Statistical Foundations of Data Science

Continuous Distributions

University of the Witwatersrand

2025

Review Question

- A five card poker hand is drawn. What's the probability that all five cards are the same suit.

Lesson Plan

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- Continuous distributions

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- Exponential distribution

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- Preview of the normal and t-distributions

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- $\frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48} = 0.00198079231$

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- Lots of variables like this.
- Instead we can sensibly ask. "What's the probability the Junior's weight is between 3.2 and 3.3 kg".
- More generally for a continuous random variable we'll have an integrable function $f(x)$ and $\mathbb{P}(X \in (a, b)) = \int_a^b f(x)dx$
- $f(x)$ is called a "density function"

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- Variance formulas hold.
- We're also interested in $F(c) = \mathbb{P}(X \leq c) = \int_{-\infty}^{c} f(x)dx$

Example

- Find c such that

$$f(x) = \begin{cases} cx & x \in (0,2) \\ 0 & elsewhere \end{cases}$$

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- $c = \frac{1}{2}$

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- $V(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = 2 - \frac{4}{3}^2 = \frac{2}{9}$

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- $V(X) = \frac{(b-a)^2}{12}$

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- Let $y = \lambda x$ so $\frac{dy}{dx} = \lambda$ and $\mathbb{E}[X] = \int_0^\infty y e^{-y} \frac{dy}{\lambda} = \frac{1}{\lambda}$

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- Usually it's generate $\alpha + \beta - 1$ uniforms and choose the α smallest or β largest.
- $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ is a scaling factor.
- It's also the continuous analog of the choose function.

Beta distribution

- $\mathbb{E}[X] = \int_0^1 x \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} dx = \int_0^1 \frac{x^{\alpha+1-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} dx = \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{\alpha}{\alpha+\beta}$

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- $$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt, \quad \text{for } x,y > 0. \qquad (1)$$

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$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{2}$$

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- The Jacobian is given by:

$$J = \begin{vmatrix} \frac{\partial u}{\partial s} & \frac{\partial v}{\partial s} \\ \frac{\partial u}{\partial t} & \frac{\partial v}{\partial t} \end{vmatrix} = \begin{vmatrix} t & s \\ 1-t & -s \end{vmatrix} = s. \tag{5}$$

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- This gives us:

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- Showing that

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}. \quad (7)$$

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- The leads to the celebrated Central Limit Theorem. Which roughly says that if we add some i.i.d random variables together that the sample mean will be approximately distributed normally.
- Cavaets exist here. We need finite moments and how good the approximation is will depend on how many X_i we're adding.

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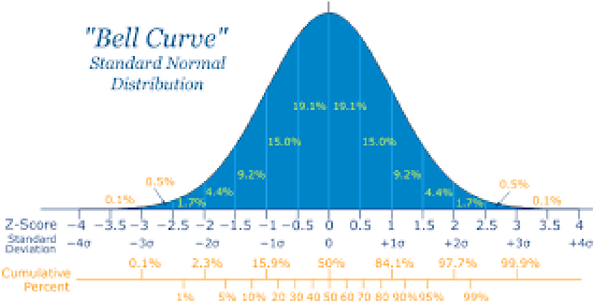
- $f(x) = \frac{1}{\sqrt{2\sigma\pi}} e^{(\frac{x-\mu}{\sigma})^2}$
- $F(X)$ doesn't have a nice closed form. We'll use a table to look up values.

Normal distribution



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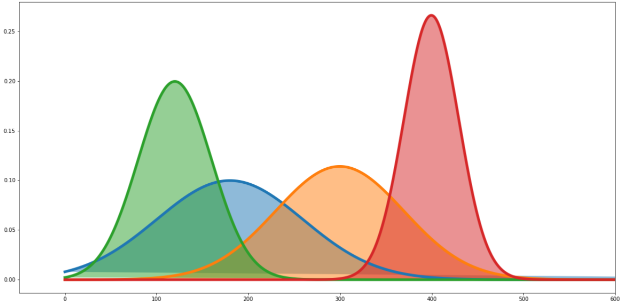

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- $f(X)$ is really ugly, and in practice we'll use a table.

Student T-distribution