

“Space Data Processing: Making Sense of Experimental Data”

Topic 5

“Model construction at state space under uncertainty”

Part I. Noise statistics identification

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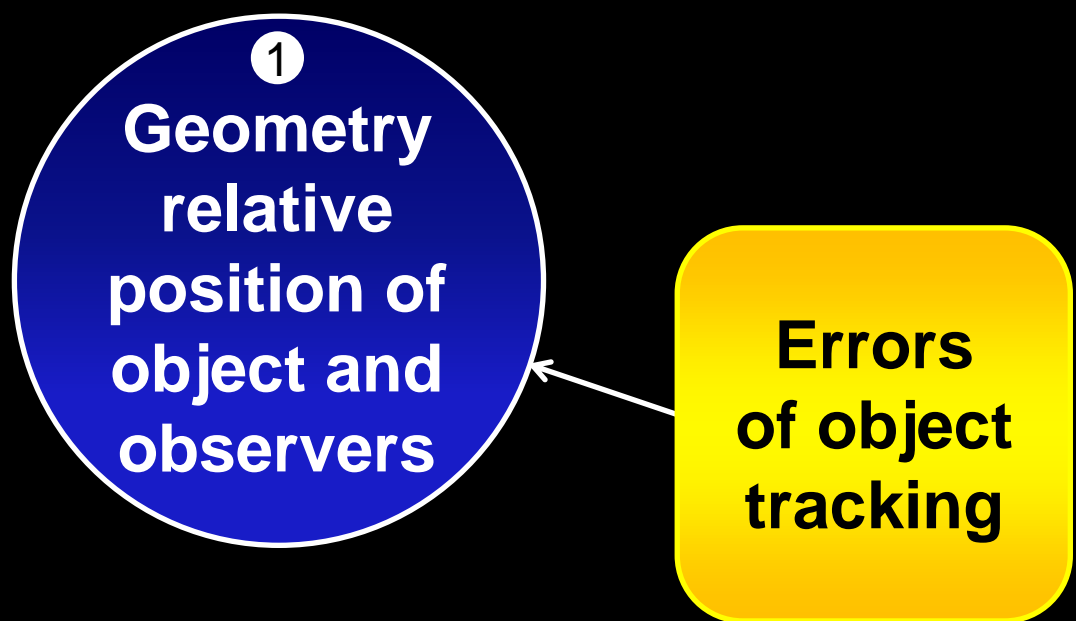
Term 4, March 28 – May 27, 2016

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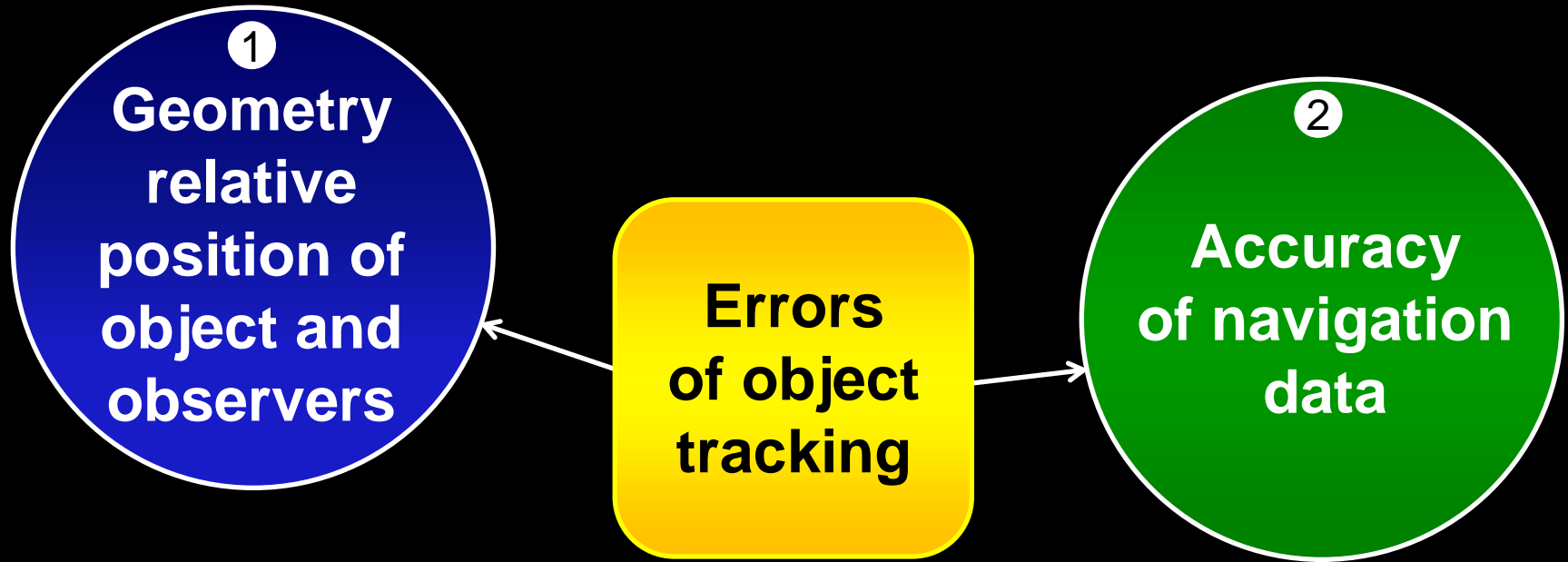
**Geometry
relative
position of
object and
observers**

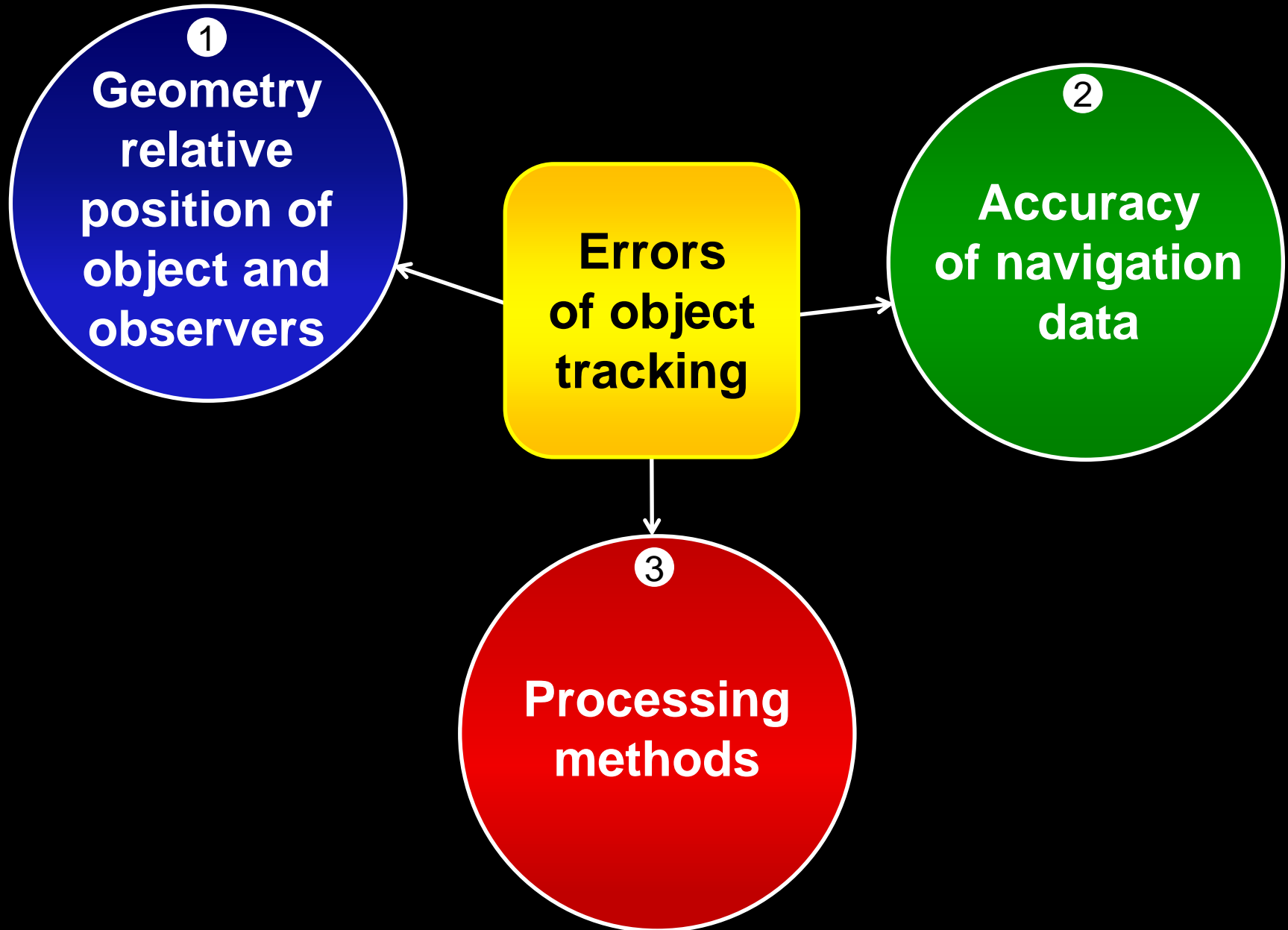
**Errors
of object
tracking**



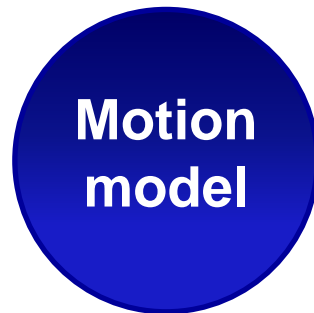
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graph LR; A[Errors of object tracking] --> B[Geometry relative position of object and observers];
```

