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Providing the VaR for an Equibalanced PortFolio of 500$ composed by PayPal's and AliBaba's assets

1 Introduction

1.1 The Aim of this report

The aim of this report is to provide the VaR (value at risk) for an equibalanced portfolio composed by PayPal's and AliBaba's assets and to verify the possibile additivity (or non additivity) of the VaR itself. In particular the VaR is computed as for the single assets as for the portfolio.
The VaR has been computed in different ways:

- Normal VaR at 99% , 99.5%, 95% with T=1,..,100 days horizon considering an estimation of sigma flat;
- Normal VaR at 99% , 99.5%, 95% with T=1,..,100 days horizon considering a sigma computed following Riskmetrics EWMA with λ=0.94;
- VaR computed with MonteCarlo simulation at levels 99%,99.5%, 95%;
- VaR computed with the method of the historical simulation

Furthermore the Historical VaR is been computed as-well according to the historical value of the returns of the portfolio.
In this report I also shown that there is not subadditivity for the VaR for my portfolio. In fact one of the problems related to VaR is that this risk measure is not-subadditive: this is equivalent to saying that, given two portfolios X and Y it could be that $VaR(X+Y) > VaR(X)+VaR(Y)$. This result means that diversification (achieved with the two assets) does not necessarily reduce risk.
By the way when we say VaR is not sub-additive, we mean that it is possible to find cases where it fails. That doesn't mean it always fails in fact there are particular cases where it can be. Such cases occur for portfolios containing elliptically distributed risk factors (for example the normal distribution is among the elliptical distributions family).

1.2 PayPal company profile

PayPal Holdings, Inc. operates a technology platform that enables digital payments on behalf of merchants and consumers worldwide. It provides payment solutions under the PayPal, PayPal Credit, Braintree, Venmo, Xoom, Zettle, Hyperwallet, Honey, and Paidy names. The company's payments platform allows consumers to send and receive payments in approximately 200 markets and in approximately 100 currencies, withdraw funds to their bank accounts in 56 currencies, and hold balances in their PayPal accounts in 25 currencies. PayPal Holdings, Inc. was founded in 1998 and is headquartered in San Jose, California. Some other informations are listed below:

- adress: 2211 North First Street San Jose, CA 95131,United States
- number: 408 967 1000

- website: https://www.paypal.com
- Sector(s): Financial Services
- Industry: Credit Services
- Full Time Employees: 30,900

1.3 Alibaba company profile

Alibaba Group Holding Limited, through its subsidiaries, provides technology infrastructure and marketing reach to merchants, brands, retailers, and other businesses to engage with their users and customers in the People's Republic of China and internationally. It operates through four segments: Core Commerce, Cloud Computing, Digital Media and Entertainment, and Innovation Initiatives and Others. The company operates Taobao Marketplace, a social commerce platform; Tmall, a third-party online and mobile commerce platform for brands and retailers; Alimama, a monetization platform; 1688.com and Alibaba.com, which are online wholesale marketplaces; AliExpress, a retail marketplace; Lazada, Trendyol, and Daraz that are e-commerce platforms; and Tmall Global and Kaola, which are import e-commerce platforms. It also operates Lingshoutong that connects FMCG manufacturers and their distributors to small retailers; Cainiao Network logistic services platform; Ele.me, an on-demand delivery and local services platform; Koubei, a restaurant and local services guide platform; and Fliggy, an online travel platform. In addition, the company offers pay-for-performance, in-feed, and display marketing services; and Taobao Ad Network and Exchange, a real-time online bidding marketing exchange. Further, it provides elastic computing, database, storage, virtualization network, large-scale computing, security, management and application, big data analytics, machine learning platform, and Internet of Things services. Additionally, the company operates Youku, an online video platform; Alibaba Pictures and other content platforms that provide online videos, films, live events, news feeds, literature, music, and others; Amap, a mobile digital map, navigation, and real-time traffic information app; DingTalk, a business efficiency app; and Tmall Genie, an AI-enabled smart speaker. The company was incorporated in 1999 and is based in Hangzhou, the People's Republic of China. Some other informations are listed below:

- adress :969 West Wen Yi Road Yu Hang District Hangzhou, 311121 China
- number: 86 571 8502 2088
- website: https://www.alibabagroup.com
- Sector(s): Consumer Cyclical
- Industry: Internet Retail
- Full Time Employees: 259,316

1.4 The Var

Value at Risk (VaR) is an attempt to provide a single number summarizing the total risk in a portfolio of financial assets. It has become widely used by corporate treasurers and fund managers as well as by financial institutions. Bank regulators have traditionally used VaR in determining the capital a bank is required to keep for the risks it is bearing. When using the value-at-risk measure, an analyst is interested in making a statement of the following form:

" I am X percent certain there will not be a loss of more than V dollars in the next N days."

