Skoltech

Skolkovo Institute of Science and Technology

“Space Data Processing: Making Sense of Experimental Data”

Development of tracking filter of a moving object when measurements and motion model are in different coordinate systems

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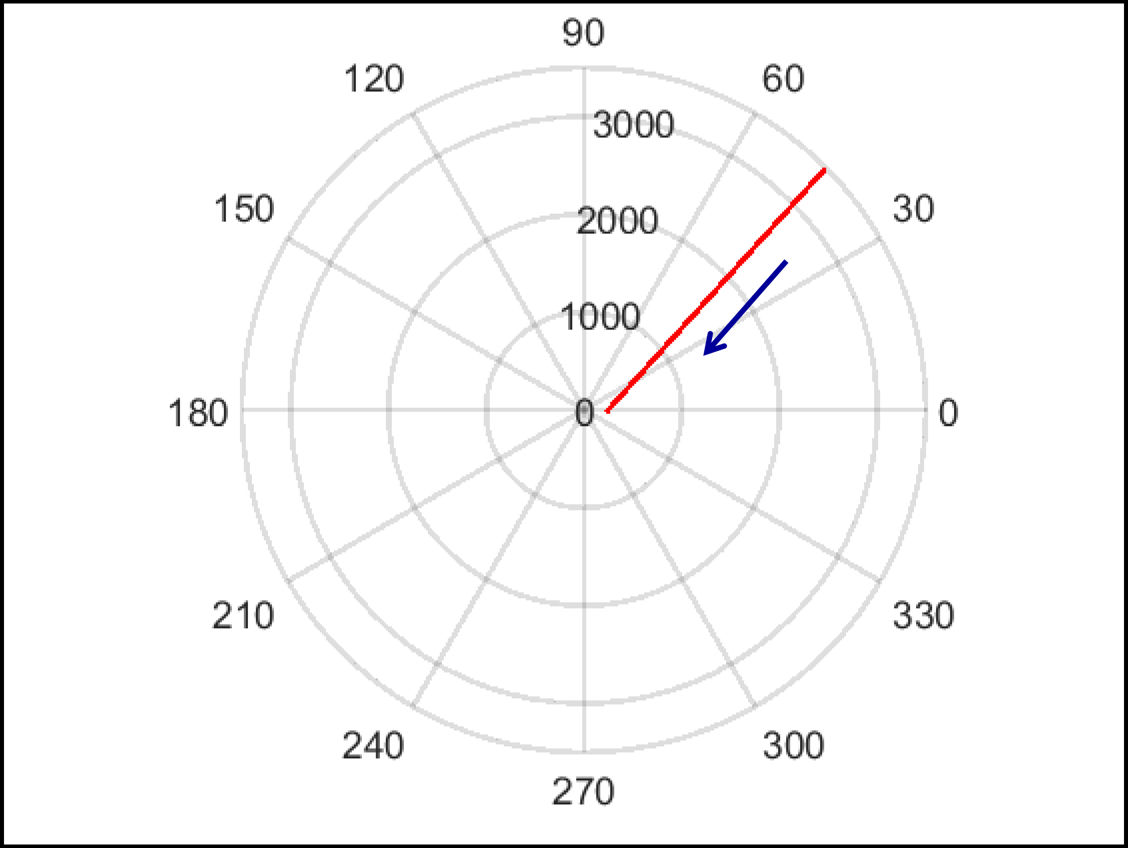
Measurements of navigation parameters are available in polar coordinate system



Range D → **Distance from an observer to a moving object**

Azimuth β → **Angle between direction to North and direction to a moving object**

Motion model is in Cartesian coordinate system

Model of uniform motion →

$$x_i = x_{i-1} + V_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T$$
$$V_i^x = V_{i-1}^x$$
$$V_i^y = V_{i-1}^y$$

