
(ii) Use the function `boxcox` on the package `MASS` with the argument set to the model you fitted in (i). If the maximum value in the graph is close to an integer value, use a power transformation with exponent equal to the integer value for `Pressure` and fit a new model. Obtain a summary of the new regression and compare with the previous one. Draw the diagnostic plots and compare with the previous results.

```r
library(MASS)
boxcox(model3)
```




The maximum is close to -1, so we make the power transformation $\texttt{Pressure}^{-1} = 1/\texttt{Pressure}$, and fit a new model

```r
model4 <- lm(1/Pressure ~ Height, data = data2)
plot(1/Pressure ~ Height, data = data2, pch = 16)
abline(model4)
```



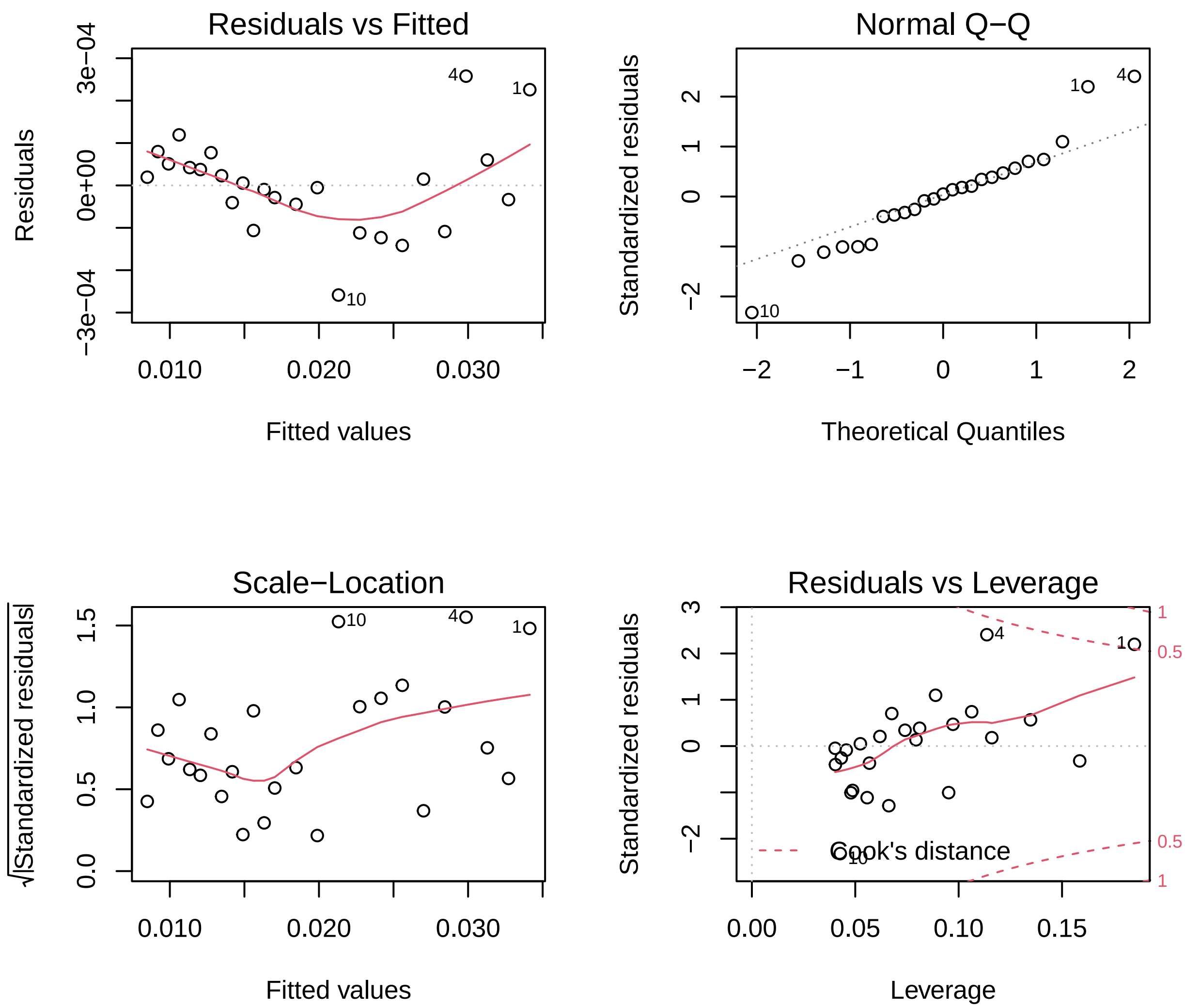

We see that the fit is excellent. Next, look at the summary table:

```
summary(model4)
```

```
##
## Call:
## lm(formula = 1/Pressure ~ Height, data = data2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -2.586e-04 -4.452e-05  5.514e-06  5.090e-05  2.578e-04
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -6.746e-05  6.073e-05  -1.111    0.278
## Height       7.126e-04  2.076e-06 343.248   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0001138 on 23 degrees of freedom
## Multiple R-squared:  0.9998, Adjusted R-squared:  0.9998
## F-statistic: 1.178e+05 on 1 and 23 DF,  p-value: < 2.2e-16
```

