GameStop: A study in the Black-Scholes Formula

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Abstract

This is a simple paragraph at the beginning of the document. A brief introduction to the main subject.

1 Introduction

In the first section of my essay, I plan to outline what happened in February with GameStop and the internet, showing graphs of the situation, for example in GameStop, in order to motivate the reader in our analysis of the Black-Scholes formula under the bizarre situation of GameStop. I can also talk about similar situations in the past of similar significant "short squeezes", and consider how analysing GameStop could allow us to predict future short squeezes with different companies by analysing their financial situation.

I would then move on to explaining the Black-Scholes Model, and could attempt a simple derivation of the Black-Scholes Formula. I can then move on to solving the formula for different intial conditions and variables, using my tools from Programming for Scientists last year to generate graphs of the predicted stock value (and value of options) in Python, where suitable. I will then take a conclusion from my calculations, deciding whether the prelevence of GameStop on messageboards and the internet is what caused the large spike, or if the high short interest in GameStop meant that it was inevitable to happen anyway.

I would expect to use Jupyter notebook and Python for graph sketching, and I plan to reference "Stochastic Differential Equations: An Introduction with Applications" by Bernt Karsten Øksendal, and "Options, Futures and Other Derivatives" by John C. Hull as part of writing my essay, as they contain critical information relevant to my topic. Below is the Black-Scholes formula:

$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$ Where t is time in years, r is the interest rate, S is the price of the underlying asset and σ is the standard deviation of the stock's returns.

I can also talk about other, more complicated financial models and how they compared to Black-Scholes, for example which assumptions they start to calculate which are ignored in Black-Scholes.

$A(t) = rA(t)$

Let's cite! Einstein's journal paper [2] and Dirac's book [1] are physics-related items.

2 What are shares, options, and short selling?

3 What happened to GameStop?

4 Asset dynamics

4.1 A risk-free asset

A risk-free asset, as defined in The Black-Scholes Model [4] can be thought of as a money-market account with zero risk, for example a bank account with an interest rate. It is described by the deterministic function

$$dA(t) = rA(t)\mathrm{d}t \tag{1}$$

Where in 1, r represents the risk-free rate and A the value of the asset. We typically let $r > 0$ and $A(0) = 1$ for convenience. This can be rewritten as the ordinary differential equation $A'(t) = rA(t)$, which has the unique solution

$$A(t) = e^{rt} \tag{2}$$

4.2 Ito processes

According to Ito process [3], an Ito process is the stochastic (random) process $X = \{X_t, t >= 0\}$ which solves

$$X_t = X_0 + \int_0^t a(X_s, s)\mathrm{d}s + \int_0^t b(X_s, s)\mathrm{d}W_s \tag{3}$$

Here X_0 is the "scalar starting point", and a, b are stochastic processes defined for $\{a(X_t, t) : t \geq 0\}$ and $\{b(X_t, t) : t \geq 0\}$. Then we call a the drift of the variable, and b is the diffusion, or more precisely in a financial context, volatity. 3 is commonly written as

$$X_t = a(X_t, t)\mathrm{d}t + b(X_t, t)\mathrm{d}W_t \tag{4}$$

or even

$$X_t = a_t\mathrm{d}t + b_t\mathrm{d}W_t \tag{5}$$

This converges at some time T to Brownian motion with instantaneous drift a_T and variance b_T^2. We let dW be normally distributed with mean zero and variance $\mathrm{d}t$. Then we can further simplify 3 as

$$\mathrm{d}X_t = a_t\mathrm{d}t + b_t\sqrt{\mathrm{d}t}\xi \tag{6}$$

where

$$\xi \sim \mathcal{N}(\mu, \sigma^2) \tag{7}$$

As defined in [3], Ito's lemma is

Lemma 4.1 (Ito's Lemma) *Suppose $f : R \to R$ is twice continuously differentiable and $\mathrm{d}X = a_t\mathrm{d}t + b_t\mathrm{d}W$. Then $f(X)$ is the Ito process,*

$$f(X_t) = f(X_0) + \int_0^t f'(X_s)a_s\,\mathrm{d}s + \int_0^t f'(X_s)b_s\,\mathrm{d}W + \frac{1}{2}\int_0^t f''(X_s)b_s^2\,\mathrm{d}s \tag{8}$$

for $t \geq 0$

4.3 A risky asset

A risk asset, as defined in The Black-Scholes Model [4], can be thought of as a stock, is represented as the Ito process

$$\mathrm{d}S(t) = \mu S(t)\mathrm{d}t + \sigma S(t)\mathrm{d}W(t) \tag{9}$$

with $S(0)$ a given starting price of the stock, $\mu \in \mathbb{R}$ is again the drift and $\sigma > 0$ is the volatility of the stock price S.

5 Options

6 The Black-Scholes Model

7 Modelling GameStop

8 Extensions to the Black-Scholes Model

9 Conclusion

References

[1] Paul Adrien Maurice Dirac. *The Principles of Quantum Mechanics*. International series of monographs on physics. Clarendon Press, 1981. ISBN: 9780198520115.

[2] Albert Einstein. "Zur Elektrodynamik bewegter Körper. (German) [On the electrodynamics of moving bodies]". In: *Annalen der Physik* 322.10 (1905), pp. 891–921. DOI: http://dx.doi.org/10.1002/andp.19053221004.

[3] Prof. Yuh-Dauh Lyuu. *Ito Process*. URL: https://www.csie.ntu.edu.tw/~lyuu/finance1/2012/20120418.pdf. (accessed: 15.04.2022).

[4] Ekkehard Kopp Marek Capinski. *The Black-Scholes Model*. Cambridge University Press, 2012. ISBN: 9780521173001.