The variable V is the VaR of the portfolio. It is a function of two parameters: the time horizon (N days) and the confidence level ($X\%$). It is the loss level over N days that has a probability of only (100 -X)% of being exceeded. Bank regulators require banks to calculate VaR for market risk with $N = 10$ and $X = 99$.
When N days is the time horizon and X% is the confidence level, VaR is the loss corresponding to the (100-X)th percentile of the distribution of the gain in the value of the portfolio over the next N days. (Note that, when we look at the probability distribution of the gain, a loss is a negative gain and VaR is concerned with the left tail of the distribution. When we look at the probability distribution of the loss, a gain is a negative loss and VaR is concerned with the right tail of the distribution.) For example, when N=5 and X=97, VaR is the third percentile of the distribution of gain in the value of the portfolio over the next 5 days.
VaR is an attractive measure because it is easy to understand. In essence, it asks the simple question "How

bad can things get?" This is the question all senior managers want answered. They are very comfortable with the idea of compressing all the Greek letters for all the market variables underlying a portfolio into a single number.
VaR has two parameters: the time horizon N, measured in days, and the confidence level X. In practice, analysts almost invariably set N=1 in the first instance when VaR is estimated for market risk. This is because there is not usually enough data available to estimate directly the behavior of market variables over periods of time longer than 1 day. The usual assumption is:

$$N - day\ VaR = 1 - day\ VaR \cdot \sqrt{N}$$

This formula is exactly true when the changes in the value of the portfolio on successive days have independent identical normal distributions with mean zero. In other cases it is an approximation. There are three methods to calculate Value at Risk which are : Historical Simulation , The Model-Building Approach and Monte Carlo Simulation.

1.4.1 Historical Simulation

In the Historical Simulation we create a database of the daily movements in all market variables. The first simulation trial assumes that the percentage changes in all market variables are as on the first day, the second simulation trial assumes that the percentage changes in all market variables are as on the second day and so on for all the other days.
In particular:

- Suppose we use m days of historical data
- Let v_i be the value of a variable on day i;
- There are m-1 simulation trials;
- The ith trial assumes that the value of the market variable tomorrow (i.e., on day $m+1$) is:

$$v_m \cdot \frac{v_i}{v_{i-1}}$$

1.4.2 The Model-Building Approach

The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically This is known as the model building approach or the variance-covariance approach.
In VaR calculations we measure volatility "per day" $\sigma_{day} = \frac{\sigma_{year}}{\sqrt{252}}$, in particular strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day, in practice we assume that it is the standard deviation of the percentage change in one day.

1.4.3 The Monte Carlo Method

To calculate VaR using M.C. simulation we:

- Value portfolio today;
- Sample once from the multivariate distributions of the Δx_i;
- Use the Δx_i to determine market variables at end of one day;
- Revalue the portfolio at the end of day;
- Calculate ΔP;
- Repeat many times to build up a probability distribution for ΔP;
- VaR is the appropriate fractile of the distribution times square root of N

2 Data analysis

2.1 Equibalanced Portfolio

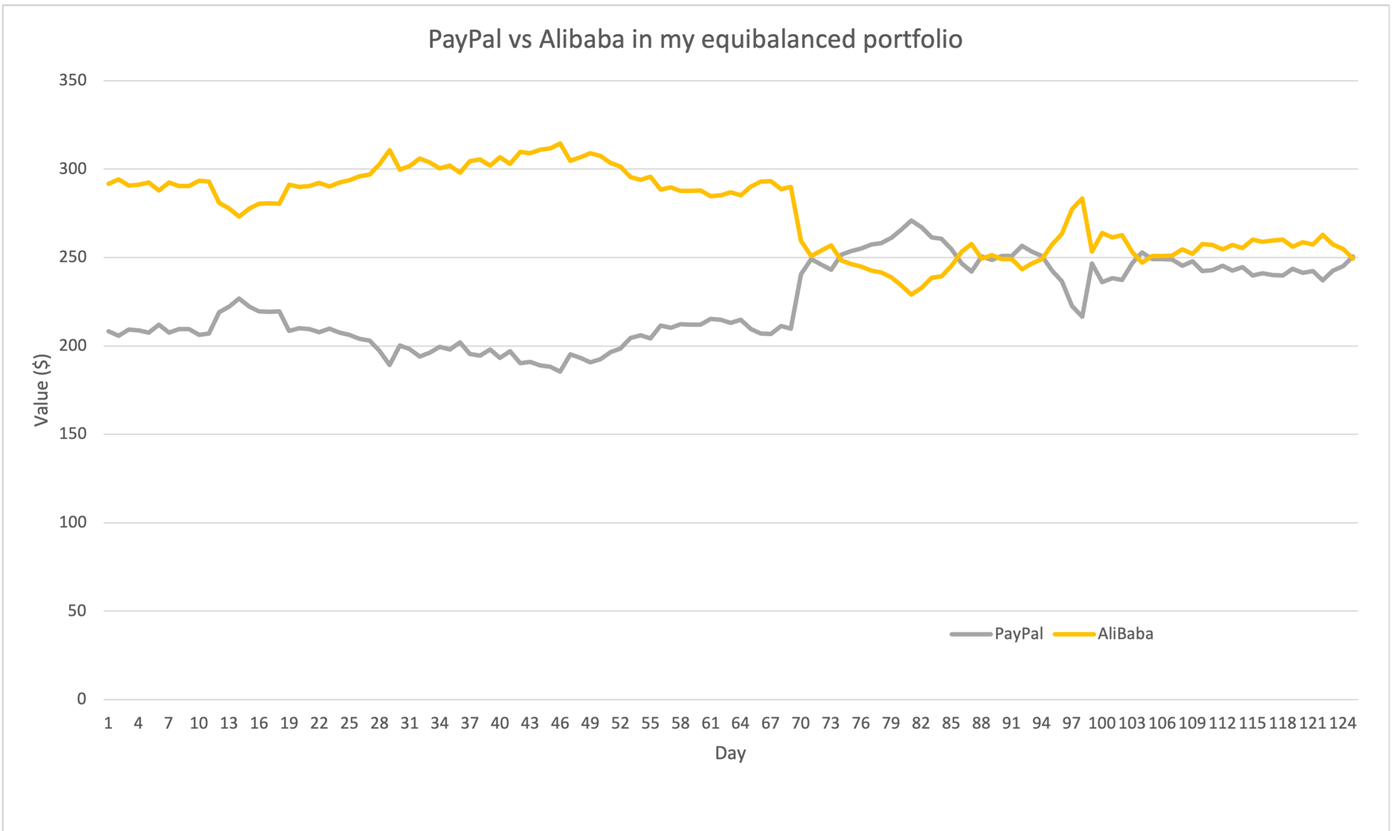
First of all I chose two assets (PayPal and AliBaba) with the same close price (86.03$ for PayPal and 86.49$ for AliBaba in date 23/04/2022) and I downloaded the historical data of the previous 6 months from $yahoo\ finance \longrightarrow PayPal/Alibaba\ Historical\ Data$.
In order to build a equibalanced portfolio I decideded to consider the following equation:

$$p \cdot S_1 + (1-p) \cdot S_2 = c$$

where p is the weight of the asset while c is the value of the portfolio (in this case 500$) and $S_{1,2}$ the historical price for the two assets.
In particular I have a balance portfolio if $p \in [0,1]$, $p \cdot S_1 = \frac{c}{2}$ and $(1-p) \cdot S_2 = \frac{c}{2}$. By this consideration I got the following system of two equations:

$$\begin{cases} p = \frac{S_2}{S_1+S_2} \\ (1-p) = \frac{S_1}{S_1+S_2} \end{cases} . \tag{1}$$

So for each asset's daily close price I got its weight (p) in the portfolio and setting $S \cdot weight = \frac{c}{2}$ I got the constraint that keeps the balance between the two assets.
Hence, by these considerations and using the historical data I could compute p and $(1-p)$ for each day of the last 6 months and I could compute the assets' price for my equibalanced portfolio of 500$ for each day by these formulas $S_1 = 250 \cdot p$ and $S_2 = 250 \cdot (1-p)$. The value of the two assets in my portfolio are reported in the graph below. In particular day=1 is the first day of the historical data that corresponds to 25/10/2021 while day=124 to 23/04/2022.



1. Plot of the value of the 2 assets in the balanced portfolio.

As shown in the graph, as one of the two assets increases the other decreases in order to keep the balance in the portfolio. I want to underline the fact that I consider an equibalanced portfolio for each day i.e. at the end of the day the 500$ are recessed and in the next day they are invested again. This means that there isn't at the end a cumulative sum of the money invested (500$ $\cdot$ 6 months) but just 500$. I decided to following this way to make the computation of equibalanced portfolio simpler.

Then I computed the mean and the variance for the two assets in the portfolio. The values are reported in table below.

	$PayPal$	$Alibaba$	$Portfolio$
$E[X](\$)$	223.95	276.05	500
$\sigma^2(\$)$	529.13	529.13	0

Table with the mean and the variance for each asset and for the portfolio.

As expected the mean of the portfolio is 500$ and the variance equal to 0; this is due to the fact that there is a balance between the two assets.

2.2 VaR computation with the Model-Building Approach

2.2.1 σ flat

First of all I computed the returns of the closing price $Returns = \frac{S_t - S_{t-1}}{S_{t-1}}$ for both assets and for the portfolio (considering the weight p), then the mean of the returns and the standard deviation. In particular the standard deviation for the portfolio has been computed by the following formula:

$$\sigma_{portfolio} = \sqrt{\sigma^2_{PayPal} + \sigma^2_{Alibaba} + 2 \cdot \rho \cdot \sigma_{Alibaba} \cdot \sigma_{PayPal}}$$

where $\rho = -0.988$ is the correlation between the two assets. The value are reported below in table below:

	$PayPal$	$Alibaba$	$Portfolio$
$E[X](\$)$	0.002	-0.001	0.0006
$\sigma(\$)$	0.027	0.021	0.0069

Table with the mean and the variance for each asset's close return and for the portfolio's ones.