The slope has a $p$-value equal to zero while the intercept has a large $p$-value, and we cannot reject the null hypothesis that it is equal to zero.

```
par(mfrow = c(2,2))
plot(model4)
```




```
par(mfrow = c(1,1))
```

(iii) If the $p$-value for the intercept is large, fit a model without intercept by adding `+ 0` at the end of the regression equation in the call to the `lm` function. Use this model to write down an equation for the relation between pressure and height for a gas. What would be the predicted `Pressure` for a point with `Height` $= 32$? Draw a scatterplot of `Pressure` against `Height` and add the regression line for the first model and the curve you obtained with the second regression.

```
model5 <- lm(1/Pressure ~ Height + 0, data = data2)
summary(model5)
```

```
##
## Call:
## lm(formula = 1/Pressure ~ Height + 0, data = data2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -2.619e-04 -6.542e-05 -1.286e-05  2.861e-05  2.801e-04
##
## Coefficients:
##         Estimate Std. Error t value Pr(>|t|)
## Height 7.105e-04  7.821e-07   908.5   <2e-16 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0001144 on 24 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 8.253e+05 on 1 and 24 DF,  p-value: < 2.2e-16
```

The equation is

$$\frac{1}{P} = 7.105 \times 10^{-4} \times H$$

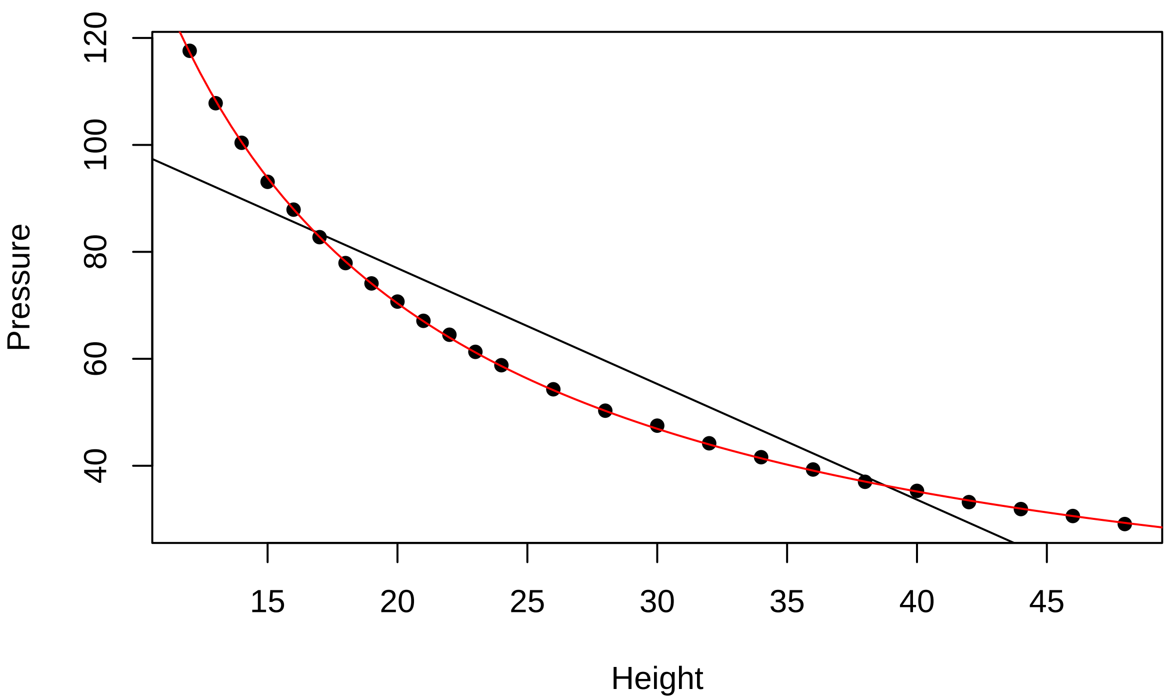
and inverting we get

$$P = \frac{1}{7.105} 10^4 \times \frac{1}{H} = 1407.46 \times \frac{1}{H}$$

If we set $H = 32$ with this model, the pressure is

$$P = 1407.46/32 = 43.983$$

```
plot(Pressure ~ Height, data = data2, pch = 16)
abline(model3)
curve(1407.46/x, 10, 50, add =T, col = 'red')
```




### Note

Observe that in this experiment, volume is proportional to height because the gas is enclosed in a cylinder with variable height but fixed radius. The relation between pressure and volume for a gas was investigated by Robert Boyle, who proved that pressure is inversely proportional to volume:

$$P = \propto \frac{1}{V}.$$

This is known as Boyle's law. The data that we used for this problem is Boyle's data and the equation we obtained is Boyle's law.