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Providing the Greeks for a BS model using a VBA script, providing the implied volatility surface and the Greeks for TESLA's call options and pricing with the BS formula the ATM calls for TESLA

1 Introduction

1.1 The Aim of this report

This report consists in 3 different parts with differents aims. In the first part I calculated the Greeks letters for a BS model using a VBA script, in the second one I computed the implied volatility surface and the Greeks for TESLA's call options and I provided the Greeks using these TESLA's options and in the third one I computed the price of the ATM options using BS formula again and I compared the implied volatility with the historical one.
The general aim is to compare the theoretical computation with the empirical one to verify if I get the results that I expect to get from the theory.

2 Part A

2.1 The Greeks for a BS model

A financial institution that sells an option to a client in the over-the-counter markets is faced with the problem of managing its risk. If the option happens to be the same as one that is traded on an exchange, the financial institution can neutralize its exposure by buying on the exchange the same option as it has sold. But when the option has been tailored to the needs of a client and does not correspond to the standardized products traded by exchanges, hedging the exposure is far more difficult. One way to approach this problem is using the "Greek letters" (or "Greeks").
Each Greek letter measures a different dimension to the risk in an option position and the aim of a trader is to manage the Greeks so that all risks are acceptable.

2.1.1 Delta

The delta Δ of a portfolio of an option is defined as

$$\Delta = \frac{\delta p}{\delta S} = \Phi(d_1)$$

where p is the price of the portfolio , S is the price of the underlying asset , $\Phi(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy$ and $d_1 = \frac{ln(\frac{S}{K})+(R+\frac{1}{2}\sigma^2)\cdot(T-t)}{\sigma\sqrt{T-t}}$. In particular d_1 is given by BS formula.

2.1.2 Gamma

The gamma Γ of a portfolio of an option is defined as

$$\Gamma = \frac{\delta^2 p}{\delta^2 S} = \frac{\delta \Delta}{\delta S} = \frac{1}{S \cdot \sigma \cdot \sqrt{T-t}} \cdot \phi(d_1)$$

where p is the price of the portfolio , σ the volatility,S is the price of the underlying asset, $\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ is the gaussian density distribution and $d_1 = \frac{ln(\frac{S}{K})+(R+\frac{1}{2}\sigma^2)\cdot(T-t)}{\sigma\sqrt{T-t}}$. In particular d_1 is given by BS formula.

2.1.3 Rho

The rho ρ of a portfolio of an option is defined as

$$\rho = \frac{\delta p}{\delta R} = K \cdot (T-t) \cdot e^{-R \cdot (T-t)} \cdot \Phi(d_2)$$

where p is the price of the portfolio , T is matuirty time, S is the price of the underlying asset, K is the strike price, R is the interest rate and $d_2 = d_1 - \sigma\sqrt{T-t}$ and $\Phi(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy$. In particular d_1 and d_2 are given by BS formula.

2.1.4 Vega

The vega ν of a portfolio of an option is defined as

$$\nu = \frac{\delta p}{\delta \sigma} = S \cdot (T-t) \cdot \cdot \phi(d_1)$$

where p is the price of the portfolio , S is the price of the underlying asset, $\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ is the gaussian density distribution, σ is the volatility and $d_1 = \frac{ln(\frac{S}{K})+(R+\frac{1}{2}\sigma^2)\cdot(T-t)}{\sigma\sqrt{T-t}}$. In particular d_1 is given by BS formula.

2.1.5 Theta

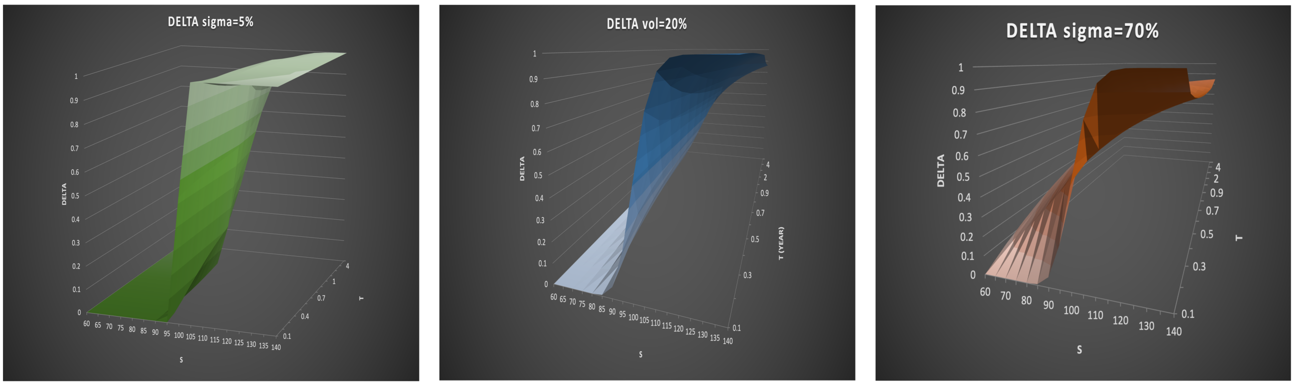
The theta Θ of a portfolio of an option is defined as

$$\Theta = \frac{\delta p}{\delta t} = -\frac{S \cdot \phi(d_1) \cdot \sigma}{2\sqrt{T-t}} - R \cdot K \cdot e^{-R \cdot (T-t)} \cdot \Phi(d_2)$$

