Example of preliminary exam, April 2022

1. **Question 1 (3 points)**

   What does the incorrelation between X and Y suggest about model $Y = \beta_0 + \beta_1 X + \varepsilon$? (3 points; 1 correct answer; penalty 33.3%; no answer: 0 points)

   (a) the estimate of β_0 is 0
   (b) the p-value associated to the significance test for β_1 is close to 1 ✓
   (c) $R^2 = 1$
   (d) the residual standard error is 0.5
   (e) no answer

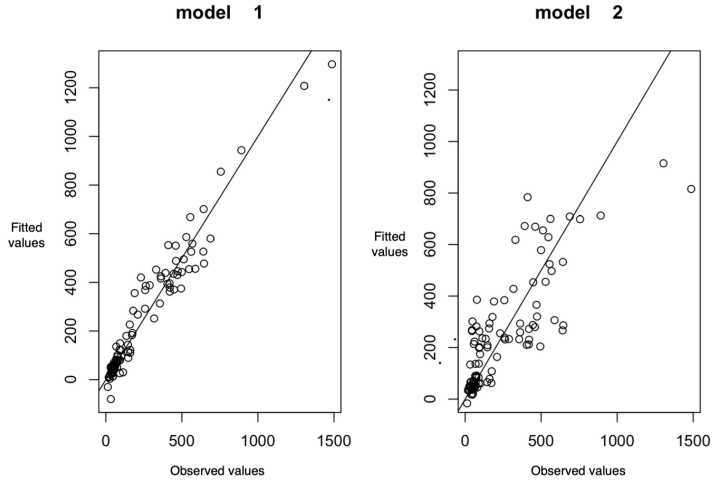
2. **Question 2 (3 points)**

   Which one of the following is an assumption about the response variable Y in the linear regression model?(3 points; 1 correct answer; penalty 33.3%; no answer: 0 points)

   (a) minimum distance from residuals
   (b) variance that depends on the covariates
   (c) mean equal to zero
   (d) Normal distribution ✓
   (e) no answer

3. **Question 3 (3 points)**

   The following graph compares the observed values and the fitted values from two linear regression models. Model 2 is a sub-model of model 1 (nested model). The line in both the graphs is the bisector of the first and third quadrants. What can we conclude? (3 points; 1 correct answer; penalty 33.3%; no answer: 0 points)



(a) model comparison based on test F suggests to reject model 2, at significance level 0.05 ✓
(b) the explained deviance for model 2 is larger than explained deviance for model 1
(c) model comparison based on test F suggests not to reject model 2, at significance level 0.05
(d) R^2 is larger for model 2 than for model 1
(e) no answer

4. **Question 4, 8 points**

What kind of information does a numerical interval with confidence level equal to 95% suggest? What can we expect from the interval in case of big datasets?

5. **Question 5 (8 points)**

Consider the analysis of the features of 120 houses for sale. The following output refers to a model relating the logarithm of the sale price (in thousands of dollars, `Lprice`) to the size of the house (in square feet, `Size`) and the State (CA, NJ, NY, PA, `State`).

```
Call:
lm(formula = Lprice ~ Size + State)

Residuals:
     Min       1Q   Median       3Q      Max
-1.77915 -0.27947 -0.03962  0.29892  1.69701

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.84306    0.13469  35.957  < 2e-16 ***
Size         0.52416    0.04409  11.888  < 2e-16 ***
StateNJ     -0.13825    0.13835  -0.999 0.319735
StateNY     -0.01309    0.13842  -0.095 0.924802
StatePA     -0.47680    0.13860  -3.440 0.000811 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5358 on 115 degrees of freedom
Multiple R-squared:  0.5864, Adjusted R-squared:  0.572
F-statistic: 40.76 on 4 and 115 DF,  p-value: < 2.2e-16
```

Describe how variable `State` is treated and how the sale price changes with respect to `State`.

6. **Question 6 (8 points)**

Model `houses1` is extended to include other information. Consider the following models

`houses2` : *with covariates* `Size`, `Baths`, `Beds`

`houses3` : *with covariates* `Size`, `State`, `Beds`

`houses4` : *with covariates* `Size`, `State`, `Baths`

where `Baths` and `Beds` indicate the number of bathrooms and the number of bedrooms in the houses, respectively. Consider the following comparisons between models.

```
> anova(houses1, houses2)
Analysis of Variance Table

Model 1: Lprice ~ Size + Baths + Beds
Model 2: Lprice ~ Size + State
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)
1    116 36.245
2    115 33.009  1    3.2362 11.274 0.001066 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> anova(houses1, houses3)
Analysis of Variance Table

Model 1: Lprice ~ Size + State
Model 2: Lprice ~ Size + State + Beds
  Res.Df    RSS Df  Sum of Sq      F Pr(>F)
1    115 33.009
2    114 33.009  1 0.00050993 0.0018 0.9666
> anova(houses1, houses4)
Analysis of Variance Table

Model 1: Lprice ~ Size + State
Model 2: Lprice ~ Size + State + Baths
  Res.Df    RSS Df Sum of Sq     F  Pr(>F)
1    115 33.009
2    114 31.515  1    1.4947 5.407 0.02183 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Which kind of comparison is used? What can be inferred? Are the comparisons useful?