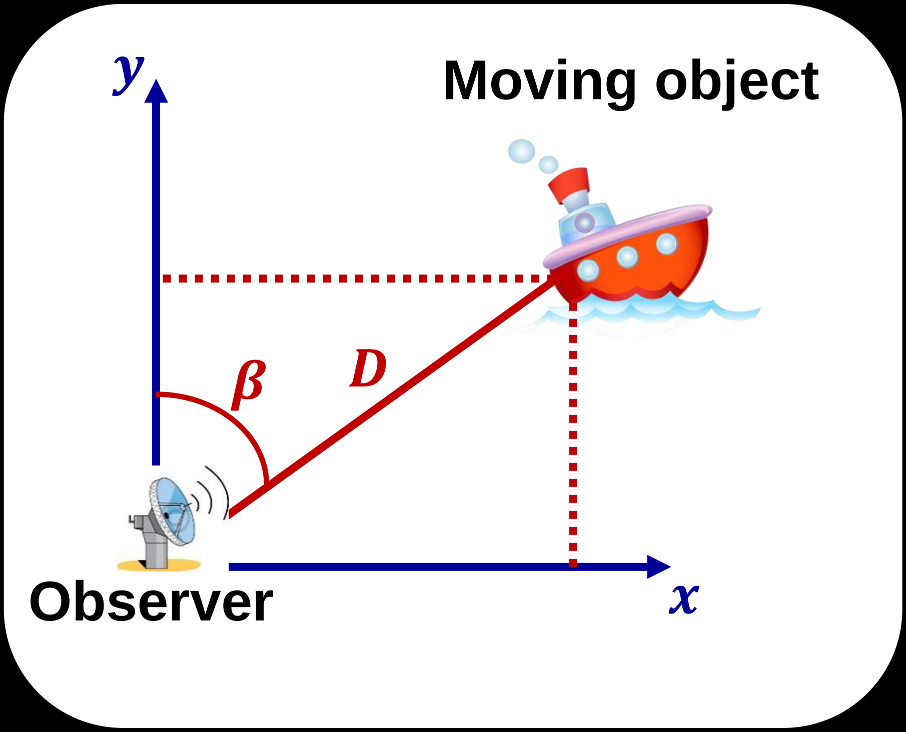
→ Cartesian coordinate system

x_i, y_i → Cartesian coordinates

V_i^x, V_i^y → Components of velocity V_i

$$V_i = \sqrt{\left(V_i^x\right)^2 + \left(V_i^y\right)^2}$$

Transformation from polar to Cartesian coordinate system



$$D = \sqrt{x^2 + y^2}$$

$$\beta = arctg\left(\frac{x}{y}\right)$$

$$x = D\sin\beta$$
$$y = D\cos\beta$$

State-space model, state equation

State equation

$$X_i = \Phi_{i,i-1} X_{i-1}$$

State vector

$$X_i = \begin{vmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \end{vmatrix}$$

Transition matrix

$$\Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → **Measurements of range D**

β_i^m → **Measurements of azimuth β**

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → **Nonlinear relation**

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → **Nonlinear relation**

$$Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p \Rightarrow Z_i^p = \begin{vmatrix} \sqrt{x_i^2 + y_i^2} \\ arctg\left(\dfrac{x_i}{y_i}\right) \end{vmatrix} + \eta_i^p$$

State-space model, measurement equation

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$

D_i^m → Measurements of range D

β_i^m → Measurements of azimuth β

Measurement equation → $Z_i^p = h(X_i) + \eta_i^p$ → **Nonlinear relation**

$$Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p \rightarrow Z_i^p = \begin{vmatrix} \sqrt{x_i^2 + y_i^2} \\ arctg\left(\dfrac{x_i}{y_i}\right) \end{vmatrix} + \eta_i^p$$

$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$ → Vector of measurement errors of range D and azimuth β

Noises η_i^D, η_i^β are uncorrelated with each other and have variances $\sigma_D^2, \sigma_\beta^2$

Measurement equation
From nonlinear to linear equation

Transform polar measurements D_i^m and β_i^m to Cartesian coordinates → $x_i^m = D_i^m sin\beta_i^m$