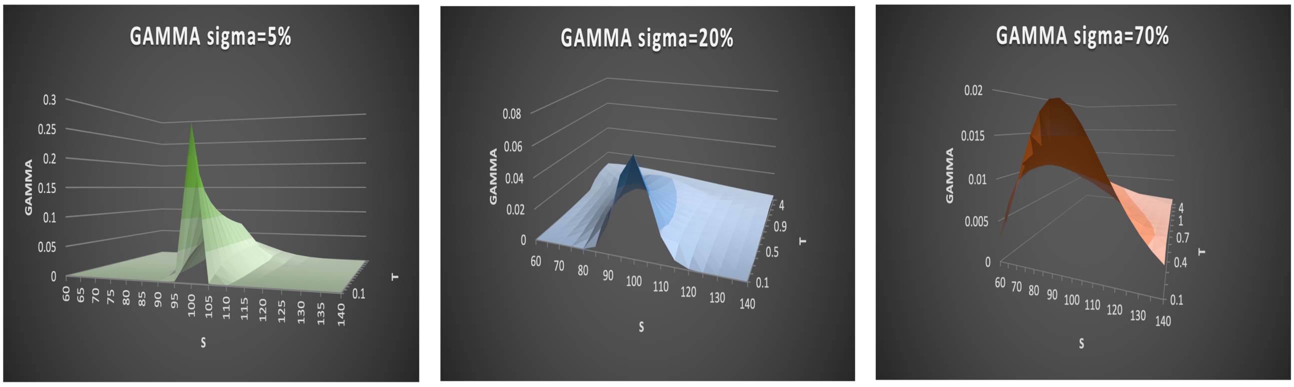
where p is the price of the portfolio , S is the price of the underlying asset, $\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ is the gaussian density distribution, σ is the volatility, $\Phi(x) = \int_{-\infty}^{x} \frac{e^{\frac{-y^2}{2}}}{\sqrt{2\pi}} dy$, K the strike price, R the interest rate, $d_1 = \frac{ln(\frac{S}{K})+(R+\frac{1}{2}\sigma^2)\cdot(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$. In particular d_1 and d_2 are given by BS formula.

2.2 Data analysis

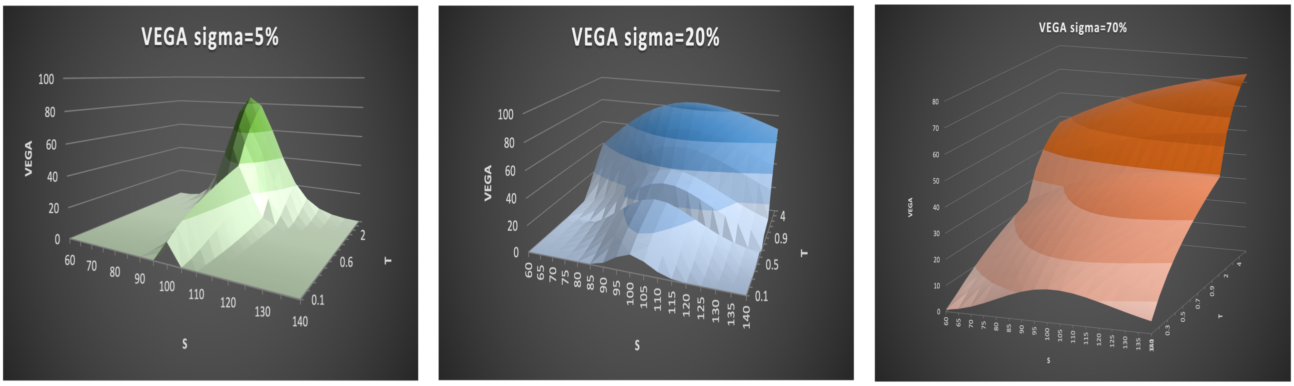
In the first part I calculated the Greeks using VBA script with fixed values for S, K, t, σ, R. In particular S=60,70,..,140, K=10 R=1%, $\sigma = 20\%, 5\%, 70\%$ and t=0.1,0.2,..1,2,3,4,5 years. I want also to underline that in these formulas I consider the continuous dividend yield (or foreign interest rate with currency options) equal to 0. The VBA script that I used to compute the Greeks can be found in $https://sites.google.com/view/vinegarhill-financelabs/black-scholes-merton/black-scholes-greeks$.
Then I plotted the different results for each Greek letter as function of time t and underlying S to show how incisive is the value of σ and how the shape changes dramatically when σ itself changes. This is why I decided to take σ=5% and σ=70% that are extreme cases.
The plots are reported below.



