A LEVEL H2 PHYSICS (9749)

MUST KNOW DERIVATIONS AND PROOFS

Chapter 2: Kinematics

Equations of Motion

1. Derive v = u + at

Derivation

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at$$

2. Derive s = ½ (u + v) t

Method 1:

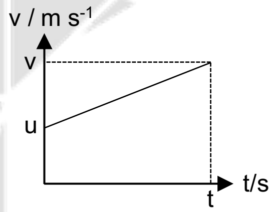
s = average velocity × time

= ½ (u + v) × t

Assumption: object is travelling with **uniform acceleration** in a **straight line**.

OR

Method 2:



s = area under v-t graph

= area of trapezium

= ½ (u + v) t

3. Derive s = ut + ½ at²

Substitute equation 1 into equation 2:

$$s = \tfrac{1}{2}(u + v) \times t$$

$$= \tfrac{1}{2}(u + u + at) \times t$$

$$= \tfrac{1}{2}(2ut + at^2)$$

$$= ut + \tfrac{1}{2}at^2$$

4. Derive v² = u² + 2as

From equation 1, substitute $t = \frac{v - u}{a}$ into equation 2:

$$s = \tfrac{1}{2}(u + v) \times t$$

$$s = \tfrac{1}{2}(u + v) \times \frac{v - u}{a}$$

$$s = \tfrac{1}{2}\frac{(v + u)(v - u)}{a}$$

$$2as = v^2 - u^2$$

$$\therefore v^2 = u^2 + 2as$$

Condition: The equations of motion only apply to objects travelling with **uniform acceleration** in **rectilinear motion**. "Rectilinear" means motion along a straight line.

Chapter 3: Dynamics

Newton's 2nd Law

5. Prove that Newton's 2nd Law ($F_R = \frac{dp}{dt}$) simplifies to F_R = ma when mass is constant.

$$F_R = \frac{dp}{dt} \qquad \text{(N2L)}$$

$$F_R = \frac{d(mv)}{dt} \qquad \text{apply chain rule}$$

$$F_R = m\frac{dv}{dt} + \cancel{v\frac{dm}{dt}}^{\,0} \qquad \because \text{m constant in time}$$

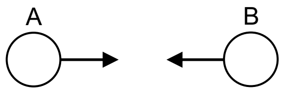
$$F_R = ma$$

Condition: F_R = ma is only true when m is constant.

Collisions

6. Use Newton's Law to prove the principle of conservation of momentum

When 2 objects collide, they exert an equal and opposite force on each other.



$F_{AB} = -F_{BA}$ (N3L)

$m_B \frac{\Delta v_B}{\Delta t} = -m_A \frac{\Delta v_A}{\Delta t}$ ($\because$ mass of A & B constant)

$m_B \Delta v_B = -m_A \Delta v_A$ ($\because$ by N3L, duration that action-reaction pair acts for is the same)

$m_B (v_B - u_B) = -m_A (v_A - u_A)$

$m_B v_B - m_B u_B = -m_A v_A + m_A u_A$

$m_A u_A + m_B u_B = m_A v_A + m_B v_B$

Let A be object 1, B be object 2,

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

7. Prove that impulse is equivalent to change in momentum, Δ_P

Impulse

$= \int F_R \, . \, dt$ } by definition

$=$ net area under F_R - time graph.

$=$ average resultant force $\langle F_R \rangle \times$ duration Δt

$=$ change in momentum, Δp by N2L $\langle F_R \rangle = \frac{\Delta p}{\Delta t}$

8. Derive, for elastic collisions

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \quad \text{and} \quad v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1$$

(1) $u_1 - u_2 = v_2 - v_1$ (relative velocity of approach $= -$ relative velocity of separation)

(2) $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$ (POCOLM)

From (1), Sub $v_2 = u_1 - u_2 + v_1$ into (2)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 - u_2 + v_1)$

$(m_1 - m_2) u_1 + 2m_2 u_2 = (m_1 + m_2) v_1$

$\therefore v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2$

From (1), Sub $v_1 = v_2 + u_2 - u_1$ into (2)

$m_1 u_1 + m_2 u_2 = m_1 (v_2 + u_2 - u_1) + m_2 v_2$

$2m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) v_2$

$v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1$

9. Analysis of v_1 and v_2

Case 1:

If both bodies have the same mass (ie. $m_1 = m_2$)

$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 = 0 + \frac{2m_2}{2m_2} u_2 = u_2$

$v_2 = \frac{m_2 - m_1}{m_1 + m_2} u_2 + \frac{2m_1}{m_1 + m_2} u_1 = 0 + \frac{2m_1}{2m_1} u_1 = u_1$

$\therefore v_1 = u_2$ & $v_2 = u_1$ The objects swap velocities after collisions

Example

- cue ball hitting target ball in billiard.

