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Comparison of the price of a call option with three different methods calculated using VBA script : Binomial Model, Leisen and Reimer method, Black-Scholes Formula

1 Introduction

1.1 The Aim of this report

There are different methods to compute the price of an option, some of them are calculated in discrete time while others in continuous time.
In particular in this report I'm going to use Binomial Model, Leisen-Reimer Model and Black-Scholes formula. The first two are formulas in discrete time while the Black-Scholes in continuous; in fact this last one is given by the convergence of discrete model to continuous.
For each of this model it's possible to write a VBA script that can compute the price of the option given the values of initial asset price S, interest rate r, strike price K $= x$, maturity time T, volatily σ and the number of steps n; in particular in this report I'm going to assume $S = 100$, $x = 100$, $r = 0.01$, $T = 1$ *year* $\sigma = 0.2$.
The first aim of this report is to compute the price of the call option with these initial values changing the number of n with these three different methods using VBA script, the second one is to investigate the rate of convergence and the third one is to show that Leisen and Reimer is more accurate than Binomial Model and it reproduces results very close to Black-Scholes' ones.

1.2 Binomial Model

The Binomial Model is a stochastic model in discrete time.
The price of an option is given by:

$$p_0(Call) = e^{-R\cdot T} \cdot (f^u \cdot q + f^d \cdot (1-q))$$

where the risk neutral probability Q $= \binom{q}{1-q}$ with risk neutral weight $q = \frac{e^{R\cdot T}-d}{u-d}$, the parameters $u, d = e^{\pm\sigma_{daily}\cdot\sqrt{252\cdot T}}$ and the value of the payoff equal to $\begin{pmatrix} f^u = (S\cdot u - K)^+ \\ f^d = (S\cdot d - K)^+ \end{pmatrix}$.

The VBA script that I used for this model can be found in $http://www.anthony-vba.kefra.com/vba/vba7.htm$

1.3 Black-Scholes Formula

The Black-Scholes Formula is a stochastic model in continuous time. In fact it is the convergence to continuous model solutions of the discrete time binomial model.
The price of the option is given by :

$$p_0(Call) = S \cdot \Phi(d_1) - K \cdot e^{-R \cdot T} \cdot \Phi(d_2)$$

where

- $\Phi(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy$
- $d_1 = \frac{ln(\frac{S}{K})+(r+\frac{1}{2}\sigma^2)\cdot T}{\sigma\sqrt{T}}$
- $d_2 = d_1 - \sigma\sqrt{T}$

I want to underline that in these formulas I consider the continuous dividend yield (or foreign interest rate with currency options) equal to 0.
The VBA script that I used for this model can be found in $http://www.anthony{-}vba.kefra.com/vba/vba6.htm$.

1.4 Leisen-Reimer method

The idea of this model is quite similar to Binomial Model's one. In fact its main idea is that the underlying price binomial tree is centered around the option's strike price at expiration. The logic and calculation of tree nodes and option price is the same as in other binomial models. The difference is only in the calculation of tree parameters (u, d and q), which I explain below.
In Leisen-Reimer model, probabilities must be calculated before move sizes, because the former are inputs for the latter. But first it's necessary to calculate d_1 and d_2 that are the same of Black-Scholes formula.
Hence the risk neutral weight is equal to :

$$q = h^{-1}(d_2)$$

where $h^{-1}(z)$ is the Peizer-Pratt inversion function, which provides (discrete) binomial estimates for the (continuous) normal cumulative distribution function.

Once we got q we can compute u and d:

- $u = e^{r \cdot \Delta T} \cdot \frac{q'}{q}$
- $d = e^{r \cdot \Delta T} \cdot \frac{1-q'}{1-q}$

where exponent term can be interpreted as net cost of holding the underlying security over one step, as ΔT is the duration of one step in years, calculated as $\frac{T}{n}$ and $q' = h^{-1}(d_1)$.
Also in this case I consider the continuous dividend yield equal to 0.
The VBA script that I used for this model can be found in $https://onedrive.live.com/view.aspx?resid = 5C5883DBCC1B9CC5!27877ithint = file\%2cdocxauthkey =!AEbR - N5mZ6QJyu0$.