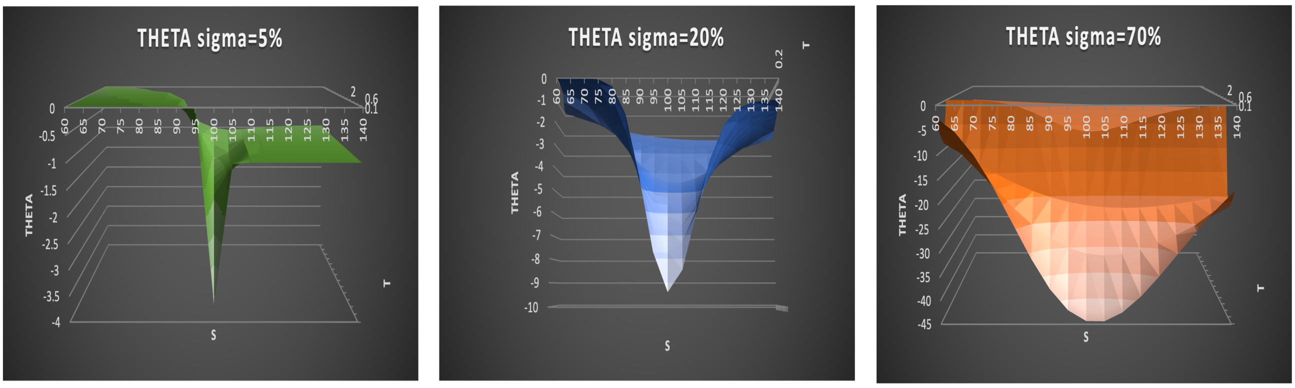
1. Plot of Δ with different σ.



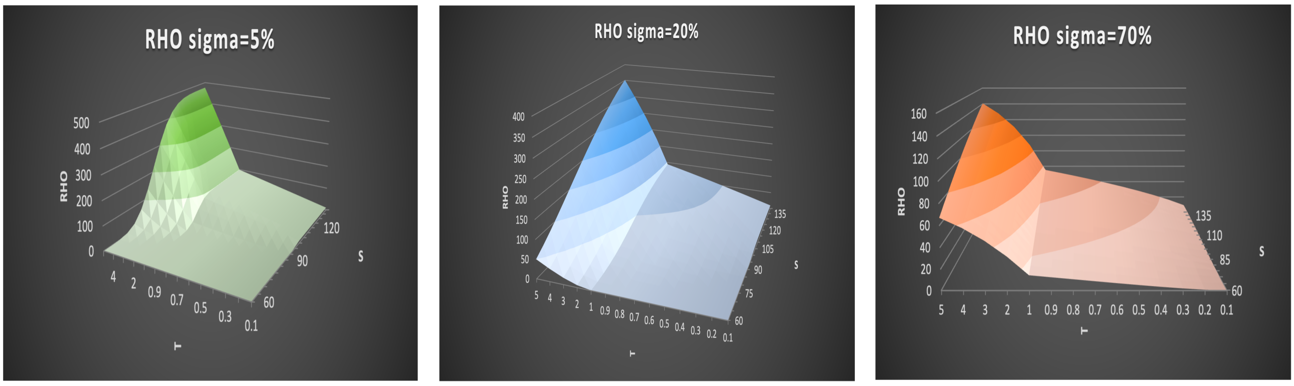
2. Plot of Γ with different σ.



3. Plot of ν with different σ.



4. Plot of Θ with different σ.



5. Plot of ρ with different σ.

2.3 Conclusions

So when σ changes the graphs change as-well as predicted. As shown in the plots, there is an enlargement when σ increases its value. In particular it's interesting how Γ and ν for $\sigma = 5\%$ are quite similar in their shape. Also for Δ there is differences between the case with $\sigma = 5\%$ and $\sigma = 70\%$, for example the higher part is flat for $\sigma = 5\%$ while it is concave for $\sigma = 70\%$. As for Θ when σ changes the width of the parabola (we can visualize it in 2D) changes, demonstrating that sigma is crucial in the computation. Regarding vega, as sigma increases the graph becomes more flat.

3 Part B

3.1 Tesla, Inc.: The company

Tesla, Inc. is an American electric vehicle and clean energy company based in Austin, Texas. Tesla designs and manufactures electric cars, battery energy storage from home to grid-scale, solar panels and solar roof tiles, and related products and services. Tesla is one of the world's most valuable companies and remains the world's most valuable automaker with a market capitalization of nearly US$ 1 trillion. The company had the most worldwide sales of battery electric vehicles and plug-in electric vehicles, capturing 23 %of the battery-electric (purely electric) market and 16 % of the plug-in market (which includes plug-in hybrids) in 2020. Through its subsidiary Tesla Energy, the company develops and is a major installer of photovoltaic systems in the United States. Tesla Energy is also one of the largest global suppliers of battery energy storage systems, with 3.99 gigawatt-hours (GWh) installed in 2021.
Founded in July 2003 by Martin Eberhard and Marc Tarpenning as Tesla Motors, the company's name is a tribute to inventor and electrical engineer Nikola Tesla. In February 2004, via a $6.5 million investment, X.com co-founder Elon Musk became the largest shareholder of the company and its chairman. He has served as CEO since 2008.
Tesla began production of its first car model, the Roadster sports car, in 2009. This was followed by the Model S sedan in 2012, the Model X SUV in 2015, the Model 3 sedan in 2017, and the Model Y crossover in 2020. The Model 3 is the all-time best-selling plug-in electric car worldwide, and, in June 2021, became the first electric car to sell 1 million units globally. Tesla's global sales were 936,222 cars in 2021, a 87% increase over the previous year, and cumulative sales totaled 2.3 million cars at the end of 2021. In October 2021, Tesla's market capitalization reached $1 trillion, the sixth company to do so in U.S. history.
For the fiscal (and calendar) year 2021, Tesla reported a net income of $5.52 billion. The annual revenue was $53.8 billion, an increase of 71 % over the previous fiscal year. Tesla ended 2020 with over $19 billion of cash on hand after having raised approximately $12 billion in stock sales. At the end of 2019 it had $6.3 billion cash on hand. Of the revenue number in 2021, $314 million came from selling regulatory credits to other automakers to meet government pollution standards. That number has been a smaller percentage of revenue for multiple quarters.
The quarter ending June 2021 was the first time Tesla made a profit independent of Bitcoin and regulatory credits. In 2021 the revenue was 53,823 (million USD), the netcome was 5,519(million USD), the total asset was 62,131(million USD) and the employees were 99,290.