Case 2:

If body 2 was initially at rest. (ie. $u_2 = 0$)

And $m_1 \ll m_2$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}\cancelto{0}{u_2}$$

$$= -\frac{m_2}{m_2}u_1$$

$$= -u_1$$

$$v_2 = \frac{m_2 - m_1}{m_1 + m_2}\cancelto{0}{u_2} + \frac{2m_1}{m_1 + m_2}u_1$$

$$= \frac{2m_1}{m_2}u_1$$

$$\approx 0$$

Lighter object rebounds with same speed in opposite direction. Heavy object remains stationary.

Example

- gas molecule hitting wall of container
- tennis ball bouncing off a wall

And $m_1 \gg m_2$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}\cancelto{0}{u_2}$$

$$= \frac{m_1}{m_1}u_1$$

$$= u_1$$

$$v_2 = \frac{m_2 - m_1}{m_1 + m_2}\cancelto{0}{u_2} + \frac{2m_1}{m_1 + m_2}u_1$$

$$= \frac{2m_1}{m_2}u_1$$

$$= 2u_1$$

Heavy object continues with same speed and direction but light object moves off with twice the speed of heavier object.

Example

- bowling ball hitting pins
- truck hitting a man

Chapter 4: Force

Hooke's Law

10. Prove that when a spring-mass system is at equilibrium => $mg = kx_o$, where x_o is the extension at equilibrium.

At equilibrium, $\Sigma F = 0$

Taking downwards as positive,

$$mg + (-kx_0) = 0$$

$$\therefore mg = kx_0$$

11. Prove that if spring-mass system is stretched FURTHER downwards by x_1 from equilibrium, then the resultant force upon release is given by $F_R = -kx_1$

Then the resultant resisting force is given by $F_R = -kx_1$

Taking downwards as positive,

$$F_R = mg + (-k)(x_0 + x)$$

$$= mg - kx_0 - kx$$

$$= 0 - kx$$

$$= -kx$$

Where the –ve sign represents an upward resultant restoring

12. Derive that EPE in Hooke's spring is given by $E_P = \frac{1}{2}kx^2$ and not $E_P = kx^2$

By Hooke's Law, $F = kx$, where k is a positive constant,

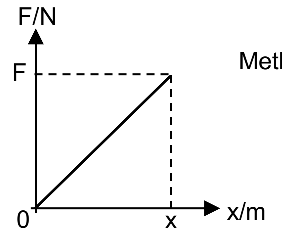
(negative sign omitted because it only represents direction).

Method 1: E_P = area under F – x graph

$$= \tfrac{1}{2} Fx$$

$$= \tfrac{1}{2}(kx)(x)$$

$$= \tfrac{1}{2}kx^2$$



Method 2: E_P = average force × displacement

$$= (\tfrac{1}{2}kx) \times x$$

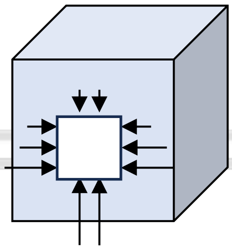
$$= \tfrac{1}{2}kx^2$$

13. Derivation that upthrust on an object partially/fully submerged in a fluid is given by $U_T = \rho Vg$
Where ρ = density of fluid displaced
V = volume of fluid displaced
g = gravitational field strength

Upthrust = $\Delta p \times A$

Where Δp = different in pressure at top and bottom

and A = cross-sectional area



Upthrust = $\Delta p \times A$ (recall: $F_R = \Delta p \times A$)

= $\rho g \underbrace{\Delta h \times A}$ (recall: $\Delta p = \rho g \Delta h$)

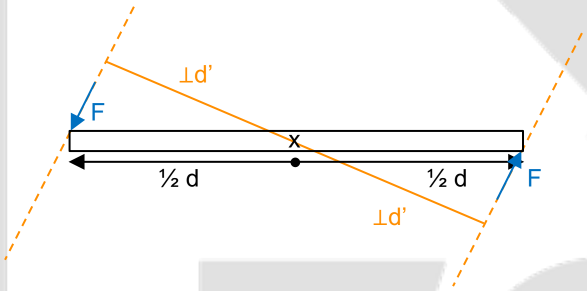
= ρVg

For floating objects

Upthrust = weight of object (∵ object is in equilibrium)

∴ $\rho_{Fluid} V_{Fluid} g = m_{object} g$

14. Prove that the torque of a couple, is the product of one force of the couple and the perpendicular between the couple. Ie. $\tau = F \times$ hd



Take CG as pivot

$\tau = F \times \perp d' + F \times \perp d'$

$= F \times 2\perp d'$

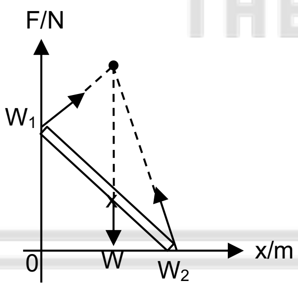
Where F = one force of the couple

$\perp d'$ = perpendicular distance from line of action of each force to the pivot

