6

6.1.1)

$$T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+y \\ x-2y \\ -x-y \end{bmatrix} = x\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + y\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 1 \\ 1 & -2 \\ -1 & -1 \end{bmatrix}$$ linear ✓

$$T_2\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y^2 \\ (x+y)z \\ 0 \end{bmatrix}$$ $u+v = (0,2,0)$, $T_2(u+v) = (0+4, 0+2\cdot0, 0) = (4,0,0)$
$T_2(u) = (1,0,0)$, $T_2(v) = (1,0,0)$ $T_2(u)+T_2(v) = (2,0,0)$

<u>Not Linear</u>

$$T_3\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ $T_3(u+v) = 0+0+0 = T_3(u) + T_3(w)$ <u>Linear</u>

T_1 = linear T_3 = linear T_2 = not linear

6.2.1) a)

$$(T(v))_k = \begin{cases} v_k & \text{if } k \neq j \\ bv_j & \text{if } k = j \end{cases}$$

For $k \neq j$, $(T(u+v)_k) = u_k + v_k = (T(u))_k + (T(v))_k$
For $k = j$, $(T(u+v))_j = b(u_j+v_j) = bu_j + bv_j = (T(u))_j + (T(v))_j$
T is linear

$$A = I + (b-1)e_j e_j^T \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b)
For $k \neq i$, $T(u+v)_k = u_k + v_k = T(u)_k + T(v)_k$
F $k = i$, $T(u+v)_i = (u_i+v_i) + b(u_j+v_j) = (u_i + bu_j) + (v_i + bv_j) = T(u)_i + T(v$
<u>T is linear</u>

$$A = I + be_i e_j^T$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) $T(v)_k = v_k$ if $k \neq i, j$, $T(v)_i = v_j$, $T(v)_j = v_i$

$$A_{kl} = \begin{cases} \delta_{kl} & \text{if } l \neq i,j \\ 1 & \text{if } (k,l) = (i,j) \text{ or } (k,l) = (j,i) \\ 0 & \text{if } (k,l) = (i,i) \text{ or } (j,j) \text{ when swapped} \end{cases}$$

$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad Ae_i = e_j, Ae_j = e_i, Ae_k = e_k$

6.2.3)

$u_1 = \begin{bmatrix} 1 \\ 2 \\ -6 \end{bmatrix}$ $u_2 = \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix}$ $u_3 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

$v_1 = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ $v_3 = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & -1 \\ -6 & 5 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 5 & 1 & 5 \\ 1 & 1 & 3 \\ 3 & 5 & -2 \end{bmatrix}$

$\det(A) = \det\begin{bmatrix} -1 & -1 \\ 5 & 2 \end{bmatrix} - (-1)\det\begin{bmatrix} 2 & -1 \\ -6 & 2 \end{bmatrix}$

$1 \cdot (-1(2) - (-1)(5)) + 1((2)(2) - (-1)(6))$

$3 - 2 = 1$

$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 & 1 & 0 \\ -6 & 5 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 6R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -1 & 2 & 6 & 0 & 1 \end{array}\right]$

$R_3 + R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 + R_3 \\ R_1 + R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 1 \\ 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 4 & 1 & 1 \end{array}\right]$ $\quad A^{-1} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 4 & 1 & 1 \end{bmatrix}$

$BA^{-1} = \begin{bmatrix} 5 & 1 & 5 \\ 1 & 1 & 3 \\ 3 & 5 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 4 & 1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 37 & 17 & 11 \\ 17 & 7 & 5 \\ 11 & 14 & 6 \end{bmatrix}}$

6.3.1) $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\theta = \pi/3$

$R(\frac{\pi}{3}) = \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 1/2 \end{bmatrix}$

6.4.1) $R_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $R_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_y R_x = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}}$

6.4.3) $R_{\pi/6} = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

$F_y F_x = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ $M = (F_y F_x) R_{\frac{\pi}{6}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \boxed{\begin{bmatrix} -\frac{\sqrt{3}}{2} & 1/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}}$

6.4.7) $Av = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 + 1 \cdot (-1) \\ -1 \cdot 2 + 2 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$B \cdot Av = \begin{bmatrix} 0 & -2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \cdot 5 + (-2)(-4) \\ 4 \cdot 5 + 2(-4) \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \end{bmatrix}$

$\boxed{(S \circ T)(v) = \begin{bmatrix} 8 \\ 12 \end{bmatrix}}$

6.4.10)

$\det(A) = (2)(2) - (1)(5) = -1$

$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 2 & -1 \\ -5 & 2 \end{bmatrix}$

$\boxed{T^{-1} = \begin{bmatrix} -2 & 1 \\ 5 & -2 \end{bmatrix}}$