The diagram consists of two main components on a black background. On the left is a blue circle with a white border, containing the text '1 Geometry relative position of object and observers'. On the right is a yellow rounded rectangle with a black border, containing the text 'Errors of object tracking'. A white arrow points from the right side of the yellow rectangle to the right side of the blue circle.

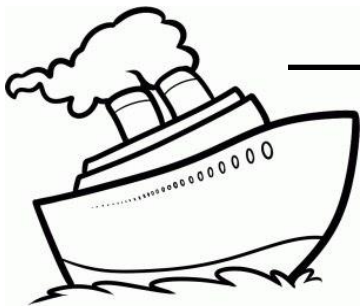




Process noise should not be filtered



$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$
$$V_i = V_{i-1} + a_{i-1}T$$



**Unintentional maneuver can be
described by random acceleration a_i**

ship pitching or undercurrents

x

a_i

**Noise intrinsic
to the process
itself that should
not be filtered**

Stochastic model

State
equation



$$X_i = \Phi_{i,i-1}X_{i-1} + Ga_i$$

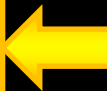
Measurement
equation



$$z_i = H_iX_i + \eta_i$$

Ga_i

Noise intrinsic to
the process itself
that should not
be filtered



State space
model separates
noises in
contrast to linear
regression



η_i

Measurement
noise that
should be
filtered

Noise statistics identification

State
equation



$$X_i = \Phi_{i,i-1}X_{i-1} + Ga_i$$

Measurement
equation



$$z_i = H_iX_i + \eta_i$$

$$Gq$$

Bias of state
noise a_i
 $q = E[a_i]$

$$Q = GG^T \sigma_a^2$$

Covariance matrix
of state noise w_i
 $\sigma_a^2 = E[(a_i - q)^2]$

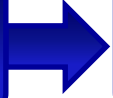
$$R = \sigma_\eta^2$$

Covariance matrix of
measurement noise η_i
 $E[\eta_i] = 0, \sigma_\eta^2 = E[\eta_i^2]$

How to identify q, σ_a^2 , and σ_η^2
using measurements?