2 Data analysis

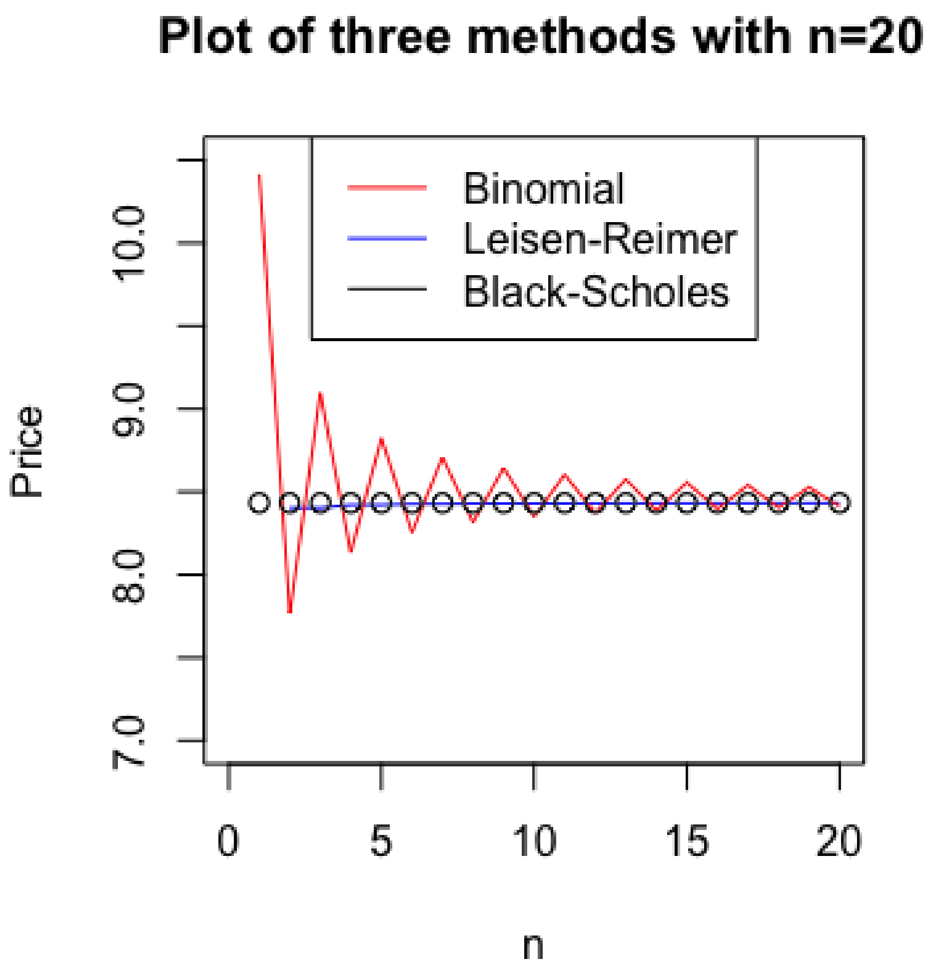
First of all I computed the value of call's price using Black-Scholes' VBA:

$$p_{BS} = 8.433327$$

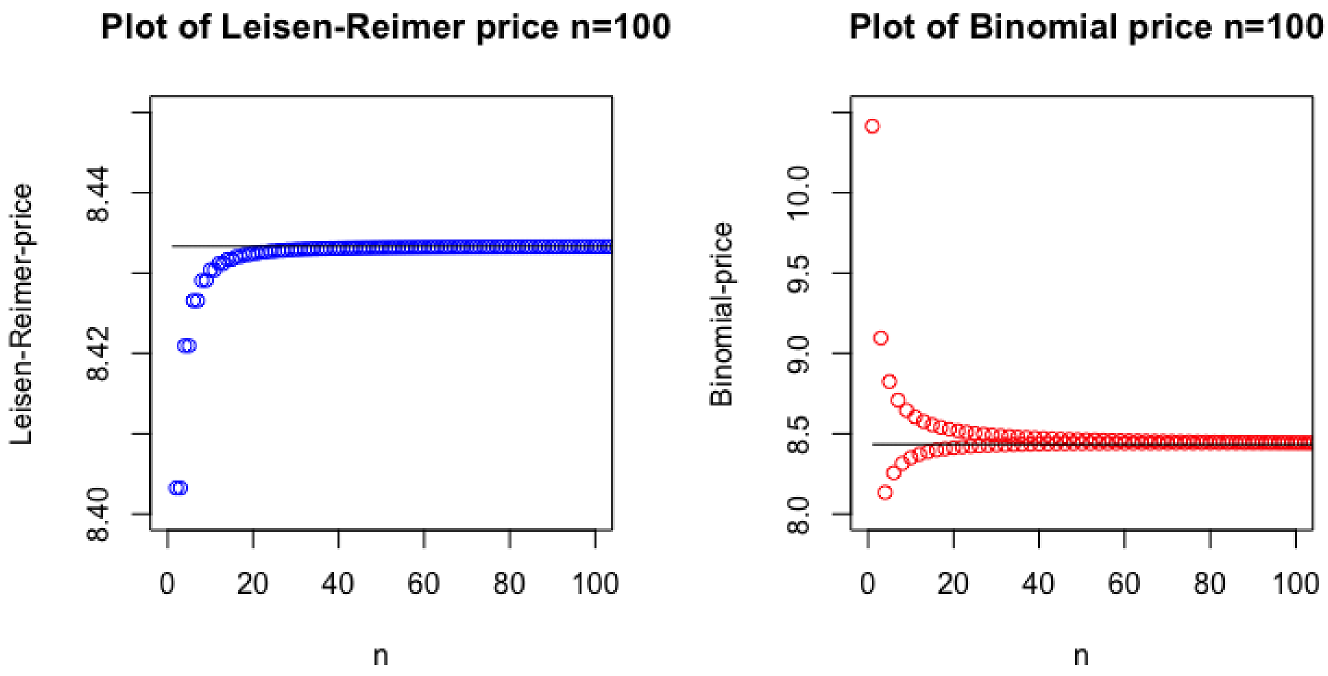
This is my "real" price and it is the target for the other discrete models. I decided to keep 6 significant figures (after the dot) to show how Leisen-Reimer method is more precise than Binomial Model and how its accuracy is good.
Once I got p_{BS} I computed the Binomial Model's p_{BM} price and Leisen-Reimer' s price p_{LR} using different number of steps. The results for different n are reported in table below.

