Chaotic Variations on Poetry

Luke Meszar

Abstract

This paper considers the effects of using tools from chaos theory to generate variations on existing poems. Chunks of each poem were mapped onto the points in a trajectory from the Lorenz system. Then a new trajectory was computed and a nearest neighbor search was done for each point in the new trajectory to find the closest point in the initial trajectory. Next, the chunk associated with the nearest point was added to the poem. A user survey was conducted to test the quality of the generated poems. The survey provided evidence that chaotic variations could hold up relatively well compared to original poems. There was also evidence that the longer the chunk sizes used, the better the method worked.

1 Introduction

Chaos theory has been used in numerous areas to generate variations on existing works. Examples include music [4], dance [13], and rock climbing routes [10]. In this paper, I explore the effects of using these chaotic variations on poetry. The highly sensitive dependence on initial conditions in chaotic systems was the basis for the variations. The chaotic system I used was the Lorenz Attractor [9]. Work has been done in the past using simpler mathematical techniques to create poetry. These included using a mathematical sequence to determine the number of syllables in a line or laying out poems to have certain geometric shapes [7]. The goal of my research was to determine whether coherent and enjoyable poetry can be generated using chaotic variations on existing poetry. The quality of the generated poetry was evaluated with a user survey.

2 Method for Generating Variations

To generate the variations, there were a few factors to consider. They were the mathematical background of chaos, the structure of the poetry, and the computational aspect. These three factors will be discussed below.

2.1 Mathematical Design

The sensitive dependence on initial conditions of chaotic systems was the key component in generating the variations of poetry. The Lorenz system was used:

$$\dot{x} = a(y - x),$$
$$\dot{y} = x(r - z) - y,$$
$$\dot{z} = xy - bz.$$

The values for the parameters were $a = 16$, $r = 45$, and $r = 4$. These values were chosen since they are known to result in a chaotic solution to the Lorenz system. As described in [12, p. 320] the Lorenz system displays sensitive dependence on initial conditions. This is useful because two trajectories beginning from different initial conditions will have different long term behavior. The other important aspect of the Lorenz system is that the trajectories stay on a bounded attractor as $t \to \infty$. The important fact is that two trajectories from different initial conditions will traverse the same attractor in a different order.

To generate the variations, an initial trajectory was generated. Then each chunk of the poem was mapped onto a point in the trajectory. Next, a Voronoi diagram [15] was constructed using the points in the initial trajectory such that the entire attractor was covered. Then a new trajectory was generated using different initial conditions. For each point in the new trajectory, the Voronoi cell in which it lay was calculated. Then the chunk associated with the point from the initial trajectory in this cell was appended to the new poem.

2.2 Structure of Poetry

A challenge for this project was deciding how to break up a poem into individual pieces that could be mapped onto a trajectory. Human language is entirely dependent on the context in which words are placed. Since the generation of new poetry depends on switching the order in which chunks of the original poem are written, the context and structure of the new poem will inherently be different. The size of chunks used as the atomic components of the poem will greatly affect how the structure is changed. I decided that breaking a poem apart at the word level would be too small. Even though poetry lacks a lot of the grammatical rules other forms of written English follow, generating variations at the word level would break the fundamentals of English structure and produce something entirely unreadable.

I ultimately decided to break up each poem into chunks using one of four criteria. A chunk of a poem was defined as one of the following: a stanza, a line, a phrase separated with a comma, or a sentence ending with a period. Since the structure of different pieces of poetry varies significantly, the method of creating chunks varied from poem to poem. For example, some poems have no stanzas so breaking the poem up into stanzas wouldn't make any sense. I also made the choice that once a chunk had been used, it couldn't be used again. Without this restriction, there were often repeated lines which produced obviously computer-generated poetry.

For most of the poems I looked at, there were at least two reasonable ways the poem could be broken up. The different variations using different chunk sizes were compared in the user survey. A variety of styles of poetry from Ancient Greek to 20th century poetry was used in this project.

2.3 Computational Methods

The computational method for generating the variations consisted of three main parts. The first was breaking the original poem into a list of chunks. The second was generating the initial trajectory and the new trajectory. The final step was creating the new poem using the original poem and the two trajectories.

Breaking down the original poem into a list of chunks was a task in file and string manipulation. Each poem was stored as a plain text file that was read in and parsed based on the desired chunk size. The method for all four chunk sizes were relatively similar with a few details of implementation changed for the different chunk sizes. The poem's chunks were then stored as strings in an array.

The initial and new trajectories were both calculated using a 4th order Runge-Kutta ODE solver (RK4). The length of the trajectories were 30,000 plus the number of chunks in the poem. The first 30,000 points were thrown out to make sure that any transient effects of the dynamics had died out. The initial trajectory was always created using the initial conditions $[x, y, z] = [10, -5, 2]$. The new trajectory was generated using the initial conditions $[x, y, z] = [U(-20, 20), U(-20, 20), U(-20, 20)]$ where $U(-20, 20)$ is the uniform distribution function on the interval $[-20, 20]$. Although these initial conditions can be far from the original initial conditions of $[10, -5, 2]$, it is not important since the transient is allowed to die out and the two trajectories will both lie on the same bounded attractor. The two trajectories won't diverge wildly even from a large difference in initial conditions, but they will still result in different trajectories. The differences in two trajectories can be seen in Figure 1.

(a) Two trajectories each containing 316 points
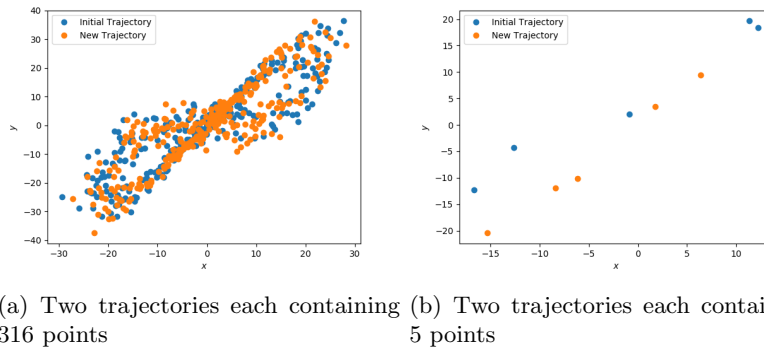
(b) Two trajectories each containing 5 points

Figure 1: Differences in two trajectories with different initial conditions

The chunks in the poem were mapped to the initial trajectory by the indices in their corresponding arrays. That is, the first chunk was associated to the first point in the trajectory by the fact that they both had index 0 in their respective arrays. Next, for each point in the new trajectory, we computed its nearest neighbor in the original trajectory. The nearest neighbor search is done in $O(n)$ time. This computational method simulates creating a

Voronoi diagram. Once the nearest neighbor was located, the chunk of the poem associated with that point was appended to the new poem. In this manner, a new poem was generated from an existing one.

