4.7.1)

$x^T Y = \begin{bmatrix}-1\\-1\\1\end{bmatrix}\begin{bmatrix}0 & 1 & 2\end{bmatrix}$

$0\begin{bmatrix}-1\\-1\\1\end{bmatrix} \quad 1\begin{bmatrix}-1\\-1\\1\end{bmatrix} \quad 2\begin{bmatrix}-1\\-1\\1\end{bmatrix} = \begin{bmatrix}0&-1&-2\\0&-1&-2\\0&1&2\end{bmatrix} = x^TY$

Possible

$xY^T = \begin{bmatrix}-1 & -1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\2\end{bmatrix}$

$-1(0) + -1(1) + 1(2) = 0-1+2 = 1 = xY^T$

Possible

4.7.2) a) $-3A^T = \begin{bmatrix}1&3&1\\2&2&-1\end{bmatrix}(-3) = \begin{bmatrix}-3&-9&-3\\-6&-6&3\end{bmatrix}$

b) $3B - A^T = \begin{bmatrix}6&-15&6\\-9&6&3\end{bmatrix} - \begin{bmatrix}1&3&1\\2&2&-1\end{bmatrix} = \begin{bmatrix}5&-18&5\\-11&4&4\end{bmatrix}$

c) $E^TB = \begin{bmatrix}1 & 3\end{bmatrix}\begin{bmatrix}2&-5&2\\-3&2&1\end{bmatrix}$

$1(2) + 3(-3) = -7$
$1(-5) + 3(2) = 1$
$1(2) + 3(1) = 5$

$\begin{bmatrix}-7 & 1 & 5\end{bmatrix} = E^TB$

4.9.1)

Rendezvous at dawn

[17 4 13], [3 4 25], [21 14 20], [18 0 19], [3 0 22], [13 23 23]

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 17 \\ 4 \\ 13 \end{bmatrix} \quad \begin{aligned} 2(17)+1(4)+1(13) &= 51 \,\%\, 29 = 22 \\ 1(17)+3(4)+1(13) &= 42 \,\%\, 29 = 13 \\ 1(17)+1(4)+4(13) &= 73 \,\%\, 29 = 15 \end{aligned}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 25 \end{bmatrix} \quad \begin{aligned} 2(3)+1(4)+1(25) &= 35 \,\%\, 29 = 6 \\ 1(3)+3(4)+1(25) &= 40 \,\%\, 29 = 11 \\ 1(3)+1(4)+4(25) &\quad 107 \,\%\, 29 = 20 \end{aligned}$$

$$A\begin{bmatrix} 21 \\ 14 \\ 20 \end{bmatrix} \quad \begin{aligned} 2(21)+1(14)+1(20) &= 76 \,\%\, 29 = 18 \\ 1(21)+3(14)+1(20) &= 83 \,\%\, 29 = 25 \\ 1(21)+1(14)+4(20) &= 115 \,\%\, 29 = 28 \end{aligned}$$

$$A\begin{bmatrix} 18 \\ 26 \\ 0 \end{bmatrix} \quad \begin{aligned} 2(18)+1(26)+1(0) &= 62 \,\%\, 29 = 4 \\ 1(18)+3(26)+1(0) &= 96 \,\%\, 29 = 9 \\ 1(18)+1(26)+4(0) &= 44 \,\%\, 29 = 15 \end{aligned}$$

$$A\begin{bmatrix} 19 \\ 26 \\ 3 \end{bmatrix} \quad \begin{aligned} 2(19)+1(26)+1(3) &= 67 \,\%\, 29 = 9 \\ 1(19)+3(26)+1(3) &= 100 \,\%\, 29 = 13 \\ 1(19)+1(26)+4(3) &= 57 \,\%\, 29 = 28 \end{aligned}$$

$$A\begin{bmatrix} 0 \\ 22 \\ 13 \end{bmatrix} \quad \begin{aligned} 2(0)+1(22)+1(13) &= 35 = 6 \\ 1(0)+3(22)+1(13) &= 79 = 21 \\ 1(0)+1(22)+4(13) &= 74 = 16 \end{aligned}$$

