Solutions Guide

1. a)

Linear Algebra $\longrightarrow$ 12, 9, 14, 5, 1, 18, 0, 1, 12, 7, 5, 2, 18, 1, 0

$$\begin{bmatrix}12\\9\\14\end{bmatrix} \begin{bmatrix}5\\1\\18\end{bmatrix} \begin{bmatrix}0\\1\\12\end{bmatrix} \begin{bmatrix}7\\5\\2\end{bmatrix} \cancel{\begin{bmatrix}2\\18\\1\end{bmatrix}} \begin{bmatrix}18\\1\\0\end{bmatrix}$$

multiply each by A and mod by 29

$$\begin{bmatrix}2&4&1\\3&1&5\\1&3&2\end{bmatrix}\begin{bmatrix}12\\9\\14\end{bmatrix} = \begin{bmatrix}16\\28\\9\end{bmatrix}$$

$$\begin{bmatrix}16\\28\\9\end{bmatrix}, \begin{bmatrix}3\\19\\15\end{bmatrix}, \begin{bmatrix}16\\3\\27\end{bmatrix}, \begin{bmatrix}7\\7\\26\end{bmatrix}, \begin{bmatrix}11\\26\\21\end{bmatrix}$$

P,JCTPPD.GGZLZV

~~b)~~

$$\cancel{A^{-1} \bmod 29 = \begin{bmatrix}23&20&11\\4&17&28\\26&8&11\end{bmatrix}}$$

~~WQFMHUXPXH~~ $\cancel{\longrightarrow 23, 17, 6, 13, 8, 21, 24, 16, 24, 8}$

$$\cancel{\begin{bmatrix}23\\17\\6\end{bmatrix}, \begin{bmatrix}13\\8\\21\end{bmatrix}, \begin{bmatrix}24\\16\\24\end{bmatrix}, \begin{bmatrix}8\\0\\0\end{bmatrix}}$$

2.

a) $U+V=0 \Rightarrow U=-V \Rightarrow V=-U$

$U-2W=0 \Rightarrow W=U/2$

$$W=\left\{\begin{bmatrix} u \\ -u \\ u/2 \end{bmatrix} : u \in \mathbb{R}\right\}$$

$$= u\begin{bmatrix} 1 \\ -1 \\ 1/2 \end{bmatrix} \Rightarrow \left\{\begin{bmatrix} 1 \\ -1 \\ 1/2 \end{bmatrix}\right\}$$ basis of W

dimension 1

b)

$$\begin{bmatrix} 2u+6v+7w \\ -3u-9v-12w \\ 2u+6v+6w \\ u+3v+3w \end{bmatrix} = u\begin{bmatrix} 2 \\ -3 \\ 2 \\ 1 \end{bmatrix} + v\begin{bmatrix} 6 \\ -9 \\ 6 \\ 3 \end{bmatrix} + w\begin{bmatrix} 7 \\ -12 \\ 6 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 7 \\ -3 & -9 & -12 \\ 2 & 6 & 6 \\ 1 & 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\left\{\begin{bmatrix} 2 \\ -3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ -12 \\ 6 \\ 3 \end{bmatrix}\right\}$$ form a basis of S

$\dim(S) = 2$

3.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 6 & 0 & 2 \\ 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 13/2 \\ 0 & 1 & 0 & 2 & -5/2 \\ 0 & 0 & 1 & -1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccccc} \uparrow & \uparrow & \uparrow & & \end{array}$$

Column space:

basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \\ 1 \\ 2 \end{bmatrix} \right\}$ rank 3

Row space:

$$\left\{ [1\ 0\ 0\ 0\ 13/2], [0\ 1\ 0\ 2\ -5/2], [0\ 0\ 1\ -1\ 1/2] \right\}$$

Null space:

$x = -13/2\, s$
$y = -2t + 5/2\, s$
$z = t - 1/2\, s$
$w = t$
$l = s$

$$\begin{bmatrix} x \\ y \\ z \\ w \\ l \end{bmatrix} = \begin{bmatrix} -13/2\, s \\ -2t + 5/2\, s \\ t - 1/2\, s \\ t \\ s \end{bmatrix} = s \begin{bmatrix} -13/2 \\ 5/2 \\ -1/2 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

basis $\left\{ \begin{bmatrix} -13/2 \\ 5/2 \\ -1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

