Memory has Many Faces: Simplicial Complexes as Agent Memory Layers

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***Abstract*—Current memory architectures for large language model agents employ vector embeddings and graph-based representations, with both approaches exhibiting significant performance gaps. While retrieval- augmented generation achieves strong results on benchmarks like 86% accuracy on LongMemEval, graph-based methods significantly underperform at 71%. This suggests that existing graph representations fail to capture essential memory structures. This paper proposes simplicial complexes as a unifying representation for agent memory that preserves higher-order relationships without the information loss inherent in pairwise graph projections. We introduce a database-backed simplex tree architecture that enables efficient storage and retrieval of multi-way interactions while supporting standard memory operations. Through a category-theoretic lens, we demonstrate that simplicial complexes occupy a privileged position in the hierarchy of knowledge representations, with structure-preserving functors to both graphs and vector spaces. Our approach enables the representation of contextual co-occurrence patterns (entities appearing together in conversations, or sessions) as geometric objects whose dimensional structure naturally encodes confidence and relationship strength.**

***Index Terms*—Agent Memory, Simplicial Complexes, Knowledge Graphs, RAG**

I. Introduction

Agent memory systems have converged on two paradigms: retrieving memories as vectors embeddings in semantic space, and knowledge graphs representing entity relationships explicitly. Despite recent advances, including hybrid architectures, performance gaps persist. On LongMemEval, RAG systems achieve 86% accuracy while graph-based methods reach only 71% [1].

This gap reflects a representational limitation. Vector embeddings collapse co-occurrence structure into continuous distances, losing discrete relationships. Knowledge graphs preserve structure but are constrained to pairwise interactions. When three or more entities co-occur meaningfully, such as multiple concepts in a conversation, or events in a session, pairwise graphs either discard this information [2] or attempt reconstruction through secondary inference [3]. Recent attempts at formalising this information loss include Wang and Kleinberg (2024), which proved the combinatorial impossibility of recovering higher-order structures from graph projections [4].

We propose simplicial complexes as a unifying representation that addresses these limitations. Simplicial complexes occupy a privileged position in the knowledge representation hierarchy, with structure-preserving functors to graphs and vectors while avoiding information loss from projection. The simplex tree data structure provides efficient database-backed implementation. This enables memory architectures preserving contextual co-occurrence patterns, yielding measurable improvements on multi-hop reasoning and temporal queries.

II. Motivation: A Category Theoretic Lens

Category. A category C consists of

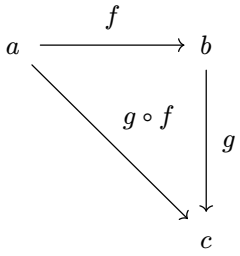
- A collection of objects $a, b, c, ...$ denoted formally as the **class** $\mathrm{ob}(C)$.
- A collection of **morphisms** (arrows) $\hom_C(a, b)$ for each object pair a, b in $\mathrm{ob}(C)$. The expression $f : a \to b$ indicates that f is a morphism that maps a to b, as depicted in the commutative diagram below. The collection of $\hom_C(a, b)$ for all object pairs a, b in $\mathrm{ob}(C)$ is denoted as $\hom(C)$.

$$a \xrightarrow{\quad f \quad} b$$

- The composition binary operator $\circ$; that is, for any three objects a, b, c in $\mathrm{ob}(C)$,

$$\circ : \hom_C(a, b) \times \hom_C(b, c) \to \hom_C(a, b) \quad (1)$$

That is, given morphisms $f : a \to b$ and $g : b \to c$, the morphisms $g \circ f : a \to c$ exists, depicted below.



Furthermore, the composition operation satisfies **associativity** and **identity** rules.

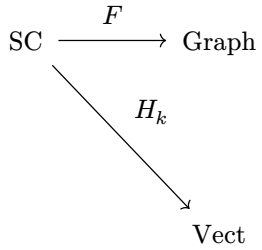
1) **Associativity**: For four objects a, b, c, d in $\mathrm{ob}(C)$ and morphisms $f : a \to b,\ g : b \to c,\ h : c \to d$, we have
$$(h \circ g) \circ f = h \circ (g \circ f) \quad (2)$$
2) **Identity**: For every object x in $\mathrm{ob}(C)$, there exists an identity morphism $\mathrm{id}_x : x \to x$ such that for every morphism $f : a \to b$, we have
$$\mathrm{id}_b \circ f = f = f \circ \mathrm{id}_a \quad (3)$$

Some examples of categories and their morphisms are sets and functions, groups and group homeomorphisms, vector spaces and linear maps, and topologies and continuous maps.

