Statistical Foundations of Data Science

How to lie with statistics

University of the Witwatersrand

2023

Quotes

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- "My mother is the greatest statistician in the world, she can reach a conclusion with only one data point" -Anon.

Review Question

- I flip a fair coin three times. Find the probability that I get exactly two heads.

Lesson Plan

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- Examples of places where statistical thinking helps

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- Solution 2: There are $\frac{1}{8}$ chances of getting 0 or 3 heads so a $\frac{3}{4}$ chance of getting one or two heads. These are symmetrical so equally likely. Hence the chance of getting exactly two heads is $\frac{3}{8}$.

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- The moral here is that averages can be misleading when we have outliers.

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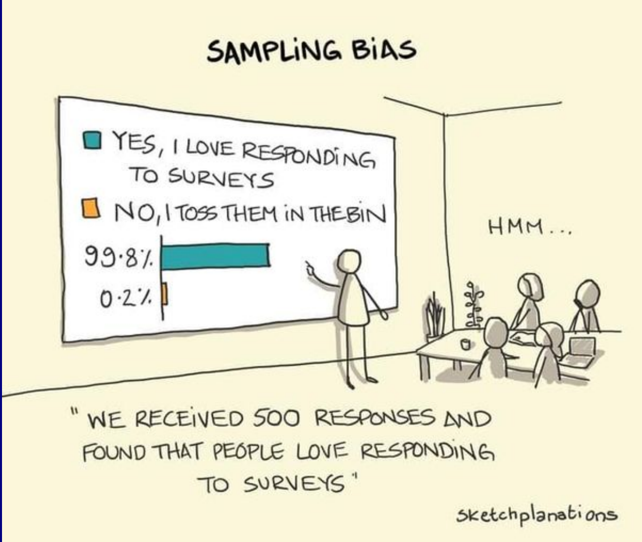

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- Well let's compute the probability that no two have the same birthday and let's start with fewer people
- One person: Probability 1 doesn't have the same birthday as himself.
- Two people: Probability $1(1-\frac{1}{365})$ don't share a birthday.

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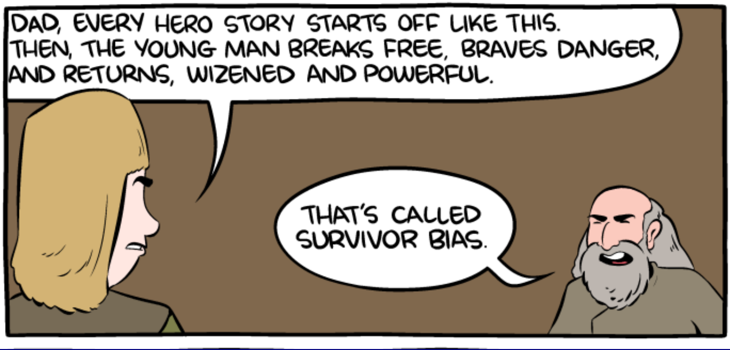

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- Twenty three people: Probability $1(1-\frac{1}{365})(1-\frac{2}{365})...(1-\frac{22}{365})$ don't share a birthday.

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- Twenty three people: Probability $1(1-\frac{1}{365})(1-\frac{2}{365})...(1-\frac{22}{365})$ don't share a birthday.
- Don't share a birthday with probabilty: $1-1(1-\frac{1}{365})(1-\frac{2}{365})...(1-\frac{22}{365}) = 1-\prod_{i=0}^{22}(1-\frac{i}{365}) = 0.507297$

How many people live in your house

- We want to know how many people live in the average house so we collect a sample of 100 people. 50 say they live alone. The other 50 say they live in a house of 5 people. We conclude that the average house has 3 people in it, and that the split is even between houses with 1 person and houses with 5 people.

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- That's wrong. **People are evenly split between living alone and five-person houses here. But it takes only a fifth as many five-person-houses to house the half of the population that lives together as it does one person houses that so that the mean house contains** $1 \times \frac{5}{6} + 5 \times \frac{1}{6} = 1.666$ **people.**

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- You probably go to the dentist at a more crowded than average time. Most people do. That's what makes those times crowded.

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- Then $p_k = \frac{k h_k}{\sum_{i=1}^{\infty} i h_i}$

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- Yes, given certain knowns (which we have). That Monty knows where the car, that he always offers the chance to switch and that when the player chooses the car he chooses the door with the goat uniformly.