3   Survey Design

To test the effectiveness of the method, a feedback survey was designed. Ten different poems were chosen for respondents to read and rate in three different categories. The categories were "coherence of the poem," "flow of the poem," and "overall enjoyment of the poem." The "coherence of the poem" was to gauge how much sense the poem made. Obviously, changing the poem from the original could easily lead to passages that made no sense and were just a random collection of words and phrases with no discernible meaning. The "flow of the poem" was a very subjective question. However, it was essentially asking whether there seemed to be any continuity between parts of the poem. In general, a human author will try to make each part of their poem merge into the next as seamlessly as possible. However, with these chaotic variations, the algorithm has no knowledge of what "flow" is and thus won't make any decisions to maximize it. The last question, "overall enjoyment," was again a very subjective question. It was designed to see whether or not a respondent enjoyed reading a poem in a holistic manner. These questions were also chosen to test in an implicit manner whether or not the respondents believed the poem could have been written by a human. Respondents were asked to rate each of the three questions on a scale from 1 to 5. This scale was chosen because it was a large enough number that sufficient differentiation between respondents could be achieved without giving them meaningless choices which could happen with a larger scale like 1-10 [14].

The following poems and the respective chunk sizes were used in the survey:

Poem	Chunk Method
Linear Algebra Sonnet - Michael Zahniser [5, p. 175]	Lines, Commas
Sonnet 1 - William Shakespeare [11]	Lines, Commas
IV O Pan of the evergreen forest - Sappho [1]	Lines, Stanzas
If - e e cummings [3]	Commas, Original
VI Peer of the gods he seems - Sappho [1]	Original
My Love - e e cummings [3]	Original

Table 1: Poems in Survey

The chief reason these poems were chosen was for their length. They are not incredibly short (e.g. four lines) and they are not pages long. Length was a big consideration since reading poetry takes time to do well. I feared that if respondents had to spend hours reading all the poems, they would become frustrated and either give up or give inaccurate responses. Poems that were too short were not considered since I believed the method for generating variations worked better on longer poems. I used multiple chunk methods for the same poems to try and get data on which method worked best. I included different styles of poetry from different time periods to see if this had any influence on the effectiveness of the chaotic technique. Three poems in their original form were also included in the survey to see how the chaotic variations compared to works created by people.

The difficulty in designing this survey was the limited amount of data that could be collected. Ideally, I would have tested hundreds of poems with multiple variations and chunk methods against a large number of original poems. This would have enabled a more in depth look at which types of poems and chunk sizes work best with this method. However, the limiting factor was the time needed to take the survey. It would be unreasonable to ask respondents to read hundreds of poems, some potentially long, and rate all of them.

4   Survey Results

In the end, the survey received 21 responses. The survey was sent to over 100 people so a less than 20% response rate is poor. This small response size makes it harder to determine any meaningful results since there isn't much data to work with. This was likely due to the survey having a high time investment. For each poem, an average score was calculated based on the responses for each of the three questions described in section 3. An additional value was calculated for each poem based on the mean results of the three questions. The mean of the three results was calculated to give an idea of how well the poem fared in all three categories. Each score is a value between 1 and 5 with 5 being the highest. The results for each poem are as follows:

Poem	Method	Coherence	Flow	Overall Enjoyment	Mean
Linear Algebra Sonnet	Lines	2.90	2.76	3.33	3.00
Linear Algebra Sonnet	Commas	3.20	3.45	3.10	3.25
Sonnet 1	Lines	2.85	2.85	2.80	2.83
Sonnet 1	Commas	2.62	3.00	2.43	2.68
IV O Pan of the evergreen forest	Stanzas	3.85	3.95	3.50	3.77
IV O Pan of the evergreen forest	Lines	3.26	3.21	2.95	3.14
If	Commas	3.67	3.86	3.86	3.80
If	Original	4.50	4.45	3.95	4.30
VI Peer of the gods he seems	Original	3.68	3.63	3.58	3.63
My Love	Original	3.47	3.37	3.42	3.42

Table 2: Results of Survey

5 Analysis

The results can be summarized by listing the best and worst poems in all four categories as seen in Table 3

Category	Best Overall	Best Variation	Worst Poem
Coherence	*If* (Original) 4.50/5.00	*IV O Pan of the evergreen forest* (Stanzas) 3.85/5.00	*Sonnet 1* (Commas) 2.62/5.00
Flow	*If* (Original) 4.45/5.00	*IV O Pan of the evergreen forest* (Stanzas) 3.95/5.00	*Linear Algebra Sonnet* (Lines) 2.76/5.00
Overall Enjoyment	*If* (Original) 3.95/4.00	*If* (Commas) 3.86/5.00	*Sonnet 1* (Commas) 2.43/5.00
Mean	*If* (Original) 4.30/5.00	*If* (Commas) 3.80/5.00	*Sonnet 1* (Commas) 2.68/5.00

Table 3: Summary of Results

The most obvious result is that the best poem using the mean score metric was an original poem. This indicates that the method of chaotic variations cannot compete with human creativity. However, the second and third best poems using the mean score metric were poems generated chaotically, beating out the other two original poems in the survey (see Table 4). The poem *If* (Commas) came second in both the "Flow" and "Overall Enjoyment" categories and came third in "Coherence." So, although chaotic variations did not produce the outright best poem in any category, they were able to beat some original poems in all categories.

Poem	Method
If	Original
If	Commas
IV O Pan of the evergreen forest	Stanzas
VI Peer of the gods he seems	Original
My Love	Original
Linear Algebra Sonnet	Commas
IV O Pan of the evergreen forest	Lines
Linear Algebra Sonnet	Lines
Sonnet 1	Lines
Sonnet 1	Commas

Table 4: Overall Ranking Based on Mean Score

Another interesting result was how the same poem was rated differently depending on the method for creating the chunks. The most notable example was *IV O Pan of the evergreen forest*. When the chunks were based on stanzas, the mean score was 0.63 higher than when the chunks were based on lines. The most likely reason for this is that this poem only has 3 stanzas whereas it has 36 lines. When only 3 large pieces are varied, not a whole lot is going to change, while there is a lot of potential for the poem to turn into a jumbled mess when varying 36 pieces.