Functor. A functor $F : C \to D$ is a **structure-preserving** map between categories C and D. In particular,

- it assigns each object a in $\mathrm{ob}(C)$ an object $F(a)$ in $\mathrm{ob}(D)$.
- it assigns each morphism $f : a \to b$ in $\mathrm{hom}(C)$ a morphism $F(f) : F(a) \to F(b)$ in $\mathrm{hom}(D)$, such that $F(\mathrm{id}_x) = \mathrm{id}_{F(x)}$ and $F(f \circ g) = F(f) \circ F(g)$.

With that in mind, we can view representations as categories: vector embeddings as **Vect**, directed labelled graphs as **Vect**, and simplicial complexes as **SC**. Mapping from simplicial complexes to graphs happens via the **1-skeleton functor** F. Likewise, mapping from simplicial complexes to vector spaces happens via the **simplicial homology functor** H_k.



However, as seen from the above diagram, no natural functor exists from graphs back to simplicial complexes. Given three mutually connected entities $\{a, b, c\}$, the graph doesn't indicate whether these entities co-occurred together (a filled triangle) or merely pairwise across contexts (an empty triangle) as seen in Fig. 1. This presents an implication for agent memory, When users discuss multiple concepts together, these co-occurrences carry semantic significance beyond pairwise projections. Graph-based systems either discard this at storage or attempt reconstruction at retrieval, both degrading performance.

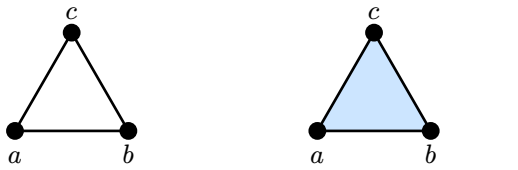


Fig. 1. A graph represents only pairwise edges, while a simplicial complex captures the 2-simplex $\{a, b, c\}$ as a filled face, encoding joint co-occurrence.

Simplicial complexes avoid this by representing higher-order co-occurences as first class objects. The functor hierarchy establishes representational power: complexes project to graphs and to embeddings, but systems at lower levels cannot recover richer structure. This points towards how memory systems should store information in the most structured form and project to coarser representations only when required.

III. Simplicial Complex for Knowledge

Simplicial Complex. A simplicial complex (Fig. 2) is a pair $R = (V, S)$ where

- V is the **vertex set** $V = \{v_1, v_2, ..., v_n\}$
- S is the **simplex/face set**. It is the set of non-empty subsets of V that satisfies the following properties by construction:
  1) $v \in V \Rightarrow \{v\} \in S$
  2) $s_1 \in S, s_2 \subseteq s_1 \Rightarrow s_2 \in S$. Here, s_2 is called the **face** of the simplex s_1. Furthermore, if $s_2 \subset s_1$, s_2 is called the **proper face** of the simplex s_1.

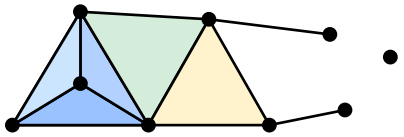


Fig. 2. A simplicial complex containing a 3-simplex (blue tetrahedron), 2-simplices (filled triangles), 1-simplices (edges), and 0-simplices (vertices). By downward closure, all faces of higher-dimensional simplices are automatically included.

The second property of **downward closure** for simplicial complexes distinguishes simplicial complexes from other possible candidates of knowledge representation like hypergraphs. If entities $\{v_1, v_2, v_3\}$ exists, then $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\{v_1, v_3\}$, $\{v_1\}$, $\{v_2\}$, $\{v_3\}$ must exist. This captures semantic intuition: joint co-occurrence implies all sub-co-occurrences occurred. If three concepts were discussed together, each pair was discussed, and each individually. The way to describe such co-occurence is baked into the structure of simplicial complexes. If $s \in S$ has $k + 1$ elements, where $k \geq 0$, s is said to be a ***k*-simplex** of dimension k. A point is a 0-simplex, a line a 1-simplex, a filled triangle a 2-simplex, and a tetrahedron a 3-simplex (Fig. 3).

