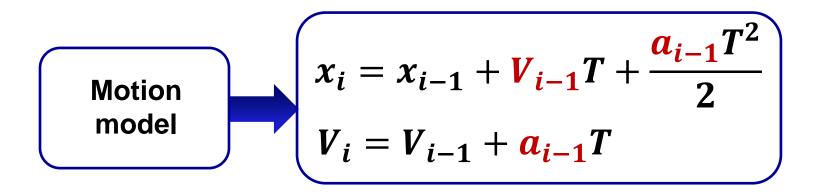


# "Space Data Processing: Making Sense of Experimental Data"

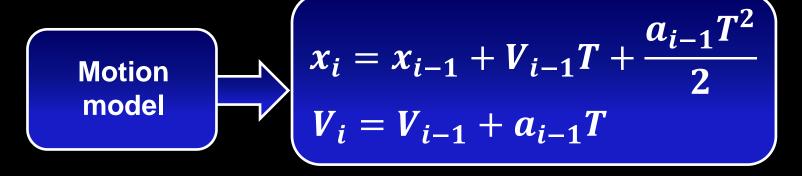
Laboratory work 6
Analysis of accuracy decrease of filtration in conditions of correlated biased state and measurement noise

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# Moving object which trajectory is disturbed by random acceleration



### State equation



$$X_i = \begin{vmatrix} x_i \\ V_i \end{vmatrix}$$
 State inform state of

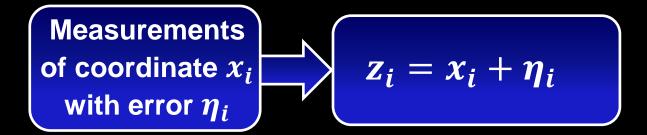
It contains full information about the state of system at time *i* 

$$X_i = \Phi X_{i-1} + Ga_{i-1}$$

$$\Phi = \begin{vmatrix} 1 & T \\ 0 & 1 \end{vmatrix}$$
 Transition matrix

$$G = \begin{vmatrix} T^2/2 \\ T \end{vmatrix}$$
 Input matrix

### Measurement equation



Measurement equation

$$z_i = HX_i + \eta_i$$

$$X_i = \left| \begin{matrix} x_i \\ V_i \end{matrix} \right|$$

## Prediction procedure in Kalman filter

### **1** Prediction (extrapolation)

Prediction of state vector at time i using i-1 measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1}$$

**Prediction error covariance matrix** 

$$P_{i,i-1} = \Phi_{i,i-1}P_{i-1,i-1}\Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

 $X_{i,i-1}$ 

First subscript *i* denotes time on which the prediction is made



Second subscript i-1 represents the number of measurements to get  $X_{i,i-1}$ 

## Filtration procedure in Kalman filter

# **2** Filtration Adjustment of predicted estimate

Improved estimate by incorporating a new measurement

$$X_{i,i} = X_{i,i-1} + K_i(z_i - HX_{i,i-1})$$
Residual

Filter gain, weight of residual

$$K_{i} = P_{i,i-1}H_{i}^{T}(H_{i}P_{i,i-1}H_{i}^{T} + R_{i})^{-1}$$

Filtration error covariance matrix

$$P_{i,i} = (I - K_i H_i) P_{i,i-1}$$

$$P_{i,i} = E[(X_i - X_{i,i})(X_i - X_{i,i})^T]$$

# Standard Kalman filter provides optimal estimate

State noise and measurement noise are uncorrelated and unbiased

In practice these assumptions are often not true

Analysis and modifications of Kalman filter

#### **Biased state noise**



Random acceleration is biased 
$$E[a_i] = q \neq 0$$

How to take bias of acceleration into account in Kalman filter algorithm?

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# Prediction procedure in Kalman filter taking into account bias of state noise

**1** Prediction (extrapolation)

Prediction of state vector at time i using i-1 measurements

$$X_{i,i-1} = \Phi_{i,i-1}X_{i-1,i-1} + Gq$$

**Prediction error covariance matrix** 

$$P_{i,i-1} = \Phi_{i,i-1}P_{i-1,i-1}\Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T]$$

 $X_{i,i-1}$ 

First subscript *i* denotes time on which the prediction is made



Second subscript i-1 represents the number of measurements to get  $X_{i,i-1}$ 

### Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration 
$$a_i = e^{-\lambda T} a_{i-1} + \zeta_i$$

Uncorrelated noise with variance 
$$\sigma_{\zeta}^2 = \sigma_a^2 (1 - e^{-2\lambda T})$$

Value that is inverse to correlation interval

$$\lambda = 1000$$
  $a_i$  - uncorrelated noise  $\lambda = 0.1$   $a_i$  - correlated noise

$$\sigma_a^2$$
 Variance of acceleration