It makes sense that the flow would be 0.74 higher when using stanzas over lines since there is guaranteed to be some flow within each stanza. The same difference can be seen with both *Linear Algebra Sonnet* and *Sonnet 1.* In the case of *Linear Algebra Sonnet*, there are more line breaks than commas so it makes sense that the commas method would be more effective than the lines method. For *Sonnet 1*, the opposite is true; there are more commas than lines. For this poem, the lines method was more effective than the commas method. Thus, it seems that larger chunks lead to better results of the chaotic variation.

As mentioned in Section 3, there were issues with the amount and length of poems used in the survey. Due to concerns of the survey taking too long otherwise, only 10 short poems were included. This meant it was impossible to get a full picture of how this method behaves with longer poems. For instance, there might be a poem with the correct method of creating chunks and the right initial conditions that could have produced a stunning poem. It was also impossible to compare how longer pieces compared to shorter ones. And finally, it was impossible to make any conclusions as to which authors, styles, or time periods produced the best results. Another factor to consider is how people's personal feelings about poetry affected the results. One respondent mentioned really disliking the variations of Shakespeare's *Sonnet 1* because of the fact that it was Shakespeare. This sort of bias was not accounted for in the survey design and thus makes it harder to analyze the results.

As an interesting aside, I considered how this method would work with short stories. I hypothesized that the strong temporal nature of most short stories would lead to poor results. When varying a short story, all of the underlying temporal structure would likely be destroyed. Due to the inherent length of short stories, I decided not to include them in the survey. The following short stories were chosen:

Short Story	Chunk Method
The Overcoat - Nikola Gogol [6]	Sentences, Paragraphs
The Old Man at the Bridge - Ernest Hemingway [8]	Sentences
Nine Billion Names of God - Arthur C. Clarke [2]	Sentences

Table 5: Short stories that were chaotically varied

I evaluated the quality of the variations of the short stories using my own judgment. As expected, all the short stories tested became completely unreadable. From completely nonsensical dialog in *The Old Man at the Bridge* and the *Nine Billion Names of God* to random sentences strewn throughout *The Overcoat* (Sentences), the variations were incoherent and unenjoyable. I think it is safe to say this method does not work in any capacity for short stories.

6   Conclusion

In this paper I explored the effects of creating variations of poems using the Lorenz attractor. I considered and tested how breaking the original poem up into different chunk sizes would affect the success of this method. Using the sensitive dependence on initial conditions of the Lorenz system, I was able to generate variations of a poem. A survey was created to test the quality of the generated poems. Due to the potential length of the survey, a fully comprehensive study could not be conducted. Only preliminary results could be obtained about the quality of chaotically generated poems with different chunk sizes. It was determined that while chaotically generated poems were not the outright best, some were successful. There also appeared to be some correlations between the sizes of the chunks and the quality of the poems. The method of chaotic variations provides an interesting way to take existing work and create something new and unique from it. It doesn't always produce the best results, but it does produce interesting results.

7   Appendix

7.1   Poetry

Below are all the poems included in the survey.

Linear Algebra Sonnet

This sort of math annoys a busy man,
When he has lots of precious time to kill:
"The space the vectors fill is called the span,

the span's the space the set of vectors fill.
A pivot is an isolated one
With zeroes up above and down below.
Row echelon is when your work is done,
With ohs before the ones in every row.
A transformation is a funny beast
Which maps a set of vectors to a range."
I understand this much of it at least,
And I can't help it if the rest seems strange.
Nor should I be beset by pangs of guilt,
When matrices turn out to have been tilt.

Michael Zahniser

Linear Algebra Sonnet - Lines

And I can't help it if the rest seems strange.
With zeroes up above and down below.
"The space the vectors fill is called the span,
With ohs before the ones in every row.
This sort of math annoys a busy man,
When matrices turn out to have been tilt.
A pivot is an isolated one
the span's the space the set of vectors fill.
Which maps a set of vectors to a range."
When he has lots of precious time to kill:
Row echelon is when your work is done,
Nor should I be beset by pangs of guilt,
A transformation is a funny beast
I understand this much of it at least,

Chaotic Method

Linear Algebra Sonnet - Commas

When he has lots of precious time to kill:
"The space the vectors fill is called the span
With ohs before the ones in every row.
A transformation is a funny beast
Which maps a set of vectors to a range."
I understand this much of it at least
This sort of math annoys a busy man
the span's the space the set of vectors fill.
A pivot is an isolated one
With zeroes up above and down below.
Row echelon is when your work is done
When matrices turn out to have been tilt.
And I can't help it if the rest seems strange.
Nor should I be beset by pangs of guilt

Chaotic Method

Sonnet 1

From fairest creatures we desire increase,

That thereby beauty's rose might never die,
But as the riper should by time decease,
His tender heir might bear his memory:
But thou contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Making a famine where abundance lies,
Thy self thy foe, to thy sweet self too cruel:
Thou that art now the world's fresh ornament,
And only herald to the gaudy spring,
Within thine own bud buriest thy content,
And tender churl mak'st waste in niggarding:
Pity the world, or else this glutton be,
To eat the world's due, by the grave and thee.

William Shakespeare

Sonnet 1 - Lines

Making a famine where abundance lies,
Thy self thy foe, to thy sweet self too cruel:
And tender churl mak'st waste in niggarding:
Within thine own bud buriest thy content,
But as the riper should by time decease,
His tender heir might bear his memory:
To eat the world's due, by the grave and thee.
Pity the world, or else this glutton be,
From fairest creatures we desire increase,
But thou contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Thou that art now the world's fresh ornament,
That thereby beauty's rose might never die,
And only herald to the gaudy spring,

Chaotic Method

Sonnet 1 - Commas

And only herald to the gaudy spring
by the grave and thee.
Feed'st thy light's flame with self-substantial fuel
That thereby beauty's rose might never die
But as the riper should by time decease
That thereby beauty's rose might never die
by the grave and thee.
Making a famine where abundance lies
From fairest creatures we desire increase
His tender heir might bear his memory:
But thou contracted to thine own bright eyes
But as the riper should by time decease
That thereby beauty's rose might never die

Chaotic Method

IV O Pan of the evergreen forest

O Pan of the evergreen forest,

Protector of herds in the meadows,
Helper of men at their toiling,—
Tillage and harvest and herding,—
How many times to frail mortals
Hast thou not hearkened!
Now even I come before thee
With oil and honey and wheat-bread,
Praying for strength and fulfilment
Of human longing, with purpose
Ever to keep thy great worship
Pure and undarkened.