Then I computed the VaR for 99.5%, 95%, and 99% confidence level using $NORM.INV function$ in Excel. In particular the value are:

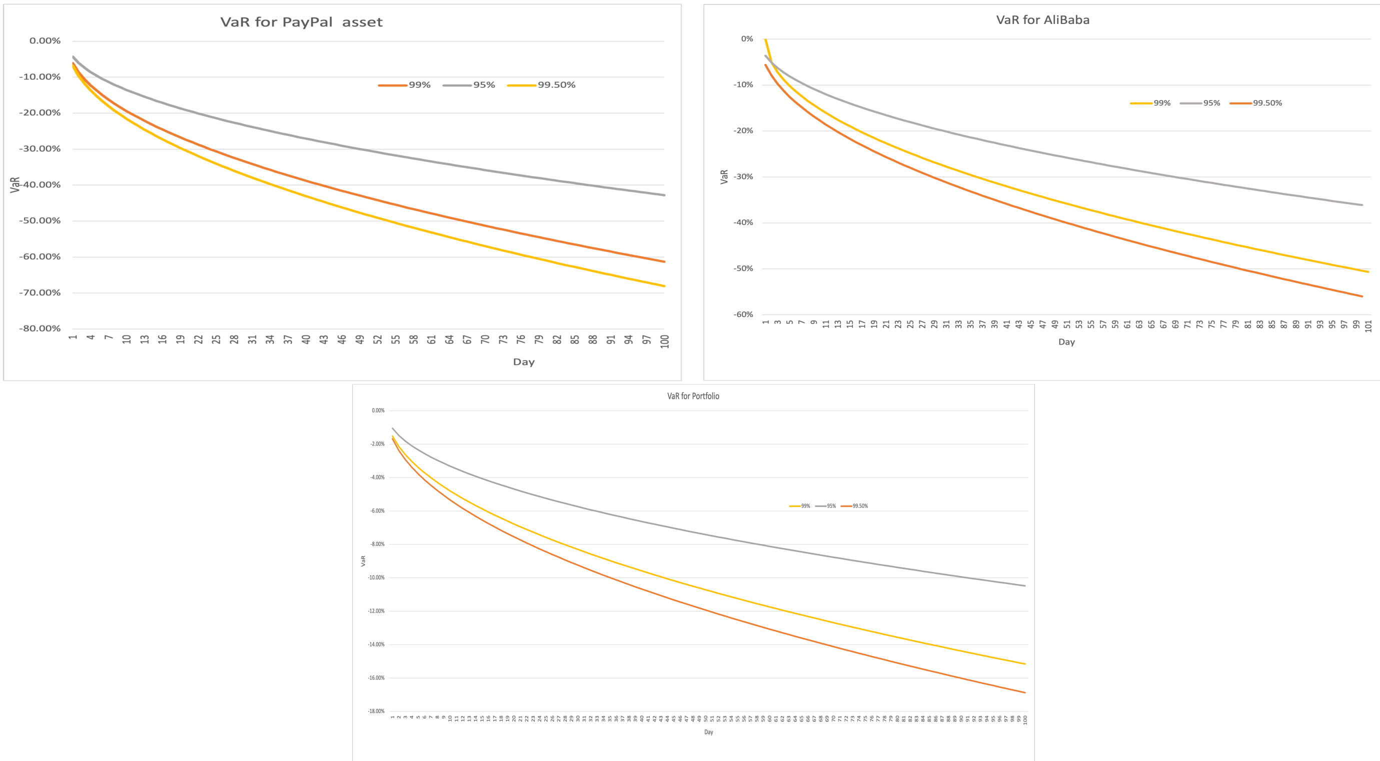
	$N(-X) = 0.01$	$N(-X) = 0.05$	$N(-X) = 0.005$
X	2.326	1.645	2.576

Hence, I could compute the VaR for one day as $VaR = E[X] - X \cdot \sigma$. The computed values are reported below.

$ConfidenceLevel$	$PayPal$	$Alibaba$	$Portfolio$
99%	-6.1%	-5.1%	-1.5%
95%	-4.3%	-3.6%	-1%
99.5%	-6.8%	-5.6%	-1.7%

Table with the VaR for each asset and for the portfolio.

In order to compute the VaR for T=1,..,100 days I used the formula $VaR_{day_N} = VaR_{day_1} \cdot \sqrt{N}$. The results are reported in the graphs below.



2. Plot of VaR for the different assets for N=1,..,100 days .

2.2.2 σ estimated following Riskmetrics EWMA with $\lambda = 0.94$

In this case I computed the σ used to calculated the VaR following Riskmetrics EWMA with $\lambda = 0.94$. In particular the standard deviation of one asset is given by the formula:

$$\sigma = \sqrt{\lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2}$$

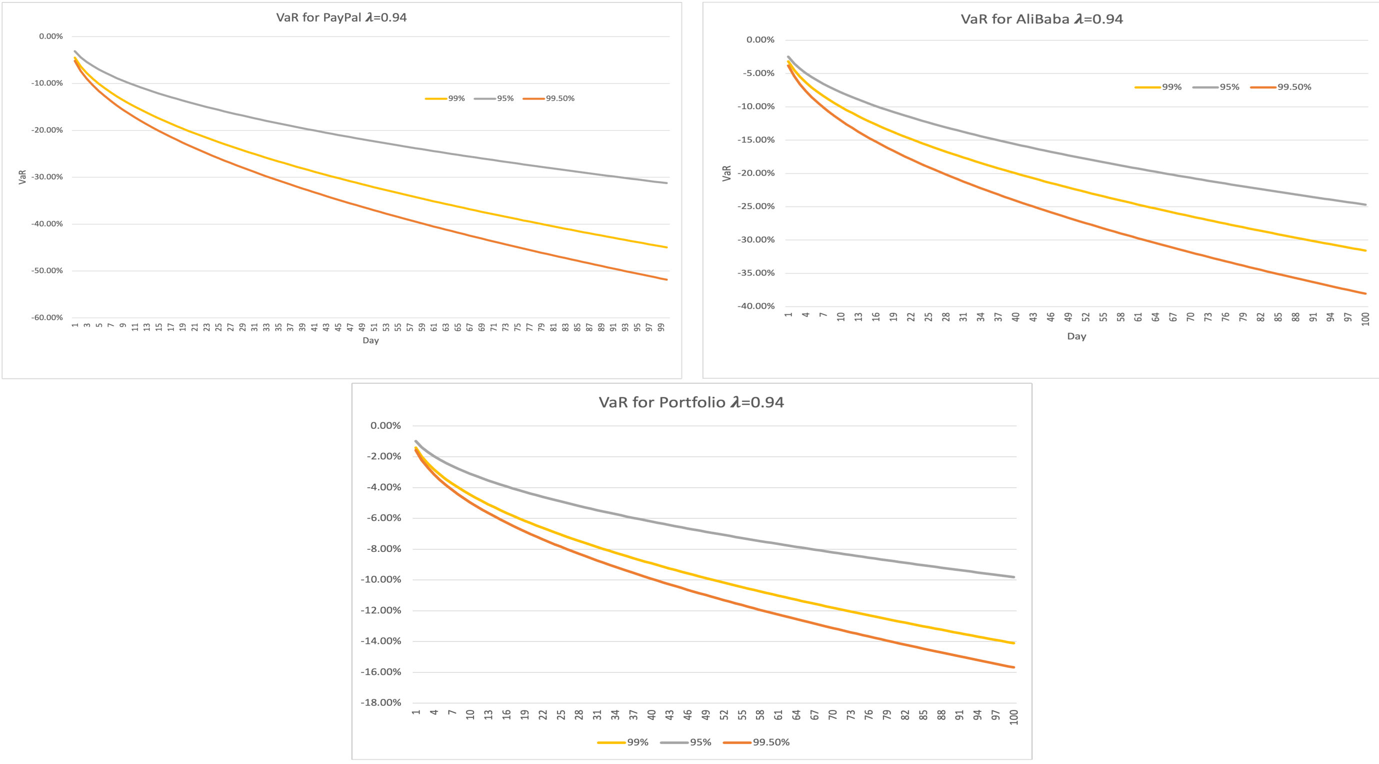
where the estimate, σ_n, of the volatility of a variable for day n (made at the end of day $n-1$) is calculated from σ_{n-1} (the estimate that was made at the end of day $n-2$ of the volatility or day $n-1$) and u_{n-1} (the most recent daily percentage change in the variable). The standard deviation of the portfolio and the VaR has been computed in the same way explained in the chapter before using the new standard deviations for the two assets.
The results are reported in tables below.

	$PayPal$	$AliBaba$	$Portfolio$
$\sigma(\$)$	0.02	0.01	0.006

$ConfidenceLevel$	$PayPal$	$Alibaba$	$Portfolio$
99%	-4.5%	-3.2%	-1.4%
95%	-3.1%	-2.5%	-1%
99.5%	-5.2%	-3.8%	-1.5%

Tables with the standard deviation and the VaR for each asset and for the portfolio.

Again, in order to compute the VaR for T=1,..,100 days I used the formula $VaR_{day_N} = VaR_{day_1} \cdot \sqrt{N}$. The results are reported in the graphs below.



3. Plot of VaR for the different assets with $\lambda = 0.94$ for N=1,..,100 days .

2.2.3 Conclusions

In σ-flat's case there is an non additivity for the VaR. In fact the VaR of the portfolio for all the three confidence levels is given (at first approximation) by the subtraction of the two assets' VaRs .
The same for EWMA's case.
It also interesting to see that tha VaR computed by using $\lambda = 0.94$ are less than the ones with σ-flat. One explanation of this could be the fact that the volatility of the two assets in the last 6 months is particularly high so this could have engraved the computation of the σs used to computed the VaR.
By the way, even if for the two assets the VaR is different, the portfolio's VaR, as expected, is the same for the two cases (considering the first significant digit).

