

## “Space Data Processing: Making Sense of Experimental Data”

Laboratory work 7

Development of forward-backward Kalman filter  
in conditions of correlated state noise

**Tatiana Podladchikova   Rupert Gerzer**

**Term 4, March 28 – May 27, 2016**

**t.podladchikova@skoltech.ru**

# Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random  
acceleration

$$a_i = e^{-\lambda T} a_{i-1} + \zeta_i$$

$\zeta_i$

Uncorrelated noise with variance

$$\sigma_{\zeta}^2 = \sigma_a^2 (1 - e^{-2\lambda T})$$

$\lambda$

Value that is inverse  
to correlation interval

$$\lambda = 1000$$

$a_i$  - uncorrelated noise

$$\lambda = 0.1$$

$a_i$  - correlated noise

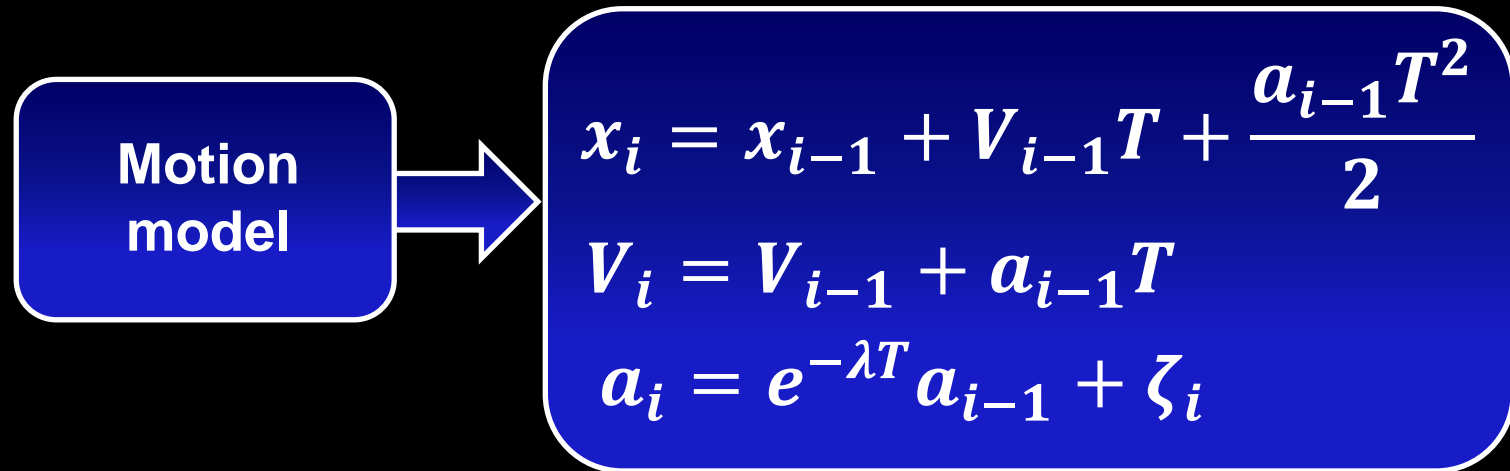
$T$

Time interval between  
measurements

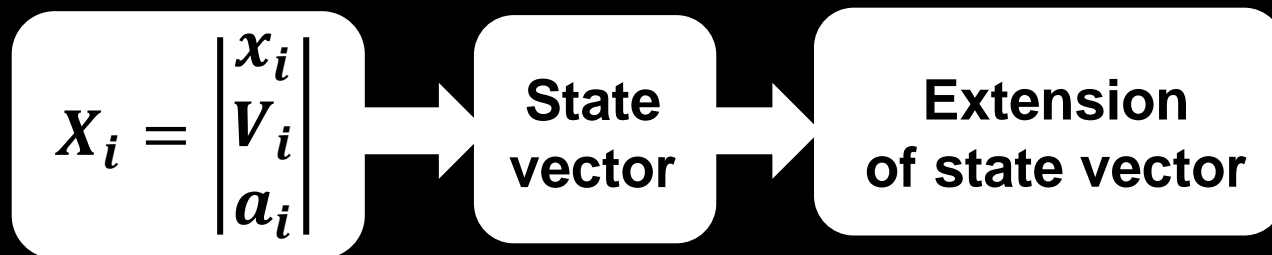
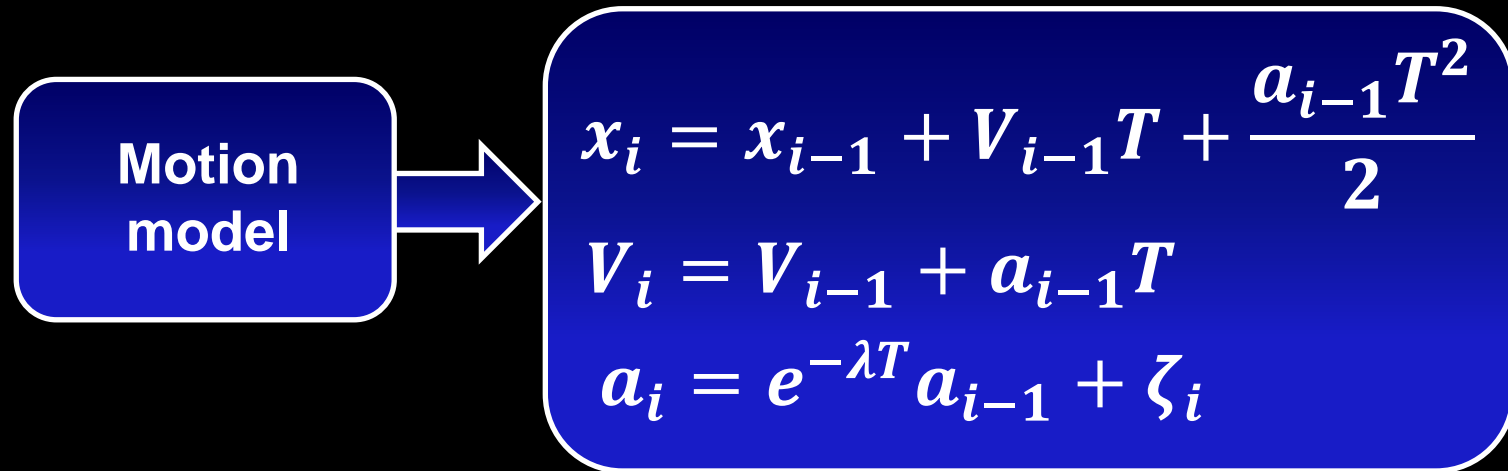
$\sigma_a^2$

Variance  
of acceleration

# Moving object which trajectory is disturbed by correlated random acceleration



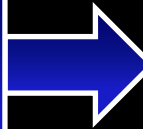
# Moving object which trajectory is disturbed by correlated random acceleration



Beside estimation of coordinate  $x_i$  and velocity  $V_i$ ,  
