“Space Data Processing:
Making Sense of Experimental Data”

Laboratory work 7
Development of forward-backward Kalman filter in conditions of correlated state noise

Tatiana Podladchikova Rupert Gerzer
Term 4, March 28 – May 27, 2016
t.podladchikova@skoltech.ru

Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration → $a_i = e^{-\lambda T} a_{i-1} + \zeta_i$

ζ_i → **Uncorrelated noise with variance** $\sigma_\zeta^2 = \sigma_a^2 (1 - e^{-2\lambda T})$

λ → **Value that is inverse to correlation interval**

$\lambda = 1000$ → a_i **- uncorrelated noise**

$\lambda = 0.1$ → a_i **- correlated noise**

T → **Time interval between measurements**

σ_a^2 → **Variance of acceleration**

Moving object which trajectory is disturbed by correlated random acceleration

Motion model

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

$$a_i = e^{-\lambda T}a_{i-1} + \zeta_i$$

Moving object which trajectory is disturbed by correlated random acceleration

Motion model

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

$$a_i = e^{-\lambda T}a_{i-1} + \zeta_i$$

$$X_i = \begin{vmatrix} x_i \\ V_i \\ a_i \end{vmatrix}$$

State vector

Extension of state vector

Beside estimation of coordinate x_i and velocity V_i, Kalman filter will also estimate the dynamics of correlated acceleration a_i

State space model

State equation → $X_i = \Phi X_{i-1} + G\zeta_i$

Φ → **Transition matrix**

G → **Input matrix**

Measurement equation → $z_i = HX_i + \eta_i$

$$X_i = \begin{vmatrix} x_i \\ V_i \\ a_i \end{vmatrix}$$

H → **Observation matrix**

Smoothing with fixed interval

Smoothing is performed in backward in time

$$X_{i,N} = X_{i,i} + A_i(X_{i+1,N} - \Phi_{i+1,\mathrm{i}} X_{i,i})$$

$$i = N-1, N-2, \cdots 1$$

Coefficient $A_i = P_{i,i}\Phi_{i+1,i}^T P_{i+1,i}^{-1}$

Smoothing error covariance matrix

$$P_{i,N} = P_{i,i} + A_i\left(P_{i+1,N} - P_{i+1,i}\right)A_i^T$$

$X_{i,i}$ - filtered estimate, $X_{N,N}$ - initial estimate

$P_{i,i}$ - filtration error covariance matrix

$P_{i+1,i}$ - prediction error covariance matrix

Smoothing takes into account both current and future measurements and therefore provides improved estimation