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Providing the discount factor and the implicit dividend for maturity T of Apple's call and put option

1 Introduction

1.1 Apple Inc.: The company

Apple Inc. designs, manufactures, and markets smartphones, personal computers, tablets, wearables, and accessories worldwide. It also sells various related services. In addition, the company offers iPhone, a line of smartphones; Mac, a line of personal computers; iPad, a line of multi-purpose tablets; AirPods Max, an over-ear wireless headphone; and wearables, home, and accessories comprising AirPods, Apple TV, Apple Watch, Beats products, HomePod, and iPod touch. Further, it provides AppleCare support services; cloud services store services; and operates various platforms, including the App Store that allow customers to discover and download applications and digital content, such as books, music, video, games, and podcasts. Additionally, the company offers various services, such as Apple Arcade, a game subscription service; Apple Music, which offers users a curated listening experience with on-demand radio stations; Apple News+, a subscription news and magazine service; Apple TV+, which offers exclusive original content; Apple Card, a co-branded credit card; and Apple Pay, a cashless payment service, as well as licenses its intellectual property. The company serves consumers, and small and mid-sized businesses; and the education, enterprise, and government markets. It distributes third-party applications for its products through the App Store. The company also sells its products through its retail and online stores, and direct sales force; and third-party cellular network carriers, wholesalers, retailers, and resellers. Apple Inc. was incorporated in 1977 and is headquartered in Cupertino, California.
Apple is the world's largest information technology company by revenue, the world's largest technology company by total assets, and the world's second-largest mobile phone manufacturer after Samsung. In its fiscal year ending in September 2011, Apple Inc. reported a total of $108 billion in annual revenues—a significant increase from its 2010 revenues of $65 billion—and nearly $82 billion in cash reserves. On March 19, 2012, Apple announced plans for a $2.65-per-share dividend beginning in fourth quarter of 2012, per approval by their board of directors. The company's worldwide annual revenue in 2013 totaled $170 billion.[In May 2013, Apple entered the top ten of the Fortune 500 list of companies for the first time, rising 11 places above its 2012 ranking to take the sixth position. As of 2016, Apple has around US$234 billion of cash and marketable securities, of which 90% is located outside the United States for tax purposes. Apple amassed 65% of all profits made by the eight largest worldwide smartphone manufacturers in quarter one of 2014, according to a report by Canaccord Genuity. In the first quarter of 2015, the company garnered 92% of all earnings. On April 30, 2017, The Wall Street Journal reported that Apple had cash reserves of $250 billion, officially confirmed by Apple as specifically $256.8 billion a few days later. As of August 3, 2018, Apple was the largest publicly traded corporation in the world by market capitalization. On August 2, 2018, Apple became the first publicly traded U.S. company to reach a $1 trillion market value. Apple was ranked No. 4 on the 2018 Fortune 500 rankings of the largest United States corporations by total revenue.

1.2 The Aim of this report

The aim of this report is providing the discount factor and the implicit dividend for maturity T of Apple's call and put option considering T= 2 months T= 3 months T=6 and T=1 year as the maturity time.
In particular I used boxe spread and call-put parity strategy .
The financial Apple's data that I'm going to use are from $https : //finance.yahoo.com/quote/AAPL/options?p = AAPL$.

1.3 Box Spread

A box spread is a combination of a bull call spread with strike prices K1 and K2 and a bear put spread with the same two strike prices. The payoff from a box spread is always $K2 - K1$. The value of a box spread is therefore always the present value of this payoff or $(K_2 - K_1) \cdot e^{-r \cdot T}$, where r is the interested rate. If it has a different value there is an arbitrage opportunity. If the market price of the box spread is too low, it is profitable to buy the box. This involves buying a call with strike price K1, buying a put with strike price K2, selling a call with strike price K2, and selling a put with strike price K1. If the market price of the box spread is too high, it is profitable to sell the box. This involves buying a call with strike price K2, buying a put with strike price K1, selling a call with strike price K1, and selling a put with strike price K2.

1.4 Pull-Call Parity

Put–call parity is a relationship between the price, c, of a European call option on a stock and the price, p, of a European put option on a stock. For a non-dividend-paying stock, it is $c + K \cdot e^{-r \cdot T} = p + S_0$ For a dividend-paying stock, the put–call parity relationship is $c + K \cdot e^{-r \cdot T} + D = p + S_0$.
Put–call parity does not hold for American options. However, it is possible to use arbitrage arguments to obtain upper and lower bounds for the difference between the price of an American call and the price of an American put.

2 Data analysis

2.1 Discount Rate

First of all I chose to take one share of Apple at the price of $S = S_0 = 164,69\$$ (this is the price of one share in date 21/03/2022, 4:10PM -Rome) and four call and put options with strike $K_1 << K_2$, for T= 2 months T= 3 months T=6 and T=1 year. The information of the options is given in the table below.