Characteristics of residuals of optimal filter

Filtration



$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$

Residual

$$v_i = z_i - HX_{i,i-1}$$



**Mismatch between
measurement and prediction**

Characteristics of residuals of optimal filter

Filtration

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$

Residual

$$v_i = z_i - HX_{i,i-1}$$

Mismatch between
measurement and prediction

① Mathematical
expectation of v_i

$$E[v_i] = 0$$

Characteristics of residuals of optimal filter

Filtration

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$

Residual

$$v_i = z_i - HX_{i,i-1}$$

Mismatch between
measurement and prediction

① Mathematical
expectation of v_i

$$E[v_i] = 0$$

② Covariance
of v_i

$$E[v_i v_i^T] = HP_{i,i-1}H^T + R$$

Characteristics of residuals of optimal filter

Filtration

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$

Residual

$$v_i = z_i - HX_{i,i-1}$$

Mismatch between
measurement and prediction

① Mathematical
expectation of v_i

$$E[v_i] = 0$$

② Covariance
of v_i

$$E[v_i v_i^T] = HP_{i,i-1}H^T + R$$

③ Correlation
moment of v_i

$$E[v_i v_j^T] = 0, i \neq j$$

Consistent identification methods

- ① **R. Mehra (1970), On the identification of variances and adaptive Kalman filtering in IEEE Transactions on Automatic Control, vol. 15, no. 2, pp. 175-184**



Difficult to implement in real -time

- ② **Anderson, W. N. et al (1969), Consistent estimates of the parameters of a linear system, Ann. Math. Stat., 40(3), 2064–2075.**



All components of state vector should be measured

Stochastic model

State
equation



$$X_i = \Phi_{i,i-1} X_{i-1} + G a_i$$

Measurement
equation



$$z_i = H_i X_i + \eta_i$$

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

State
vector

$$\Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

Transition
matrix

$$G = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$$

Input
matrix

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Observation
matrix

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Let's create
pseudo-
measurement
vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Let's create
pseudo-
measurement
vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider
difference

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^*$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Let's create
pseudo-
measurement
vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider
difference

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^*$$

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-1} \\ \frac{z_{i-1} - z_{i-2}}{T} \end{bmatrix}$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Let's create
pseudo-
measurement
vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider
difference

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^*$$

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-1} \\ \frac{z_{i-1} - z_{i-2}}{T} \end{bmatrix}$$

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^* = \begin{bmatrix} z_i - 2z_{i-1} + z_{i-2} \\ \frac{z_i - 2z_{i-1} + z_{i-2}}{T} \end{bmatrix}$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Let's create
pseudo-
measurement
vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider
difference

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^*$$

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-1} \\ \frac{z_{i-1} - z_{i-2}}{T} \end{bmatrix}$$

$$Z_i^* - \Phi_{i,i-1} Z_{i-1}^* = \begin{bmatrix} z_i - 2z_{i-1} + z_{i-2} \\ \frac{z_i - 2z_{i-1} + z_{i-2}}{T} \end{bmatrix}$$

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$z_i = x_i + \eta_i$$

