Lecture 7: Training Neural Networks, Part I

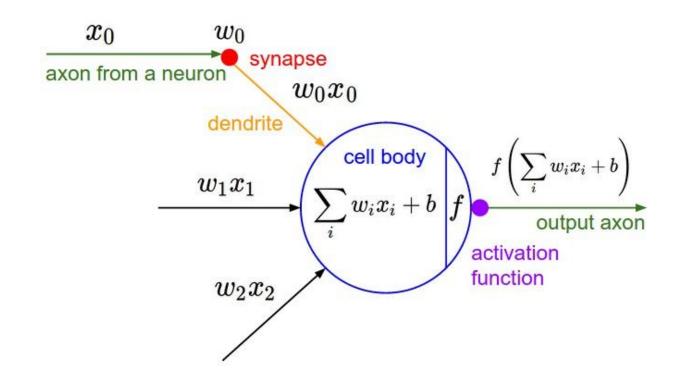
Next: Training Neural Networks

Overview

- 1. One time setup activation functions, preprocessing, weight initialization, regularization, gradient checking
- 2. Training dynamics babysitting the learning process, parameter updates, hyperparameter optimization
- 3. Evaluation model ensembles, test-time augmentation

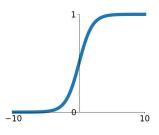
Part 1

- Activation Functions
- Data Preprocessing
- Weight Initialization
- Batch Normalization
- Babysitting the Learning Process
- Hyperparameter Optimization

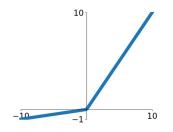


Sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

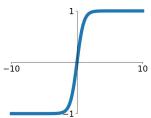


Leaky ReLU max(0.1x, x)



tanh

tanh(x)

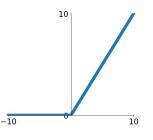


Maxout

 $\max(w_1^T x + b_1, w_2^T x + b_2)$

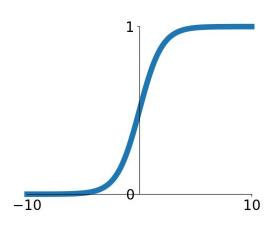
ReLU

 $\max(0,x)$



ELU

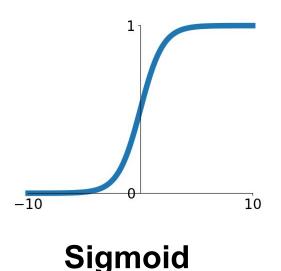
$$\begin{cases} x & x \ge 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

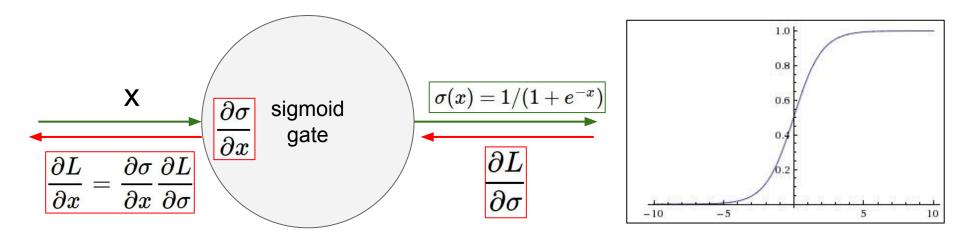


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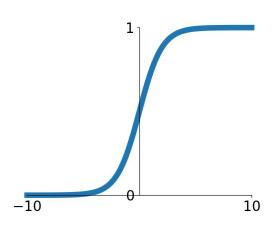
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3 problems:

 Saturated neurons "kill" the gradients



What happens when x = -10? What happens when x = 0? What happens when x = 10?



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

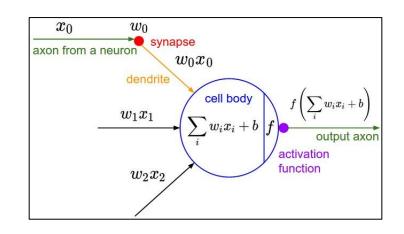
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3 problems:

- Saturated neurons "kill" the gradients
- Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_{i}w_{i}x_{i}+b
ight)$$



What can we say about the gradients on **w**?