3.2 Volatility Smile

A plot of the implied volatility of an option with a certain life as a function of its strike price is known as a volatility smile. The implied volatility of a European call option is always the same as the implied volatility of a European put option when the two have the same strike price and maturity date. To put this another way, for a given strike price and maturity, the correct volatility to use in conjunction with the Black–Scholes–Merton model to price a European call should always be the same as that used to price a European put. This means that the volatility smile (i.e., the relationship between implied volatility and strike price for a particular maturity) is the same for European calls and European puts. More generally, it means that the volatility surface (i.e., the implied volatility as a function of strike price and time to maturity) is the same for European calls and European puts. These results are also true to a good approximation for American options. This is a particularly convenient result. It shows that when talking about a volatility smile we do not have to worry about whether the options are calls or puts. For a dimonstration of this it possible to consult chapter 20.1 of $Hull J.C.\ Options, Futures\ and\ Other\ Derivatives$ 9th *Edition*.

The implied volatility is relatively low for at-the-money options. It becomes progressively higher as an option moves either into the money or out of the money. So far we have defined the volatility smile as the relationship between implied volatility and strike price. The relationship depends on the current price of the asset. For example, the lowest point of the volatility smile is usually close to the current exchange rate. If the exchange rate increases, the volatility smile tends to move to the right; if the exchange rate decreases, the volatility smile tends to move to the left.

Traders allow the implied volatility to depend on time to maturity as well as strike price. Implied volatility tends to be an increasing function of maturity when short-dated volatilities are historically low. This is because there is then an expectation that volatilities will increase. Similarly, volatility tends to be a decreasing function of maturity when short-dated volatilities are historically high. This is because there is then an expectation that volatilities will decrease. Volatility surfaces combine volatility smiles with the volatility term structure to tabulate the volatilities appropriate for pricing an option with any strike price and any maturity.The shape of the volatility smile depends on the option maturity.

3.3 Data Analysis

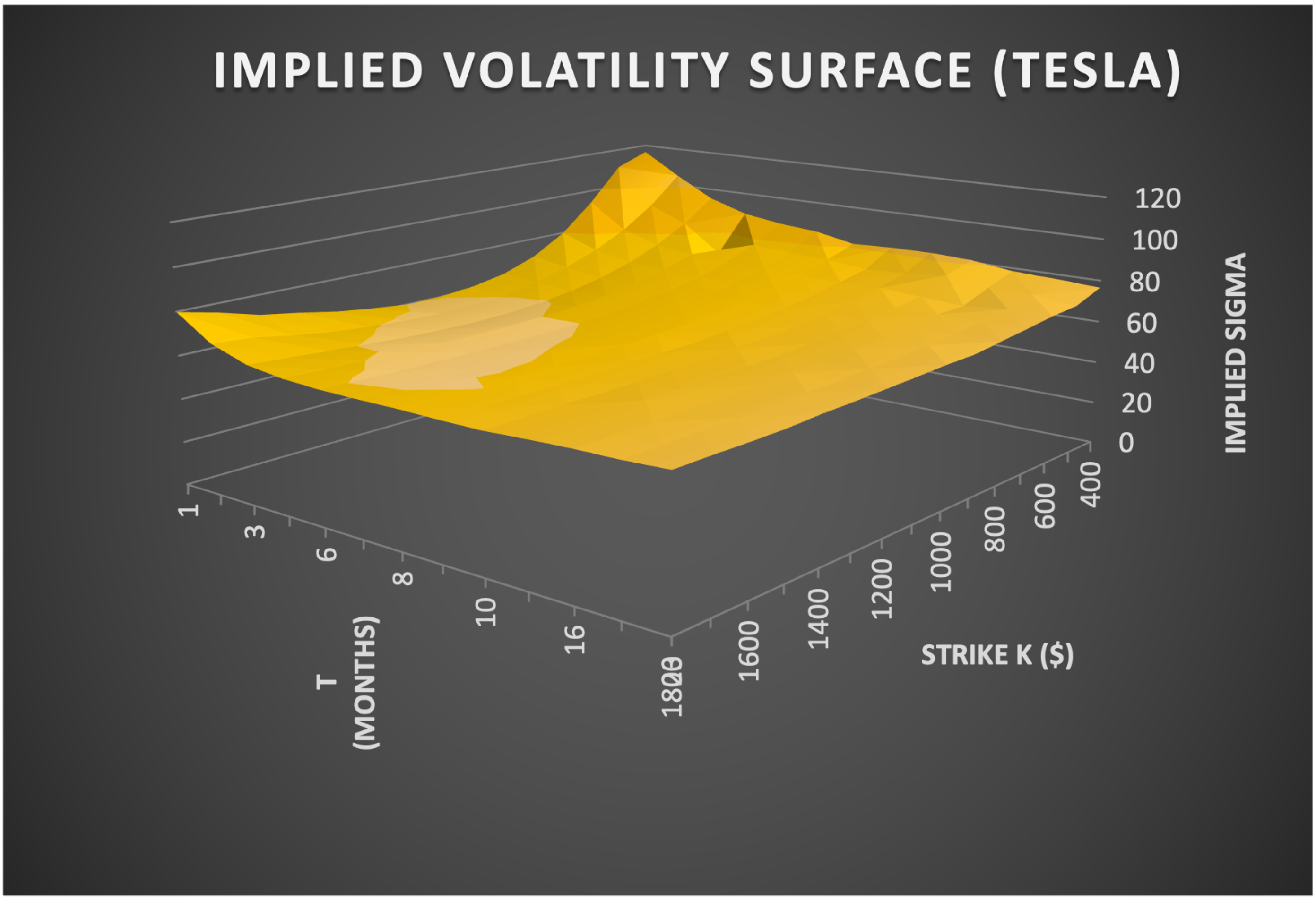
First of all I chose a Tesla's asset that doesn't provide dividend. In particular the value of one share of Tesla is $S = 1084.59\$$ in date 01-04-2022. Then I chose different call options with different K, T and implied volatility σ' to provide the implied volatility surface. In particular all the data can be found on $https://finance.yahoo.com/quote/TSLA/options?p = TSLAdate = 1650585600$.