2.3 The Monte Carlo Method

First of all to compute the Monte Carlo simulation I generated a seed value that correspondes to a possible ending value for the asset. In order to calculate this I used a derivation from the $Black - Scholes - Models$:

$$Final\ value = e^{((r-0.5 \cdot \sigma^2)\frac{T}{252} + \sigma \cdot \sqrt{\frac{T}{252}} \cdot NORM.S.INV(RAND()))}$$

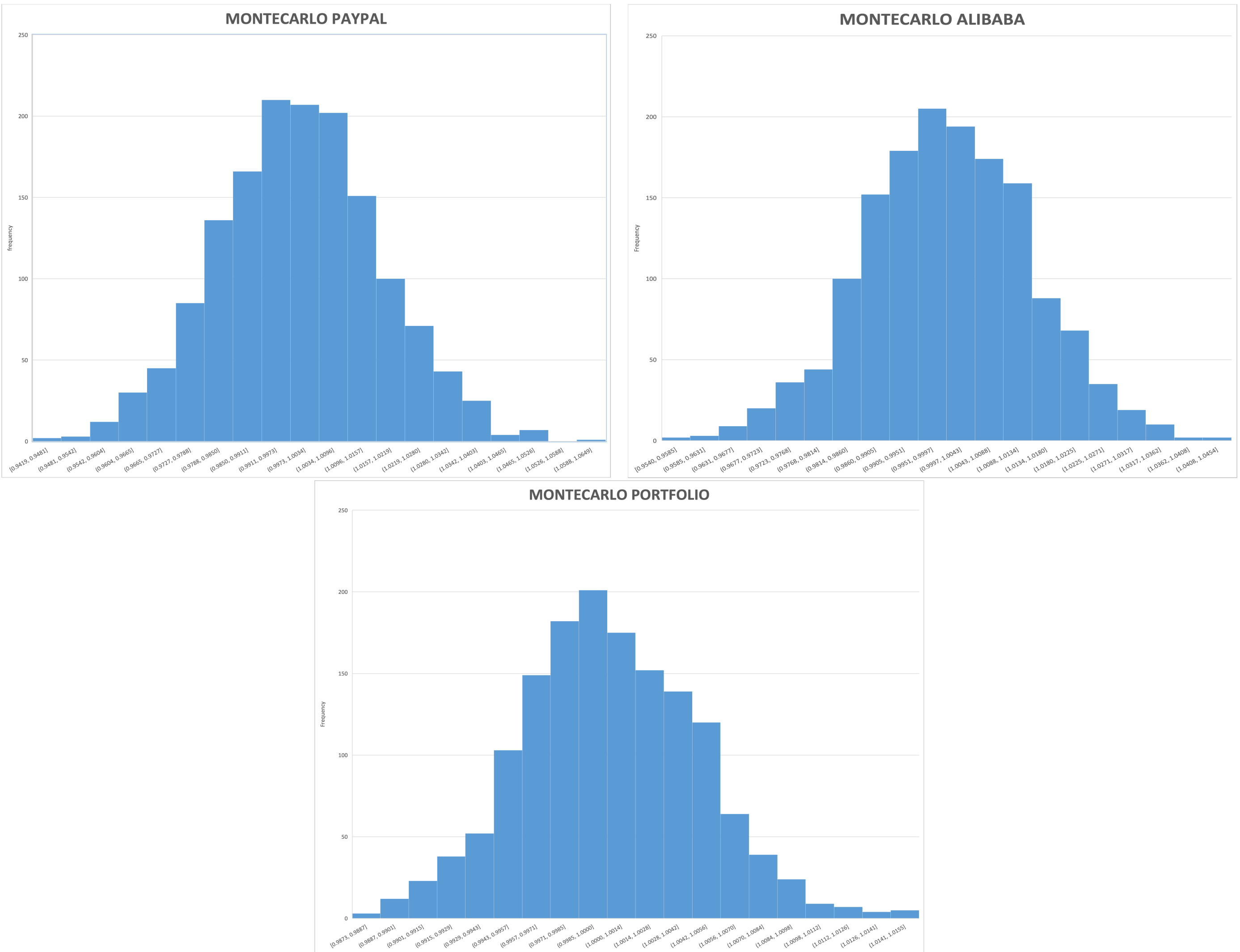
where σ is the expected volatility (I used the σ-flat computed before),T is the time for the VaR, r is the riskfree rate(=0 in this case) and NORM.S.INV(RAND()) is a function of Excel that generates a random variable.
Then from this seed I simulated 1500 ending values for T=100 days and using the PERCENTILE.EXC() function I could compute the VaRs. In particular the formula is :

$$VaR = 1 - PERCENTILE.EXC(1th\ seed\ value : 1500th\ seed\ value, 1 - confidence\ level)$$

The results from the simulations are reported in table and in the graphs below.

$ConfidenceLevel$	$PayPal$	$Alibaba$	$Portfolio$
99%	-4.1%	-3.2%	-1%
95%	-2.9%	-2.3%	-0.7%
99.5%	-4.5%	-3.5%	-1.1%

Table with VaR values for each asset and for the portfolio computed with Monte Carlo simulation.



4. Plot of Monte Carlo simulations for the two assets and for the portfolio considering T=100 days .

In particular the VaR is calculated as the appropriate percentile of the probability distribution of ΔP. So In this case for example with 1500 steps the 100-day 99% VaR is the value of ΔP for the 15-th worst outcome. This procedure gives the same result computed using the formula above the previous table.