$y_i^m = D_i^m cos\beta_i^m$

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$ → Pseudo-measurement vector → $Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$

Measurement equation
From nonlinear to linear equation

Transform polar measurements D_i^m and β_i^m to Cartesian coordinates → $x_i^m = D_i^m sin\beta_i^m$
$y_i^m = D_i^m cos\beta_i^m$

Measurement vector → $Z_i^p = \begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix}$ → Pseudo-measurement vector → $Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$

Measurement equation → $Z_i^c = HX_i + \eta_i^c$ → Linear relation

$H = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$

$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$ → Vector of pseudo-measurement errors of x_i^m and x_i^m

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector
of measurement errors
of range D_i^m and azimuth β^m

$$\begin{vmatrix} D_i^m \\ \beta_i^m \end{vmatrix} = \begin{vmatrix} D_i \\ \beta_i \end{vmatrix} + \eta_i^p$$

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector
of pseudo-measurement
errors of x_i and y_i

$$\begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix} = \begin{vmatrix} x_i \\ x_i \end{vmatrix} + \eta_i^c$$

Full differential, example 1

x
Function

$x = f(u)$

Full differential, example 1

x
Function

$$x = f(u)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u) - f(u)$$

Full differential, example 1

x
Function

$$x = f(u)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u) - f(u)$$

Taylor series

$$f(u + \Delta u) = f(u) + \frac{1}{1!}\frac{dx}{du}\Delta u + \frac{1}{2!}\frac{d^2x}{du^2}(\Delta u)^2 + \frac{1}{3!}\frac{d^3x}{du^3}(\Delta u)^3 + \ \ldots +$$

Full differential, example 1

x
Function

$$x = f(u)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u) - f(u)$$

Taylor series

$$f(u + \Delta u) = f(u) + \frac{1}{1!}\frac{dx}{du}\Delta u + \frac{1}{2!}\frac{d^2x}{du^2}(\Delta u)^2 + \frac{1}{3!}\frac{d^3x}{du^3}(\Delta u)^3 + \ \ldots +$$

$$\Delta x = f(u + \Delta u) - f(u) \approx \frac{dx}{du}\Delta u$$

$\Delta u \rightarrow 0$

Differential dx
Infinitely small increment of function

$$dx = \frac{dx}{du}du$$

Full differential, example 2

x
Function

$$x = f(u, v)$$

Δx
Small increment of function

$$\Delta x = f(u + \Delta u, v + \Delta \mathrm{v}) - f(u, v)$$

$$\Delta x \approx \frac{dx}{du}\Delta u + \frac{dx}{dv}\Delta v$$

$$\Delta u \to 0$$
$$\Delta v \to 0$$

Full differential dx
Infinitely small increment of function

$$dx = \frac{dx}{du}du + \frac{dx}{dv}\mathrm{dv}$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix} \Rightarrow$$

Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} \Rightarrow$$

Vector of pseudo-measurement errors of x_i^m and y_i^m

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector of pseudo-measurement errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$

$$y_i^m = D_i^m cos\beta^m$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector
of measurement errors
of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector
of pseudo-measurement
errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2

$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$

Vector
of measurement errors
of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$

Vector
of pseudo-measurement
errors of x_i^m and y_i^m

1

$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2

$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

3

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\eta_i^p = \begin{vmatrix} \eta_i^D \\ \eta_i^\beta \end{vmatrix}$$ ⇒ Vector of measurement errors of range D_i^m and azimuth β

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix}$$ ⇒ Vector of pseudo-measurement errors of x_i^m and y_i^m

