ENCM 5100
Computational Fluid Dynamics I
Fall 2011
Computing Project #4
Due: 10/12/2011 after class

Recall that the integral (finite volume) form of the inviscid Burgers equation is given by

$$\frac{\partial u_i}{\partial t} + \delta_i f = 0$$

Implicit Euler time differencing gives this finite volume discretization in functional form as:

$$F\left(u^{n+1}\right) = u_i^{n+1} - u_i^n + \frac{\Delta t}{\Delta x}\left(f_{i+1/2}^{n+1} - f_{i-1/2}^{n+1} \right)$$

where $f = u^2/2$.

1. Using simple upwind flux extrapolations as shown in the sketch below below, derive a Newton-based methodology in delta-form for numerical solution.
2. Compute the numerical solution using initial and boundary conditions as follows: at time $t = 0$ the initial waveform is given by:

$$\begin{aligned} u^{(0)}\left(x\right) = u\left(x,0\right) &= u_L \quad , \quad x \le 1 \\ &= u_R \quad , \quad x > 1 \end{aligned}$$

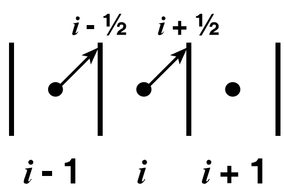
   where $\left(u_L, u_R\right) = \left(1.5, 0.5\right)$. Use the lagging boundary condition approach (i.e., $\Delta u_0^{n+1,m} = \Delta u_M^{n+1,m} = 0$) and phantom cells to enforce boundary conditions at the entrance and exit; that is, use $u_0^{n+1,m+1} = u_L$ at the left-most phantom cell, and $u_M^{n+1,m+1} = u_{M-1}^{n+1,m+1}$ at the right-most phantom cell.

Execute your program for 40 time steps, using 10 Newton iterations per time step and a Courant number of 1.0. Present your results as "*u* versus *x*" plots after 5, 10, 15, and 40 time steps. Use $\Delta x = 0.05$, which yields 61 equally spaced mesh points. On each plot, compare the numerical solutions with the exact solution after 40 time steps and label your curves appropriately. You should clearly describe the method you used to plot results from the finite volume formulation.

One method to measure the magnitude of change in the dependent variables from one time step to the next is to combine the computed differences in a way that in some sense represents a measure of the “overall” change. A common mathematical term that encompasses such a measure is the so-called L_2 -norm and for our case can be written as

$$\left\| L_2 \right\| = \frac{1}{M-2} \left[\sum_{i=1}^{M-1} \left(\Delta u_i^{n+1,m} \right)^2 \right]^{1/2}$$

Note the summation above encompasses only the interior cells of the computational domain. The above parameter is computed at the conclusion of each time step and then plotted on a linear-log scale (i.e., the horizontal axis is linear, whereas the vertical axis is log scale). Using this approach, make plots of $\left\| L_2 \right\|$ (vertical axis) versus Newton iteration (horizontal axis) for the 5th, 10th, 15th, and 40th time steps.



Upwind Flux Extrapolations