n	p_{BM}	p_{LR}
10	8.349950	8.430399
50	8.439501	8.433169
100	8.443050	8.433279
250	8.440478	8.433312
500	8.437136	8.433317
750	8.435235	8.433318
1000	8.433965	8.433318

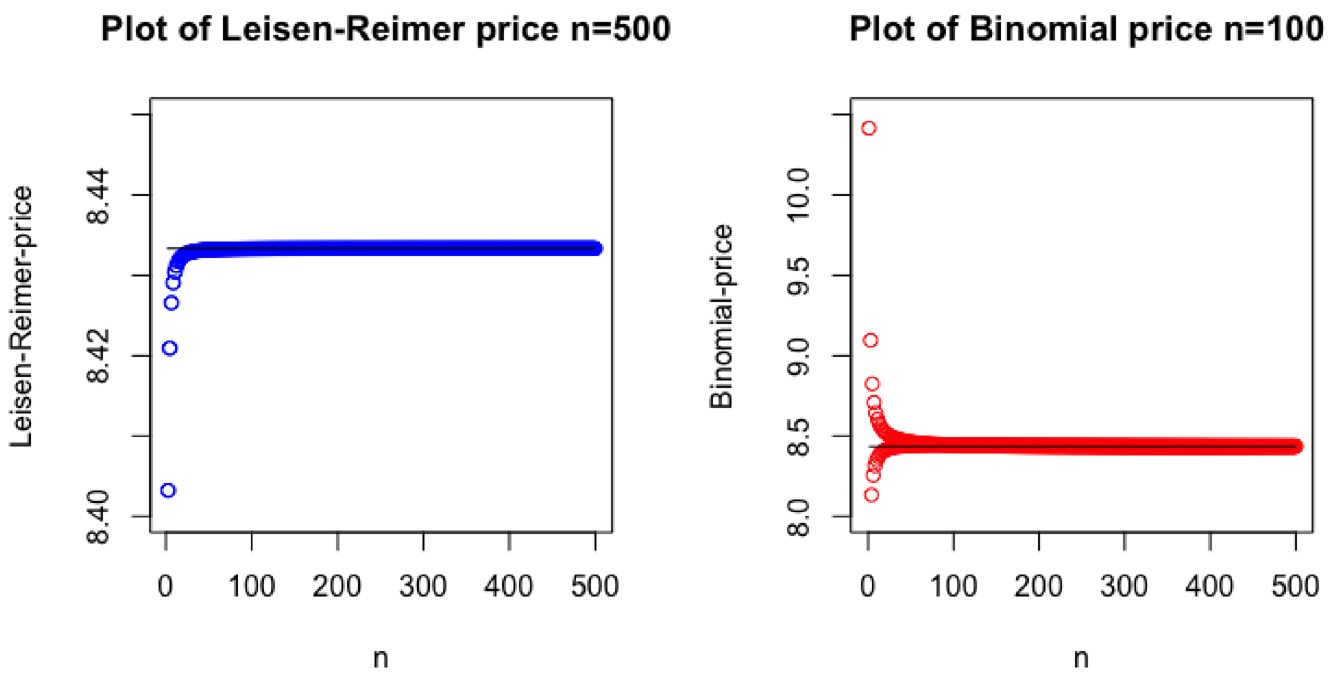
Then I plotted the value of the $p(n)$ for the 2 methods.



1. Plot of 3 methods with n=20.



2. Plot of two methods with n=100.



3. Plot of two methods with n=500.

3 Conclusions

So comparing the 2 different methods it's clear that Leisen-Reimer method is more accurate than Binomial model. Particularly, its main benefit is greater precision with smaller number of steps, compared to Binomail model; for example just with n=10 the first two significant figures are the same of BS'formula as reported in table above.
As n grows both Binomial Model's results convergence to BS' formula as-well but it keep being less accurate: for n=1000 we got just three significant figures while with LR four.
By the way both of them, as shown in plots, converge to BS for $n >> 1$.
Hence, if we have to choose one discrete method to describe the call's option it's better to choose Leisen-Reimer model than Binomial as because it's need less computational resources (it gives good results with small n) as because it's more accurate.
Naturally if we could take $n = \infty$ steps they would give the same results but computationally this is not possibile.

4 Bibliography

-Hull J.C. « Options,Futures and Other Derivatives ninth Edition» (15th January 2014)

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-https ://www.researchgate.net/profile/Dietmar-Leisen-2/publication/227603938$_Binomial_Models_for_Option_Valuation-Examining_and_Improving_Convergence/links/00b49532955e5ac4cb000000/Binomial-Models-for-Option-Valuation-Examining-and-Improving-Convergence.pdf?origin=publication_detail$