MTH 309 - Practice Exam 2 Questions

Problem 1

Encrypt the message “ `Linear Algebra` ” using the Hill cipher with block size 3 and encryption matrix

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 1 & 5 \\ 1 & 3 & 2 \end{bmatrix}.$$

Problem 2

Find a basis for the following sets and determine the dimension.

a.

$$W = \left\{ \begin{bmatrix} u \\ v \\ w \end{bmatrix} \in \mathbb{R}^3 : u + v = 0 \text{ and } u - 2w = 0 \right\}.$$

b.

$$S = \left\{ \begin{bmatrix} 2u + 6v + 7w \\ -3u - 9v - 12w \\ 2u + 6v + 6w \\ u + 3v + 3w \end{bmatrix} : u, v, w \in \mathbb{R} \right\}.$$

Problem 3

Find a basis of the row space, column space, and null space of the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 6 & 0 & 2 \\ 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 & 0 \end{bmatrix}$$

Problem 4

Compute the determinant of the following matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 1 & 3 & 2 \end{bmatrix}$$

a.

By co factor expansion along the bottom row.

b.

By co factor expansion along the first column.

Problem 5

Consider the following linear transformations $T : \mathbb{R}^4 \to \mathbb{R}^3$. For each, determine the matrix A such that

$$T(x) = Ax.$$

$$T \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3x + 2y + z + w \\ x + 2y + 6z - 2w \\ 3y + w \end{bmatrix}$$

Problem 6

a.

Find the matrix for the linear transformation that rotates every vector in $\mathbb{R}^2$ by an angle of $\pi/3$.

b.

Find the matrix for the linear transformation that rotates every vector in $\mathbb{R}^2$ by an angle of $2\pi/3$ and then reflects about the x-axis.

c.

Find the matrix of the linear transformation that rotates every vector in $\mathbb{R}^3$ counterclockwise about the z-axis, when viewed from the positive z-axis, by an angle of $30°$, and then reflects about the xy-plane.

Problem 7

Find the characteristic polynomial, all eigenvalues, and a basis for each corresponding eigen space of the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 1 \\ 2 & -2 & 0 \end{bmatrix}$$

Problem 8

Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 3 & -1 & 0 \\ 1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}.$$

One eigenvalue is 3. Diagonalize the matrix if possible.

Problem 9

Compute A^{10} for

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -1 & 5 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

Problem 10

Consider the sequence of numbers defined by the recurrence

$$b_0 = 1, \quad b_1 = 2, \quad b_2 = 3, \quad b_{n+3} = 2b_{n+2} + b_{n+1} - 2b_n, \quad \text{for all } n \geq 0.$$

1. Find the first 5 members of this sequence.
2. Solve the recurrence and find b_{20}.