∴ $\tau = F \times \perp d$

Where $\perp d$ = perpendicular distance between line of action of forces in a couple

15. Prove that when 3 non-parallel force act on the some object that is in rotational equilibrium, then the line of action of the 3 forces must intersect at a single point



if the 3 non-parallel forces don't meet at a single point, then any 2 forces will meet at a common point since they are not parallel. Choose this common point as pivot, then the moments due to the 2 forces intersecting at this point is zero, ∵ their perpendicular distance are zero. However, the 3[rd] force would cause non-zero moments about the pivot, which suggest that the object is not in rotational equilibrium and this contradicts the premise ∴ all 3 forces need to intersect at a common point.

Chapter 5: Work, Energy and Power

16. Prove that $E_K = ½\,mv^2$

Consider an external force that is used to accelerate object horizontally along frictionless floor.

$$\Delta E_K = W = F \times s$$

$$= mas \qquad \text{(since F = ma for an accelerating mass)}$$

$$= m\left(\frac{v^2 - u^2}{2}\right) \qquad \left(\text{since } v^2 = u^2 + 2as \Rightarrow as = \frac{v^2 - u^2}{2}\right)$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Since the change in E_K is $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$, the generic form of $E_K = \frac{1}{2}mv^2$

17. Prove that $E_P = mg\Delta h$

Consider an external force that is used to lift an object vertically without any acceleration (ie. change in KE)

$$\Delta E_P = F \times s$$

$$= mg(\Delta h) \qquad (\text{no acceleration} \Rightarrow \sum F = 0 \therefore F = mg)$$

$$= mg\Delta h$$

18. Prove that $P = Fv$

$$\therefore P = \frac{W}{t} = \frac{F \times s}{t}$$

$$= Fv \qquad \text{(constant F \& v)}$$

19. Prove that efficiency = $\frac{\text{useful energy output}}{\text{total energy input}}$ is equivalent to $\frac{\text{useful power output}}{\text{total power input}}$

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

$$= \frac{\text{useful energy output / time}}{\text{total energy input / time}}$$

$$= \frac{\text{useful power output}}{\text{total power input}}$$

20. Prove that $v = r\omega$

$$v = \frac{ds}{dt} \qquad \text{(rate of change of displacement)}$$

$$= \frac{d\,r\theta}{dt}$$

$$= r\frac{d\theta}{dt} + \theta\cancelto{0}{\frac{dr}{dt}} \qquad (\because \text{ for circular motion of constant radius})$$

$$= r\omega \qquad \left(\because \omega = \frac{d\theta}{dt}\right)$$

Chapter 6,7 (hee hee): Circular Motion and G-field

21. Prove Kepler's Law that an object in orbit obeys $T^2 \propto r^3$

In space, the gravitational force provides the centripetal force for the orbiting object.

$$F_G = F_C$$

$$\frac{GMm}{r^2} = mr\omega^2$$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

Since $\frac{4\pi^2}{GM}$ is a constant, $T^2 \propto r^3$

22. Prove that total energy of an object in orbit is given by $E_k = \frac{GMm}{2r}$ and $E_{Tot} = -\frac{GMm}{2r}$

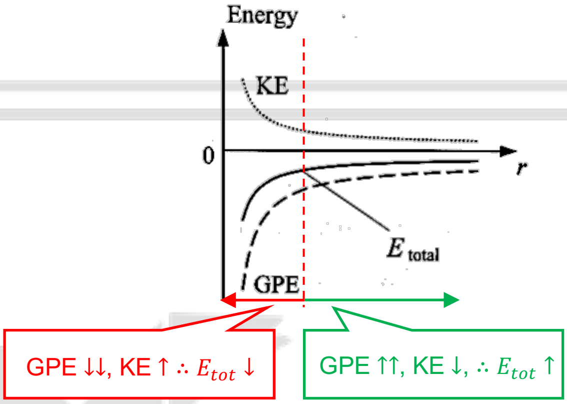
In space, the gravitational force provides the centripetal force for the orbiting object

$F_G = F_C$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{1}{2}\frac{GMm}{r} = \frac{1}{2}mv^2$$

$$\therefore E_k = \frac{GMm}{2r}$$



$E_{Tot} = E_p + E_k$

$$= -\frac{GMm}{r} + \frac{GMm}{2r}$$

$$= -\frac{GMm}{2r}$$

23. Prove that the minimum speed to escape Earth's from its surface is given by $v_{esc} = \sqrt{\frac{2GM}{r}}$

To escape Earth, the object must be able to reach infinity. And to escape with minimum speed means object must just come to rest at infinity. At infinity, E_K and $E_P = 0$ ie. total energy = 0.