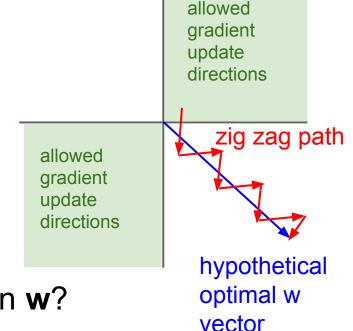
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Always all positive or all negative :(



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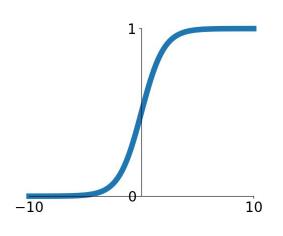
$$f\left(\sum_{\pmb{i}} w_{\pmb{i}} x_{\pmb{i}} + b
ight)$$

What can we say about the gradients on w?

Always all positive or all negative :(

(For a single element! Minibatches help)

allowed gradient update directions zig zag path allowed gradient update directions hypothetical optimal w vector



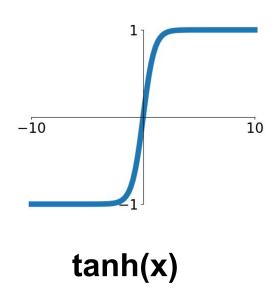
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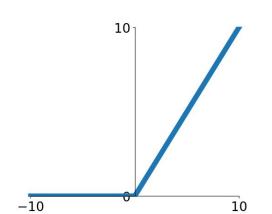
3 problems:

- Saturated neurons "kill" the gradients
- 2. Sigmoid outputs are not zero-centered
- 3. exp() is a bit compute expensive



- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

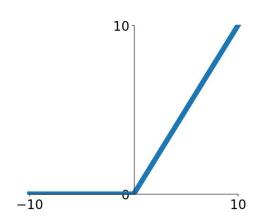
[LeCun et al., 1991]



- Computes f(x) = max(0,x)
- Does not saturate (in +region)
 - Very computationally efficient
 - Converges much faster than sigmoid/tanh in practice (e.g. 6x)

ReLU (Rectified Linear Unit)

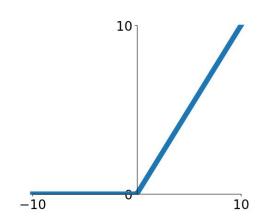
[Krizhevsky et al., 2012]



ReLU (Rectified Linear Unit)

- Computes f(x) = max(0,x)
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Not zero-centered output

