

CPSC 335 Project 2: Crane Unloading

**Exhaustive Optimization Algorithm:**

Pseudocode:

```
path crane_unloading_exhaustive(setting)
    assert(setting.rows > 0)
    assert(setting.columns > 0)
    size_t max_steps = setting.rows + setting.column - 2
    assert(max_steps < 64)
    path best(setting)

    for size_t steps = 0 in max_steps do
        msk = uint64(1)
        for uint64_t bits = 0 in msk do
            path candidate(setting)
            Bool valid = TRUE
            for size_t i = 0 in steps do
                Size_t bit = (bits >> i) & 1
                if bit == 1 do
                    if (candidate.is_step_valid(STEP_DIRECTION_EAST))
                        do
                            candidate.add_step(STEP_DIRECTION_EAST)
                        else do
                            valid = FALSE
                        endif
                    else do
                        if (candidate.is_step_valid(STEP_DIRECTION_SOUTH))
                            do
                                candidate.add_step(STEP_DIRECTION_SOUTH)
                            elsedo
                                valid = false;
                            Endif
                        Endif
                    endif
                endif
            endfor
            If (valid == true and (candidate.total_cranes > best.total_cranes) do
```

```

        best = candidate
    endif
endfor
Endfor
return best
DONE

```

#### Time Complexity Step Count:

$$SC = 3 + 3 + 5 + 2 + 1 + (\sum_{s=1}^n) [\sum_{b=0}^{2^s-1} [1 + 1 + 1 + \sum_{i=0}^{s-1} [3+3] + 5]]$$

$$SC = 14 + (\sum_{s=1}^n) [\sum_{b=0}^{2^s-1} [3 + 6(s - 1 + 1) + 5]]$$

$$SC = 14 + (\sum_{s=1}^n) [\sum_{b=0}^{2^s-1} [6s]] + (\sum_{s=1}^n) [\sum_{b=0}^{2^s-1} [8]]$$

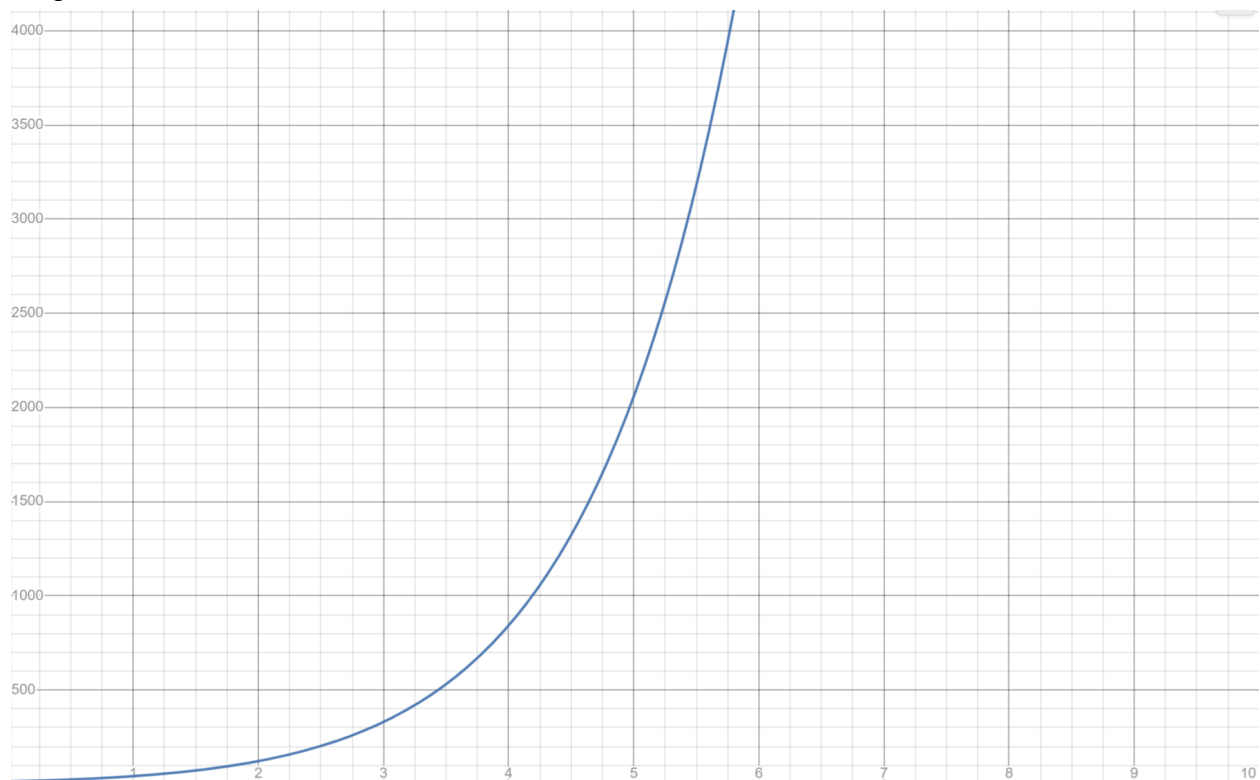
$$SC = 14 + (12(1 - 2^n + 2^{n+1})) + 16(2^n - 1)$$