By conservation of energy, initial total energy must be equal to final total energy: $E_{tot_i} = E_{tot_f}$

$KE_i + GPE_i = KE_f + GPE_f$

$$\frac{1}{2}mv_{esc}^2 + \left(-\frac{GMm}{r}\right) = 0 + 0$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

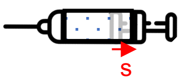
Chapter 8: Ideal Gas Law

24. Prove that work done **by** a gas, $W_{by} = p\Delta V$

$W_{by} = F \times \Delta s$

$= pA \times \Delta s$ (A = cross sectional area)

$= p\,\Delta V$ (isobaric cases)



Note: $W_{on} = -p\,\Delta V$

25. Given that pV = nRT and pV = NkT derive $p = \frac{1}{3}\rho V^2_{rms}$ and <KE> = $\frac{3}{2}$kT for monoatomic ideal gas.

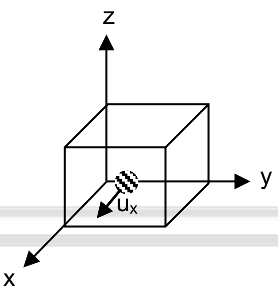
For 1 molecule moving in x direction,

$$F = \frac{\Delta p}{\Delta t}$$

$$= \frac{m(\Delta v)}{\Delta t} \quad (\because \text{mass of particle constant})$$

$$= \frac{m(u_x - (-u_x))}{2L/u_x}$$

$$= \frac{mu_x^2}{L}$$



For N molecules with varying speeds in x direction,

$$F = \frac{Nmu^2_{x\,rms}}{L}$$

For N molecules moving with varying speeds in all directions

$$F = \frac{Nm}{L}\left[\frac{1}{3}v^2_{rms}\right]$$

Since pressure of gas p is given by

$$p = \frac{F}{A} = \frac{Nm}{AL}\left[\frac{1}{3}v^2_{rms}\right]$$

$$p = \frac{Nm}{V}\left[\frac{1}{3}v^2_{rms}\right]$$

$$p = \frac{1}{3}\rho v^2_{rms}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v^2 = 3v_x^2$$

$$v_x^2 = \frac{1}{3}v^2$$

(ie. 1D $v^2 = \frac{1}{3}$ that of 3D v^2)

Similarly,

$$\langle v_x^2 \rangle = \frac{1}{3}\langle v^2 \rangle$$

$$v^2_{x\,rms} = \frac{1}{3}v^2_{rms}$$

$$\because pV = nRT \;\;\&\;\; pV = \frac{1}{3}\rho V v^2_{rms}$$

$$\frac{1}{3}\rho V v^2_{rms} = nRT$$

$$\frac{1}{3}\frac{Nm}{V}V v^2_{rms} = NkT$$

$$\frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$$

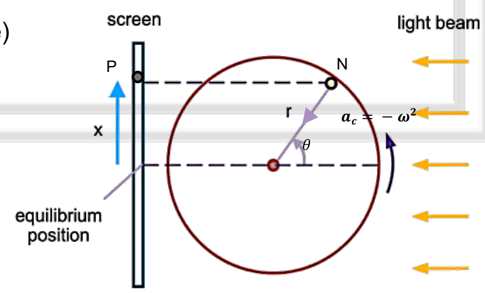
$$\therefore \langle KE \rangle = \frac{3}{2}kT \quad \text{(for monoatomic ideal gas)}$$

Chapter 10: Oscillations

26. Relationship between SHM and uniform circular motion

Consider a point, N moving in uniform circular motion, then its projection, P, along a vertical axis is in SHM.

- vertical displacement of P is given by $x = r\sin\theta$ (from centre of circle)
- centripetal acceleration of N $a_c = -\omega^2 r$ (towards the centre of the circle)



This means P will experience vertical component of a_c which is given by

$$a_{c_{vertical}} = a_c\sin\theta = -\omega^2 r\sin\theta = -\omega^2 x$$

Since $\omega = 2\pi f$ is a positive constant,

$$\therefore a_{c_{vertical}} \propto -x \;(SHM)$$

This means the vertical acceleration of the projection is directly proportional to its displacement x from the fixed equilibrium point, and is directed towards the equilibrium.