O Hermes, master of knowledge,
Measure and number and rhythm,
Worker of wonders in metal,
Moulder of malleable music,
So often the giver of secret
Learning to mortals!
Now even I, a fond woman,
Frail and of small understanding,
Yet with unslakable yearning
Greatly desiring wisdom,
Come to the threshold of reason
And the bright portals.

And thou, sea-born Aphrodite,
In whose beneficent keeping
Earth, with her infinite beauty,
Colour and fashion and fragrance,
Glows like a flower with fervour
Where woods are vernal!
Touch with thy lips and enkindle
This moon-white delicate body,
Drench with the dew of enchantment
This mortal one, that I also
Grow to the measure of beauty
Fleet yet eternal.

<div align="center">Sappho</div>

IV O Pan of the evergreen forest - Lines

O Pan of the evergreen forest,
Of human longing, with purpose
Now even I, a fond woman,
Frail and of small understanding,
With oil and honey and wheat-bread,
And thou, sea-born Aphrodite,
Colour and fashion and fragrance,
Earth, with her infinite beauty,
So often the giver of secret
Moulder of malleable music,
Glows like a flower with fervour
Fleet yet eternal.
Helper of men at their toiling,—
O Hermes, master of knowledge,
Pure and undarkened.

Ever to keep thy great worship
Drench with the dew of enchantment
Tillage and harvest and herding,—
Worker of wonders in metal,
This moon-white delicate body,
Touch with thy lips and enkindle
In whose beneficent keeping

Greatly desiring wisdom,
Measure and number and rhythm,
Grow to the measure of beauty
Now even I come before thee
Hast thou not hearkened!
Praying for strength and fulfilment

Yet with unslakable yearning
Protector of herds in the meadows,
Where woods are vernal!
And the bright portals.
Come to the threshold of reason
How many times to frail mortals
This mortal one, that I also
Learning to mortals!

<center>Chaotic Method</center>

IV O Pan of the evergreen forest - Stanzas

And thou, sea-born Aphrodite,
In whose beneficent keeping
Earth, with her infinite beauty,
Colour and fashion and fragrance,
Glows like a flower with fervour
Where woods are vernal!
Touch with thy lips and enkindle
This moon-white delicate body,
Drench with the dew of enchantment
This mortal one, that I also
Grow to the measure of beauty
Fleet yet eternal.

O Hermes, master of knowledge,
Measure and number and rhythm,
Worker of wonders in metal,
Moulder of malleable music,
So often the giver of secret
Learning to mortals!
Now even I, a fond woman,
Frail and of small understanding,
Yet with unslakable yearning
Greatly desiring wisdom,
Come to the threshold of reason
And the bright portals.

O Pan of the evergreen forest,
Protector of herds in the meadows,
Helper of men at their toiling,—

<center>9</center>

Tillage and harvest and herding,—
How many times to frail mortals
Hast thou not hearkened!
Now even I come before thee
With oil and honey and wheat-bread,
Praying for strength and fulfilment
Of human longing, with purpose
Ever to keep thy great worship
Pure and undarkened.

<div align="center">Chaotic Method</div>

If

If freckles were lovely, and day was night,
And measles were nice and a lie warn't a lie,
Life would be delight,—
But things couldn't go right
For in such a sad plight
I wouldn't be I.
If earth was heaven and now was hence,
And past was present, and false was true,
There might be some sense
But I'd be in suspense
For on such a pretense
You wouldn't be you.
If fear was plucky, and globes were square,
And dirt was cleanly and tears were glee
Things would seem fair,—
Yet they'd all despair,
For if here was there
We wouldn't be we.

<div align="center">e e cummings</div>

If - Commas

There might be some sense
But I'd be in suspense
For on such a pretense
You wouldn't be you.
If fear was plucky
If freckles were lovely
For if here was there
We wouldn't be we.
Life would be delight
And measles were nice and a lie warn't a lie
Yet they'd all despair
and day was night
and globes were square
And past was present
and false was true
But things couldn't go right
For in such a sad plight
I wouldn't be I.
If earth was heaven and now was hence

And dirt was cleanly and tears were glee
Things would seem fair

VI Peer of the god she seems

Peer of the gods he seems,
Who in thy presence
Sits and hears close to him
Thy silver speech-tones
And lovely laughter.
Ah, but the heart flutters
Under my bosom,
When I behold thee
Even a moment;
Utterance leaves me;
My tongue is useless;
A subtle fire
Runs through my body;
My eyes are sightless,
And my ears ringing;
I flush with fever,
And a strong trembling
Lays hold upon me;
Paler than grass am I,
Half dead for madness.
Yet must I, greatly
Daring, adore thee,
As the adventurous
Sailor makes seaward
For the lost sky-line
And undiscovered
Fabulous islands,
Drawn by the lure of
Beauty and summer
And the sea's secret.

Sappho

My Love

my love
thy hair is one kingdom
the king whereof is darkness
thy forehead is a flight of flowers

thy head is a quick forest
filled with sleeping birds
thy breasts are swarms of white bees
upon the bough of thy body
thy body to me is April
in whose armpits is the approach of spring

thy thighs are white horses yoked to a chariot
of kings

they are the striking of a good minstrel
between them is always a pleasant song

my love
thy head is a casket
of the cool jewel of thy mind
the hair of thy head is one warrior
innocent of defeat
thy hair upon thy shoulders is an army
with victory and with trumpets

thy legs are the trees of dreaming
whose fruit is the very eatage of forgetfulness

thy lips are satraps in scarlet
in whose kiss is the combinings of kings
thy wrists
are holy
which are the keepers of the keys of thy blood
thy feet upon thy ankles are flowers in vases
of silver

in thy beauty is the dilemma of flutes

thy eyes are the betrayal
of bells comprehended through incense

e e cummings

7.2 Short Stories

The results of the chaotic variations of the short stories are below. Only an excerpt is included from each story. The originals are not included for space considerations.

The Overcoat - Sentences

The Germans invented them in order to make more money. Now they were even more dim and lonely: the lanterns began to grow rarer, oil, evidently, had been less liberally supplied. Then came wooden houses and fences: not a soul anywhere; only the snow sparkled in the streets, and mournfully veiled the low-roofed cabins with their closed shutters. He approached the spot where the street crossed a vast square with houses barely visible on its farther side, a square which seemed a fearful desert. He paid him, thanked him, and set out at once in his new cloak for the department. Petrovitch followed him, and, pausing in the street, gazed long at the cloak in the distance, after which he went to one side expressly to run through a crooked alley, and emerge again into the street beyond to gaze once more upon the cloak from another point, namely, directly in front.