$$v_i = z_i - 2z_{i-1} + z_{i-2} = x_i + \eta_i - 2(x_{i-1} + \eta_{i-1}) + (x_{i-2} + \eta_{i-2})$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$z_i = x_i + \eta_i$$

$$v_i = z_i - 2z_{i-1} + z_{i-2} = x_i + \eta_i - 2(x_{i-1} + \eta_{i-1}) + (x_{i-2} + \eta_{i-2})$$

$$v_i = (x_i - x_{i-1}) - (x_{i-1} - x_{i-2}) + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$z_i = x_i + \eta_i$$

$$v_i = z_i - 2z_{i-1} + z_{i-2} = x_i + \eta_i - 2(x_{i-1} + \eta_{i-1}) + (x_{i-2} + \eta_{i-2})$$

$$v_i = (x_i - x_{i-1}) - (x_{i-1} - x_{i-2}) + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$v_i = V_{i-1}T + \frac{a_{i-1}T^2}{2} - V_{i-2}T - \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$z_i = x_i + \eta_i$$

$$v_i = z_i - 2z_{i-1} + z_{i-2} = x_i + \eta_i - 2(x_{i-1} + \eta_{i-1}) + (x_{i-2} + \eta_{i-2})$$

$$v_i = (x_i - x_{i-1}) - (x_{i-1} - x_{i-2}) + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$v_i = V_{i-1}T + \frac{a_{i-1}T^2}{2} - V_{i-2}T - \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

$$V_{i-1}T = V_{i-2}T + a_{i-2}T^2$$

$$v_i = \frac{a_{i-1}T^2}{2} + \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

v_i depends
only on noises
 a_i, η_i

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$v_i = \frac{a_{i-1}T^2}{2} + \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$v_i = \frac{a_{i-1}T^2}{2} + \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Mathematical
expectation of v_i

$$E[v_i] = qT^2$$

$$E[v_i] \approx \frac{1}{N-2} \sum_{i=3}^N v_i$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$v_i = \frac{a_{i-1}T^2}{2} + \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Mathematical
expectation of v_i

$$E[v_i] = qT^2$$

$$E[v_i] \approx \frac{1}{N-2} \sum_{i=3}^N v_i$$

Bias q

$$q = \frac{E[v_i]}{T^2}$$

Noise statistics identification

Let's consider

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$v_i = \frac{a_{i-1}T^2}{2} + \frac{a_{i-2}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Mathematical
expectation of v_i

$$E[v_i] = qT^2$$

$$E[v_i] \approx \frac{1}{N-2} \sum_{i=3}^N v_i$$

Bias q

$$q = \frac{E[v_i]}{T^2}$$

Variance of v_i

$$E[(v_i - qT^2)^2] = \frac{1}{2} \sigma_a^2 T^4 + 6\sigma_\eta^2$$

$$E[(v_i - qT^2)^2] \approx \frac{1}{N-2} \sum_{i=3}^N (v_i - qT^2)^2$$

σ_a^2 - variance of a_i

σ_η^2 - variance of η_i

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Pseudo-measurement vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider difference

$$Z_i^* - \Phi_{i,i-2} Z_{i-2}^*$$

$$\Phi_{i,i-2} = \Phi_{i,i-1} \Phi_{i-1,i-2}$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Pseudo-measurement vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider difference

$$Z_i^* - \Phi_{i,i-2} Z_{i-2}^*$$

$$\Phi_{i,i-2} = \Phi_{i,i-1} \Phi_{i-1,i-2}$$

$$Z_i^* - \Phi_{i,i-2} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-2} \\ \frac{z_{i-2} - z_{i-3}}{T} \end{bmatrix}$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Pseudo-measurement vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider difference

$$Z_i^* - \Phi_{i,i-2} Z_{i-2}^*$$

$$\Phi_{i,i-2} = \Phi_{i,i-1} \Phi_{i-1,i-2}$$

$$Z_i^* - \Phi_{i,i-2} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-2} \\ \frac{z_{i-2} - z_{i-3}}{T} \end{bmatrix}$$

$$Z_i^* - \Phi_{i,i-2} Z_{i-1}^* = \begin{bmatrix} z_i - 3z_{i-2} + 2z_{i-3} \\ \frac{z_i - z_{i-1} - z_{i-2} + z_{i-3}}{T} \end{bmatrix}$$

Noise statistics identification

State vector

$$X_i = \begin{bmatrix} x_i \\ V_i \end{bmatrix}$$

Pseudo-measurement vector

$$Z_i^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix}$$

Measurements

$$z_i$$

Let's consider difference

$$Z_i^* - \Phi_{i,i-2} Z_{i-2}^*$$

$$\Phi_{i,i-2} = \Phi_{i,i-1} \Phi_{i-1,i-2}$$