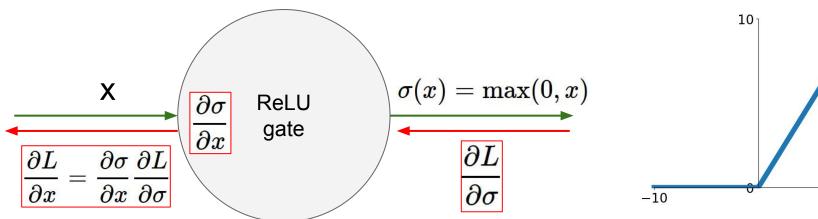


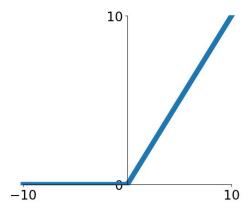
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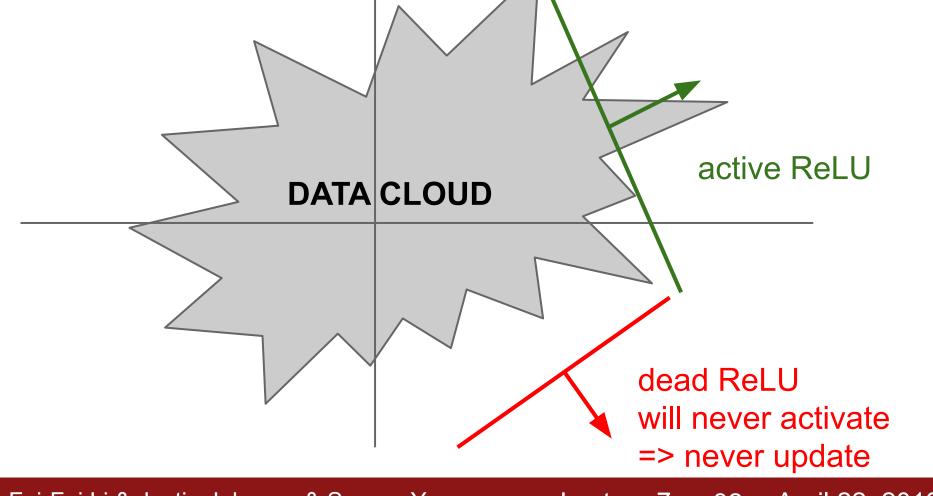
- Not zero-centered output
- An annoyance:

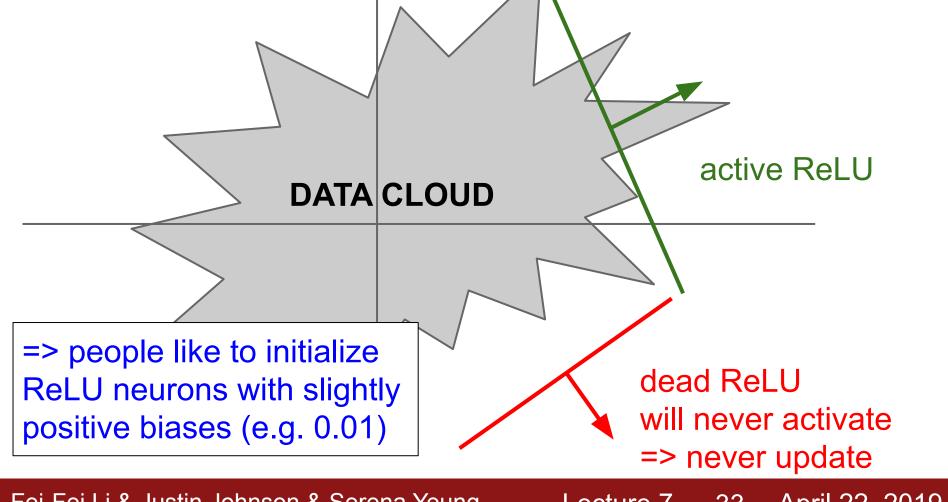
hint: what is the gradient when x < 0?



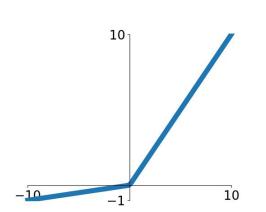


What happens when x = -10? What happens when x = 0? What happens when x = 10?





[Mass et al., 2013] [He et al., 2015]

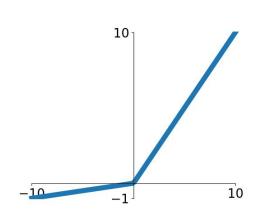


- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- will not "die".

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

[Mass et al., 2013] [He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
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Leaky ReLU

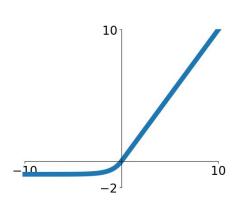
$$f(x) = \max(0.01x, x)$$

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha / (parameter)

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise

- Computation requires exp()

Maxout "Neuron"