Meantime Akakiy Akakievitch went on in holiday mood. He glanced back and on both sides, it was like a sea about him. "No, it is better not to look," he thought, and went on, closing his eyes. When he opened them, to see whether he was near the end of the square, he suddenly beheld, standing just before his very nose, some bearded individuals of precisely what sort he could not make out. All grew dark before his eyes, and his heart throbbed.

"But, of course, the cloak is mine. said one of them in a loud voice, seizing hold of his collar. Akakiy Akakievitch was about to shout "watch," when the second man thrust a fist, about the size of a man's head, into his mouth, muttering, "Now scream. He took the cloak out of the pocket handkerchief in which he had brought it. The handkerchief was fresh from the laundress, and he put it in his pocket for use. Taking out the cloak, he gazed proudly at it, held it up with both hands, and flung it skilfully over the shoulders of Akakiy Akakievitch. In despair, but without ceasing to shout, he started at a run across the square, straight towards the watchbox, beside which stood the watchman, leaning on his halberd, and apparently curious to know what kind of a customer was running towards him and shouting. Akakiy Akakievitch ran up to him, and began in a sobbing voice to shout that he was asleep, and attended to nothing, and did not see when a man was robbed. The watchman replied that he had seen

two men stop him in the middle of the square, but supposed that they were friends of his; and that, instead of scolding vainly, he had better go to the police on the morrow, so that they might make a search for whoever had stolen the cloak.

Akakiy Akakievitch ran home in complete disorder; his hair, which grew very thinly upon his temples and the back of his head, wholly disordered; his body, arms, and legs covered with snow. Such is the habit of tailors; it is the first thing they do on meeting one. "But I, here, this – Petrovitch – a cloak, cloth – here you see, everywhere, in different places, it is quite strong – it is a little dusty, and looks old, but it is new, only here in one place it is a little – on the back, and here on one of the shoulders, it is a little worn, yes, here on this shoulder it is a little – do you see. that is all. And a little work –".

Petrovitch took the cloak, spread it out, to begin with, on the table, looked hard at it, shook his head, reached out his hand to the window-sill for his snuff-box, adorned with the portrait of some general, though what general is unknown, for the place where the face should have been had been rubbed through by the finger, and a square bit of paper had been pasted over it. Having taken a pinch of snuff, Petrovitch held up the cloak, and inspected it against the light, and again shook his head once more. After which he again lifted the general-adorned lid with its bit of pasted paper, and having stuffed his nose with snuff, closed and put away the snuff-box, and said finally, "No, it is impossible to mend it; it's a wretched garment.

Akakiy Akakievitch's heart sank at these words.

"Why is it impossible, Petrovitch. he said, almost in the pleading voice of a child; "all that ails it is, that it is worn on the shoulders. You must have some pieces –".

"Yes, patches could be found, patches are easily found," said Petrovitch, "but there's nothing to sew them to. The thing is completely rotten; if you put a needle to it – see, it will give way. He seemed fully sensible that he had done no small deed, and crossed a gulf separating tailors who only put in linings, and execute repairs, from those who make new things. But the reader must first be told where the first half came from.

Petrovitch worked at the cloak two whole weeks, for there was a great deal of quilting: otherwise it would have been finished sooner. He charged twelve rubles for the job, it could not possibly have been done for less. It was all sewed with silk, in small, double seams; and Petrovitch went over each seam afterwards with his own teeth, stamping in various patterns.

It was – it is difficult to say precisely on what day, but probably the most glorious one in Akakiy Akakievitch's life, when Petrovitch at length brought home the cloak. What is to be said.

Akakiy Akakievitch went out into the street as if in a dream. "Such an affair. Once, in the course of every month, he had a conference with Petrovitch on the subject of the cloak, where it would be better to buy the cloth, and the colour, and the price. He always returned home satisfied, though troubled, reflecting that the time would come at last when it could all be bought, and then the cloak made.

At the word "new," all grew dark before Akakiy Akakievitch's eyes, and everything in the room began to whirl round. The only thing he saw clearly was the general with the paper face on the lid of Petrovitch's snuff-box. Whether he suspected that Akakiy Akakievitch needed a cloak, or whether it was merely chance, at all events, twenty extra rubles were by this means provided. This circumstance hastened matters. Two or three months more of hunger and Akakiy Akakievitch had accumulated about eighty rubles. His heart, generally so quiet, began to throb. I will make you a capital new one, so let us settle about it now.

Akakiy Akakievitch was still for mending it; but Petrovitch would not hear of it, and said, "I shall certainly have to make you a new one, and you may depend upon it that I shall do my best. It may even be, as the fashion goes, that the collar can be fastened by silver hooks under a flap. Petersburg climate, the malady progressed more rapidly than could have been expected: and when the doctor arrived, he found, on feeling the sick man's pulse, that there was nothing to be done, except to prescribe a fomentation, so that the patient might not be left entirely without the beneficent aid of medicine; but at the same time, he predicted his end in thirty-six hours. After this he turned to the landlady, and said, "And as for you, don't waste your time on him: order his pine coffin now, for an oak one will be too expensive for him. Where was the money to come from. In the department of – but it is better not to mention the department. There is nothing more irritable than departments, regiments, courts of justice, and, in a word, every branch of public service. Each individual attached to them nowadays thinks all society insulted in his person. Quite recently a complaint was received from a justice of the peace, in which he plainly demonstrated that all the imperial institutions were going to the dogs, and that the Czar's sacred name was being taken in vain; and in proof he appended to the complaint a romance in which the justice of the peace is made to appear about once every ten lines, and sometimes in a drunken condition. Therefore, in order to avoid all unpleasantness, it will be better to describe the department in question only as a certain department.

The Overcoat - Paragraphs

Petrovitch took the cloak, spread it out, to begin with, on the table, looked hard at it, shook his head, reached out his hand to the window-sill for his snuff-box, adorned with the portrait of some general, though what general is unknown, for the place where the face should have been had been rubbed through by the finger, and a square bit of paper had been pasted over it. Having taken a pinch of snuff, Petrovitch held up the cloak, and inspected it against the light, and again shook his head once more. After which he again lifted the general-adorned lid with its bit of pasted paper, and having stuffed his nose with snuff, closed and put away the snuff-box, and said finally, "No, it is impossible to mend it; it's a wretched garment!"

Akakiy Akakievitch's heart sank at these words.