27. Prove that $x = \pm x_0 \sin(wt - \theta)$ and $x = \pm x_0 \cos(wt - \theta)$ are possible generic solutions to SHM

$x = \pm x_0 \sin(\omega t - \theta)$
$v = \pm \omega x_0 \cos(\omega t - \theta)$
$a = \mp \omega^2 x_0 \sin(\omega t - \theta)$
$\therefore a = -\omega^2 x$ (SHM)

$x = \pm x_0 \cos(\omega t - \theta)$
$v = \mp \omega x_0 \sin(\omega t - \theta)$
$a = \mp \omega^2 x_0 \cos(\omega t - \theta)$
$\therefore a = -\omega^2 x$ (SHM)

28. Derive $v = \pm \omega\sqrt{x_0^2 - x^2}$.

Without loss of generality, suppose $x = x_0 \sin wt$

$v = \omega x_0 \cos \omega t$

$v^2 = \omega^2 x_0^2 \cos^2 \omega t$
$= \omega^2 x_0^2 (1 - \sin^2 \omega t)$
$= \omega x_0^2 - w^2 x_0^2 \sin^2 \omega t$
$= \omega^2 x_0^2 - x^2$
$= \omega^2 (x_0^2 - x^2)$
$\therefore v = \pm\omega\sqrt{x_0^2 - x^2}$

29. Prove that $E_K = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$ & $E_P = \frac{1}{2} m\omega^2 x^2$

$\therefore v = \pm\omega\sqrt{x_0^2 - x^2}$

Proof:

$E_K = \frac{1}{2} mv^2$
$= \frac{1}{2} m\omega^2 (x_0^2 - x^2)$
$E_{Total} = E_{Kmax}$
$= \frac{1}{2} m\omega^2 x_0^2$
$E_P = E_{Total} - E_K$
$= \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 (x_0^2 - x^2)$
$= \frac{1}{2} m\omega^2 x^2$

30. Prove that spring mass system, $T = \frac{1}{2\pi}\sqrt{\frac{m}{k}}$, where k is the spring constant

First, prove that spring mass system is SHM.

At equilibrium, $mg + (-kx_0) = 0 \therefore mg = kx_0$

When displaced further by x,

$F_R = mg + (-k(x_0 + x))$
$= mg - kx_0 - kx \quad (\because mg = kx_0)$
$= -kx$

$a = \frac{F_R}{m}$
$= -\frac{k}{m} x$

$\because$ k, m are positive constant, $a \propto -x$. (SHM)

By comparing $a = -\frac{k}{m} x$ to $a = -\omega^2 x \Rightarrow \omega^2 = \frac{k}{m}$

$\therefore \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\therefore T = \frac{1}{2\pi}\sqrt{\frac{m}{k}}$

31. Prove that the motion of a simple pendulum can be approximated to simple harmonic motion, & that $T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$

For small angle θ

$\tan\theta = \sin\theta = \theta$

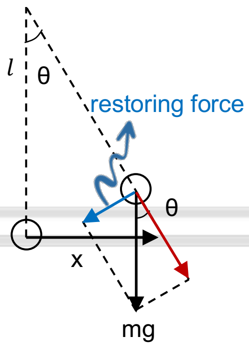
Restoring force $= -mg\sin\theta$

$\approx -mg\tan\theta$

$= -mg(\frac{x}{l})$

$\therefore a = \frac{F}{m}$

$= -\frac{g}{l}x$



$\therefore$ g, l are positive constants, $a \propto -x$ (SHM) where $\omega^2 = \sqrt{\frac{g}{l}}$

$\therefore \omega = \sqrt{\frac{g}{l}} \therefore \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$

$T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$

Chapter 11: Wave Motion

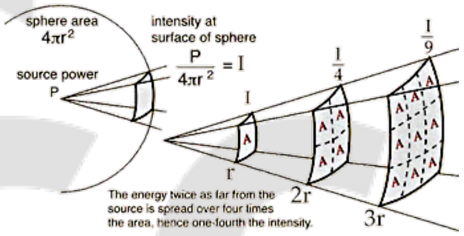
32. Prove that for 3-D Power Propagation Case, Intensity $\propto \frac{1}{r^2}$

Intensity $= \frac{\text{Power}}{\text{Area}}$

$= \frac{P}{4\pi r^2}$

$\because$ P, 4π are constant $\therefore$ Intensity $\propto \frac{1}{r^2}$

$\because$ Intensity $\propto$ Amplitude^2 $\therefore$ Amplitude $\propto \frac{1}{r}$



33. Prove that for 2-D Propagation Case, Intensity $\propto \frac{1}{r}$

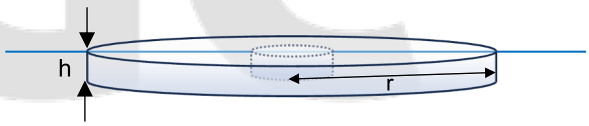
$I = \frac{\text{Power}}{\text{Area}}$

$= \frac{P}{4\pi rh}$

$\because$ P, 2π, h constant (and small)

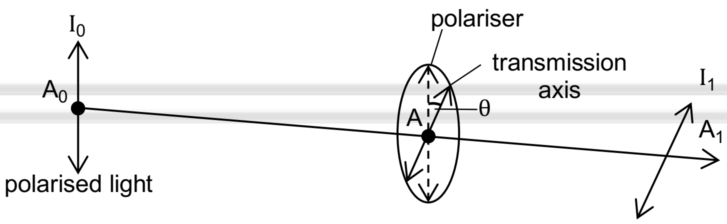
$\therefore$ Intensity $\propto \frac{1}{r}$

$\because$ Intensity $\propto$ Amplitude^2 $\therefore$ Amplitude $\propto \frac{1}{\sqrt{r}}$



34. Prove Malus' Law.

Note: Malus' Law only applies to **already polarised light**.