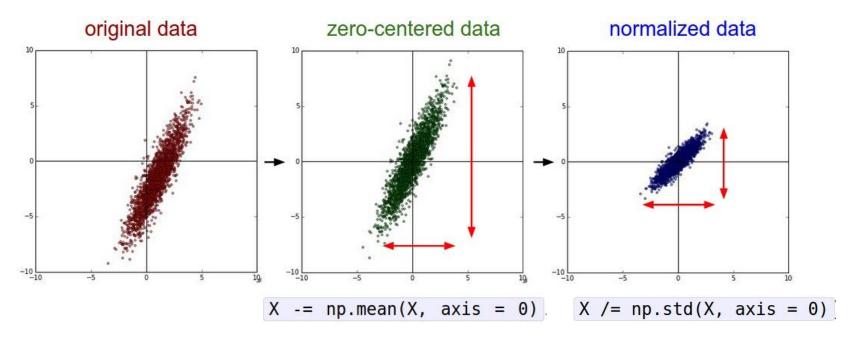
- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

Problem: doubles the number of parameters/neuron:(

TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Try out Leaky ReLU / Maxout / ELU
- Try out tanh but don't expect much
- Don't use sigmoid



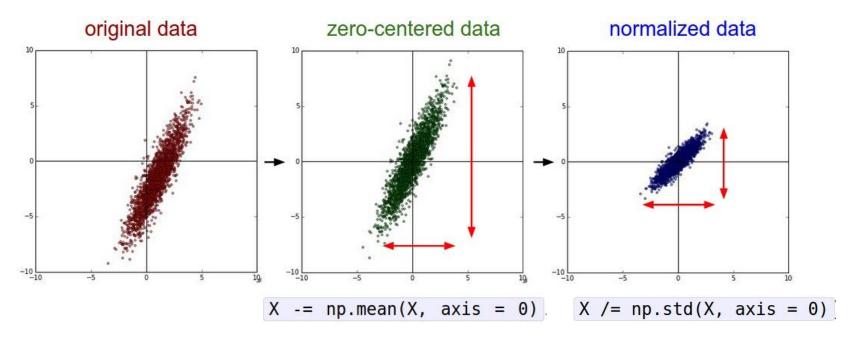
(Assume X [NxD] is data matrix, each example in a row)

Remember: Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_{\pmb{i}} w_{\pmb{i}} x_{\pmb{i}} + b
ight)$$

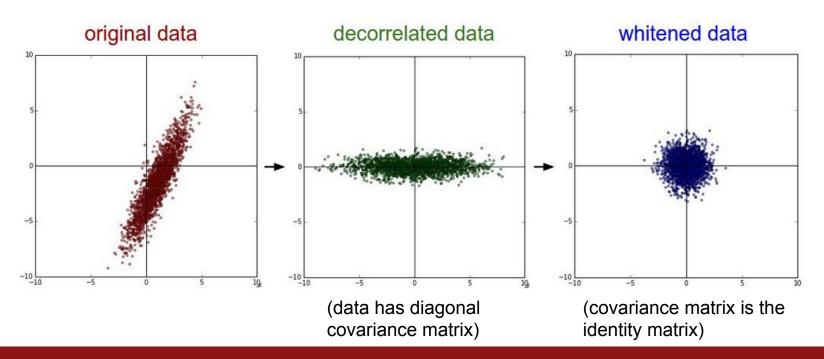
What can we say about the gradients on **w**? Always all positive or all negative :((this is also why you want zero-mean data!)

allowed gradient update directions zig zag path allowed gradient update directions hypothetical optimal w vector



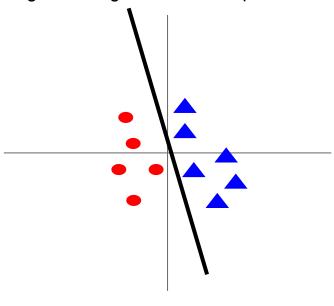
(Assume X [NxD] is data matrix, each example in a row)

In practice, you may also see PCA and Whitening of the data



Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize

After normalization: less sensitive to small changes in weights; easier to optimize