4. a)

$$\begin{vmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 1 & 3 & 2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 3 & 2 \\ 9 & 6 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 \\ 3 & 9 \end{vmatrix}$$

$$= 1 \cdot (18-18) - 3(6-6) + 2(9-9) = 0$$

b)

$$\begin{vmatrix} 1 & 3 & 2 \\ 3 & 9 & 6 \\ 1 & 3 & 2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 9 & 6 \\ 3 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 2 \\ 9 & 6 \end{vmatrix}$$

$$= 1 \cdot (18-18) - 3(6-6) + 1(18-18) = 0$$

5.

$$T\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 2 \cdot 0 + 0 + 0 \\ 1 + 2 \cdot 0 + 6 \cdot 0 - 2 \cdot 0 \\ 3 \cdot 0 + 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

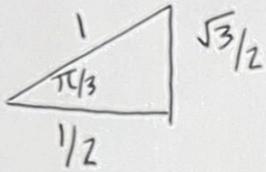
$$T\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

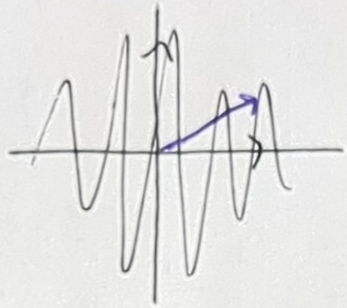
$$T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix} \qquad A = \begin{bmatrix} T\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} & T\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} & T\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & T\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$$

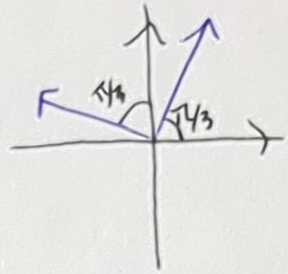
$$T\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 2 & 1 & 1 \\ 1 & 2 & 6 & -2 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

6.

a)





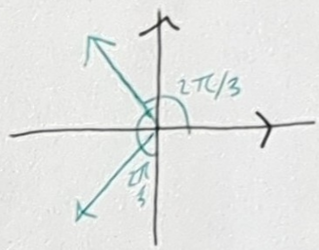


$$\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}1/2\\ \sqrt{3}/2\end{bmatrix}, \quad \begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}-\sqrt{3}/2\\ 1/2\end{bmatrix}$$

$$A = \begin{bmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{bmatrix}$$

b)

rotate by $2\pi/3$



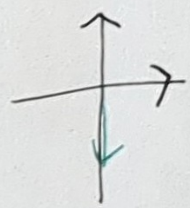
$$\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}-1/2\\ \sqrt{3}/2\end{bmatrix}$$

$$\begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}-\sqrt{3}/2\\ -1/2\end{bmatrix}$$

$$\begin{bmatrix}-1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & -1/2\end{bmatrix}$$

reflects about x axis

$$\begin{bmatrix}1\\0\end{bmatrix} \to \begin{bmatrix}1\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\end{bmatrix} \to \begin{bmatrix}0\\-1\end{bmatrix}$$

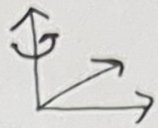


$$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

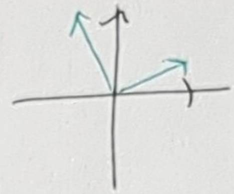
$$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}-1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & -1/2\end{bmatrix} = \underline{\begin{bmatrix}-1/2 & -\sqrt{3}/2\\ -\sqrt{3}/2 & 1/2\end{bmatrix}}$$

c) rotates about z axis $30° = \pi/6$

- same as rotating xy plane about origin by $\pi/6$

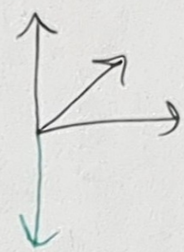


$$\begin{bmatrix}1\\0\\0\end{bmatrix} \to \begin{bmatrix}\sqrt{3}/2\\1/2\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}-1/2\\\sqrt{3}/2\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \to \begin{bmatrix}0\\0\\1\end{bmatrix}$$