When polarised light passes through a polariser, the amplitude of the wave, $\mathbf{A_0}$ is reduced to the component parallel to the transmission axis, i.e.:

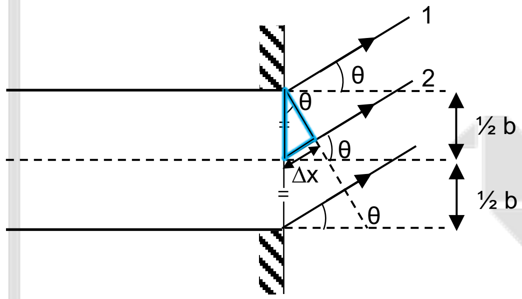
$A_1 = A_0\cos\theta$

$\because$ Intensity $\propto A^2 \Rightarrow \dfrac{\text{Intensity}_1}{\text{Intensity}_0} = \dfrac{A_1{}^2}{A_0{}^2}$

$\therefore$ Intensity$_1 = \dfrac{A_0{}^2\cos^2\theta}{A_0{}^2} \times I_0 = I_0\cos^2\theta$

Chapter 12: Superposition

35. Prove the Single Slit Experiment Formula ($b\sin\theta = n\lambda$), b = slit width, θ = angle to nth order minima, n = order of minima,



$\sin\theta = \dfrac{\Delta x}{\frac{1}{2}b}$

At n^{th} minima, destructive interference occurs. If we treat the single slit as 2 equal halves, and consider the top rays of each half (ray 1 and 2) then ray 1 and 2 must surely interfere destructively. Consequently, every ray below ray 1 will also have a corresponding ray below ray 2 that destructively interferes with it, so the overall effect is indeed a minima.

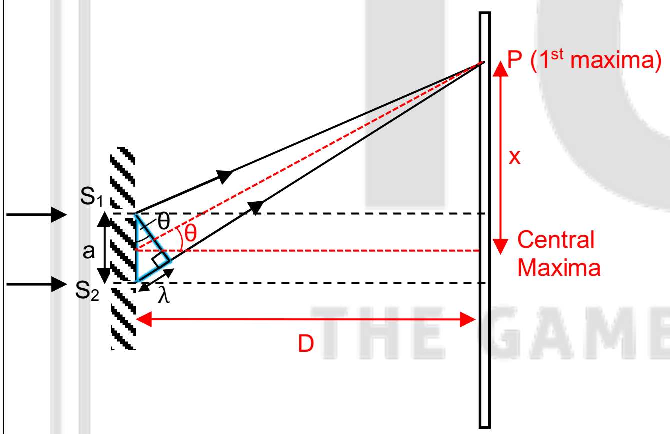
Hence, for ray 1 and 2, their path difference, Δx is some $n\left(\frac{1}{2}\lambda\right)$

$\Delta x = n\left(\frac{1}{2}\lambda\right)$ (see blue triangle)

$\therefore \frac{1}{2}b\sin\theta = n\left(\frac{1}{2}\lambda\right)$

$b\sin\theta = n\lambda$

36. Prove the Double Slit Experiment Formula ($x = \frac{D\lambda}{a}$), x = fringe separation, D = screen to double slit distance, a = slit separation.



Using 1st maxima,

$\tan\theta = \dfrac{x}{D}$ (see red lines)

For a << D,

- S_1P and S_2P are approximately parallel, and

$\sin\theta = \dfrac{\lambda}{a}$

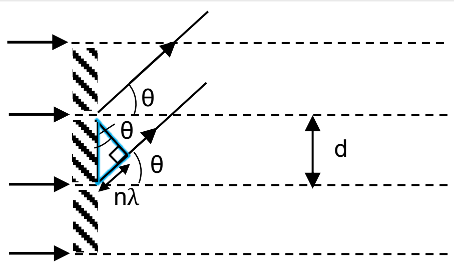
For small θ,

$\sin\theta \approx \tan\theta \approx \theta$

$\therefore \dfrac{\lambda}{a} = \dfrac{x}{D}$

$x = \dfrac{D\lambda}{a}$

37. Prove the Diffraction Grating Formula ($d\sin\theta = n\lambda$), d = slit separation, θ = angle to nth order maxima, n = order of maxima



