Stochastic Methods for Finance

Exam 27 June 2016

Exercice 1 *Consider a Black&Scholes market and a derivative contract with payoff at the maturity T given by*

$$(K_2 - 2 * S_T) * 1_{K_1 < S_T < K_2},$$

where 1_A denotes the indicator function of the event A and $0 < K_1 < K_2$.

i) Compute the price of the contract at any time $t \in [0, +\infty)$;

ii) Compute the Delta of the contract;

iii) Illustrate graphically the change of price and Delta for a upward shift of the volatility.

Exercice 2 *Consider a Black&Scholes market and a quadratic-CALL contract, that is a derivative with payoff at the maturity T given by*

$$(S_T^2 - K)^+$$

i) Compute the price of the contract at any time $t \in [0, +\infty)$;

ii) Compute the Delta of the contract;

iii) Illustrate graphically the change of price and Delta for a upward shift of the volatility.

Exercice 3 *i) Provide an example of a derivative contract that has a negative Gamma.*

ii) Give a financial interpretation of potential hedging consequences of a negative Gamma.

Exercice 4 *Solve the following PDE for $t \leq T$:*

$$\begin{aligned} \frac{\partial F}{\partial t} + x^2 \frac{\partial^2 F}{\partial x^2} + F + e^x &= 0 \\ F(T, x) &= x. \end{aligned}$$

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Exercice 5 *In the Black-Scholes model, find the price at time $t \leq T$ of a Digital Put DOWN-AND-OUT with strike price K, where the owner receives a unitary payoff at the maturity T only if the asset has not reached the lower barrier L. Compare this price with the one of a Digital Put (without barrier). Finally, find the Delta of both options.*

Exercice 6 *Questions on the theory.*

i) State and prove the Farkas Lemma.

ii) Find the moment generating function of the first hitting time for the standard Brownian motion at level $a \in \mathbb{R}$.

iii) Find the dynamics of the exchange rate process X in a 2-currencies Black-Scholes model in both the domestic and foreign economies under the corresponding risk neutral probability measures.

iv) Show the backward equation satisfied by the optimal value of an American option in a (discrete time) binomial model and prove it.