$$\cancel{\begin{bmatrix}\sqrt{3}/2\\1/2\\0\end{bmatrix}} \quad \begin{bmatrix}\sqrt{3}/2 & -1/2 & 0\\1/2 & \sqrt{3}/2 & 0\\0 & 0 & 1\end{bmatrix}$$

reflect about xy plane



$$\begin{bmatrix}1\\0\\0\end{bmatrix} \to \begin{bmatrix}1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\end{bmatrix} \to \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\end{bmatrix} \to \begin{bmatrix}0\\0\\-1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}\sqrt{3}/2 & -1/2 & 0\\1/2 & \sqrt{3}/2 & 0\\0 & 0 & 1\end{bmatrix} = \underline{\begin{bmatrix}\sqrt{3}/2 & -1/2 & 0\\1/2 & \sqrt{3}/2 & 0\\0 & 0 & -1\end{bmatrix}}$$

7.

$$\det\left(\begin{bmatrix} 2-\lambda & 0 & 0 \\ -1 & 3-\lambda & 1 \\ 2 & -2 & -\lambda \end{bmatrix}\right) = (2-\lambda)(3-\lambda)(-\lambda) - (-2)(2-\lambda)$$

$$= (6-5\lambda+\lambda^2)(-\lambda) + 4 - 2\lambda$$

$$= -6\lambda + 5\lambda^2 - \lambda^3 + 4 - 2\lambda$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda + 4$$

Characteristic polynomial

$$-\lambda^3 + 5\lambda^2 - 8\lambda + 4$$

$$-\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0$$

$$\lambda^3 - \lambda^2 - 4\lambda^2 + 4\lambda + 4\lambda - 4 = 0$$

$$\lambda^2(\lambda-1) - 4\lambda(\lambda-1) + 4(\lambda-1) = 0$$

$$(\lambda-1)(\lambda^2 - 4\lambda + 4)$$

$$(\lambda-1)(\lambda-2)^2$$

Eigen values $\boxed{\lambda = 1, 2}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & -2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1/2 & 0 \end{array}\right]$$

$x = 0$
$y = -1/2\,t$
$z = t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ -1/2 \\ 1 \end{bmatrix}$$

Basis of $\lambda = 1$ eigen space

$$\left\{ \begin{bmatrix} 0 \\ 1/2 \\ 1 \end{bmatrix} \right.$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 2 & -2 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 2 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = t+s \\ y = t \\ z = s \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

basis of $\lambda = 2$ eigen space

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

8.

$$\det\left(\begin{bmatrix} 3-\lambda & -1 & 0 \\ 1 & 4-\lambda & -1 \\ -1 & -1 & 4-\lambda \end{bmatrix} \right) = (3-\lambda)(4-\lambda)^2 - 1 - (3-\lambda) + (4-\lambda)$$

$$= (3-\lambda)(4-\lambda)^2 - 1 - 3 + \lambda + 4 - \lambda$$

$$= (3-\lambda)(4-\lambda)^2$$

Eigen values: $\lambda = 3, 4$

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = t \\ y = 0 \\ z = t \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \leftarrow \text{eigen vectors of eigen value } \lambda = 3$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x = -t \\ y = 0 \\ z = t \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \leftarrow \text{eigen vectors of } \lambda = 4$$

not diagonalizable because there is only 2 linearly independent eigen vectors.

