Stochastic Methods for Finance

Exam July 5, 2022

Exercice 1 *(For everybody) Consider a Black-Scholes market and a derivative contract with payoff at the maturity $T > 0$ given by*

$$F(n, S_T) = (S_T - K) * 1_{S_T > max\{K,n\}},$$

where 1_A denotes the indicator function of the event A, $K > 0$ and $n \in \mathbb{N}$.

i) Compute the price of the contract $F(n, S_T)$ at any time $t \in [0, T)$ and any $n = 0, 1, 2, \ldots$. Compute the limit of the price for $n \to \infty$; 1

ii) Compute the Delta of the contract $F(n, S_T)$ and the limit of the Delta for $n \to \infty$; 1

iii) Illustrate graphically the change of price and Delta of $F(n, S_T)$ for an upward shift of the volatility ; 1

iv) Assume $K \in \mathbb{N}$ and compute the price of the portfolio F given by

$$F(S_T) = F(2K, S_T) - F(K, S_T);$$

0

v) Compute the amount of Put options with strike price K one has to buy/sell in order to get a Delta neutral (global) portfolio.

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Exercice 2 *(For everybody)*

Solve the following PDE for $t \le T$:

$$\begin{aligned} \frac{\partial F}{\partial t} + x^2 \frac{\partial^2 F}{\partial x^2} + y^2 \frac{\partial^2 F}{\partial y^2} + xy &= 0 \\ F(T, x, y) &= \frac{x}{y}. \end{aligned}$$

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Exercice 3 *(For MathEng and Data Science, NOT for Mathematics)*

Consider a (stationary) binomial model for the evolution of a risky asset S, starting from the initial price $S_0 = 100$, and increasing (resp. decreasing) factor $u = 1,1$ (resp. $d = 0,9$). The interest rate is flat at $0,1\%$ per period (1 period = 1 year). 8

3 *i) Price 5 long positions in a European Call on S with maturity $T = 2$ years and strike price $K_1 = 100$;*

2 *ii) Find the price of 5 long positions in a American Put on S with maturity $T = 3$ years and strike price $K_2 = 110$;*

2 *iii) Find the position that the trader has to take in a European Call with maturity $T = 1$ year and strike price $K_3 = 95$ in order to obtain a Delta-neutral portfolio at time 0 involving the positions at points i) and ii).*

Exercice 4 *(For MathEng and Mathematics, NOT for Data Science)*

In the Black-Scholes model, find the price at time $t \leq T$ of an UP-AND-IN contract where the owner receives the payoff

$$F(n, S_T) = (S_T - K) * 1_{S_T > max\{K,n\}},$$

at the maturity T only if the asset has reached the upper barrier $L > 0$. Provide the price of the contract when $n \to +\infty$. Finally, find the Delta of the contract.