Skoltech
Skolkovo Institute of Science and Technology

“Space Data Processing: Making Sense of Experimental Data”

Laboratory work 6
Analysis of accuracy decrease of filtration in conditions of correlated biased state and measurement noise

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Term 4, March 28 – May 27, 2016
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Moving object which trajectory is disturbed by random acceleration

Motion model

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

State equation

Motion model →

$$x_i = x_{i-1} + V_{i-1}T + \frac{a_{i-1}T^2}{2}$$

$$V_i = V_{i-1} + a_{i-1}T$$

$$X_i = \begin{vmatrix} x_i \\ V_i \end{vmatrix}$$ → **State vector** → **It contains full information about the state of system at time i**

State equation →

$$X_i = \Phi X_{i-1} + G a_{i-1}$$

$$\Phi = \begin{vmatrix} 1 & T \\ 0 & 1 \end{vmatrix}$$ → **Transition matrix**

$$G = \begin{vmatrix} T^2/2 \\ T \end{vmatrix}$$ → **Input matrix**

Measurement equation

Measurements of coordinate x_i with error η_i → $z_i = x_i + \eta_i$

Measurement equation → $z_i = HX_i + \eta_i$

$$X_i = \begin{vmatrix} x_i \\ V_i \end{vmatrix}$$

$H = |1 \quad 0|$ → Observation matrix

Prediction procedure in Kalman filter

❶ Prediction (extrapolation)

Prediction of state vector at time i using $i-1$ measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1}$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1} P_{i-1,i-1} \Phi_{i,i-1}^{T} + Q_i$$

$$P_{i,i-1} = E\left[(X_i - X_{i,i-1})(X_i - X_{i,i-1})^T\right]$$

$X_{i,i-1}$ → **First subscript i denotes time on which the prediction is made** → **Second subscript $i-1$ represents the number of measurements to get $X_{i,i-1}$**

Filtration procedure in Kalman filter

❷ Filtration
Adjustment of predicted estimate

Improved estimate by incorporating a new measurement

$$X_{i,i} = X_{i,i-1} + K_i\underbrace{(z_i - HX_{i,i-1})}_{\text{Residual}}$$

Filter gain, weight of residual

$$K_i = P_{i,i-1}H_i^T\left(H_iP_{i,i-1}H_i^T + R_i\right)^{-1}$$

Filtration error covariance matrix

$$P_{i,i} = (I - K_iH_i)P_{i,i-1}$$

$$P_{i,i} = E[(X_i - X_{i,i})\left(X_i - X_{i,i}\right)^T]$$

Standard Kalman filter provides optimal estimate

State noise and measurement noise are uncorrelated and unbiased

In practice these assumptions are often not true

Analysis and modifications of Kalman filter

Biased state noise

a_i → Random acceleration

Random acceleration is biased → $E[a_i] = q \neq 0$

How to take bias of acceleration into account in Kalman filter algorithm?

Prediction procedure in Kalman filter

❶ Prediction (extrapolation)

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$X_{i,i-1}$ → **First subscript i denotes time on which the prediction is made** → **Second subscript $i-1$ represents the number of measurements to get $X_{i,i-1}$**

Prediction procedure in Kalman filter taking into account bias of state noise

❶ Prediction (extrapolation)

Prediction of state vector at time i using $i-1$ measurements

$$X_{i,i-1} = \Phi_{i,i-1} X_{i-1,i-1} + Gq$$

Prediction error covariance matrix

$$P_{i,i-1} = \Phi_{i,i-1} P_{i-1,i-1} \Phi_{i,i-1}^T + Q_i$$

$$P_{i,i-1} = E\left[(X_i - X_{i,i-1})\left(X_i - X_{i,i-1}\right)^T\right]$$

$X_{i,i-1}$ → **First subscript i denotes time on which the prediction is made** → **Second subscript $i-1$ represents the number of measurements to get $X_{i,i-1}$**

Correlated state noise

In practice correlated noise is often presented as a Gauss-Markov first-order process

Random acceleration → $a_i = e^{-\lambda T} a_{i-1} + \zeta_i$

ζ_i → **Uncorrelated noise with variance** $\sigma_\zeta^2 = \sigma_a^2(1 - e^{-2\lambda T})$

λ → **Value that is inverse to correlation interval**

$\lambda = 1000$ → a_i **- uncorrelated noise**

$\lambda = 0.1$ → a_i **- correlated noise**

T → **Time interval between measurements**

σ_a^2 → **Variance of acceleration**