Akakiy Akakievitch ran home in complete disorder; his hair, which grew very thinly upon his temples and the back of his head, wholly disordered; his body, arms, and legs covered with snow. The old woman, who was mistress of his lodgings, on hearing a terrible knocking, sprang hastily from her bed, and, with only one shoe on, ran to open the door, pressing the sleeve of her chemise to her bosom out of modesty; but when she had opened it, she fell back on beholding Akakiy Akakievitch in such a state. When he told her about the affair, she clasped her hands, and said that he must go straight to the district chief of police, for his subordinate would turn up his nose, promise well, and drop the matter there. The very best thing to do, therefore, would be to go to the district chief, whom she knew, because Finnish Anna, her former cook, was now nurse at his house. She often saw him passing the house; and he was at church every Sunday, praying, but at the same time gazing cheerfully at everybody; so that he must be a good man, judging from all appearances. Having listened to this opinion, Akakiy Akakievitch betook himself sadly to his room; and how he spent the night there any one who can put himself in another's place may readily imagine.

Early in the morning, he presented himself at the district chief's; but was told that this official was asleep. He went again at ten and was again informed that he was asleep; at eleven, and they said: "The superintendent is not at home;" at dinner time, and the clerks in the ante-room would not admit him on any terms, and insisted upon knowing his business. So that at last, for once in his life, Akakiy Akakievitch felt an inclination to show some spirit, and said curtly that he must see the chief in person; that they ought not to presume to refuse him entrance; that he came from the department of justice, and that when he complained of them, they would see.

The clerks dared make no reply to this, and one of them went to call the chief, who listened to the strange story of the theft of the coat. Instead of directing his attention to the principal points of the matter, he began to question Akakiy Akakievitch: Why was he going home so late? Was he in the habit of doing so, or had he been to some disorderly house? So that Akakiy Akakievitch got thoroughly confused, and left him without knowing whether the affair of his cloak was in proper train or not.

All this, the noise, the talk, and the throng of people was rather overwhelming to Akakiy Akakievitch. He simply did not know where he stood, or where to put his hands, his feet, and his whole body. Finally he sat down by the players, looked at the cards, gazed at the face of one and another, and after a while began to gape, and to feel that it was wearisome, the more so as the hour was already long past when he usually went to bed. He wanted to take leave of the host; but they would not let him go, saying that he must not fail to drink a glass of champagne in honour of his new garment. In the course of an hour, supper, consisting of vegetable salad, cold veal, pastry, confectioner's pies, and champagne, was served. They made Akakiy Akakievitch drink two glasses of champagne, after which he felt things grow livelier.

Still, he could not forget that it was twelve o'clock, and that he should have been at home long ago. In order that the host might not think of some excuse for detaining him, he stole out of the room quickly, sought out, in the ante-room, his cloak, which, to his sorrow, he found lying on the floor, brushed it, picked off every speck upon it, put it on his shoulders, and descended the stairs to the street.

In the street all was still bright. Some petty shops, those permanent clubs of servants and all sorts of folk, were open. Others were shut, but, nevertheless, showed a streak of light the whole length of the door-crack, indicating that they were not yet free of company, and that probably some domestics, male and female, were finishing their stories and conversations whilst leaving their masters in complete ignorance as to their whereabouts. Akakiy Akakievitch went on in a happy frame of mind: he even started to run, without knowing why, after some lady, who flew past like a flash of lightning. But he stopped short, and went on very quietly as before, wondering why he had quickened his pace. Soon there spread before him those deserted streets, which are not cheerful in the daytime, to say nothing of the evening. Now they were even more dim and lonely: the lanterns began to grow rarer, oil, evidently, had been less liberally supplied. Then came wooden houses and fences: not a soul anywhere; only the snow sparkled in the streets, and mournfully veiled the low-roofed cabins with their closed shutters. He approached the spot where the street crossed a vast square with houses barely visible on its farther side, a square which seemed a fearful desert.

At the word "new," all grew dark before Akakiy Akakievitch's eyes, and everything in the room began to whirl round. The only thing he saw clearly was the general with the paper face on the lid of Petrovitch's snuff-box. "A new one?" said he, as if still in a dream: "why, I have no money for that."

Even at the hour when the grey St. Petersburg sky had quite dispersed, and all the official world had eaten or

dined, each as he could, in accordance with the salary he received and his own fancy; when all were resting from the departmental jar of pens, running to and fro from their own and other people's indispensable occupations, and from all the work that an uneasy man makes willingly for himself, rather than what is necessary; when officials hasten to dedicate to pleasure the time which is left to them, one bolder than the rest going to the theatre; another, into the street looking under all the bonnets; another wasting his evening in compliments to some pretty girl, the star of a small official circle; another – and this is the common case of all – visiting his comrades on the fourth or third floor, in two small rooms with an ante-room or kitchen, and some pretensions to fashion, such as a lamp or some other trifle which has cost many a sacrifice of dinner or pleasure trip; in a word, at the hour when all officials disperse among the contracted quarters of their friends, to play whist, as they sip their tea from glasses with a kopek's worth of sugar, smoke long pipes, relate at times some bits of gossip which a Russian man can never, under any circumstances, refrain from, and, when there is nothing else to talk of, repeat eternal anecdotes about the commandant to whom they had sent word that the tails of the horses on the Falconet Monument had been cut off, when all strive to divert themselves, Akakiy Akakievitch indulged in no kind of diversion. No one could ever say that he had seen him at any kind of evening party. Having written to his heart's content, he lay down to sleep, smiling at the thought of the coming day – of what God might send him to copy on the morrow.

Thus flowed on the peaceful life of the man, who, with a salary of four hundred rubles, understood how to be content with his lot; and thus it would have continued to flow on, perhaps, to extreme old age, were it not that there are various ills strewn along the path of life for titular councillors as well as for private, actual, court, and every other species of councillor, even for those who never give any advice or take any themselves.

Akakiy Akakievitch had felt for some time that his back and shoulders suffered with peculiar poignancy, in spite of the fact that he tried to traverse the distance with all possible speed. He began finally to wonder whether the fault did not lie in his cloak. He examined it thoroughly at home, and discovered that in two places, namely, on the back and shoulders, it had become thin as gauze: the cloth was worn to such a degree that he could see through it, and the lining had fallen into pieces. You must know that Akakiy Akakievitch's cloak served as an object of ridicule to the officials: they even refused it the noble name of cloak, and called it a cape. In fact, it was of singular make: its collar diminishing year by year, but serving to patch its other parts. The patching did not exhibit great skill on the part of the tailor, and was, in fact, baggy and ugly. Seeing how the matter stood, Akakiy Akakievitch decided that it would be necessary to take the cloak to Petrovitch, the tailor, who lived somewhere on the fourth floor up a dark stair-case, and who, in spite of his having but one eye, and pock-marks all over his face, busied himself with considerable success in repairing the trousers and coats of officials and others; that is to say, when he was sober and not nursing some other scheme in his head.

