Statistical Foundations of Data Science

Hypothesis testing

University of the Witwatersrand

2025

Review Question

- Three players roll dice. What's the probability that they all get different numbers?

Lesson Plan

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- Hypothesis Tests

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- If the data does fit the hypothesis we'll say we "fail to reject". Don't trust the random guy claiming to have a magical cure!

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- At the point where we have 9 out of 10 we do not.

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- Does random chance explain the potion curing 67 out of 100 patients when we usually see 58 recover?
- Compare this to the Bayesian approach where we'd have a distribution on the coins probability of coming up heads and adjust.

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- Compute Z-score and p-value

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- Yeah they lied!

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- $s^2 = \frac{(x_i-\overline{X})^2}{n-1}$

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- $\overline{X} = 170$. $s^2 = \frac{100+25+100}{3} = 75$
- $T = \frac{170-175}{\frac{\sqrt{75}}{\sqrt{3}}} = -1$
- p-value is around about 0.19 We don't have evidence to comclude our guys are too small

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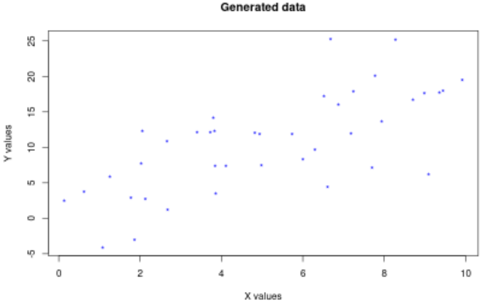

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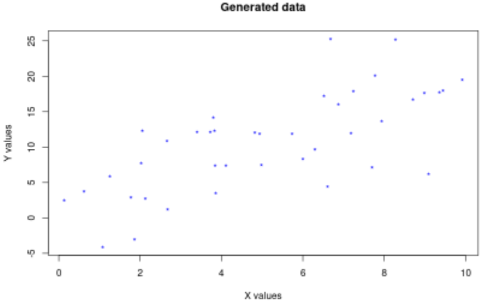

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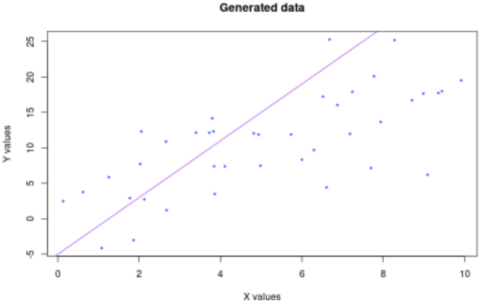

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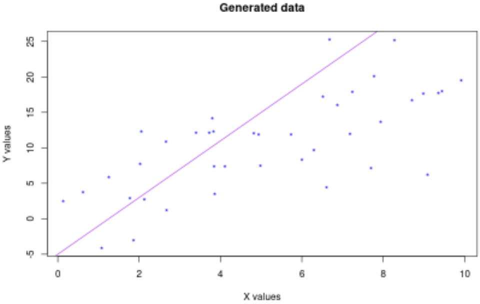

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- Idea is we'll predict Y from X. As no line is exact we'll really use $Y = \beta_0 + \beta_1 X + \epsilon$. Where the ϵ are distributed normal, independent with mean zero and common variance.

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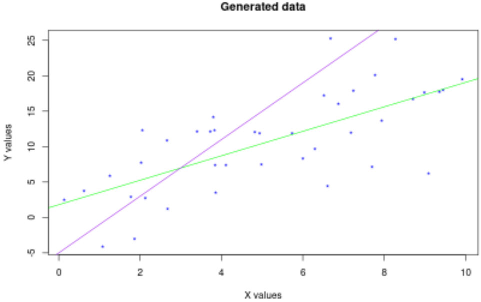

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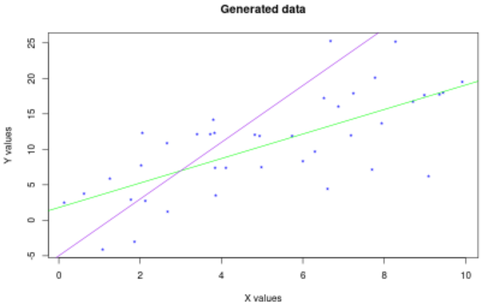

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- In the case of single variable linear regression we have mathematical tools to get exact formulas. Which provide interruptibility and intuition for what's going on.

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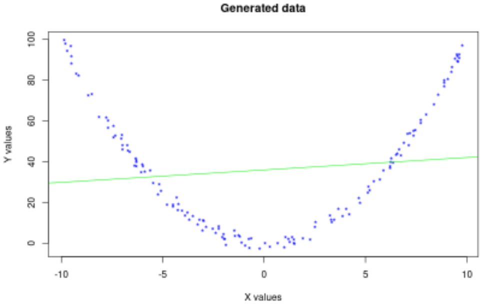

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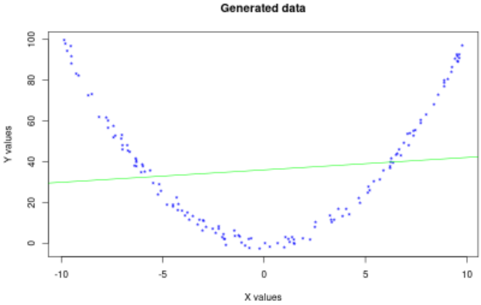

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- Quadratic signal. Slope insignificant! But you can still get a good estimate from x just not with a linear model.

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- Important bit here is that we care if a variable is doing significantly better than random.