1
$$x_i^m = D_i^m sin\beta^m$$
$$y_i^m = D_i^m cos\beta^m$$

2
$$x_i^m = x_i + \eta_i^x$$
$$y_i^m = y_i + \eta_i^y$$

3
$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

Rewrite (1) using (2) and (3) for x_i

$$x_i + \eta_i^x = \left(D_i + \eta_i^D\right) sin\left(\beta_i + \eta_i^\beta\right)$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = \left(D_i + \eta_i^D\right) sin(\beta_i + \eta_i^\beta)$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = \left(D_i + \eta_i^D\right) sin(\beta_i + \eta_i^\beta)$$

$$\Downarrow$$

$$\eta_i^x = \left(D_i + \eta_i^D\right) sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = (D_i + \eta_i^D) sin(\beta_i + \eta_i^\beta)$$

$$\eta_i^x = (D_i + \eta_i^D) sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

$$\eta_i^x = (D_i + \eta_i^D) sin\left(\beta_i + \eta_i^\beta\right) - D_i sin\beta_i$$

$$x = Dsin\beta$$

How to determine dependence of η_i^c on η_i^p?

$$x_i + \eta_i^x = (D_i + \eta_i^D)sin(\beta_i + \eta_i^\beta)$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - x_i$$

$$\eta_i^x = (D_i + \eta_i^D)sin\left(\beta_i + \eta_i^\beta\right) - D_i sin\beta_i$$

$$x = Dsin\beta$$

Or

$$\Delta\mathrm{x} = (D + \Delta\mathrm{D})sin(\beta + \Delta\beta) - Dsin\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta x = (D + \Delta D)sin(\beta + \Delta\beta) - Dsin\beta$$

$$x = Dsin\beta$$

$$\Delta y = (D + \Delta D)cos(\beta + \Delta\beta) - Dsin\beta$$

$$y = Dcos\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta x = (D + \Delta D) sin(\beta + \Delta\beta) - Dsin\beta$$

$$x = Dsin\beta$$

$$\Delta y = (D + \Delta D) cos(\beta + \Delta\beta) - Dsin\beta$$

$$y = Dcos\beta$$

Taylor series

$$(D + \Delta D) sin(\beta + \Delta\beta) \approx Dsin\beta + \frac{dx}{dD}\Delta D + \frac{dx}{d\beta}\Delta\beta$$

$$(D + \Delta D) cos(\beta + \Delta\beta) \approx Dcos\beta + \frac{dy}{dD}\Delta D + \frac{dy}{d\beta}\Delta\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta \mathrm{x} = (D + \Delta \mathrm{D})sin(\beta + \Delta\beta) - Dsin\beta$$

$$\mathrm{x} = Dsin\beta$$

$$\Delta \mathrm{y} = (D + \Delta \mathrm{D})cos(\beta + \Delta\beta) - Dsin\beta$$

$$\mathrm{y} = Dcos\beta$$

Taylor series

$$(D + \Delta \mathrm{D})sin(\beta + \Delta\beta) \approx Dsin\beta + \frac{dx}{dD}\Delta \mathrm{D} + \frac{dx}{d\beta}\Delta\beta$$

$$(D + \Delta \mathrm{D})cos(\beta + \Delta\beta) \approx Dcos\beta + \frac{dy}{dD}\Delta \mathrm{D} + \frac{dy}{d\beta}\Delta\beta$$

$$\Delta \mathrm{x} = \frac{dx}{dD}\Delta \mathrm{D} + \frac{dx}{d\beta}\Delta\beta$$

$$\Delta \mathrm{y} = \frac{dy}{dD}\Delta \mathrm{D} + \frac{dy}{d\beta}\Delta\beta$$

How to determine dependence of η_i^c on η_i^p?

