Skoltech

Skolkovo Institute of Science and Technology

“Space Data Processing:
Making Sense of Experimental Data”

Continuation of topic 2 "Quasi-optimal approximation under uncertainty”

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Exponential smoothing

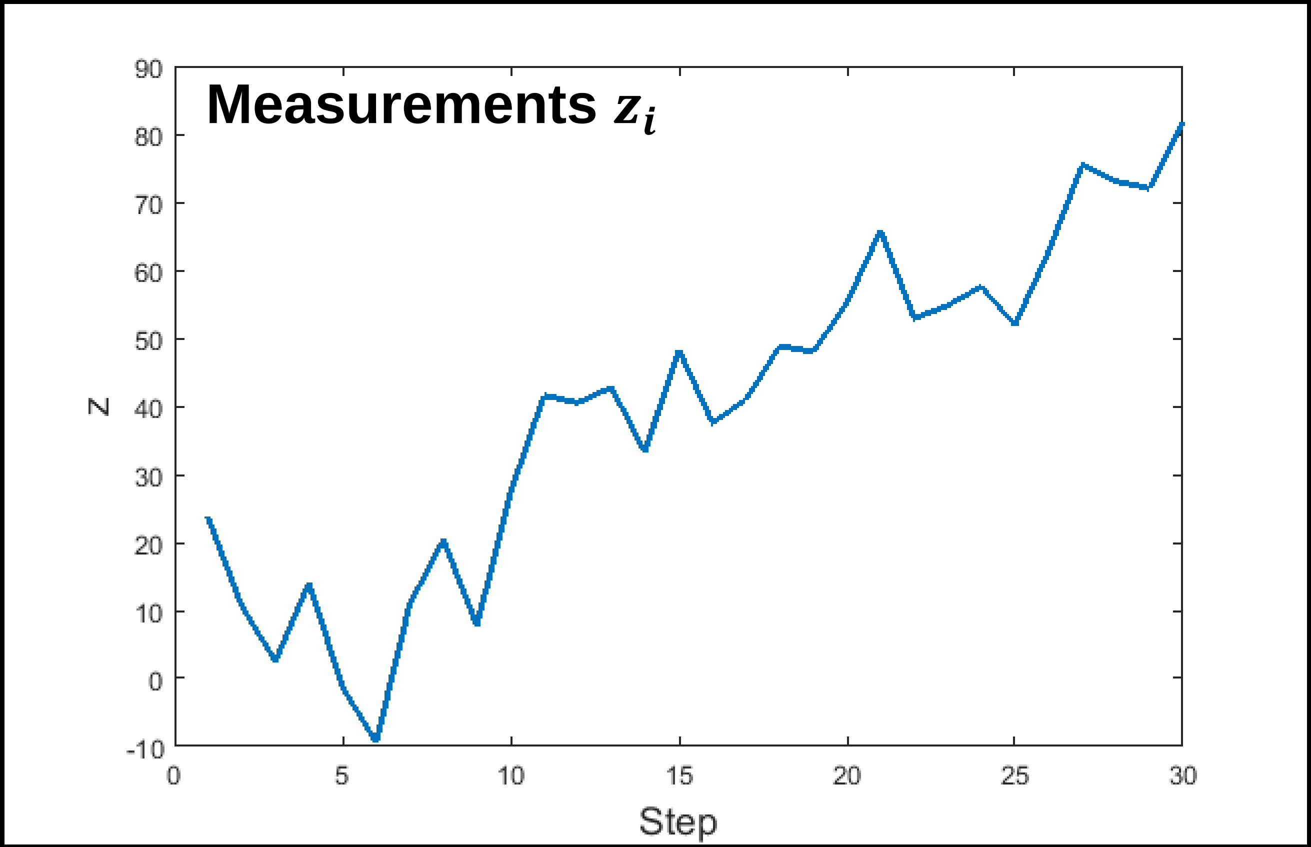
$$\hat{X}_i = \hat{X}_{i-1} + \alpha(z_i - \hat{X}_{i-1})$$

Errors of exponential smoothing due to measurement errors

$$\sigma^2_{\hat{X}} = \sigma^2_{\eta} \frac{\alpha}{2 - \alpha}$$

Optimal choice of smoothing constant α

Process X is characterized by sudden and unpredictable changes



Optimal choice of smoothing constant α

Process X is characterized by sudden and unpredictable changes

↓

$$X_i = X_{i-1} + w_i$$

↓

w_i - unbiased random noise with variance σ_w^2

↓

Random walk model

Optimal choice of smoothing constant α

Optimal α for random walk model

$$\alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2}$$

$$\chi = \frac{\sigma_w^2}{\sigma_\eta^2}$$

σ_η^2 - variance of measurement noise

Muth J.F. (1960), Optimal properties of exponentially weighted forecasts of time series with permanent and transitory components, J.Amer. Statist. Ass.01960.-Vol.55.-p.299.

Full error of smoothing error

$$\sigma_\eta^2 a$$

$$a\sigma_\eta^2 > \sigma_\eta^2 \frac{\alpha}{2 - \alpha}$$

Optimal choice of smoothing constant α

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Full error of smoothing error

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Variances σ_w^2 σ_η^2 should be identified

Identification of noise statistics σ_w^2 and σ_η^2

Process X_i	$X_i = X_{i-1} + w_i$	1
Measurements	$z_i = X_i + \eta_i$	2
Residual ν_i	$\nu_i = z_i - z_{i-1}$	3
Residual ρ_i	$\rho_i = z_i - z_{i-2}$	4
Residual ν_i	$\nu_i = w_i + \eta_i - \eta_{i-1}$	5
Residual ρ_i	$\rho_i = w_i + w_{i-1} + \eta_i - \eta_{i-2}$	6
Math. expectation	$E[\nu_i^2] = \sigma_w^2 + 2\sigma_\eta^2$	7
Math. expectation	$E[\rho_i^2] = 2\sigma_w^2 + 2\sigma_\eta^2$	8

Rewrite Eq. 3 using Eq. 1 and 2

Anderson, W. N., G. B. Kleindorfer, P. R. Kleindorfer, and M. B. Woodroofe (1969), Consistent estimates of the parameters of a linear system, Ann. Math. Stat., 40(3), 2064–2075.

Identification of noise statistics σ_w^2 and σ_η^2

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Rewrite Eq. 3 using Eq. 1 and 2

$$E[\nu_i^2] \approx \frac{1}{N-2}\sum_{k=2}^{N} \nu_k^2$$

$$E[\rho_i^2] \approx \frac{1}{N-3}\sum_{k=3}^{N} \rho_k^2$$

Consistent estimates σ_w^2 and σ_η^2 are obtained by solving system of equations (7,8)