The table with data used in the surface is reported below.

		K ($)														
		400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800
T (months)	1	116.41	110.45	93.73	79.93	70.54	64.69	61.59	60.19	59.97	61.8	64.24	67.6	70.56	75.64	79.86
	2	105.18	91.63	81.21	73.44	67.62	63.4	60.23	58.88	58.54	59.15	60.32	61.88	64.36	67.59	68.98
	3	95.02	84.09	75.81	69.57	64.98	61.67	59.49	58.18	57.74	57.95	58.55	59.67	60.82	62.08	63.4
	5	89.03	70.85	73.55	68.7	65.21	62.74	61.05	60.03	59.47	59.27	59.32	59.59	60.01	60.55	61.15
	6	86.24	78.09	72.29	67.97	64.96	62.68	61.09	60.00	59.38	59.17	59.13	59.28	59.50	59.83	60.36
	7	84.65	76.68	72.02	67.89	65.21	63.34	61.94	60.89	60.30	60.00	59.78	59.74	59.83	59.99	60.28
	8	80.89	75.87	71.26	68.14	65.6	63.77	62.78	61.72	61.00	60.23	60.29	60.19	59.98	60.08	60.44
	9	81.71	74.02	70.82	67.79	65.73	63.88	62.88	61.36	61.16	60.69	60.39	60.21	60.16	60.22	60.23
	10	81.88	75.14	71.3	68.25	65.96	64.30	63.11	62.46	61.61	61.09	60.77	60.52	60.63	60.22	60.21
	13	81.09	77.09	70.05	68.44	66.44	64.95	63.80	62.69	62.44	62.03	61.79	62.02	61.37	61.23	61.22
	16	78.46	73.97	68.8	68.58	66.55	65.66	64.79	63.66	63.63	62.74	62.42	62.21	62.31	62.01	61.98
	19	77.75	73.67	70.76	68.69	66.74	66.18	64.83	64.63	63.62	63.63	63.30	62.95	62.63	62.38	62.25
	23	76.55	71.52	70.59	68.85	67.25	65.61	65.53	64.93	64.48	64.03	63.8	63.07	63.03	63.24	63.20

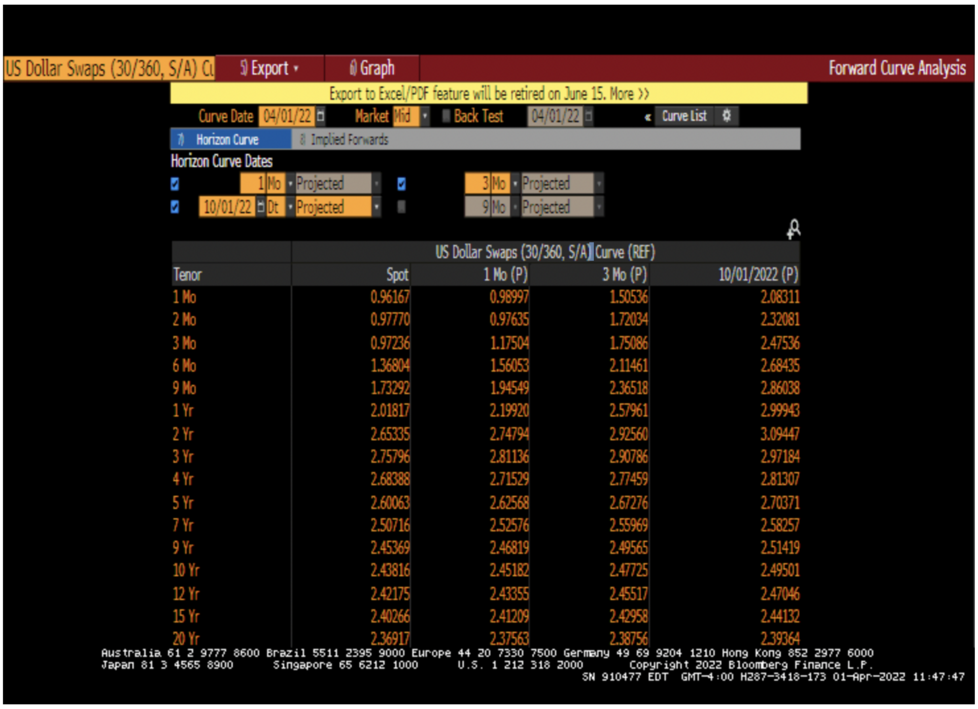
Table of implied volatility with respect to time T and K. All the values of implied volatility are in percentage.