9.

$$A - \lambda Id = \begin{bmatrix} 1-\lambda & 3 & 3 \\ -1 & 5-\lambda & 3 \\ 1 & -1 & 1-\lambda \end{bmatrix}$$

characteristic polynomial

$$(1-\lambda)(5-\lambda)(1-\lambda) + 9 + 3 - 3(5-\lambda) + 3(1-\lambda) + 3(1-\lambda)$$

$$(1-\lambda)(5-\lambda)(1-\lambda) + 12 - 15 + 3\lambda + 3 - 3\lambda + 3 - 3\lambda$$

$$(1-\lambda)(5-\lambda)(1-\lambda) + 3 - 3\lambda$$

$$(1-\lambda)((5-\lambda)(1-\lambda) + 3)$$

$$(1-\lambda)(\lambda^2 - 6\lambda + 5 + 3)$$

$$(1-\lambda)(\lambda^2 - 6\lambda + 8)$$

$$(1-\lambda)(\lambda - 2)(\lambda - 4)$$

eigen values

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 4$$

$$A - \lambda_1 Id = \left[\begin{array}{ccc|c} 0 & 3 & 3 & 0 \\ -1 & 4 & 3 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 3 & 3 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} x_1 &= -t \\ x_2 &= -t \\ x_3 &= t \end{aligned} \qquad t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

eigen vector for λ_1 $\quad v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

other eigen vectors

$$v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad P = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^{10} = PD^{10}P^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{10} & 0 \\ 0 & 0 & 4^{10} \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 \\ 2^{10} & -2^{10} & 0 \\ 0 & 4^{10} & 4^{10} \end{bmatrix} = \begin{bmatrix} 1 & -1+4^{10} & -1+4^{10} \\ 1+2^{10} & -1+2^{10}+4^{10} & -1+4^{10} \\ -1+2^{10} & 1-2^{10} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1048575 & 1048575 \\ -1023 & 1049599 & 1048575 \\ 1023 & -1023 & 1 \end{bmatrix}$$

10.

1)

$b_0 = 1 \quad b_1 = 2 \quad b_2 = 3 \quad b_3 = 2b_2 + b_1 - 2b_0 = 6$

$b_4 = 2b_3 + b_2 - 2b_1 = 11$

$$\boxed{b_0 = 1,\ b_1 = 2,\ b_2 = 3,\ b_3 = 6,\ b_4 = 11}$$

2)

$$X_n = \begin{bmatrix} b_{n+2} \\ b_{n+1} \\ b_n \end{bmatrix} \qquad X_{n+1} = \begin{bmatrix} b_{n+3} \\ b_{n+2} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 2b_{n+2} + b_{n+1} - 2b_n \\ b_{n+2} \\ b_{n+1} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X_n \qquad A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$X_n = A^n X_0 \qquad X_0 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

eigenvalues of A are

$$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = -1$$

eigen vectors are

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad P^{-1} = \begin{bmatrix} -1/2 & 1/2 & 1 \\ 1/3 & 0 & -1/3 \\ 1/6 & -1/2 & 1/3 \end{bmatrix}$$

$$X_n = M^n X_0 = PD^nP^{-1}X_0 =$$

$$= \begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & (-1)^n \end{bmatrix} \begin{bmatrix} -1/2 & 1/2 & 1 \\ 1/3 & 0 & -1/3 \\ 1/6 & -1/2 & 1/3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4\cdot 2^n & (-1)^n \\ 1 & 2\cdot 2^n & (-1)(-1)^n \\ 1 & 2^n & (-1)^n \end{bmatrix} \begin{bmatrix} -3/2 + 1 + 1 \\ 1 - 1/3 \\ 1/2 - 1 + 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2^{n+2} & (-1)^n \\ 1 & 2^{n+1} & (-1)^{n+1} \\ 1 & 2^n & (-1)^n \end{bmatrix} \begin{bmatrix} 1/2 \\ 2/3 \\ -1/6 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 + \frac{2}{3}(2)^{n+2} - 1/6\,(-1)^n \\ 1/2 + \frac{2}{3}(2)^{n+1} - 1/6\,(-1)^{n+1} \\ 1/2 + \frac{2}{3}(2)^n - 1/6\,(-1)^n \end{bmatrix}$$

$$\boxed{b_n = 1/2 + \frac{2}{3}(2)^n - \frac{1}{6}(-1)^n}$$

$$\boxed{b_{20} = \frac{1}{2} + \frac{2}{3}(2)^{20} - \frac{1}{6}(-1)^{20} = 699051}$$