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Providing the price of a Tesla, Inc.'s call option by using the static binomial model and comparing it with the market quotes

1 Introduction

1.1 Tesla,Inc.: The company

Tesla, Inc. is an American electric vehicle and clean energy company based in Austin, Texas. Tesla designs and manufactures electric cars, battery energy storage from home to grid-scale, solar panels and solar roof tiles, and related products and services. Tesla is one of the world's most valuable companies and remains the world's most valuable automaker with a market capitalization of nearly US$ 1 trillion. The company had the most worldwide sales of battery electric vehicles and plug-in electric vehicles, capturing 23 %of the battery-electric (purely electric) market and 16 % of the plug-in market (which includes plug-in hybrids) in 2020. Through its subsidiary Tesla Energy, the company develops and is a major installer of photovoltaic systems in the United States. Tesla Energy is also one of the largest global suppliers of battery energy storage systems, with 3.99 gigawatt-hours (GWh) installed in 2021.
Founded in July 2003 by Martin Eberhard and Marc Tarpenning as Tesla Motors, the company's name is a tribute to inventor and electrical engineer Nikola Tesla. In February 2004, via a $6.5 million investment, X.com co-founder Elon Musk became the largest shareholder of the company and its chairman. He has served as CEO since 2008.
Tesla began production of its first car model, the Roadster sports car, in 2009. This was followed by the Model S sedan in 2012, the Model X SUV in 2015, the Model 3 sedan in 2017, and the Model Y crossover in 2020. The Model 3 is the all-time best-selling plug-in electric car worldwide, and, in June 2021, became the first electric car to sell 1 million units globally. Tesla's global sales were 936,222 cars in 2021, a 87% increase over the previous year, and cumulative sales totaled 2.3 million cars at the end of 2021. In October 2021, Tesla's market capitalization reached $1 trillion, the sixth company to do so in U.S. history.
For the fiscal (and calendar) year 2021, Tesla reported a net income of $5.52 billion. The annual revenue was $53.8 billion, an increase of 71 % over the previous fiscal year. Tesla ended 2020 with over $19 billion of cash on hand after having raised approximately $12 billion in stock sales. At the end of 2019 it had $6.3 billion cash on hand. Of the revenue number in 2021, $314 million came from selling regulatory credits to other automakers to meet government pollution standards. That number has been a smaller percentage of revenue for multiple quarters.
The quarter ending June 2021 was the first time Tesla made a profit independent of Bitcoin and regulatory credits. In 2021 the revenue was 53,823 (million USD), the netcome was 5,519(million USD), the total asset was 62,131(million USD) and the employees were 99,290.

1.2 The Aim of this report

The aim of this report is providing the price of a Tesla's call option by using the static binomial model considering T= 3 months and T=6 months as the maturity time.
Once I got the price of the call option p_0 for T=3 months and T=6 months, I'm going to compare it with the mid price p_m equal to the avarage between Bid and Ask (market quote that is the target price of the model).
The financial Tesla's data that I'm going to use are from $https://finance.yahoo.com/quote/TSLA/options?p = TSLAdate = 1663286400$, while the value of Libor rate R is from $global - rates.com$.

2 Data analysis

First of all I chose to take one share of Tesla at the price of $S = S_0 = 833,59\$$ (this is the price of one share in date 16/03/2022, 5:40PM -Rome) and two call options ATM, one for T=3 months and one for T=6 months. The information of the calls is given in the table below.

	Contract Name	Last Trade Date	Strike K ($)	Last Price ($)	Bid ($)	Ask ($)	Mid Price ($)
$T = 3$	TSLA220617C00835000	2022-03-16 11:17AM EDT	835,00	102,74	102,75	104,05	103,40
$T = 6$	TSLA220916C00830000	2022-03-16 9:49AM EDT	830,00	139,63	144,70	146,85	145,78

In particular the first call is for June 17, 2022 whereas the second one is for September 16 , 2022.
The second step was to calculate daily returns from daily prices. I used historical data from 16/12/21 to 15/03/22 for T=3 months and from 16/09/21 to 15/03/ 22 for T=6 months (they can be found in $yahoo\ finance \longrightarrow Tesla\ Historical\ Data \longrightarrow Historical\ Prices$); the value S_t for each day correspond to the Adj Close Value in yahoo's data.