The Old Man at the Bridge - Sentences

I asked.

"There were three animals altogether," he explained.

It's better not to think about the others," he said.

"If you are rested I would go," I urged.

"Get up and try to walk now.

"Did you leave the dove cage unlocked.

I asked.

"Yes.

"And you have no family.

"From San Carlos," he said, and smiled.

That was his native town and so it gave him pleasure to mention it and he smiled.

"Oh," I said, not quite understanding.

"Yes," he said, "I stayed, you see, taking care of animals.

I was the last one to leave the town of San Carlos.

He did not look like a shepherd nor a herdsman and I looked at his black dusty clothes and his gray dusty face and his steel.

rimmed spectacles and said, "What animals were they.

"Various animals," he said, and shook his head.

"I had to leave them.

I was watching the bridge and the African looking country of the Ebro Delta and wondering how long now it would be.

before we would see the enemy, and listening all the while for the first noises that would signal that ever mysterious event.

called contact, and the old man still sat there.

"What animals were they.

It was a gray.

overcast day with a low ceiling so their planes were not up.

But the others.

"There were two goats and a cat and then there were four pairs of.

pigeons.

"You think so.

And you had to leave them.

I asked.

"Yes.

Because of the artillery.

The captain told me to go because of the artillery.

It was Easter Sunday and the Fascists were advancing toward the Ebro.

The trucks ground up and away heading out.

of it all and the peasants plodded along in the ankle deep dust.

"I was taking care of animals," he explained.

But the old man sat there without moving.

He was too tired to.

go any farther.

It was my business to cross the bridge, explore the bridgehead beyond and find out to what point the enemy had advanced.

I.

did this and returned over the bridge.

There were not so many carts now and very few people on foot, but the old man was still.

there.

I asked him.

The mule- drawn carts staggered up the steep.

bank from the bridge with soldiers helping push against the spokes of the wheels.

I asked, watching the far end of the bridge where a few last carts were hurrying down the slope.

of the bank.

"No," he said, "only the animals I stated.

That and the fact that cats know how to look after themselves was.

all the good luck that old man would ever have.

The cat, of course, will be all right.

"Why not," I said, watching the far bank where now there were no carts.

"But what will they do under the artillery when I was told to leave because of the artillery.

"Thank you," he said and got to his feet, swayed from side to side and then sat down backwards in the dust.

"I was taking care of animals," he said dully, but no longer to me.

"I was only taking care of animals.

There was nothing to do about him.

There was a pontoon bridge.

across the river and carts, trucks, and men, women and children were crossing it.

A cat can look out for itself, but I cannot think.

what will become of the others.

I asked.

"I am without politics," he said.

"I am seventy-six years old.

An old man with steel rimmed spectacles and very dusty clothes sat by the side of the road.

"Where do you come from.

"If you can make it, there are trucks up the road where it forks for Tortosa.

"Yes, certainly they'll fly.

Now what do you think about the others.

"What politics have you.

"Then they'll fly.

"Why they'll probably come through it all right.

I have come twelve kilometers now and I think now I can go.

no further.

”This is not a good place to stop,” I said.

But the others.

”I will wait a while,” he said, ” and then I will go.

Where do the trucks go.

”Towards Barcelona,” I told him.

There is no need to be unquiet about the cat.

”I know no one in that direction,” he said, ”but thank you very much.

He looked at me very blankly and tiredly, and then said, having to share his worry with someone, ”The cat will be all right, I.

am sure.

Thank you again very much.

Nine Billion Names of God - Sentences

”bingo.” ”Oh, I get it. ”called Chuck, pointing down into the valley. ”Ain't she beautiful.” She certainly was, thought George. The battered old DC-3 lay at the end of the. runway like a tiny silver cross. In two hours she would be bearing them away to. freedom and sanity.” George thought this over for a moment.

”That's what I call taking the Wide View,” he said presently. This was. the last place from which one could get a clear view of the lamasery. The squat, angular buildings were silhouetted against the afterglow of the sunset; here and. there lights gleamed like portholes in the sides of an ocean liner. It was due. about now.” Chuck didn't reply, so George swung round in his saddle. He could just see. Chuck's face, a white oval turned toward the sky.

”Look,” whispered Chuck, and George lifted his eyes to heaven. (There is always. a last time for everything.

Overhead, without any fuss, the stars were going out.

No one would be saying anything. But we didn't know why. It's. the craziest thing –”. ”Tell me something new,” growled George.

but old Sam's just come clean with me. You know the way he drops in every. afternoon to watch the sheets roll out. Well, this time he seemed rather excited, or. at least as near as he'll ever get to it. When I told him we were on the last cycle he. asked me, in that cute English accent of his, if I'd ever wondered what they were. trying to do. I said, 'Sure'—and he told me.” ”Go on, I'll buy it.”

”Well, they believe that when they have listed all His names—and they reckon. that there are about nine billion of them—God's purpose will have been achieved. I'm just sorry for those poor old guys up there, and I don't want to. be around when they find what suckers they've been. Wonder how Sam will take.

At least there would be no risk, thought George, of the pilot being unable to. take of because of weather conditions. That had been his only remaining worry.

He began to sing but gave it up after a while. This vast arena of mountains, gleaming like whitely hooded ghosts on every side, did not encourage such ebullience. But this sort of thing's. happened here before, you know. When I was a kid down in Louisiana we had a. crackpot preacher who said the world was going to end next Sunday. Hundreds. of people believed him— even sold their homes. Three months of this, thought George, was enough to start. anyone climbing up the wall.

”There she is. When we finish our job, it will be the end of the world.” Chuck gave a nervous little laugh.

”That's just what I said to Sam. And do you know what happened. He looked. at me in a very queer way, like I'd been stupid in class, and said, 'It's nothing as. trivial as that'. It was a thought worth savoring like a fne liqueur. K. , then all we need to do is to find something that wants replacing during. one of the overhaul periods—something that will hold up the works for a couple of. days. We'll fix it, of course, but not too quickly. ”Your Mark V computer. can carry out any routine mathematical operation involving up to ten digits. They won't be. able to catch us then.” ”I don't like it,” said George. ”This is a slightly unusual request,” said Dr. Wagner, with what he hoped was. commendable restraint. ”As far as I know, it's the first time anyone's been asked to. supply a Tibetan monastery with an automatic sequence computer. I don't wish to. be inquisitive, but I should hardly thought that your—ah—establishment had much. use for such a machine. Could you explain just what you intend to do with it.” ”Gladly,” replied the lama, readjusting his silk robe and carefully putting away. the slide rule he had been using for currency conversions.” ”Yes—but don't you see what may happen. How- ever, for our work we are interested in letters, not numbers. As we wish you to. modify the output circuits, the machine will be printing words, not columns of. figures.” ”I don't understand.”

