Lab3: Comparison of exponential and running mean for random walk model

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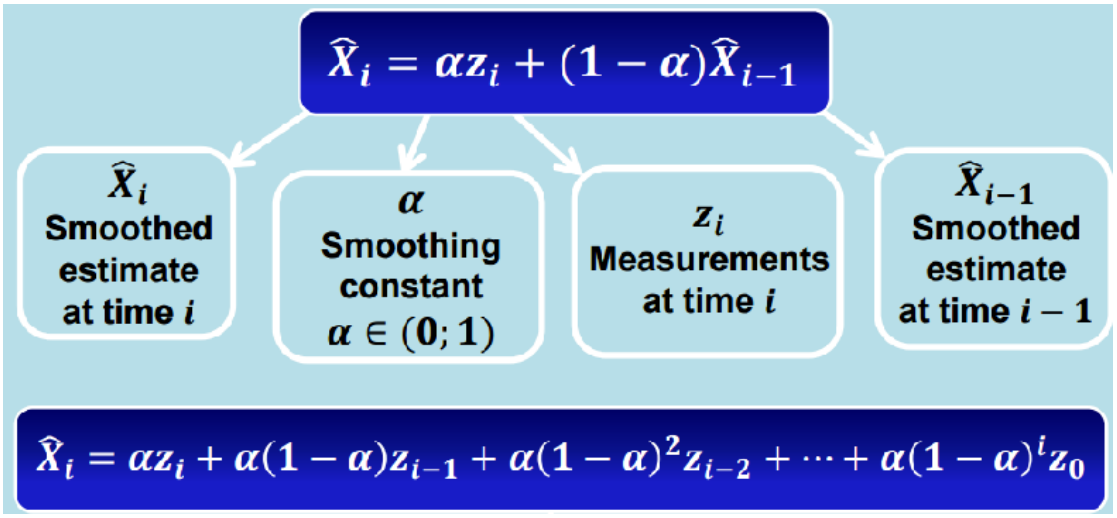
The objective of this laboratory work is to compare the errors of exponential and running mean to choose the most effective quasi-optimal estimation method in conditions of uncertainty.

Additional important outcome of this exercise is the solution of identification problem of noise statistics that is crucial for reliable estimation.

Exponential Smoothing -

The weight of measurements decreases according to geometric progression or exponential law



1)*Determination of optimal smoothing constant in exponential mean*

Let's Generate a true trajectory $X_i = X_{i-1} + W_i$ using the random walk model

and Generate measurements Z_i of the process X_i , $Z_i = X_i + N_i$,

where N_i and W_i –normally distributed random noise with zero mathematical expectation and variance σ_η^2 σ_w^2.

```
clc; clear; close all;
set(gcf,'PaperPositionMode','auto');

n=100;
[X, Z, W, N] = randomWalk(n, 28,97, 10);
```

Let's calculate noise statistics σ_η^2 and σ_w^2 and determine optimal smoothing coefficient α in exponential smoothing:

$$\sigma_n = E[v_i^2] - \frac{E[p_i^2]}{2},\ \sigma_w = E[p_i^2] - E[v_i^2],\ \chi = \frac{\sigma_w^2}{\sigma_\eta^2},\ \alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2}$$

```
% alpha = calcAlpha(W, N);
alpha = 0.25;
```

Let's Determine the window size m (use round values) that provides equality of σ_{RM}^2 and σ_{ES}^2 using determined smoothing constant α

$$\sigma_{ES}^2 = \sigma_{RM}^2 \Rightarrow m = \frac{2-\alpha}{\alpha}$$

```
m = (2-alpha)/alpha;
nearest_odd_m = 2*floor(m/2)+1;
```

Let's make running mean and exponential Smoothings and Plot the results :

```
SmoothZ = smooth(Z, nearest_odd_m);
Exp = smoothExp(Z, alpha);
ExpBack = smoothBackExp(Z, alpha);

figure('position', [0, 0, 1000, 500]) ;
plot(X,'black');
hold on;
plot(SmoothZ,'green');
plot(Exp,'yellow');
plot(ExpBack,'blue', 'LineWidth',2);
legend('X','SmoothX','Exp','ExpBack');
```