TLDR: In practice for Images: center only

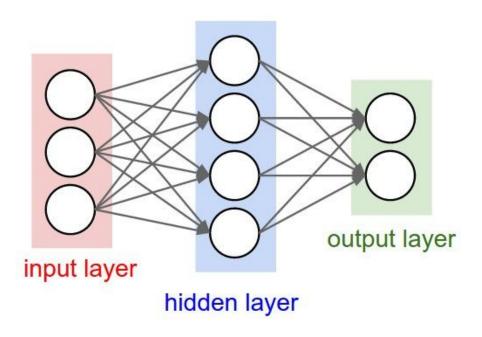
e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet) (mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
 (mean along each channel = 3 numbers)
- Subtract per-channel mean and
 Divide by per-channel std (e.g. ResNet)
 (mean along each channel = 3 numbers)

Not common to do PCA or whitening

Weight Initialization

- Q: what happens when W=constant init is used?



- First idea: **Small random numbers** (gaussian with zero mean and 1e-2 standard deviation)

W = 0.01 * np.random.randn(Din, Dout)

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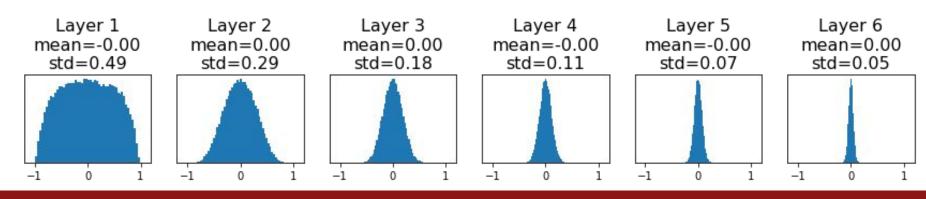
Works ~okay for small networks, but problems with deeper networks.

```
dims = [4096] * 7 Forward pass for a 6-layer
hs = [] net with hidden size 4096
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

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All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?

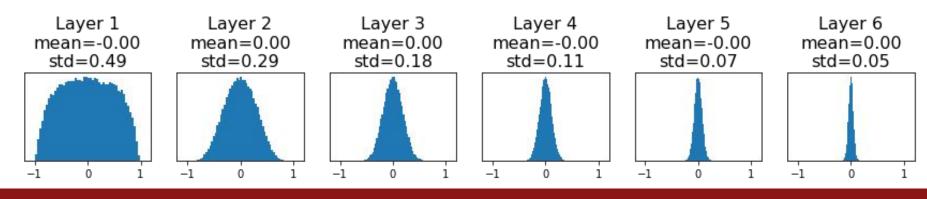


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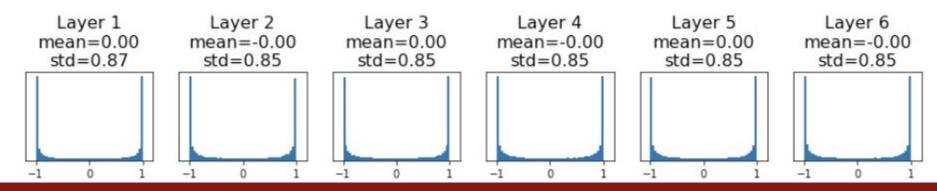
Q: What do the gradients dL/dW look like?

A: All zero, no learning =(



All activations saturate

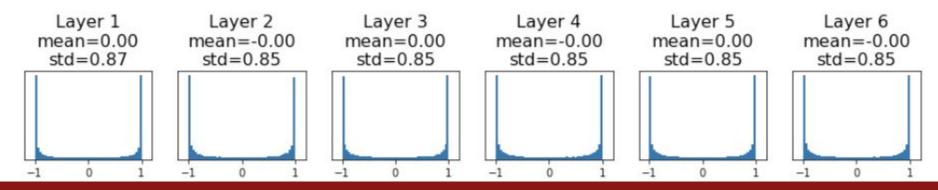
Q: What do the gradients look like?



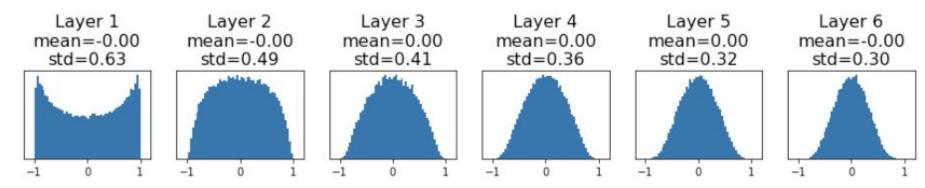
All activations saturate

Q: What do the gradients look like?

A: Local gradients all zero, no learning =(

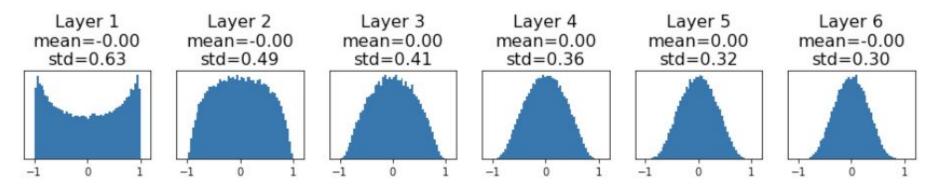


"Just right": Activations are nicely scaled for all layers!



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For conv layers, Din is kernel_size² * input_channels



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Derivation:

```
y = Wx 

h = f(y) 

Var(y<sub>i</sub>) = Din * Var(x<sub>i</sub>w<sub>i</sub>) 

= Din * (E[x<sub>i</sub><sup>2</sup>]E[w<sub>i</sub><sup>2</sup>] - E[x<sub>i</sub>]<sup>2</sup> E[w<sub>i</sub>]<sup>2</sup>) 

= Din * Var(x<sub>i</sub>) * Var(w<sub>i</sub>) 

If Var(w<sub>i</sub>) = 1/Din then Var(y<sub>i</sub>) = Var(x<sub>i</sub>)
```

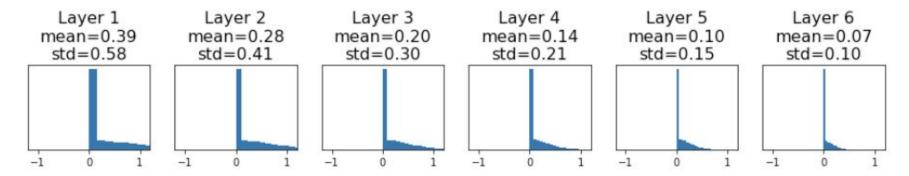
[Assume x, w are iid]
[Assume x, w independent]
[Assume x, w are zero-mean]

Weight Initialization: What about ReLU?

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Xavier assumes zero centered activation function

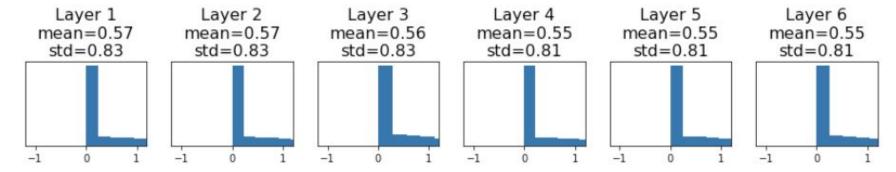
Activations collapse to zero again, no learning =(



Weight Initialization: Kaiming / MSRA Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din. Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) * np.sqrt(2/Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

"Just right": Activations are nicely scaled for all layers!



He et al, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", ICCV 2015

Proper initialization is an active area of research...

Understanding the difficulty of training deep feedforward neural networks by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

Fixup Initialization: Residual Learning Without Normalization, Zhang et al, 2019

The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks, Frankle and Carbin, 2019

Batch Normalization

Batch Normalization

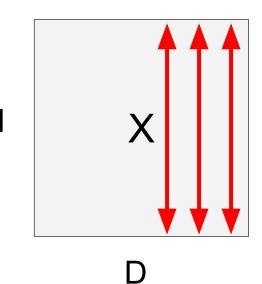
"you want zero-mean unit-variance activations? just make them so."

consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

Input: $x: N \times D$

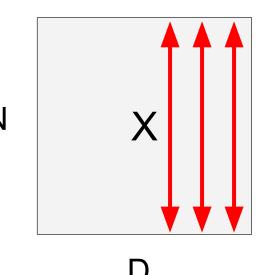


$$\mu_j = \frac{1}{N} \sum_{i=1}^N x_{i,j} \quad \text{Per-channel mean,} \\ \text{shape is D}$$

$$\sigma_j^2 = rac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2$$
 Per-channel var, shape is D

$$\hat{x}_{i,j} = rac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + arepsilon}}$$
 Normalized x, Shape is N x D

Input: $x: N \times D$



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$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}} \qquad \text{Normalized x,} \\ \text{Shape is N x D}$$

Problem: What if zero-mean, unit variance is too hard of a constraint?