	Option	Contract Name	Last Trade Date	K (\$)	Last Price (\$)	Bid (\$)	Ask (\$)	Mid (\$)
$T = 2$	Call	AAPL220520C00075000	2022-03-18 9:30AM EDT	75	85.65	90.3	90.6	90.45
	Call	AAPL220520C00205000	2022-03-21 2:41PM EDT	205	0.19	0.17	0.2	0.185
	Put	AAPL220520P00075000	2022-03-21 2:47PM EDT	75	0.05	0.04	0.07	0.055
	Put	AAPL220520P00205000	2022-03-18 10:54AM EDT	205	43.98	39.7	40	39.85
$T = 3$	Call	AAPL230317C00060000	2022-03-17 1:27PM EDT	60	99.45	105.55	105.9	105.725
	Call	AAPL230317C00190000	2022-03-25 3:59PM EDT	190	1.44	1.38	1.41	1.395
	Put	AAPL230317P00060000	2022-03-23 12:18PM EDT	60	0.03	0.03	0.04	0.035
	Put	AAPL230317P00190000	2022-03-25 3:58PM EDT	190	25.72	25.6	25.85	25.725
$T = 6$	Call	AAPL220916C00060000	2022-03-21 12:28PM EDT	60	102.4	105.4	105.8	105.6
	Call	AAPL220916C00190000	2022-03-21 3:58PM EDT	190	4.45	4.35	4.5	4.425
	Put	AAPL220916P00060000	2022-03-18 10:38AM EDT	60	0.12	0.09	0.16	0.125
	Put	AAPL220916P00190000	2022-03-17 2:24PM EDT	190	33.35	28.4	28.7	28.55
$T = 12$	Call	AAPL230317C00060000	2022-03-10 12:49PM EDT	60	98.3	106.35	107.1	106.725
	Call	AAPL230317C00190000	2022-03-21 11:12AM EDT	190	10.5	10.8	11.05	10.925
	Put	AAPL230317P00060000	2022-03-17 12:21PM EDT	60	0.6	0.45	0.55	0.5
	Put	AAPL230317P00190000	2022-03-02 11:27AM EDT	190	39	33	33.25	33.125

The second step was to calculate the discount factor $D_0(T)$ for the different T. I used the fact that the price of box spread is a multiple of the discount factor at T: $p_{boxe\ spread}\ \alpha\ D_0(T)\cdot(K_2-K_1)$. In particalur the value of $D_0(T)$ is given by :

$$D_0(T)=\frac{(p_{mid\ call\ K_1}-p_{mid\ call\ K_2}+p_{mid\ put\ K_2}-p_{mid\ put\ K_2})}{K_2-K_1}$$

where $p_{mid\ call/option\ K}$ is the mid price of the option.

I decided to take different K for T=2 months because there aren't options for $K_1=60$ and $K_2=190$ at T=2 months. By the way I kept the same difference for K_1-K_2=130.

The discount factor's values are in table below.

T	$D_0(T)$
2	1,000
3	0,997
6	1,000
12	0,988

2.2 Implicit dividend

First of all I chose two ATM call and put options with the same strike K for T=2,3,6,12 monts. The value of the option are reported in table below.

	Option	Contract Name	Last Trade Date	K ($)	Last Price ($)	Bid ($)	Ask ($)	Mid ($)
T=2	Call	AAPL220520C00165000	2022-03-22 3:59PM EDT	165	9.85	9.65	9.95	9.8
	Put	AAPL220520P00165000	2022-03-22 3:58PM EDT	165	6	5.85	6.05	5.95
T=3	Call	AAPL220325C00165000	2022-03-21 11:49AM EDT	165	9.95	9.8	9.9	9.85
	Put	AAPL220617P00165000	2022-03-21 11:42AM EDT	165	8.72	8.75	8.85	8.8
T=6	Call	AAPL220916P00165000	2022-03-21 11:54AM EDT	165	12.71	12.75	12.9	12.825
	Put	AAPL220916C00165000	2022-03-21 12:00PM EDT	165	14.45	14.3	14.5	14.4
T=12	Call	AAPL230317C00165000	2022-03-22 2:36PM EDT	165	22.7	22.45	23	22.725
	Put	AAPL230317P00165000	2022-03-22 12:13PM EDT	165	16.71	16.45	16.85	16.65

Then I used the put-call parity to compute the forward contract price $F_0(T)$. In fact, suppose it's possible to find quotes for Call/Put options with the same strike/maturity, we can set up the following strategy:

- buy a Call with strike K and expiry T;
- buy a Put with strike K and expiry T;

Hence the payoff of this strategy is $(S(T)-K)^+-(K-S(T))^+=S(T)-K$ and its price is $C_0(T,K)-P_0(T,K)=D_0(T)\cdot(F_0(T)-K)$. So, by using this strategy, $F_0(T)$ could be deduced from call price $C_0(T,K)$, put price $P_0(T,K)$ and from $D_0(T)$, in particular the formula is :

$$F_0(T)=\frac{C_0(T,K)-P_0(T,K)}{D_0(T)}+K$$

Once I got $F_0(T)$ and $D_0(T)$ I could compute the dividend rate q in exponential form, using call-parity strategy again and, in particular, the following formula:

$$e^{q\cdot T}=\frac{1}{D_0(T)}-\frac{F_0(T)}{S}$$

Then I could compute the implicit dividend D_{impl} for each T :

$$D_{impl}=S\cdot e^{q\cdot T}$$

All the computed values are reported in table below.

T	$F_0(T)(\$)$	$e^{q\cdot T}$	$D_{impl}(\$)$
2	168.85	-0.03	-4.23
3	166.05	-0.008	-1.39
6	166.58	-0.008	-1.39
12	171.15	-0.03	-4.44

3 Conclusions

So I got strange results from data analysis. In fact the value of of $D_0(T)$ should be < 1 but this is not the case, especially for T=2 months and T=3 months (for T=3 and T=6 the values are also near to 1 as-well). By these values of discount rate I also got negative implicit dividends.
One explanation of this strange result is reported in $Hull J.C.\ Chapter 12.3$. In fact, it is important to realize that a box-spread arbitrage only works with European options, but in this case I used American options so it is reasonable that the strategy doesn't work and that I got negative dividens.
In particular, as shown in $Hull J.C\ Capter 12.3\ Business Snapshot$ 12.1, inexperienced traders who treated American options as European were liable to lose money and in this report we have got an example of this.

4 Bibliography

- Hull J.C. « Options,Futures and Other Derivatives ninth Edition» (15th January 2014)
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