TSA-1 Assignment: Investigating the accuracy of the Erlang-B formula

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***Abstract*—This document details an investigation into the accuracy of the Erlang-B formula when calculating the Grade of Service(GOS) for a communication link. GOS is the probability as a percentage of a called being blocked. A comparison is made between the Predicted value from the Erlang-B formula and the Actual values calculated using the Monte Carlo method over the course of an hour. The results showed that each call holding distribution(gamma, log-normal and exponential) gave a similar GOS versus Offered Traffic curve. Each of the three curves are significantly lower than what is predicted by the Erlang-B formula for the same offered traffic values.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

The aim of this investigation is to compare the accuracy of the Erlang-B formula to the Monte Carlo simulation for a given communication link. This formula is commonly used a method for estimating the required number of channels when planning communication link. This investigation is based on a real world example [1] of a indoor base station with approximately 30 channels. This value will remain fixed at N=30 for the entirety of the investigation. The indoor channel value was chosen instead of the outdoor value of 100-150 as it would reduce the computing power necessary to complete the simulation. The mean, shape, scale and standard deviation values for the call holding distributions used have been taken from a study examining the channel holding time of public cellular telephony systems [2].

II. Simulation Model

A. Operation of simulation model

Th Monte Carlo simulation [3] operates by taking inputs randomly from distributions. These values are then used to calculate the output. When this is repeated hundreds or thousands of times the sample men approaches the theoretical mean. This method was carried out to simulate the grade of service for a communication link over the course of an hour. Call Start times are randomly chosen from a uniform distribution while call holding times are randomly chosen from either a gamma, log-normal or exponential distribution based off Figure 1 . Offered traffic is calculated as the product of number of calls and average duration of each call, it is measured in Erlangs. Offered traffic is varied by increasing the number of calls that take place in the hour time frame. The parameters underlying each distribution can be seen in Table 1. A curve is produced for each of the three chosen call holding distributions. This is done by calculating the number of blocked calls over the course of an hour. A call is deemed to be blocked if it is initiated at a time when all 30 channels are being used by other calls. The quotient of blocked calls and total calls attempted gives you GOS as a proportion. According to the Monte Carlo distribution, this should be calculated several times and the average value is found. Average GOS is plot against Offered Traffic. This can be compared to the curve calculated using the Erlang-B formula. This investigation repeats each simulation 100 times for a given Offered traffic. This is enough to smooth out the resulting curve while not being too computationally intensive.

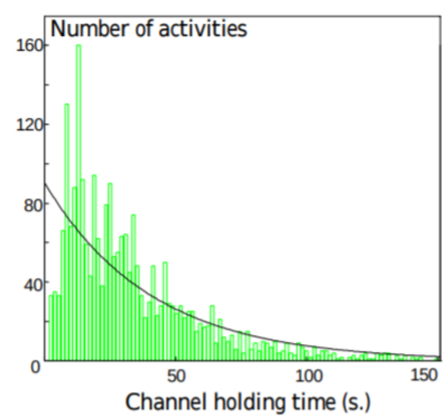

Fig. 1. Empirical vs. exponential channel holding time distribution..

TABLE I
CALL HOLDING DISTRIBUTION PARAMETERS [2]

Parameter	*Value*
Average	40.6
Scale	40.6
Shape	1.2073
Log mean	3.29
Standard Deviation	0.89

[a] Values relative to seconds

B. System used to carry out simulation

The Monte Carlo simulation and Erlang-B comparison was written in python scripting language. This script was run on a Linux operating system with 16GB RAM and Intel i5 processor. Execution time was approximately two hours.

The execution time includes the Erlang-B curve calculation along with Monte Carlo simulations for a gamma, log-normal and exponential call holding distribution. Each simulation was carried out 100 times and the average GOS found. This was repeated for 0-500 calls increasing in increments of 10.

III. Results

A. Erlang-B

Figure 2 illustrates the results of the Erlang-B calculations. The GOS begins to rise rapidly at 25 Erlangs and continues until 100 Erlangs. The rate of increase slows significantly from 150 Erlangs onwards.

