CSCI 3434 Theory of Computation Homework 3

Luke Meszar (Worked with Jeff Lipnick)

October 18, 2017

HW 5.2 Prove that the CFG

$$S \to aSb \mid bSa \mid SS \mid \varepsilon$$

generates the set of all strings over $\{a, b\}$ with equally many a's and b's. (Hint: Characterize elements of the set in terms of the graph of the function $\#b(y) - \#a(y)$ as y ranges over prefixes of x, as we did in Lecture 20 with balanced parentheses.)

HW 6.1 Prove that the following CFG G in Greibach normal form generates exactly the set of nonnull strings over $\{a, b\}$ with equally many a's and b's:

$$\begin{aligned} S &\to aB \mid bA, \\ A &\to aS \mid bAA \mid a, \\ B &\to bS \mid aBB \mid b. \end{aligned}$$

(Hint: Strengthen your induction hypothesis to describe the sets of strings generated by the nonterminals A and B: for $x \neq \varepsilon$,

$$\begin{aligned} S &\xrightarrow[G]{*} \iff \#a(x) = \#b(x), \\ A &\xrightarrow[G]{*} ???, \\ B &\xrightarrow[G]{*} ???.) \end{aligned}$$

Misc. 1 Closure properties of CFLs. For each of the following, let L_1 and L_2 be arbitrary CFLs and determine whether the given language must be a CFL (YES) or can you find a counterexample (NO), i.e., particular choices of L_1 and L_2 such that the language is not a CFL. For the YES's give a short justification involving the construction of a grammar/PDA from the given grammars/PDAs. For t he NO's, use the CFL pumping lemma or known non-context-free languages to establish your counterexample.

(a) $L_1 L_2$

(b) $L_1 \cap L_2$

(c) $L_1 \cup L_2$

(d) L_1^*

Misc. 2 For each language, decide whether it is regular, not regular but context-free, or not context-free. Justify your answer.

(a) $\{a^k b^l a^m b^n \mid k = m \text{ or } l = n\}$

(b) $\{a^k b^l a^m b^n \mid k = m \text{ and } l = n\}$

Misc. 3 Give a PDA accepting the following languages. No proof required.

(a) $\{a^n b^m c^k \mid k = n + m\}$

(b) $\{x \in \{a, b\}^* \mid \#a(x) = \#b(x)\}$.