shift parameters:
$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, β = μ will recover the identity function!

$$\mu_j = rac{1}{N} \sum_{i=1}^N x_{i,j}$$
 Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \mbox{Per-channel var,} \\ \mbox{shape is D}$$

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

$$eta_j$$
 Output, Shape is

Normalized x, Shape is N x D

Batch Normalization: Test-Time

Estimates depend on minibatch; can't do this at test-time!

Input: $x: N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = rac{1}{N} \sum_{i=1}^N x_{i,j}$$
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$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^N (x_{i,j} - \mu_j)^2 \quad \mbox{Per-channel var,} \quad \mbox{shape is D}$$

$$\hat{x}_{i,j} = rac{x_{i,j} - \mu_j}{\sqrt{\sigma_i^2 + \varepsilon}}$$
 Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$
 Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x: N \times D$

$$\mu_j = \text{ (Running) average of values seen during training}$$

Per-channel mean, shape is D

Per-channel var,

shape is D

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

During testing batchnorm becomes a linear operator!
Can be fused with the previous fully-connected or conv layer

$$\sigma_j^2 = ext{(Running) average of values seen during training} \ x_{i,j} - \mu_i$$

$$\hat{x}_{i,j} = rac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + arepsilon}}$$
 Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$
 Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x: N \times D$

$$\mu_j = 0$$

$$\mu_j = \text{ (Running) average of values seen during training}$$

Per-channel mean, shape is D

Per-channel var,

shape is D

Learnable scale and shift parameters:

$$\gamma, \beta: D$$

Learning $\gamma = \sigma$, β = μ will recover the identity function!

$$\hat{r}$$
 . . -

 $\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$

 $\sigma_j^2 = \frac{\text{(Running) average of values seen during training}}{\text{(Running)}}$

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Normalized x, Shape is N x D

Output, Shape is N x D

Batch Normalization for ConvNets

Batch Normalization for fully-connected networks

$$\mathbf{x}: \mathbf{N} \times \mathbf{D}$$
Normalize
