**Question 1.** (7 *marks*)

Consider the following input array:

$$\boldsymbol{x} = (1, 0, 2, -3, 2, -1, -4, 5, 7, 9).$$

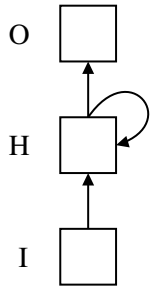
(a) (2 *marks*) Apply the kernel $\boldsymbol{w} = (\frac{1}{2}, 1, -\frac{1}{2})$ to $\boldsymbol{x}$ using a bias of 1 and *relu* activation function, and then apply max-pooling using a pooling window of size 2 and with a stride of 2.

(b) (3 *marks*) What are the advantages of using a convolutional layer in a neural network?

(c) (2 *marks*) If a kernel $\boldsymbol{w} = (w_1, \ldots, w_k)$ is applied to an input vector $\boldsymbol{x} = (x_1, \ldots, x_m)$ using stride $s$, and no padding is used, give the length of the output vector.

**Question 2.** (4 *marks*)

Consider a recurrent neural network with structure as shown in the diagram below, that has 3 input nodes, 3 nodes in the hidden layer and one output node.




Let the weight matrices and biases be as follows:

$$W_X = \begin{bmatrix} -1 & 2 & -3 \\ -2 & -4 & 2 \\ 3 & 0 & -1 \end{bmatrix}, \quad W_H = \begin{bmatrix} 2 & 0 & -3 \\ -1 & 2 & 1 \\ 0 & -4 & 3 \end{bmatrix}, \quad W_O = \begin{bmatrix} 0.3 \\ -0.2 \\ 0.1 \end{bmatrix}$$

$$\boldsymbol{b}_H = (3, 1, 2), \quad \boldsymbol{b}_O = (0.5).$$

The hidden layer uses *relu* activation function and the output layer uses *sigmoid* activation function.

Compute the outputs of the network if the sequence $\boldsymbol{x}$ is input, with $\boldsymbol{x}(1) = (2, 1, 1)$, $\boldsymbol{x}(2) = (-1, 1, 2)$.

**Question 3.** (4 *marks*)

Give pseudocode for the $k$-MEANS ALGORITHM, as applied to a dataset $S$ of points in $\mathbb{R}^m$.

**Question 4.** (5 *marks*)

(a) (4 *marks*) Use Hierarchical clustering with the **centroid linkage** on the following dataset.

(b) (1 *mark*) Draw the dendrogram for the clustering obtained.

| | $x_1$ | $x_2$ |
|---|---|---|
| $A$ | $-2$ | $-2$ |
| $B$ | $-1$ | $-1$ |
| $C$ | 0 | 4 |
| $D$ | 1 | 0 |
| $E$ | 2 | 2 |
| $F$ | 2 | 3 |

**Question 5.** (4 *marks*)

(a) (2 *marks*) With the aid of a diagram, describe the structure of an undercomplete autoencoder.

(b) (1 *marks*) Give the loss function that would typically be used to train an autoencoder.

(c) (1 *marks*) Give the loss function that would be used to train a denoising autoencoder.

**Question 6.** (6 *marks*)

(a) (2 *marks*) Give a diagram illustrating the structure of a generative adversarial network.

(b) A generative adversarial network is to be trained using the following minimax loss function:

$$\mathbb{E}_{\boldsymbol{x}}[\log(D(\boldsymbol{x}))] + \mathbb{E}_{\boldsymbol{z}}[\log(1 - D(G(\boldsymbol{z})))].$$

(i) (2 *marks*) Describe the process of discriminator training.

(ii) (2 *marks*) Describe the process of generator training.

**Question 7.** (4 *marks*)

Give pseudocode for the algorithm SARSA, as applied to an MDP with set $S$ of states and set $A$ of actions. Include a description of the $\epsilon$-greedy policy in your answer.

**Question 8.** (6 *marks*)

Consider the MDP illustrated in the diagram below.

| $s_0$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ |
|---|---|---|---|---|

The set of states is $S = \{s_0, s_1, s_2, s_3, s_4\}$ and the set of actions available at each state is $A = \{\ell, r\}$. The action $\ell$ moves the agent one state to the left with probability 0.2 and leaves the agent in the current state with probability 0.8. The action $r$ moves the agent one state to the right with probability 0.6 and leaves the agent in the current state with probability 0.4. The states $s_0$ and $s_4$ are both goal states and any action that results in getting to either goal state gives a reward of 10 and ends the episode. An action that does not move the agent into a goal state gives a reward of $-1$.

For example, $P(s_1, \ell, s_0) = 0.2$ and $P(s_1, \ell, s_1) = 0.8$, and $R(s_1, \ell, s_0) = 10$ and $R(s_1, \ell, s_1) = -1$.

The Value Iteration Algorithm is being used to find the optimal policy for the above MDP.
After 2 iterations, the values for $V$ are:

| 0 | 2.16 | 1.96 | 7.84 | 0 |
|---|---|---|---|---|

(a) (4 *marks*) Perform one more iteration of the Value Iteration Algorithm using $\gamma = 1$.
Show all your working.

(b) (2 *marks*) Calculate the action to take in state $s_1$ determined by the $V$ table obtained in (a).
Show all your working.