Then I plotted the data in table to get the implied volatility surface and the plot is reported below.



6. Plot of the implied volatility surface for TESLA.

After that I computed the greeks for the TESLA's call options with the value of K,T and σ' reported in table above. In particular I used $S = 1084.59\$$, I consider the continuous dividend yield equal to 0 and for R I used the values reported in the following image.



US Dollar Swaps (30/360, S/A) Cu | Export | Graph | Forward Curve Analysis

Export to Excel/PDF feature will be retired on June 15. More >>

Curve Date 04/01/22 Market Mid Back Test 04/01/22 Curve List

Horizon Curve | Implied Forwards

Horizon Curve Dates

1 Mo Projected | 3 Mo Projected

10/01/22 Dt Projected | 9 Mo Projected

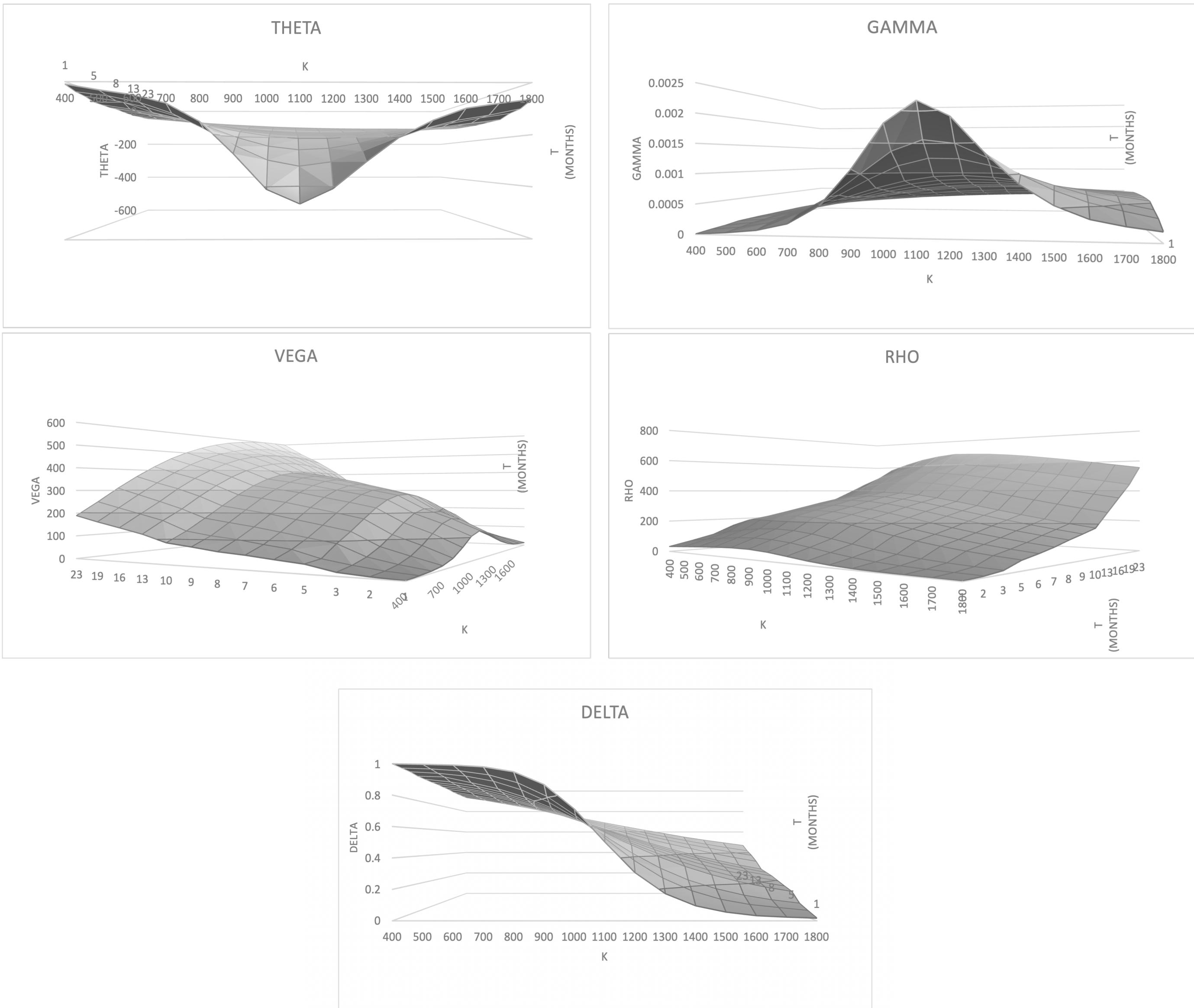
	US Dollar Swaps (30/360, S/A) Curve (REF)			
Tenor	Spot	1 Mo (P)	3 Mo (P)	10/01/2022 (P)
1 Mo	0.96167	0.98997	1.50536	2.08311
2 Mo	0.97770	0.97635	1.72034	2.32081
3 Mo	0.97236	1.17504	1.75086	2.47536
6 Mo	1.36804	1.56053	2.11461	2.68435
9 Mo	1.73292	1.94549	2.36518	2.86038
1 Yr	2.01817	2.19920	2.57961	2.99943
2 Yr	2.65335	2.74794	2.92560	3.09447
3 Yr	2.75796	2.81136	2.90786	2.97184
4 Yr	2.68388	2.71529	2.77459	2.81307
5 Yr	2.60063	2.62568	2.67276	2.70371
7 Yr	2.50716	2.52576	2.55969	2.58257
9 Yr	2.45369	2.46819	2.49565	2.51419
10 Yr	2.43816	2.45182	2.47725	2.49501
12 Yr	2.42175	2.43355	2.45517	2.47046
15 Yr	2.40266	2.41209	2.42958	2.44132
20 Yr	2.36917	2.37563	2.38756	2.39364