$$\Delta x = \frac{dx}{dD}\Delta D + \frac{dx}{d\beta}\Delta\beta \Rightarrow \eta^x = \frac{dx}{dD}\eta^D + \frac{dx}{d\beta}\eta^\beta$$

$$\Delta y = \frac{dy}{dD}\Delta D + \frac{dy}{d\beta}\Delta\beta \Rightarrow \eta^y = \frac{dy}{dD}\eta^D + \frac{dy}{d\beta}\eta^\beta$$

$$\frac{dx}{dD} = \sin\beta$$

$$\frac{dx}{d\beta} = D\cos\beta$$

$$\frac{dy}{dD} = \cos\beta$$

$$\frac{dy}{d\beta} = -D\sin\beta$$

Summary, scheme of estimation algorithm

1. Measurements are available in polar coordinate system

- D_i^m — Measurements of range D
- β_i^m — Measurements of azimuth β

2. Transform polar measurements D_i^m and β_i^m to Cartesian coordinates

$$x_i^m = D_i^m sin\beta_i^m$$
$$y_i^m = D_i^m cos\beta_i^m$$

3. Develop Kalman filter with pseudo-measurements in Cartesian coordinates

$$Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix}$$

Summary, state-space model

State equation

$$X_i = \Phi_{i,i-1} X_{i-1}$$

$$X_i = \begin{vmatrix} x_i \\ V_i^x \\ y_i \\ V_i^y \end{vmatrix} \qquad \Phi_{i,i-1} = \begin{vmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Measurement equation

$$Z_i^c = H X_i + \eta_i^c$$

$$Z_i^c = \begin{vmatrix} x_i^m \\ y_i^m \end{vmatrix} \Rightarrow \begin{aligned} x_i^m &= D_i^m sin\beta_i^m \\ y_i^m &= D_i^m cos\beta_i^m \end{aligned}$$

$$H = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} = \begin{vmatrix} \eta_i^D sin\beta_i^m + \eta_i^\beta D_i^m cos\beta_i^m \\ \eta_i^D cos\beta_i^m - \eta_i^\beta D_i^m sin\beta_i^m \end{vmatrix}$$

Noises η_i^D, η_i^β - errors of D_i^m and β_i^m

Are characterized by variances $\sigma_D^2, \sigma_\beta^2$

Covariance matrix of measurement error R

Measurement noise

$$\eta_i^c = \begin{vmatrix} \eta_i^x \\ \eta_i^y \end{vmatrix} = \begin{vmatrix} \eta_i^D sin\beta_i^m + \eta_i^\beta D_i^m cos\beta_i^m \\ \eta_i^D cos\beta_i^m - \eta_i^\beta D_i^m sin\beta_i^m \end{vmatrix}$$

R Covariance matrix

$$R = E[\eta_i^c \cdot (\eta_i^c)^T]$$

$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + (D_i^m)^2cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

σ_D^2 — Variance of measurement error of range D

σ_β^2 — Variance of measurement error of azimuth β

Goals of the laboratory work

1 **Analyze instability zone of a filter**

When object is close to observer → Navigation system may become blind → Related with ill-conditioned matrix R

Condition number of measurement error covariance matrix R

$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + (D_i^m)^2cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + (D_i^m)^2sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

σ_D^2 → Variance of measurement error of range D

σ_β^2 → Variance of measurement error of azimuth β

Eigen values of matrix R ⇒ $det(R - \lambda I) = 0$ ⇒ $\lambda_1 = \sigma_D^2$, $\lambda_2 = (D_i^m)^2\sigma_\beta^2$

Condition number of measurement error covariance matrix R

$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

σ_D^2 → Variance of measurement error of range D

σ_β^2 → Variance of measurement error of azimuth β

Eigen values of matrix R → $det(R - \lambda I) = 0$ → $\lambda_1 = \sigma_D^2$, $\lambda_2 = (D_i^m)^2\sigma_\beta^2$

Condition number → $\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{(D_i^m)^2\sigma_\beta^2}$ → $\frac{\lambda_2}{\lambda_1} = \frac{(D_i^m)^2\sigma_\beta^2}{\sigma_D^2}$

$\lambda_1 > \lambda_2$

$\lambda_2 > \lambda_1$

Condition number of measurement error covariance matrix R

Condition number → $\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$ → $\frac{\lambda_2}{\lambda_1} = \frac{(D_i^m)^2 \sigma_\beta^2}{\sigma_D^2}$