2.3.1 Conclusions

Also with Monte Carlo simulations there is non additivity for the VaR. In fact the VaR of the portfolio for all the three confidence levels is given (at first approximation) by the subtraction of the two assets' VaRs.
Altough this, in first approximation (considering just the first significant digit) the results obtained from Monte Carlo simulations are the same computed considering σ calculated following the EWMA.
This prove the fact that the σ-flat method maybe it's not the best way to compute the VaR.

2.4 Historical Simulation

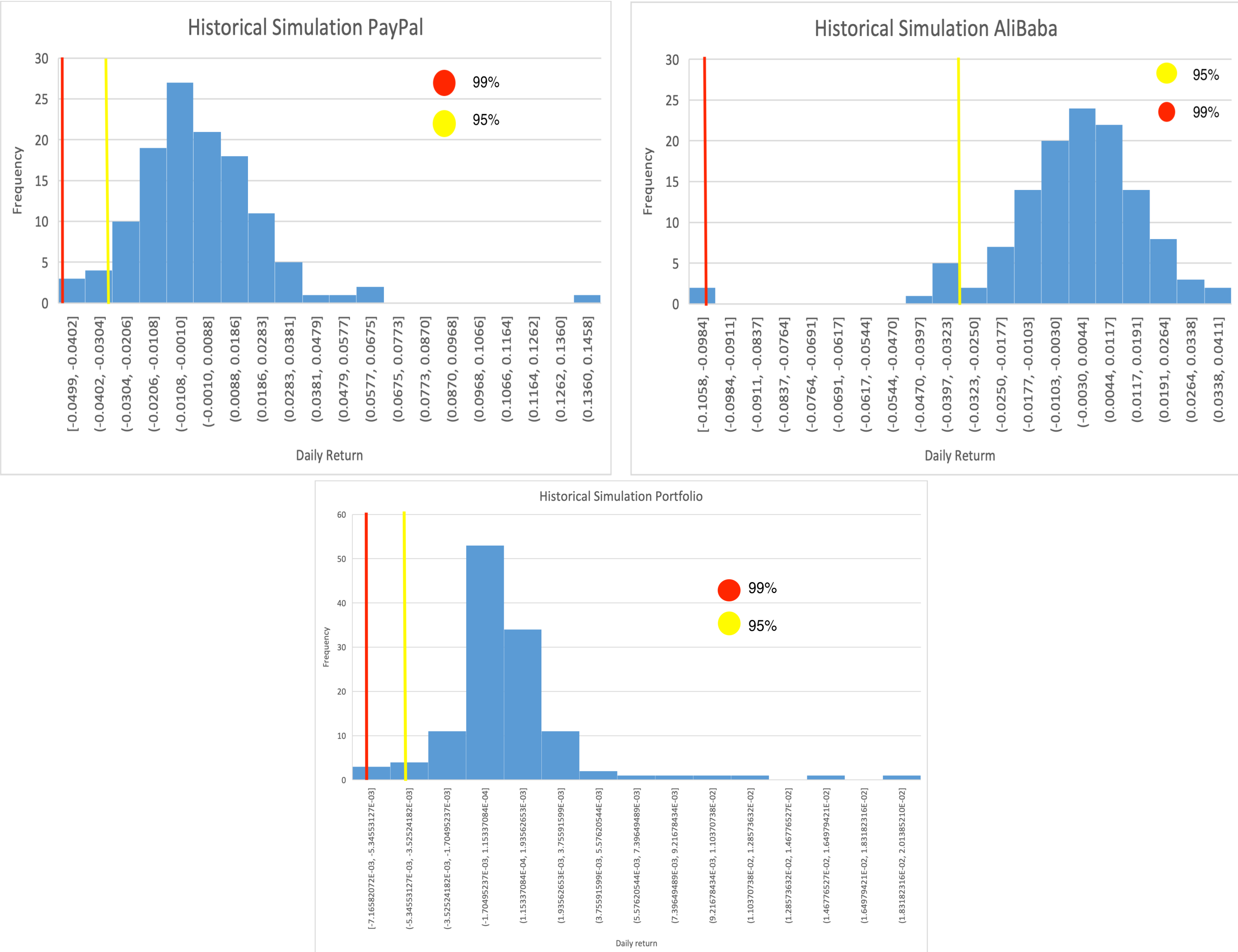
First of all I computed the close returns as explained in chapter 2.2.1, than I sorted them from the worst to the best. In order to compute the VaR I used an Excel function:

$$VaR = PERCENTILE.EXC(Worst\ Close\ Return : Best\ Close\ Return, Y)$$

where $Y = 1 - Confidence\ Level$. In particular I computed VaR(99%) and VaR(95%) and the values are reported in table below.

$ConfidenceLevel$	$PayPal$	$Alibaba$	$Portfolio$
99%	-5.7%	-10.6%	-0.7%
95%	-3.2%	-3.4%	-0.4%

Table with VaR values for the two assets and for the portfolio computed by historical simulation.