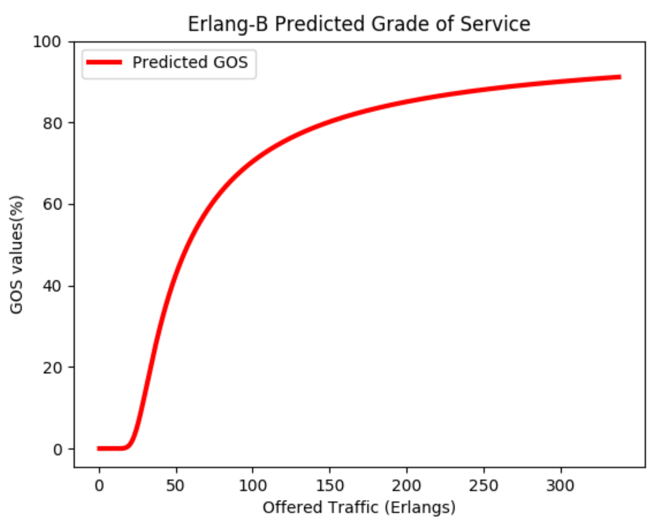

Fig. 2. Erlang-B Prediction for Grade of Service(%) for various Offered Traffic(Erlang) values.

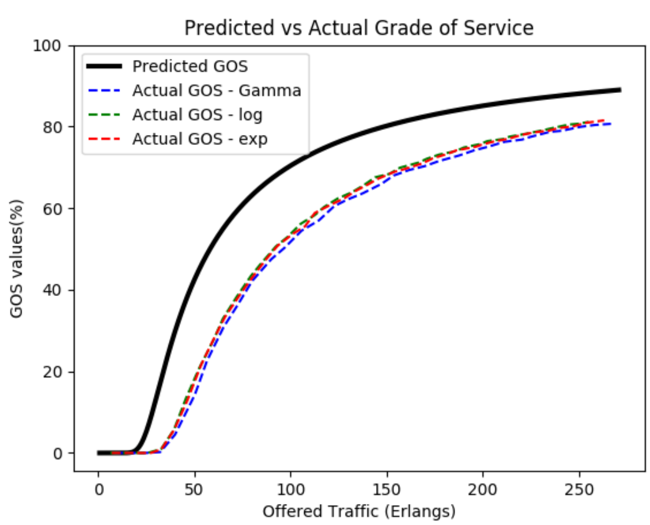

Fig. 3. Predicted vs Actual Grade of Service(%) for various Offered Traffic(Erlang) values.

B. Monte Carlo Simulation

Figure 3 illustrates the results for the simulations, using gamma, log-normal and exponential call holding distributions respectively, and compares them to the predicted GOS according to Erlang-B. It is evident that, due to the high number of simulations, there is almost negligible difference between the three simulation curves. They begin to rise at 40 Erlangs albeit at a slower rate than the Erlang-B prediction curve. This curve continues to rise until it begins to level out at 200 Erlangs.

C. Comparison

It is evident from Figure 3 that there is a significant difference between the predicted and actual GOS. For example, At an Offered Traffic of 100 Erlangs the simulation model gives a GOS value of approximately 52 percent versus a predicted value of 73 percent; that is 20 percentage points in the difference.

IV. Conclusions

It is clear from the results that the Erlang formula gives an inflated estimation. The curve is a similar shape to that of the simulations however it increases at a higher rate. The formula would still allow you to quickly design a communication channel that can handle the necessary traffic requirements. The Erlang-B line acts as an upper bound for the model. No single simulation GOS went above the value predicted. This allows for confidence when planning a resource that the actual GOS when the system goes live will be lower than what is predicted by the formula. This ensures that end users will experience ,at most, the stated GOS for he communication link. It is clear that there is a place for both the Erlang-B formula and simulations when planning and building a communication resource. The much less intensive Erlang-B formula can be used early in the project. Simulations are more time and CPU intensive so can use the Erlang-B predictions as a starting point.

References

[1] Irish Cellular Industry Association, “Frequently Asked Questions - Why do we need base stations? ” pp.2, Jan 2007.

[2] Francisco Barceló and Javier Jordán,Channel Holding Time Distribution in Public Cellular Telephony, 1999.

[3] Monte Carlo Simulation, https://www.palisade.com/risk/montecarlosimulation.asp