Laboratory work 8, Part II
Development of tracking filter of a moving object
when measurements and motion model are in different coordinate systems

Performance - Tuesday, April 28, 2016
Due to submit a performance report – Wednesday, May 11, 2016

The objective of this laboratory work is to develop a tracking filter of a moving object when measurements and motion model are in different coordinate systems. This problem is typical for radio navigation systems. Important outcome of this exercise is to detect main difficulties of practical Kalman filter implementation related with instability zone of a tracking filter, and to analyze conditions under which navigation system may become blind and filter diverges. Additional important outcome of this exercise is experience in developing algorithms to improve tracking accuracy of a moving object by taking into account available a prior information. This is important to prevent collisions and for other safety issues.

The second part of this laboratory work is performed in the class by students as in teams of 2 on April 28, 2016 and the team will submit one document reporting about the performance till Wednesday, May 11, 2016. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

Important information
Please read charts for problem formulation
Tracking_filter_cordinate_transformation_of_measurements_April_22_2016.pdf

This laboratory work consists of two parts:

I. Instability zone of a tracking filter due to ill-conditioned coordinate transformations of measurements (Tuesday, April 26, 2016)
II. How to increase tracking accuracy of a moving object by taking into account available a prior information? (Thursday, April 28, 2016)

Here is the recommended procedure for part II:
How to increase tracking accuracy of a moving object by taking into account available prior information?

1. Generate a true trajectory X_i of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V^x_{i-1}T + \frac{a^x_{i-1}T^2}{2}$$
$$V^x_i = V^x_{i-1} + a^x_{i-1}T$$
$$y_i = y_{i-1} + V^y_{i-1}T + \frac{a^y_{i-1}T^2}{2}$$
$$V^y_i = V^y_{i-1} + a^y_{i-1}T$$

   Initial conditions to generate trajectory
   (a) Size of trajectory is $N = 26$ points.

   (b) $T = 2$ – interval between measurements.

   (c) Initial coordinates

$$x_0 = \frac{13500}{\sqrt{2}};\ y_0 = \frac{13500}{\sqrt{2}}$$

(a) Initial components of velocity V
$V_x = 27; V_y = 27;$
It means that an object moves from an observer.

(b) Variance of noise a_i, $\sigma_a^2 = 0.5^2$ for both a_i^x, a_i^y

2. Generate also true values of range D and azimuth β

$$D_i = \sqrt{x_i^2 + y_i^2}$$

$$\beta_i = arctg\left(\frac{x}{y}\right)$$

Initial values $D_0 = \sqrt{x_0^2 + y_0^2}; \beta_0 = arct\left(\frac{x_0}{y_0}\right)$

3. Generate also true values of an object full velocity V and course C

$$V_i = \sqrt{(V_i^x)^2 + \left(V_i^y\right)^2}$$

$$C = arctg\left(\frac{V_i^x}{V_i^y}\right)$$

Course C – angle between velocity vector and direction toward North.

4. Generate measurements D^m and β^m of range D and azimuth β

$$D_i^m = D_i + \eta_i^D$$

$$\beta_i^m = \beta_i + \eta_i^\beta$$

Variances of measurement noises η_i^D, η_i^β are given by

$$\sigma_D^2 = 100$$

$$\sigma_\beta^2 = 0.007$$

5. Transform polar coordinates D^m and β^m to Cartesian ones and get pseudo-measurements x_i^m, y_i^m of coordinates x and y
Consult charts, page 31

6. Create the measurement vector z from pseudo-measurements x_i^m, y_i^m
Consult charts, page 31

7. Initial conditions for Kalman filter algorithm
Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} x_2^m \\ 30 \\ y_2^m \\ 24 \end{vmatrix}$$

Initial filtration error covariance matrix $P_{0,0}$
First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix}$$

When we don't know any prior information we should use great initial errors. With time filter switches off from initial conditions. But if law tracking accuracy is kept over long interval, a moving object may, for example, collide with another object. Therefore we should develop a filter that is capable to provide reliable tracking of a moving object as fast as possible. Later we will analyze how a prior information can help to increase tracking accuracy.

8. Create the transition matrix Φ and observation matrix H
   Consult charts, page 32

9. Create the measurement error covariance matrix R needed for Kalman filter algorithm
   Consult charts, page 33

10. Calculate covariance matrix Q.

    Hint how to do
    State-space equation in this case is presented as

$$X_i = \Phi X_{i-1} + w_{i-1}$$

$$w_{i-1} = \begin{vmatrix} \frac{a_{i-1}^x T^2}{2} \\ a_{i-1}^x T \\ \frac{a_{i-1}^y T^2}{2} \\ a_{i-1}^y T \end{vmatrix}$$

    Then $Q = E[w \cdot w^T] = \sigma_a^2 \begin{vmatrix} \frac{T^4}{4} & \frac{T^3}{2} & 0 & 0 \\ \frac{T^3}{2} & T^2 & 0 & 0 \\ 0 & 0 & \frac{T^4}{4} & \frac{T^3}{2} \\ 0 & 0 & \frac{T^3}{2} & T^2 \end{vmatrix}$

11. Develop Kalman filter algorithm to estimate state vector X_i (extrapolation and filtration). Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate
    (a) range D
    (b) azimuth β
    (c) full velocity V
    (d) course C

12. Run Kalman filter algorithm over $M = 500$ runs.
    Calculate true estimation errors of
    (a) Errors of extrapolation and filtration estimates of range D
    (b) Errors of extrapolation and filtration estimates of azimuth β
    (c) Errors of extrapolation and filtration estimates of full velocity V
    (d) Errors of extrapolation and filtration estimates of course C

13. Now let's try to take into account available a prior information to increase tracking accuracy of filter and get reliable tracking of a moving object as fast as possible that is important for collision prevention other safety issues.

Let's suppose that we know that our object has slow velocity of a motion, for example it is a ship, which velocity doesn't overcome on average 70 knots.
1 knot is 1852 meters per hour, or 0.514 meters per second.

And let's use this information in initial conditions for Kalman filter algorithms.
In this case let's use the following initial filtration error covariance matrix $P_{0,0}$

$$P_{0,0} = \begin{vmatrix} 2 \cdot \sigma_D^2 & 0 & 0 & 0 \\ 0 & 15^2 & 0 & 0 \\ 0 & 0 & 2 \cdot \sigma_D^2 & 0 \\ 0 & 0 & 0 & 15^2 \end{vmatrix}$$

Here $P_{0,0}(2,2)$ corresponds to variance of initial filtered estimate of V^x and $P_{0,0}(4,4)$ corresponds to variance of initial filtered estimate of V^y. It means that initial filtered estimate of velocity V is not greater than 41 knots.

$V = \sqrt{(V^x)^2 + (V^y)^2} = \sqrt{(15/0.514)^2 + (15/0.514)^2} = 41$ knots

This is logical as we know that a ship doesn't have great velocity.

14. Repeat items 11-12 with these new conditions and compare results with previous conditions.

15. Make conclusions on effectiveness of increasing tracking accuracy taking into account available a prior information.

Performance report

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2016
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.