At n^{th} maxima, constructive interference occurs. This must mean path difference between neighbouring slits, Δx is some $n\lambda$

$\sin\theta = \dfrac{n\lambda}{d}$ (see blue triangle)

$\therefore d\sin\theta = n\lambda$

Chapter 13: Electric Field

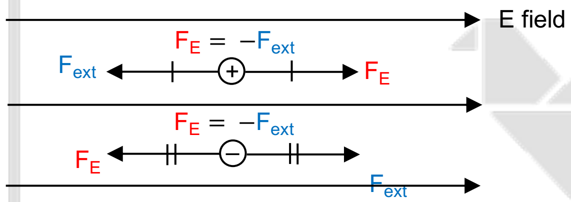
38. Prove that $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

By definition, electric field strength E is the electric force per unit positive charge placed at that point.

$$E = \frac{F_E}{q} = \frac{Qq}{4\pi\varepsilon_0 r^2} \times \frac{1}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

39. Prove that $U = \dfrac{Qq}{4\pi\varepsilon_0 r}$

Electric potential energy, U, is defined as the <u>work done by an external force</u> to bring a small positive test charge from infinity to that point without a change in kinetic energy / without acceleration.



$$U = \int_\infty^r F_{ext} \cdot ds$$

$$= \int_\infty^r -F_E \cdot dr$$

$$= -\int_\infty^r \frac{Qq}{4\pi\varepsilon_0 r^2} dr$$

$$= \frac{Qq}{4\pi\varepsilon_0 r}\Bigg|_\infty^r$$

$$= \frac{Qq}{4\pi\varepsilon_0 r}$$

40. Prove that $V = \dfrac{Q}{4\pi\varepsilon_0 r}$

By definition, Electric Potential, V, at a point is defined as the work done per unit positive charge by an external force to bring a small positive test charge from infinity to that point without change in kinetic energy / acceleration.

$$\therefore\ V = \frac{U}{q} = \frac{Qq}{4\pi\varepsilon_0 r} \times \frac{1}{q} = \frac{Q}{4\pi\varepsilon_0 r}$$

41. Prove that Electric Field strength, E within an electrical conductor is always 0, at static equilibrium

If the electric field within the electrical conductor is non-zero, then electric charges within the conductor will experience a resultant electric force and redistribute themselves. This happens until the electric charges stop moving. That implies that there is no Electric field & static equilibrium is achieved.

Chapter 14: Current of Electricity

42. Proof that I = nAVdq

$I = \dfrac{Q}{t}$ — Q → total charge

total number of charge carriers

$= \dfrac{N \times q}{t}$ — q → charge of each carrier

number density

$= \dfrac{nVq}{t}$ — V → volume

$= \dfrac{n(A \times l)\,q}{t}$

$= nAv_dq$

$\dfrac{l}{t}$ = average velocity of charged positive

= drift velocity

43. Prove that $P = VI$, $P = I^2R$ and $P = \dfrac{V^2}{R}$ are all equivalent formula

From P = VI, sub V = RI

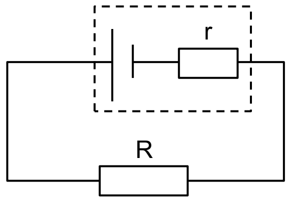
$\Rightarrow P = (RI)(I) = I^2R$

From P = VI, sub $I = \dfrac{V}{R}$

$\Rightarrow P = V\left(\dfrac{V}{R}\right) = \dfrac{V^2}{R}$

44. Prove that $V_T = \varepsilon - Ir$ using energy / power consideration

Power provided by battery = power dissipated across external circuit, R + power dissipated across internal resistance, r



$\varepsilon I = I^2R + I^2r$

$\because$ I through series circuit is the same.

$\varepsilon = IR + Ir$

$\varepsilon = V_T + Ir$

Where terminal p.d., V_T refers to the p.d. across the external circuit or the "p.d. provided by the cell to the rest of the circuit after accounting for the p.d. spent on the internal resistance, r."

Chapter 15: D.C. Circuit

45. Prove that R_{tot} of n resistors in series arrangement is given by $R_{tot} = R_1 + R_2 + ... + R_n$

In series circuit,

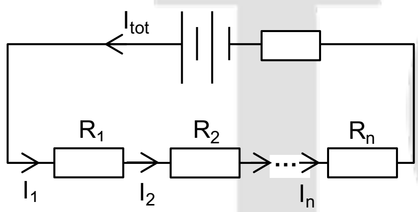
$V_T = V_1 + V_2 + ... + V_n$ (always start with what adds up)