Australia 61 2 9777 8600 Brazil 5511 2395 9000 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 4565 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2022 Bloomberg Finance L.P.
SN 910477 EDT GMT-4:00 H287-3418-173 01-Apr-2022 11:47:47

7. Interest rate US zone

In these cases where there isn't an R for T (for example T=10 months), I took the R of the nearest T and I added $0.1\% \cdot (T' - T)$ to its R (for example T'=9 months, T=10 months ,
R(9 months)=1.73292% $\longrightarrow$ R(10 months)=1.73292%+ $0.1\% \cdot (10-9)$ =1.83292%).
I want to underline that this is just a qualitative procedure (it's not the right one) but it helped me to get the R to compute the Greeks for TESLA's options.

So after that I used the same VBA described in Part A to get the Greeks.
The plots are reported below.



8. Plot of different Greeks for TESLA .

3.4 Conclusions

So, as expected, I got the implied volatility smile in plot 6. In particular it's possible to see that the curve gets higher as K>>S and K<< S while it's relative low for ATM options as expected. The lowest point is near to $S = 1084.59\$$ that is the price of the asset S. It's also possible to see that volatility tends to be a decreasing function of maturity as far as short-dated volatilies are historically high.
As for the Greeks, I got the shape that I expected.

4 Part C

4.1 Data Analysis

For the last part I chose the same asset that I used in the previous one.
As the aim is to price with BS formula the ATM Call options for the different maturities, first of all I estimated the historical volatility. Before showing the computation, I want to specify that for this part I chose just 5 T (T=1,2,3,6 and 9 months); in fact I had to find call options' prices and compare them with the market's ones so I preferred to consider maturity times with interest rate R well-known from fig. 7 and to eliminate maturity times with R computed approximately as described in paragraph 3.3.
Hence, the option's informations with this new constraint are reported in table below.

T (months)	Option	Last Date trade	K ($)	Last price ($)	Bid ($)	Ask ($)	p_{mid}($)	implied volatility (%)
1	TSLA220506C01100000	2022-04-05 9:33AM EDT	1,100.00	104.23	98.8	103.6	101.2	61.22
2	TSLA220617P01100000	2022-04-05 9:32AM EDT	1,100.00	99.75	99.75	100.85	100.3	59.07
3	TSLA220715C01100000	2022-04-05 9:30AM EDT	1,100.00	162.1	161.85	163.9	162.88	60.69
6	TSLA221021C01100000	2022-04-04 3:59PM EDT	1,100.00	229.1	223.05	227.9	225.48	62.58
9	TSLA230120C01100000	2022-04-05 9:33AM EDT	1,100.00	273.1	267.5	274.55	271.03	63.63