Kalman filter will also estimate the dynamics  
of correlated acceleration  $a_i$

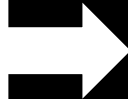
# State space model

State  
equation



$$X_i = \Phi X_{i-1} + G \zeta_i$$

$\Phi$



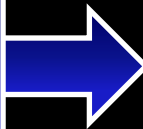
Transition  
matrix

$G$



Input  
matrix

Measurement  
equation



$$z_i = H X_i + \eta_i$$

$$X_i = \begin{bmatrix} x_i \\ V_i \\ a_i \end{bmatrix}$$

$H$



Observation  
matrix

# Smoothing with fixed interval

Smoothing is performed in backward in time

$$X_{i,N} = X_{i,i} + A_i(X_{i+1,N} - \Phi_{i+1,i}X_{i,i})$$

$$i = N - 1, N - 2, \dots 1$$

Coefficient

$$A_i = P_{i,i}\Phi_{i+1,i}P_{i+1,i}^{-1}$$

Smoothing error covariance matrix

$$P_{i,N} = P_{i,i} + A_i(P_{i+1,N} - P_{i+1,i})A_i^T$$

$X_{i,i}$  - filtered estimate,  $X_{N,N}$  - initial estimate

$P_{i,i}$  - filtration error covariance matrix

$P_{i+1,i}$  - prediction error covariance matrix

Smoothing takes into account both current and future measurements and therefore provides improved estimation