**Laboratory work 11**
**Extended Kalman filter for navigation and tracking**

Performance -Thursday, May 12, 2016
Due to submit a performance report – Tuesday, May 17, 2016

The objective of this laboratory work is to develop Extended Kalman filter for tracking a moving object when measurements and motion model are in different coordinate systems. This will bring about a deeper understanding of main difficulties of practical Kalman filter implementation for nonlinear models.

This laboratory work is performed in the class by students as in teams of 2 on May 12, 2016 and and the team will submit one document reporting about the performance till May 17, 2016. Within your group, you may discuss all issues openly, and discuss and debate until you reach a consensus.

1. ***Here is the recommended procedure:***
   Generate a true trajectory $X_i$ of an object motion disturbed by normally distributed random acceleration

$$x_i = x_{i-1} + V_{i-1}^x T + \frac{a_{i-1}^x T^2}{2}$$
$$V_i^x = V_{i-1}^x + a_{i-1}^x T$$
$$y_i = y_{i-1} + V_{i-1}^y T + \frac{a_{i-1}^y T^2}{2}$$
$$V_i^y = V_{i-1}^y + a_{i-1}^y T$$

   ***Initial conditions to generate trajectory***

   (a) Size of trajectory is $N = 500$ points.

   (b) $T = 1$ – interval between measurements.

   (c) Initial coordinates
   $x_0 = 1000;\ y_0 = 1000$

   (a) Initial components of velocity $V$
   $V_x = 10; V_y = 10;$

   (b) Variance of noise $a_i$, $\sigma_a^2 = 0.3^2$ for both $a_i^x, a_i^y$

2. Generate also true values of range $D$ and azimuth $\beta$

$$D_i = \sqrt{x_i^2 + y_i^2}$$
$$\beta_i = arctg\left(\frac{x}{y}\right)$$

3. Generate measurements $D^m$ and $\beta^m$ of range $D$ and azimuth $\beta$

$$D_i^m = D_i + \eta_i^D$$
$$\beta_i^m = \beta_i + \eta_i^\beta$$

   Variances of measurement noises $\eta_i^D, \eta_i^\beta$ are given by

$$\sigma_D^2 = 50^2$$
$$\sigma_\beta^2 = 0.004^2$$

4. Initial conditions for Extended Kalman filter algorithm
   Initial filtered estimate of state vector $X_{0,0}$

$$X_0 = \begin{vmatrix} D_i^m(1) sin\beta_i^m(1) \\ 0 \\ D_i^m(1) cos\beta_i^m(1) \\ 0 \end{vmatrix}$$

   Initial filtration error covariance matrix $P_{0,0}$
   First use great initial filtration error covariance matrix

$$P_{0,0} = \begin{vmatrix} 10^{10} & 0 & 0 & 0 \\ 0 & 10^{10} & 0 & 0 \\ 0 & 0 & 10^{10} & 0 \\ 0 & 0 & 0 & 10^{10} \end{vmatrix}$$

5. Create the transition matrix Φ
   Consult charts, page 27

6. Calculate state noise covariance matrix $Q$

$$Q = G\mathrm{G}^T\sigma_a^2$$

7. Create the measurement noise covariance matrix $R$

$$R = \begin{vmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\beta^2 \end{vmatrix}$$

8. At every filtration step in the algorithm you should linearize measurement equation by determining

$$\frac{dh(\hat{X}_{i+1,i})}{dX_{i+1}}$$

   Consult charts, page 32

9. Develop Kalman filter algorithm to estimate state vector $X_i$ (extrapolation and filtration). Using extrapolated and filtered estimates at every extrapolation and filtration step you will need to calculate
   (a) range $D$
   (b) azimuth $\beta$

10. Run Kalman filter algorithm over $M = 500$ runs.
    Calculate true estimation errors of
    (a) Errors of extrapolation and filtration estimates of range $D$
    (b) Errors of extrapolation and filtration estimates of azimuth $\beta$

11. Compare estimation results with measurement errors of $D$ and $\beta$.

***Performance report***

1. Performance report should contain all the items listed

2. The code should be commented. It should include:
   - Title of the laboratory work, for example
     % Converting a physical distance to a grid distance using least-square method
   - The names of a team, indication of Skoltech, and date, for example,
     %Tatiana Podladchikova, Skoltech, 2016
     Main procedures also should be commented, for example
     %13-month running mean
     …here comes the code

3. If your report includes a plot, then it should contain: title, title of x axis, title of y axis, legend of lines on plot.