Table with call options' informations for TESLA

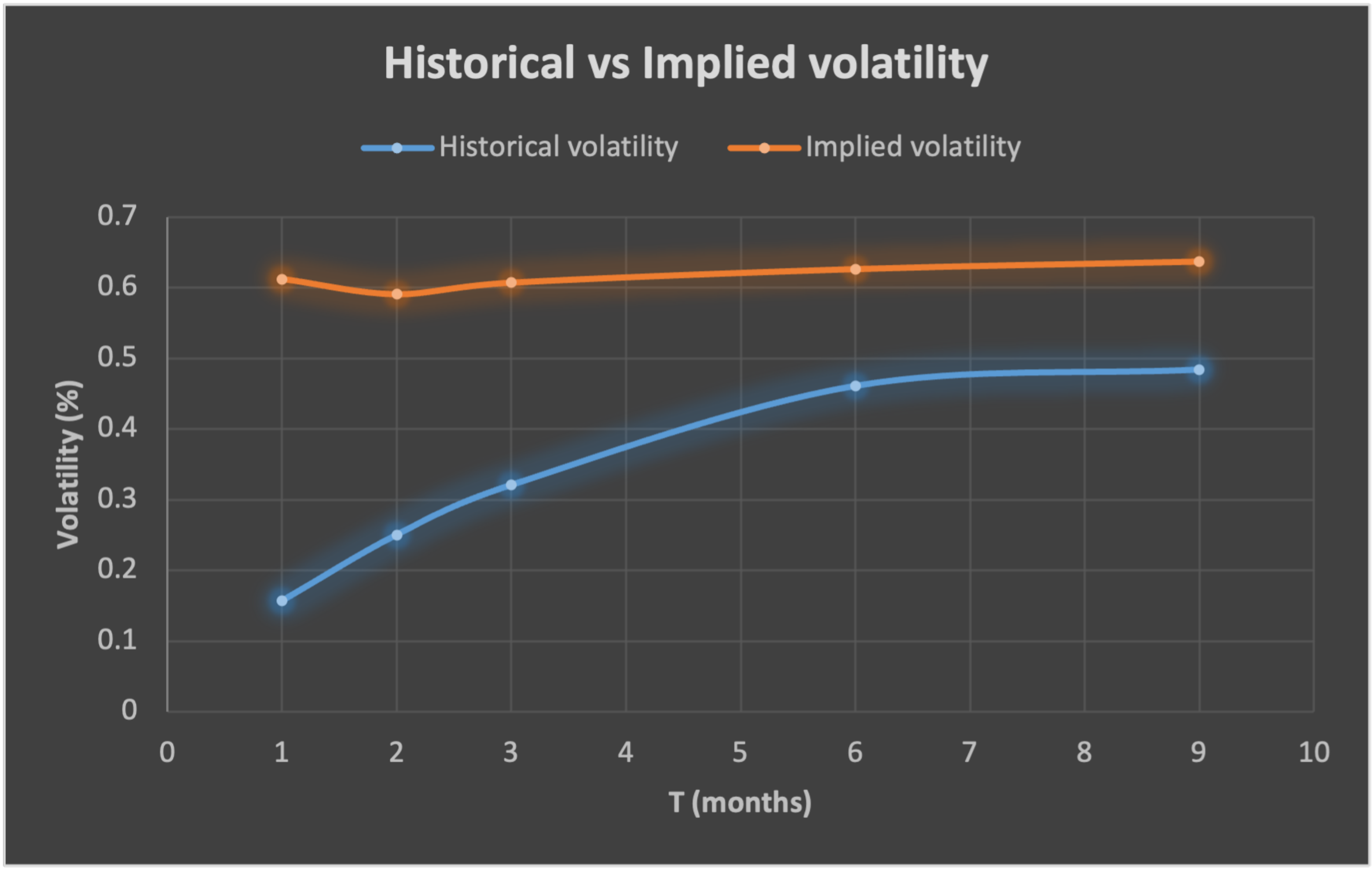
In particular p_{mid} is the option price given by the mean between Bid price and Ask price.
After these preliminary remarks I computed the historical volatility σ_T for each T . In particular I used historical data for each T that can be found in $yahoo\ finance \longrightarrow Tesla\ Historical\ Data \longrightarrow Historical\ Prices$; the value S_t for each day correspond to the Adj Close Value in yahoo's data.

Once I had gotten daily price's vector $\begin{pmatrix} S_t \\ S_{t-1} \\ . \\ . \\ S_{t-n} \end{pmatrix}$, I could compute daily returns' vector $\begin{pmatrix} \frac{S_t - S_{t-1}}{S_{t-1}} \\ . \\ . \\ \frac{S_{t-j} - S_{t-n}}{S_{t-n}} \end{pmatrix}$ with $j < n$.
Then I calculated the standard deviation of daily returns σ_{day} and the historical volatility $\sigma_T = \sigma_{day} \cdot \sqrt{T}$, where T is in the market days.
After that I computed the price for each option using BS formula; in particular the VBA that I used for the BS formula can be found in the following link: $https://sites.google.com/view/vinegarhill-financelabs/black-scholes-merton/black-scholes-greeks$. The computed values are reported in table below.

T(months)	σ_T (%)	implied volatility (%)	p_{BS} ($)	p_{mid} ($)
1	15.68	61.22	101.20	13.30
2	25.04	59.07	100.30	38.01
3	32.09	60.69	162.88	63.57
6	46.11	62.58	225.48	137.04
9	48.42	63.63	271.03	230.41

Table with historical volatility,implied volatility, BS price for the option computed with historical volatility and mid price considered as the "real price" for the option.

Then I plotted the historical and the implied volatility to check their behavior as T changes. The plot is reported in the next page.



9. Plot of Historical vs Implied volatility as function of T

As shown in graph 9 as T increases the historical volatility tends to the implied one. So I tried to compute p_{BS} with implied volatilities to check if something would changed. In particular I got these values:

T(months)	σ_T (%)	implied volatility (%)	p_{BS} (\$)	p_{mid} (\$)
1	15.68	61.22	69.83	13.30
2	25.04	59.07	121.81	38.01
3	32.09	60.69	181.01	63.57
6	46.11	62.58	231.19	137.04
9	48.42	63.63	273.22	230.41

Table with historical volatility,implied volatility, BS price for the option computed with implied volatility and mid price considered as the "real price" for the option.

4.2 Conclusions

As shown in tables the difference between the p_{BS} and p_{mid} is net. It's possibile to find one explanation of this difference in the summary chapter 20 of *HullJ.C. Options, Futures and Other Derivatives 9th Edition.* Hull writes:

"The Black–Scholes–Merton model and its extensions assume that the probability distribution of the underlying asset at any given future time is lognormal. This assumption is not the one made by traders. They assume the probability distribution of an equity price has a heavier left tail and a less heavy right tail than the lognormal distribution. They also assume that the probability distribution of an exchange rate has a heavier right tail and a heavier left tail than the lognormal distribution. Traders use volatility smiles to allow for nonlognormality."

So the reason of this net difference is related to the assumptions of the probability distribution of the underlying asset and of the exchange rate that are different for BS model and for the real world.

5 Bibliography

-Hull J.C. « Options,Futures and Other Derivatives ninth Edition» (15th January 2014)

-https://en.wikipedia.org/wiki/Tesla,Inc.