Once I had gotten daily price's vector $\begin{pmatrix} S_t \\ S_{t-1} \\ . \\ . \\ S_{t-n} \end{pmatrix}$, I could compute daily returns' vector $\begin{pmatrix} \frac{S_t - S_{t-1}}{S_{t-1}} \\ . \\ . \\ \frac{S_{t-j} - S_{t-n}}{S_{t-n}} \end{pmatrix}$ with $j < n$.
Then I calculated the standard deviation of daily returns σ_{day} and the annual volatility $\sigma_{annual} = \sigma_{day} \cdot \sqrt{252}$, where 252 are the market days in one year. Afterwards I computed the parameters u, d of the binomial model using the formula $u, d = e^{\pm \sigma_{daily} \cdot \sqrt{252 \cdot T}}$ (where T's unit of measure is year, so T = 3 months = 0,25 year and T = 6 months = 0,5 year) and the value of the payoff equal to $\begin{pmatrix} f^u = (S \cdot u - K)^+ \\ f^d = (S \cdot d - K)^+ \end{pmatrix}$.

All these values are reported in table below.

	σ_{day}	σ_{annual}	u	d	$S \cdot u(\$)$	$S \cdot d(\$)$	$f^u(\$)$	$f^d(\$)$
$T = 3$	0,04	0,7	1,42	0,71	1180,68	588,54	345,68	0
$T = 6$	0,04	0,6	1,57	0,64	1309,30	530,72	475,71	0

Then I computed the capitalisation factor using simple compounding equal to $1 \longrightarrow 1 + R \cdot T$ (in exponential form: $e^{R \cdot T}$) and simple discounting equal to $1 \longrightarrow \frac{1}{1+R \cdot T}$ (in exponential form: $e^{-R \cdot T}$). I took the R factor from $global - rates.com$ as I anticipated in previous section.

Then I calculated the risk neutral probability $Q = \binom{q}{1-q}$, with risk neutral weight $q = \frac{e^{R \cdot T} - d}{u - d}$ and I checked out that $0 < q < 1$ to avoid arbitrage opportunity.

These values are also reported in table below.

	R	$1 + R \cdot T$	$\frac{1}{1+R \cdot T}$	q	$1 - q$
$T = 3$	0,916%	1,002	0,998	0,417	0,583
$T = 6$	1,131%	1,006	0,994	0,395	0,605

Tesla,Inc., as reported in *yahoo finance* $\longrightarrow$ *Tesla Historical Data* $\longrightarrow$ *Dividend*, doesn't distribute any dividend, so all the computation is done without the dividend factor $S \cdot e^{\delta \cdot T}$.

Once I got all these values I could apply the risk neutral pricing formula for the price of a Call:

$$p_0(Call) = e^{-R \cdot T} \sum_{j=0}^{T} f^{u^j \cdot d^{T-j}} \cdot \binom{T}{j} \cdot q^j \cdot (1-q)^{T-j}$$

And in this particular case :

$$p_0(Call) = e^{-R \cdot T} \cdot (f^u \cdot q + f^d \cdot (1-q))$$

The two values of the Call options are:

- $p_0^{T=3} = 143,84\ \$ \qquad p_0^{T=6} = 186,88\ \$$

3 Conclusions

So, comparing the price computed in this report with the one presented by yahoo, it's evident that there is a difference between them: p_0 is 72% bigger then p_{mid} for T=3 months and 78% bigger for T=6 months.
These results show that binomial model isn't the best model to calculate the price for call options. Although this model is a good mathematical approximation technique and it often gives satisfactory results, in fact it's a discrete time model in a continuous time world so it is plausible this net difference between p_0 and p_{mid}.
Maybe another possible explanation of this gap , as reported in *Chapter* 13, *HullJ.C. Options, Futures and Other Derivatives 9th Edition*, is that in the binomial model we assume a world which is risk-neutral when valuing an option but in general the probability of an up movement in a risk-neutral world is not the same as the probability of an up movement in the real world.

4 Bibliography

- Hull J.C. « Options,Futures and Other Derivatives ninth Edition» (15th January 2014)