$I_{tot}(R_{tot}) = I_1R_1 + I_2R_2 + ... + I_nR_n$

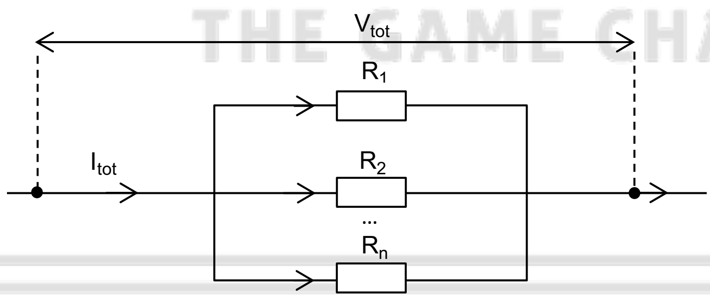
And current in a series circuit is the same throughout,

$\therefore I_{tot} = I_1 = I_2 = ... = I_n$

$\therefore R_{tot} = R_1 = R_2 = ... = R_n$



46. Prove that R_{tot} of n resistors in parallel arrangement is given by $R_{tot} = \left(\frac{1}{R_1} + \frac{1}{R_1} + ... + \frac{1}{R_1}\right)^{-1}$



In parallel circuit,

$I_{tot} = I_1 + I_2 + I_3$ (always start with what adds up)

$$\frac{V_{tot}}{R_{tot}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + ... + \frac{V_n}{R_n}$$

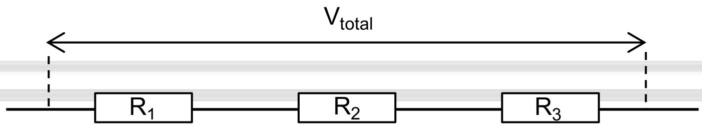
In parallel circuit,

$V_{tot} = V_1 = V_2 = V_n$

$$\therefore \frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$R_{tot} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}\right)^{-1}$$

47. Derive the Potential Divider Formula (only applicable to series circuits)



$$V_1 = R_1 I_1$$

$$= R_1 (I_{total}) \qquad \because \text{ I in a series circuit is the same throughout}$$

$$= R_1 \frac{V_{Total}}{R_{Total}}$$

$$= \frac{R_1}{R_{Total}} \times V_{Total}$$

Chapter 16: Electromagnetism

48. Prove that the force on a charged particle moving at a non-zero angle in a B-field is given by F = Bqv sin θ, where θ is the angle between B and v.

The magnetic force on a straight current-carrying conductor is given by F = BIL sin θ, where θ is the angle between the B and I.

$$F = BIL \sin\theta$$

$$= B\frac{q}{t}L \sin\theta \qquad \because \frac{L}{t} = v$$

$$= Bqv \sin\theta$$

49. Prove that in a velocity selector, the undeflected particles have $v = \frac{E}{B}$

$$F_B = F_E$$

$$qvB \sin\theta = qE$$

Since θ = angle between B and v = 90° (velocity selector uses B and E that are ⊥ to each other)

$$vB = E$$

$$v = \frac{E}{B}$$

Chapter 17: Electromagnetic Induction

50. Prove that the e.m.f. induced for a straight wire of length L moving perpendicular with velocity v to a uniform B-field of magnetic flux density is given by ε = – BLv

$$\varepsilon = -\frac{d\phi}{dt}$$

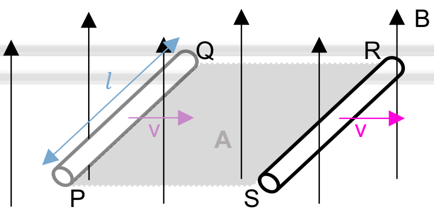
$$= -\frac{d}{dt}\left(\frac{NBA\cos\theta}{dt}\right) \qquad \because N = I \text{ and } \theta = 0^\circ$$

$$= -NB\frac{dA\cos\theta}{dt}$$

$$= -(1)B\frac{dA}{dt}$$

$$= -BLv$$

where L and v are both perpendicular to B.



Chapter 19: Quantum Physics

51. Prove that the max KE of photoelectron is given by hf – hf$_0$ where f$_0$ is the threshold frequency of the radiation.

$E_{k_{max}} = hf - \emptyset$

Where, E_K = KE of photoelectron

hf = energy of EM radiation

$\emptyset$ = work function

∵ Work function is the minimum energy required by a photon to release an electron from a metal surface, where the corresponding minimum energy possessed by the photon is given by hf_0

$E_{k_{max}} = hf - hf_0$

Chapter 20: Nuclear Physics

52. Deduce that atoms are made up of mostly empty space with a very dense positively charged nucleus

When α-particles are fired at a thin sheet of solid foil,

Observation	**Inference**
Most α-particles emerge with little or no deflection	Atoms are made up of mostly empty space
A few of the α-particles are deflected through a large angle	A large electrostatic force stems from a very small space, which suggests a dense +ve ly charged nucleus