”This is a project on which we have been working for the last three centuries—. since the lamasery was founded, in fact. It is somewhat alien to your way of. thought, so I hope you will listen with an open mind while I explain it.” ”Naturally.” ”It is really quite simple. How much longer would they share. it. wondered George. Would the

monks smash up the computer in their rage and. disappointment. Or would they just sit down quietly and begin their calculations. all over again." "Yes. We expected it would take us about fifteen thousand years to complete the. task." "Oh.

Wagner looked a little dazed. "Now I see why you wanted to hire one. of our machines. But exactly what is the purpose of this project." The lama hesitated for a fraction of a second, and Wagner wondered if he had. offended him. If so, there was no trace of annoyance in the reply.

"Call it ritual, if you like, but it's a fundamental part of our belief. All the many. names of the Supreme Being—God, Jehovah, Allah, and so on—they are only man-made labels. There is a philosophical problem of some difficulty here, which I do. not propose to discuss, but somewhere among all the possible combinations of letters, which can occur, are what one may call the real names of God. By systematic. permutation of letters, we have been trying to list them all. Wagner scribbled a note on. his desk pad. Yet nothing happened; they didn't. turn nasty, as you'd expect. "There are just two other points—". Before he could finish the sentence, the lama had produced a small slip of paper. They just decided that he'd made a mistake in his. calculations and went right on believing. I guess some of them still do." "Well, this isn't Louisiana, in case you hadn't noticed. I'm much more worried about the problem of installation and. maintenance. There are just two of us. and hundreds of these monks. I like them, and I'll be sorry for old Sam when his. lifework backfires on him. But all the same, I wish I was somewhere else." "I've been wishing that for weeks. But there's nothing we can do until the con-. tract's finished and the transport arrives to fly us out." "Of course," said Chuck thoughtfully, "we could always try a bit of sabotage. That would make things worse. If we time matters properly, we can. be down at the airfield when the last name pops out of the register." "Not the way I meant. Look at it like this. The machine will finish its run four. days from now, on the present twenty-hours-a-day basis. The transport calls in a. week. O.

"Like hell we could. There was not the slightest danger, only a certain discomfort from the bitter cold. Indeed, the very idea is something like blasphemy. The sky overhead was perfectly clear and ablaze with the familiar, friendly. stars." "It's funny," replied Chuck, "but when I said goodbye I got the idea he knew we. were walking out on him—and that he didn't care because he knew the machine. was running smoothly and that the job would soon be finished. "It will be the first time I ever walked out on a job.

Besides, it would make them suspicious. No, I'll sit tight and take what comes." "I still don't like it," she said seven days later, as the tough little mountain ponies. carried them down the winding road. "And don't you think I'm running away be-. cause I'm afraid. We have been compiling a list which shall contain all. the possible names of God. When the list's complete and the. Last Trump doesn't blow—or whatever it is that they expect—we may get the blame.

It's our machine they've been using. After that—well, of. course, for him there just isn't any After That.

George let. it roll around in his mind as the pony trudged patiently down the slope.

The swift night of the high Himalayas was now almost upon them. Fortunately. the road was very good, as roads went in this region, and they were both carrying. torches. and working up to ZZZZZZZ-. ZZ." "Then what do they expect us to do. Commit suicide. I don't like the situation one little bit." "I see," said George slowly." George turned in his saddle and stared back up the mountain road.

"But what d'ya suppose we should do about it. I don't see that it makes the. slightest difference to us. After all, we already knew that they were crazy." "I beg your pardon." "We have reason to believe," continued the lama imperturbably, "that all such. names can be written with not more than nine letters in an alphabet we have devised." "There's no need for that. When the list's completed, God steps in and simply. winds things up.

Presently George glanced at his watch.

"Should be there in an hour," he called back over his shoulder to Chuck. Then. he added, in an afterthought, "Wonder if the computer's fnished its run. Electric lights, of. course, sharing the same circuit as the Mark V. The second matter is so trivial that. I hesitate to mention it—but it's surprising how often the obvious gets overlooked.

What source of electrical energy have you." "And you have been doing this for three centuries.

He knew exactly what was happening up on the mountain at this very moment.

References

[1] Bliss Carman. *Sappho: One hundred lyrics*. A. Moring, The De La More Press, 1906.

[2] Arthur Charles Clarke. *Nine Billion Names of God*. Harcourt, 1967.

[3] Edward Estlin Cummings. *100 selected poems*. Grove Press, 1959.

[4] Diana S Dabby. Musical variations from a chaotic mapping. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 6(2):95–107, 1996.

[5] Sarah Glaz. Poetry inspired by mathematics: A brief journey through history. *Journal of Mathematics and the Arts*, 5(4):171–183, 2011.

[6] Nikola Gogol. *The overcoat, and other tales of good and evil.* Number 304. WW Norton & Company, 1965.

[7] JoAnne Growney et al. Using mathematics to make poetry. *Proceedings of Bridges Coimbra*, pages 621–625, 2011.

[8] Ernest Hemingway. *The short stories of Ernest Hemingway.* Simon and Schuster, 2002.

[9] Edward N Lorenz. Deterministic nonperiodic flow. *Journal of the atmospheric sciences*, 20(2):130–141, 1963.

[10] Caleb Phillips, Lee Becker, and Elizabeth Bradley. Strange beta: An assistance system for indoor rock climbing route setting using chaotic variations and machine learning; cu-cs-1088-11. 2011.

[11] William Shakespeare. *The Sonnets.* Thomas Thorpe, 1609.

[12] Steven H Strogatz. *Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering.* Westview press, 2014.

[13] Joshua M Stuart, Elizabeth Bradley, David Capps, and Jeffrey Luftig. Generating novel, stylistically consonant variations on human movement sequences. 2007.

[14] Dave Vannette. Three tips for effectively designing rating scales. `https://www.qualtrics.com/blog/three-tips-for-effectively-using-scale-point-questions/`. Accessed: 2017-04-29.

[15] Eric W. Weisstein. Voronoi diagram. `http://mathworld.wolfram.com/VoronoiDiagram.html`. Accessed: 2017-04-20.