$$Z_i^* - \Phi_{i,i-2} Z_{i-1}^* = \begin{bmatrix} z_i \\ \frac{z_i - z_{i-1}}{T} \end{bmatrix} - \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_{i-2} \\ \frac{z_{i-2} - z_{i-3}}{T} \end{bmatrix}$$

$$Z_i^* - \Phi_{i,i-2} Z_{i-1}^* = \begin{bmatrix} z_i - 3z_{i-2} + 2z_{i-3} \\ \frac{z_i - z_{i-1} - z_{i-2} + z_{i-3}}{T} \end{bmatrix}$$

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

Noise statistics identification

Let's consider

$$\Rightarrow \rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$



$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2} \rightarrow x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2} \rightarrow x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-2}T + a_{i-2}T^2 + \frac{a_{i-1}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-2}T + a_{i-2}T^2 + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-2}T + a_{i-2}T^2 + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_i - x_{i-2} = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$z_i = x_i + \eta_i$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3} = x_i + \eta_i - 3(x_{i-2} + \eta_{i-2}) + 2(x_{i-3} + \eta_{i-3})$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + V_{i-2}T + \frac{a_{i-2}T^2}{2} + V_{i-2}T + a_{i-2}T^2 + \frac{a_{i-1}T^2}{2}$$

$$x_i = x_{i-2} + 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_i - x_{i-2} = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_{i-2} - x_{i-3} = V_{i-3}T + \frac{a_{i-3}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i - x_{i-2} = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_{i-2} - x_{i-3} = V_{i-3}T + \frac{a_{i-3}T^2}{2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

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$$x_i - x_{i-2} = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_{i-2} - x_{i-3} = V_{i-3}T + \frac{a_{i-3}T^2}{2}$$

$$\rho_i = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} - 2V_{i-3}T - a_{i-3}T^2 + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$\rho_i = (x_i - x_{i-2}) - 2(x_{i-2} - x_{i-3}) + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$x_i - x_{i-2} = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2}$$

$$x_{i-2} - x_{i-3} = V_{i-3}T + \frac{a_{i-3}T^2}{2}$$

$$\rho_i = 2V_{i-2}T + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} - 2V_{i-3}T - a_{i-3}T^2 + \eta_i - 3\eta_{i-2} + 2\eta_{i-3}$$

$$2V_{i-2}T = 2V_{i-3}T + 2a_{i-3}T^2$$

$$\rho_i = a_{i-3}T^2 + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

ρ_i depends
only on noises
 a_i, η_i

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$\rho_i = a_{i-3}T^2 + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$\rho_i = a_{i-3}T^2 + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Mathematical
expectation of ρ_i

$$E[\rho_i] = 3qT^2$$

Noise statistics identification

Let's consider

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$

$$\rho_i = a_{i-3}T^2 + \frac{3a_{i-2}T^2}{2} + \frac{a_{i-1}T^2}{2} + \eta_i - 2\eta_{i-1} + \eta_{i-2}$$

Mathematical
expectation of ρ_i

$$E[\rho_i] = 3qT^2$$

Variance of ρ_i

$$E[(\rho_i - 3qT^2)^2] = \frac{7}{2}\sigma_a^2T^4 + 14\sigma_\eta^2$$

$$E[(\rho_i - 3qT^2)^2] \approx \frac{1}{N-3} \sum_{i=4}^N (\rho_i - 3qT^2)^2$$

σ_a^2 - variance of a_i

σ_η^2 - variance of η_i

Summary: Noise statistics identification

Bias q

$$q = \frac{E[v_i]}{T^2}$$

$$E[v_i] \approx \frac{1}{N-2} \sum_{i=3}^N v_i$$

σ_a^2 and σ_η^2 are determined
from the solution
of system of equations

$$E[(v_i - qT^2)^2] = \frac{1}{2}\sigma_a^2 T^4 + 6\sigma_\eta^2$$

$$E[(\rho_i - 3qT^2)^2] = \frac{7}{2}\sigma_a^2 T^4 + 14\sigma_\eta^2$$

$$v_i = z_i - 2z_{i-1} + z_{i-2}$$

$$\rho_i = z_i - 3z_{i-2} + 2z_{i-3}$$