$$SC = 14 + 12 - 12(2^n) + 12(2^n)(n) + 16(2^n) - 16$$

$$SC = 12(2^n)(n) + 4(2^n) + 10$$

Therefore, this algorithm falls under  $O(2^n * n^2)$  time complexity

#### Graph:



#### **Dynamic Programming Algorithm:**

##### Pseudocode:

path crane\_unloading\_dyn\_prog (setting)

```

assert(setting.rows > 0)
assert(setting.columns > 0)
using cell_type = std::optional<path>

std::vector<std::vector<cell_type>> > A(setting.rows,
                                       std::vector<cell_type>(setting.columns))

A[0][0] = path(setting)
assert(A[0][0].has_value)

for coordinate r = 0 in setting.rows do
    for coordinate c = 0 in setting.columns do
        if setting.get(r, c) not = CELL_BUILDING
            cell_type from_above
            cell_type from_left
            if r > 0 and A[r - 1][c].has_value do
                from_above = A[r - 1][c]
                if from_above->is_step_valid
                    (STEP_DIRECTION_SOUTH) Do
                        from_above->add_step(STEP_DIRECTION_SOUTH)
                    endif
                endif
            if c > 0 and A[r][c-1].has_value do
                From_left = A[r][c - 1]
                If from_left->is_step_valid
                    (STEP__DIRECTION_EAST) do
                        from_left->add_step(STEP_DIRECTION_EAST)
                    endif
                endif
            if (from_above.has_value and from_left.hasvalue do
                if from_above->total_cranes > from_left->total_cranes do
                    A[r][c] = from_above
                else
                    A[r][c] = from_left
                endif
            endif
            if from_above.has_value and not from_left.hasvalue do
                A[r][c] = from_above
            endif
            if from_left.has_value and not from_above.hasvalue do

```

```

                                A[r][c] = from_left
                            endif
                        endif
                    endfor
                endfor
cell_type best = &A[0][0]
assert(best->has_value)

for coordinate r = 0 in setting.rows do
    for coordinate c = 0 in setting.columns do
        if (A[r][c].has_value and A[r][c]->total_cranes > (*best)->total_cranes do
            best = &(A[r][c])
        endif
    endfor
Endfor

assert(best->has_value)

return **best
DONE

```

#### Time Complexity:

$$SC = 3 + 3 + 2 + 6 + 2 + 2 + n(n*(2 + \max(1 + 1 + 8 + 8 + 7 + 5 + 5, 0)))) + 3 + 5n^2$$