$$\boldsymbol{\mu}, \boldsymbol{\sigma}: \mathbf{1} \times \mathbf{D}$$

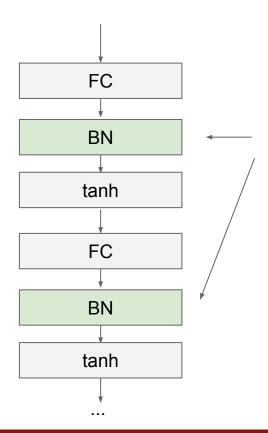
$$\mathbf{y}, \boldsymbol{\beta}: \mathbf{1} \times \mathbf{D}$$

$$\mathbf{y} = \mathbf{y}(\mathbf{x} - \boldsymbol{\mu}) / \boldsymbol{\sigma} + \boldsymbol{\beta}$$

Batch Normalization for convolutional networks (Spatial Batchnorm, BatchNorm2D)

Normalize
$$\mathbf{x}: \mathbf{N} \times \mathbf{C} \times \mathbf{H} \times \mathbf{W}$$
 $\mu, \sigma: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\gamma, \beta: \mathbf{1} \times \mathbf{C} \times \mathbf{1} \times \mathbf{1}$
 $\gamma = \gamma(\mathbf{x} - \mu) / \sigma + \beta$

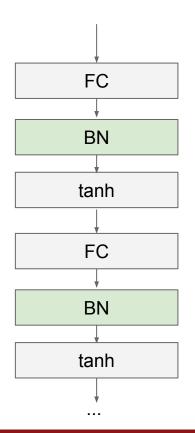
Batch Normalization



Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - E[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

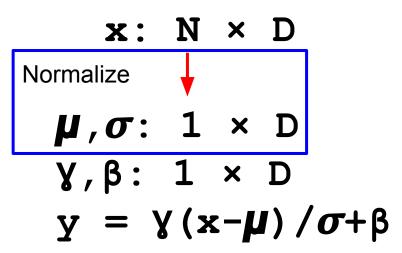
Batch Normalization



- Makes deep networks much easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Behaves differently during training and testing: this
 is a very common source of bugs!

Layer Normalization

Batch Normalization for fully-connected networks



Layer Normalization for fully-connected networks
Same behavior at train and test!
Can be used in recurrent networks

Normalize
$$\mu, \sigma: N \times D$$

$$\mu, \sigma: N \times 1$$

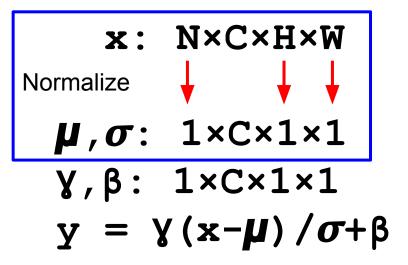
$$\gamma, \beta: 1 \times D$$

$$y = \gamma(x-\mu)/\sigma + \beta$$

Ba, Kiros, and Hinton, "Layer Normalization", arXiv 2016

Instance Normalization

Batch Normalization for convolutional networks

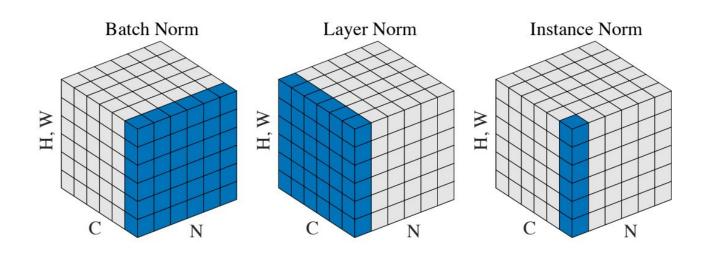


Instance Normalization for convolutional networks Same behavior at train / test!

$$x: N \times C \times H \times W$$
Normalize
 $\mu, \sigma: N \times C \times 1 \times 1$
 $y, \beta: 1 \times C \times 1 \times 1$
 $y = y(x-\mu)/\sigma + \beta$

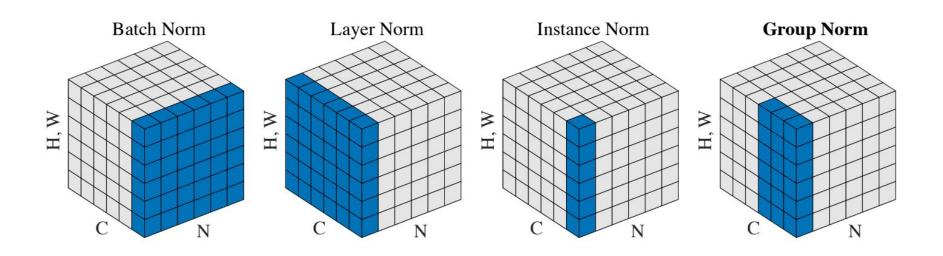
Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

Comparison of Normalization Layers



Wu and He, "Group Normalization", ECCV 2018

Group Normalization



Wu and He, "Group Normalization", ECCV 2018

Summary

TLDRs

We looked in detail at:

- Activation Functions (use ReLU)
- Data Preprocessing (images: subtract mean)
- Weight Initialization (use Xavier/He init)
- Batch Normalization (use)