5. Histograms for the historical simulation for the two assets and for the portfolio .

2.4.1 Conclusions

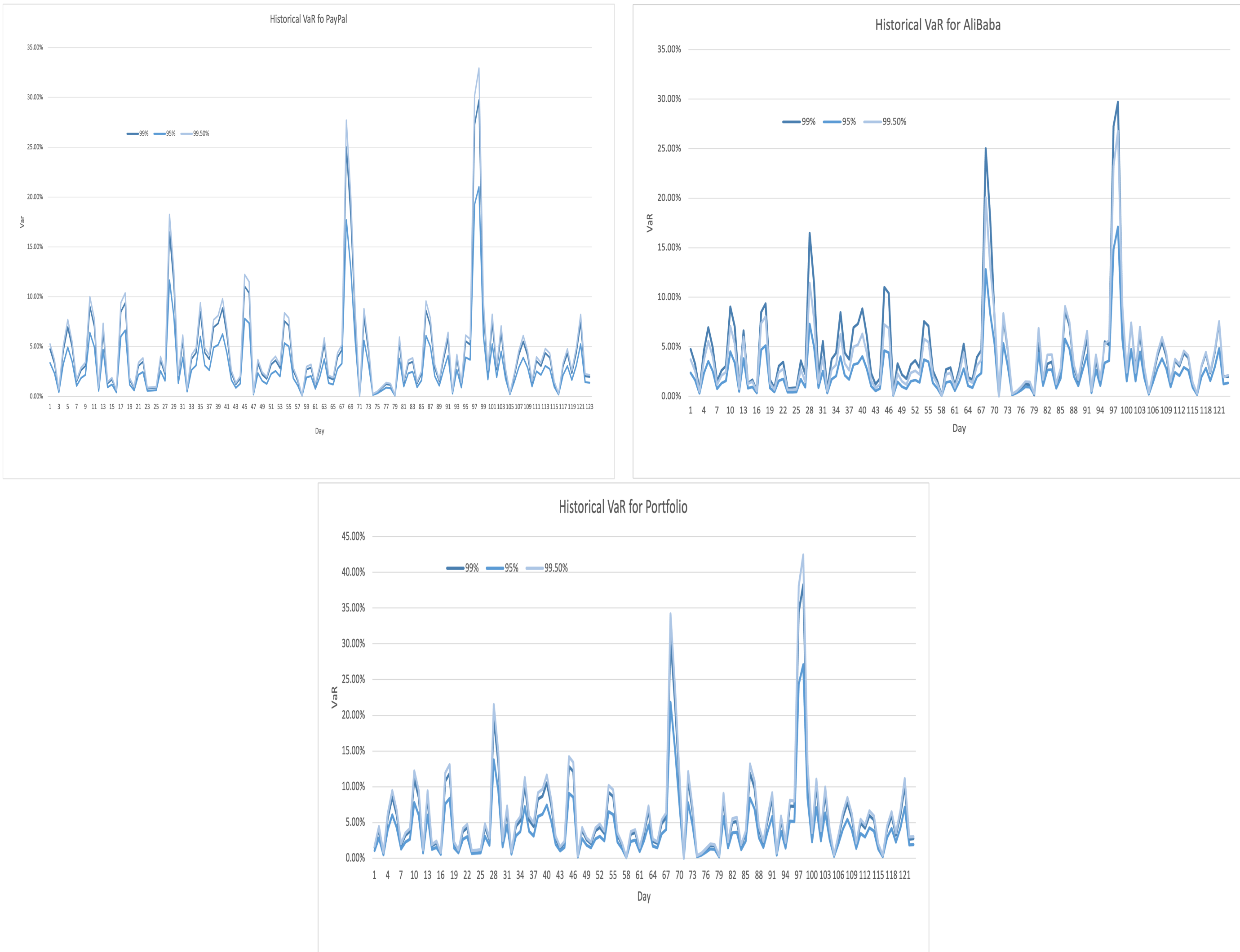
Also in this case there is non additivity.
Furthermore the VaR is equal to the one computed with Monte Carlo simulations and with the Model-Building Approach just for 95% confidence level (considering the first significant digit) for the 2 assets and not for the portfolio.This proves that the return distribution is not normal (in fact there isn't sub-additivity).

2.5 Historical VaR

First of all I computed the standard deviation between the $i-th$ and $i-th+1$ close returns for each asset and for the portfolio. Than I computed the VaR for each historical return using the formula :

$$VaR = X \cdot \sigma_i$$

where X is equal to 2.326,1.645 and 2.576 (C.L 99%,95% and 99.5%) and σ_i is the standard deviation between the $i-th$ and $i-th+1$ close returns. In particular day=1 is the first day of the historical data that corresponds to 25/10/2021 while day=124 to 23/04/2022.
The results for the last 6 months are reported in graphs below.



6. Plot of the historical VaR for the two assets and for the portfolio for the last six months .

2.5.1 Conclusions

In this case there isn't a strong additivity but the VaR for the portfolio is almost equal to the sum of the two assets' VaRs and for some days equal to the sum itself (otherwise is less than the sum). This means that in this case there is subadditivity.
It is also intersting to see how the VaR is particular high between days 95-101 and probably one explanation of that is the fact that these days corresponds to the first days of Ukraine's war that made the market unstable.

3 Bibliography

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-https://it.wikipedia.org/wiki/Valorearischio