$\lambda_1 > \lambda_2$

$\lambda_2 > \lambda_1$

If condition number of matrix $R \approx 1$ → R – well-condtioned matrix → $\sigma_D^2 \approx (D_i^m)^2 \sigma_\beta^2$

Condition number of measurement error covariance matrix R

Condition number → $\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$ → $\frac{\lambda_2}{\lambda_1} = \frac{(D_i^m)^2 \sigma_\beta^2}{\sigma_D^2}$

$\lambda_1 > \lambda_2$ $\qquad$ $\lambda_2 > \lambda_1$

If condition number of matrix R ≈ 1 → **R – well-condtioned matrix** → $\sigma_D^2 \approx (D_i^m)^2 \sigma_\beta^2$

$$R = \begin{vmatrix} sin^2\beta_i^m \sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m \sigma_\beta^2 & sin\beta_i^m cos\beta_i^m [\sigma_D^2 - (D_i^m)^2 \sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m [\sigma_D^2 - (D_i^m)^2 \sigma_\beta^2] & cos^2\beta_i^m \sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

$$\Downarrow$$

$$R = \begin{vmatrix} sin^2\beta_i^m \sigma_D^2 + (D_i^m)^2 cos^2\beta_i^m \sigma_\beta^2 & 0 \\ 0 & cos^2\beta_i^m \sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m \sigma_\beta^2 \end{vmatrix}$$

Independent assimilation for x, y

Condition number of measurement error covariance matrix R

Condition number

$$\frac{\lambda_1}{\lambda_2} = \frac{\sigma_D^2}{\left(D_i^m\right)^2 \sigma_\beta^2}$$

$\lambda_1 > \lambda_2$

$$\frac{\lambda_2}{\lambda_1} = \frac{(D_i^m)^2 \sigma_\beta^2}{\sigma_D^2}$$

$\lambda_2 > \lambda_1$

If condition number of matrix $R > 1000$

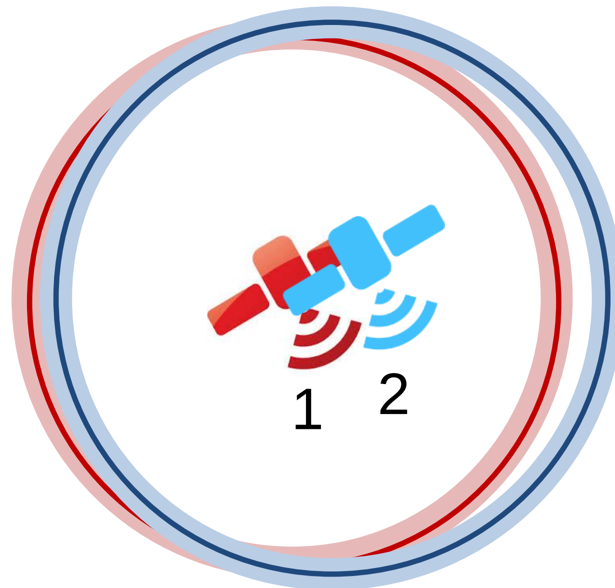
R – ill-condtioned matrix

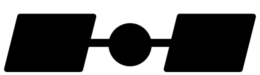
$$R = \begin{vmatrix} sin^2\beta_i^m\sigma_D^2 + (D_i^m)^2cos^2\beta_i^m\sigma_\beta^2 & sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] \\ sin\beta_i^m cos\beta_i^m[\sigma_D^2 - (D_i^m)^2\sigma_\beta^2] & cos^2\beta_i^m\sigma_D^2 + (D_i^m)^2 sin^2\beta_i^m\sigma_\beta^2 \end{vmatrix}$$

Estimation accuracy is decreased
Filter may diverge

Ill-conditioned problem

Validity of applying a technique



 Man-made satellite

 Navigation satellite

Il-conditioned problem

Satellite position is undefined!

Goals of the laboratory work

2

Use a prior information to increase tracking accuracy