$$SC = 21 + n(n*(2+35)) + 5n^2$$

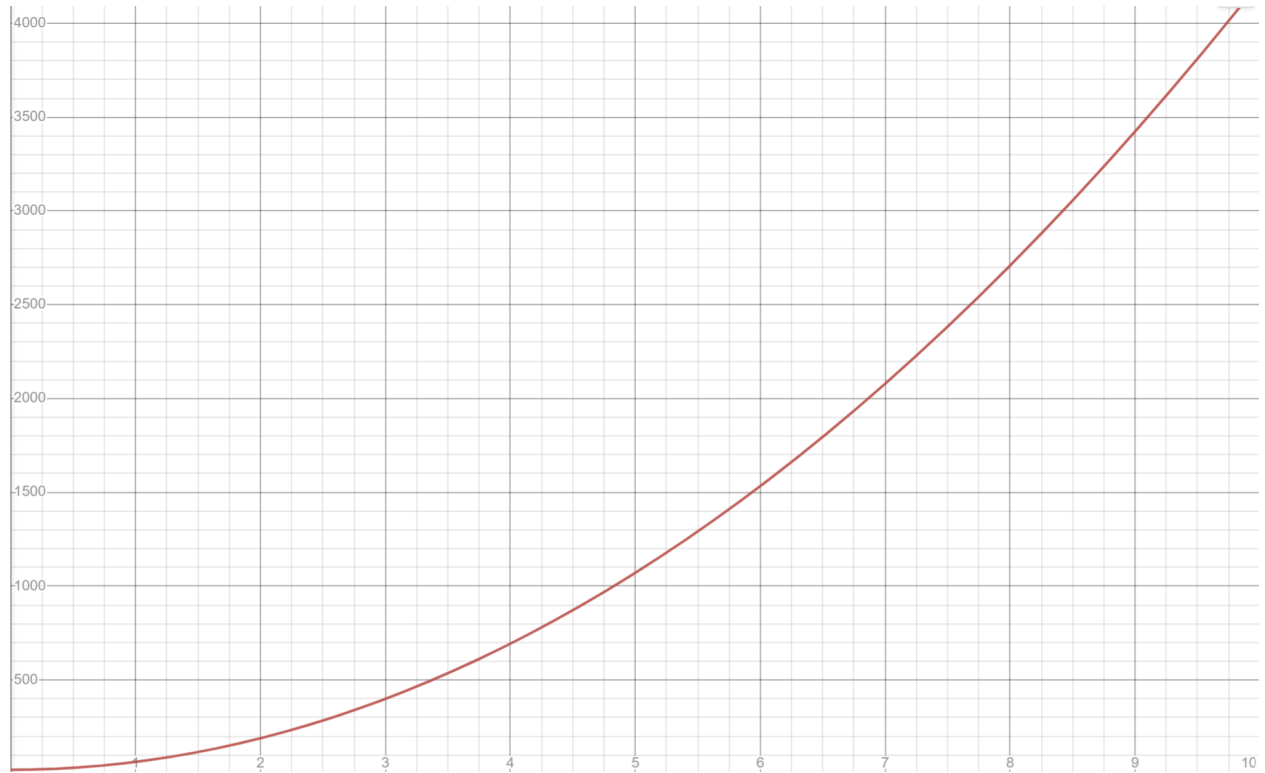
$$SC = 21 + 37n^2 + 5n^2$$

$$SC = 21 + 42n^2$$

$$SC = 42n^2 + 21$$

Therefore, this algorithm falls under  $O(n^2)$  time complexity

#### Graph:



### QUESTIONS:

1. There is a noticeable difference in the performance of the two algorithms. The dynamic programming approach is faster by quite a significant margin. This answer does not surprise me as the brute force method of the exhaustive optimization algorithm is not very efficient at all.
2. Our empirical analyses do match with our mathematical analyses. We mathematically proved that the exhaustive optimization algorithm has a time complexity of  $O(2^n * n^2)$ , making the algorithm exponential time which is very slow. We also mathematically proved that the dynamic programming algorithm belongs in  $O(n^2)$  time, which is quadratic or polynomial, significantly faster than the exponential time the exhaustive optimization runs at.
3. Our data is consistent with hypothesis 1. As stated in question 2, the polynomial time complexity of the dynamic programming algorithm is significantly faster than the exponential time complexity of the exhaustive optimization algorithm.

# Demonstration

[https://drive.google.com/file/d/1Nx7dJJ5\\_lxMKQzDZnBoL1vLKTcdFf-ND/view?usp=sharing](https://drive.google.com/file/d/1Nx7dJJ5_lxMKQzDZnBoL1vLKTcdFf-ND/view?usp=sharing)