WNPGLUSZ.EJPJN.GVQ

oo

4.9.2) $\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right]$ $R_1 \to 15R_1$, $R_1 \to R_1 \% 29$, $R_2 - R_1$, $R_3 - R_1$ $\left[\begin{array}{ccc|ccc} 1 & 15 & 15 & 15 & 0 & 0 \\ 0 & 17 & 15 & 14 & 1 & 0 \\ 0 & 15 & 18 & 14 & 0 & 1 \end{array}\right]$

$R_1 \to 12R_1$
$R_1 \to R_1 \% 29$
$R_1 - 15R_2$
$R_3 - 15R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 12 & 18 & 23 & 0 \\ 0 & 1 & 6 & 23 & 12 & 0 \\ 0 & 0 & 15 & 17 & 23 & 1 \end{array}\right]$ $R_3 \to R_3 \cdot 2$, $R_3 \to R_3 \% 29$, $R_1 - 12R_3$, $R_2 - 6R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 22 & 5 \\ 0 & 1 & 0 & 22 & 26 & 17 \\ 0 & 0 & 1 & 5 & 17 & 2 \end{array}\right]$

A^{-1}

$A^{-1} \times \begin{bmatrix} 4 \\ 17 \\ 12 \end{bmatrix}$
$16(4) + 22(17) + 5(12) = 498 \% 29 = 5$
$22(4) + 26(17) + 17(12) = 734 \% 29 \quad 9$
$5(4) + 17(17) + 2(12) = 333 \% 29 \quad 14$

$A^{-1} \begin{bmatrix} 26 \\ 3 \\ 23 \end{bmatrix}$
$16 \cdot 26 + 22 \cdot 3 + 5 \cdot 23 = 17$
$22 \cdot 26 + 26 \cdot 3 + 17 \cdot 23 = 26$
$5 \cdot 26 + 17 \cdot 3 + 2 \cdot 23 = 24$

$A^{-1} \begin{bmatrix} 24 \\ 1 \\ 9 \end{bmatrix}$
$16 \cdot 24 + 22 \cdot 1 + 5 \cdot 9 = 16$
$22 \cdot 24 + 26 \cdot 1 + 17 \cdot 9 = 11$
$5 \cdot 24 + 17 \cdot 1 + 2 \cdot 9 = 10$

$A^{-1} \begin{bmatrix} 20 \\ 22 \\ 22 \end{bmatrix}$
$16 \cdot 20 + 22 \cdot 22 + 5 \cdot 22 = 15$
$22 \cdot 20 + 26 \cdot 22 + 17 \cdot 22 = 23$
$5 \cdot 20 + 17 \cdot 22 + 2 \cdot 22 = 25$

$A^{-1} \begin{bmatrix} 27 \\ 26 \\ 9 \end{bmatrix}$
$16 \cdot 27 + 22 \cdot 26 + 5 \cdot 9 = 5$
$22 \cdot 27 + 26 \cdot 26 + 17 \cdot 9 = 2$
$5 \cdot 27 + 17 \cdot 26 + 2 \cdot 9 = 15$

$A^{-1} \begin{bmatrix} 22 \\ 16 \\ 11 \end{bmatrix}$
$16 \cdot 22 + 22 \cdot 16 + 5 \cdot 11 = 5$
$22 \cdot 22 + 26 \cdot 16 + 17 \cdot 11 = 14$
$5 \cdot 22 + 17 \cdot 16 + 2 \cdot 11 = 27$

$A^{-1} \begin{bmatrix} 28 \\ 7 \\ 4 \end{bmatrix}$
$16 \cdot 28 + 22 \cdot 7 + 5 \cdot 11 = 19$
$22 \cdot 28 + 26 \cdot 7 + 17 \cdot 11 = 28$
$5 \cdot 28 + 17 \cdot 7 + 2 \cdot 11 = 20$

FJOR YQLKPX Z FCP FO. T, U

c)

$$V_3 = \left\{ u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \middle| \; u_3 + u_1 = 2u_2 \right\}$$

zero vector: ~~...~~ $0 + 0 = 2 \cdot 0$ ✓

$[v_1, v_2, v_3]^T \quad v_3 + v_1 = 2v_2$

$(u_3+v_3)+(u_1+v_1) = (u_3+u_1)+(v_3+v_1) = 2u_2 + 2v_2 = 2(u_2+v_2)$

Closed under addition: ✓

Closed under scalar multiplication:

Let $c \in \mathbb{R}, u \in V_3$

$cu = [cu_1, cu_2, cu_3]^T$

$cu_3 + cu_1 = c(u_3+u_1) = c(2u_2) = 2(\cancel{\ }cu_2)$ ✓

V_3 is a subspace ✓

---

d)

$$V_4 = \left\{ \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \middle| \; u_3 \geq u_1 \right\}$$

zero vector: $0 = [0,0,0]^T$ satisfies $0 \geq 0$ ✓

$u = [u_1, u_2, u_3]^T \quad u_3 \geq u_1$

$v = [v_1, v_2, v_3]^T \quad v_3 \geq v_1$ ✓

Then $u_3 + v_3 \geq u_1 + u_1$

Closed under scalar multiplication: ✗

Let $c \in \mathbb{R}, u, \in V_4$

$(cu_3) \geq (cu_1) \qquad$ if $c \geq 0$ then $cu_3 \geq cu_1$

if $c < 0$, $u_3 \geq u_1 \cdot c$ reverses it: $cu_3 \leq cu_1$ so $cu \in V_4$

Not a subspace

5.3.2)

$$M = \left\{ u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; w \cdot u = 0 \right\}$$

Zero vector: ✓

$0 = [0,0,0,0]^T$

$w \cdot 0 = 0 \quad 0 \in M$ ✓

closed under addiction: ✓

Let $u, v \in M$, so $w \cdot u = 0$ and $w \cdot v = 0$

$w(u+v) = w \cdot u + w \cdot v = 0 + 0 \quad u+v \in M$ ✓

closed under scalar multiplication: ✓

Let $u \in M$ and $c \in \mathbb{R}$

$w \cdot (cu) = c(w \cdot u) = c \cdot 0 = 0 \quad cu \in M$

M is a subspace of $\mathbb{R}^4$

5.4.1) b) $V_2 = \text{span}\left\{ a_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}\ a_2 = \begin{bmatrix} -1 \\ -1 \\ -2 \\ 2 \end{bmatrix}\ a_3 = \begin{bmatrix} 2 \\ 3 \\ 5 \\ -5 \end{bmatrix}\ a_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -2 \end{bmatrix} \right\}$

$$A \rightarrow \begin{bmatrix} 0 & -1 & 2 & 0 \\ 1 & -1 & 3 & 1 \\ 1 & -2 & 5 & 2 \\ -1 & 2 & -5 & -2 \end{bmatrix} \quad \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 1 & -2 & 5 & 2 \\ -1 & 2 & -5 & -2 \end{pmatrix} \begin{matrix} R_3 - R_1 \\ R_4 + R_1 \end{matrix} \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & 2 & 1 \\ 0 & 1 & -2 & -1 \end{pmatrix}$$

$$\begin{matrix} R_3 - R_2 \\ R_4 + R_2 \end{matrix} \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} R_4 + R_3 \begin{pmatrix} 1 & -1 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

dimension = 3

basis: $\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -2 \end{bmatrix} \right\}$

c) $$\begin{bmatrix} 1 & 4 & 15 & 10 \\ -2 & -9 & -33 & -22 \\ 1 & 3 & 12 & 8 \\ -3 & -9 & -36 & -24 \end{bmatrix} \begin{matrix} R_2 + 2R_1 \\ R_3 - R_1 \\ R_4 + 3R_1 \end{matrix} \begin{bmatrix} 1 & 4 & 15 & 10 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \\ 0 & 3 & 9 & 6 \end{bmatrix} \begin{matrix} R_3 + R_2 \\ R_4 - 3R_2 \end{matrix} \begin{bmatrix} 1 & 4 & 15 & 10 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

dimension = 2 basis: $\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} -9 \\ 4 \\ 3 \\ -9 \end{bmatrix} \right\}$