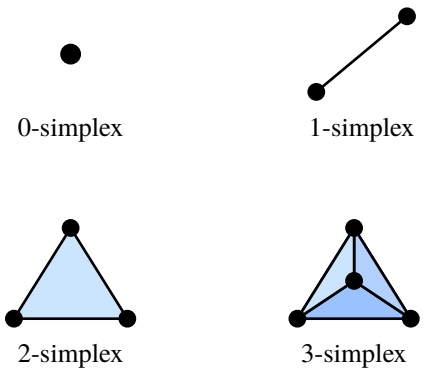


Fig. 3. Simplices of dimension 0 through 3: a vertex, edge, filled triangle, and tetrahedron.

For agent memory, this aligns with how knowledge emerges from contexts. A chat or search session referencing entities "neural networks", "backpropagation", and "gradient descent" produces a 2-simplex. By downward closure, this either creates edges for pairwise co-occurrences automatically, or implies the existence of co-occurences not yet uncovered, ensuring consistency. This idea is generalised in the structure of

the simplicial complex through **facets**, which are the maximal dimension proper faces of a simplex. This involves taking the possible combinations of k points of a k-simplex; a 3-simplex $s = \{v_1, v_2, v_3, v_4\}$ will have the facets $\{v_1, v_2, v_3\}$, $\{v_2, v_3, v_4\}$, $\{v_1, v_3, v_4\}$, $\{v_1, v_2, v_4\}$. At the same time, the same simplex itself could be contained in another higher-dimensional simplices representing deeper structural pattern. Such a collection of higher-dimensional simplices are called **cofaces**; the cofaces of a 1-simplex $s = \{v_1, v_2\}$ could be something like $\{v_1, v_2, v_3\}$, $\{v_1, v_2, v_4\}$, $\{v_1, v_2, v_3, v_4\}$. More examples are shown in Fig. 4.

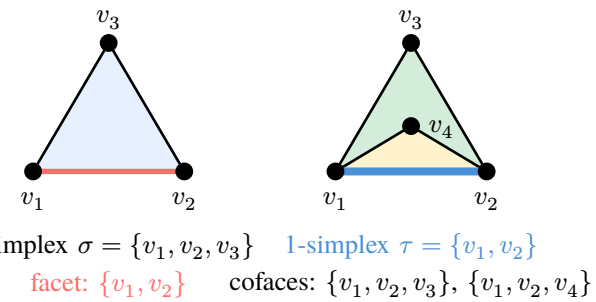


Fig. 4. Left: A facet of the 2-simplex $\{v_1, v_2, v_3\}$ is the edge $\{v_1, v_2\}$ (highlighted in red). Right: Cofaces of the 1-simplex $\{v_1, v_2\}$ (highlighted in blue) are the 2-simplices containing it.

These structures support natural memory operations. Coface lookup retrieves higher-dimensional simplices containing a query, identifying stronger contextual and narrative associations. Facet traversal descends to lower-dimensional faces, broadening the search when specificity must be relaxed.

When an agent retrieves knowledge relevant to a query, the retrieved simplices form a **subcomplex**, a simplicial complex $K' = (V', S')$ satisfying $V' \subseteq V$ and $S' \subseteq S$. However, behavioral data does not arrive in face-respecting order. A user may discuss entities $\{v_1, v_2, v_3\}$ together before ever discussing $\{v_1, v_2\}$ in isolation. The downward closure property defines a simplicial complex mathematically, but operational efficiency favours a different approach: we insert only directly observed simplices without materializing implied faces. A conversation yielding co-occurring entities $\{v_1, v_2, v_3\}$ produces a single 2-simplex insertion rather than the seven insertions required for full closure. At query time, faces that exist in the database represent independently confirmed co-occurrences, while faces computable from higher-dimensional simplices but absent from storage represent knowledge gaps—co-occurrences implied by context but never directly observed.

Given a query-induced subcomplex formed by retrieving cofaces of semantically matched vertices, we enumerate the theoretical faces of each retrieved simplex and check their existence in storage. Faces that are combinatorially required but absent from the database represent knowledge gaps local to the query context: the system has evidence that entities co-occur in some higher-order relationship, but lacks direct confirmation of the supporting lower-order structure. The agent may then pose clarifying questions to confirm these missing relationships, or discount confidence in inferences that depend on unobserved faces.

This integrates naturally with **filtration**. A filtration of simplicial complex K is an ordering of K such that all prefixes are subcomplexes of K. That is, for two simplices σ, τ in K such that $\sigma \subset \tau$, σ appears before τ in the ordering. For example, consider building a filled triangle $\{v_1, v_2, v_3\}$. A valid filtration orders the simplices as:

$$\{v_1\} \to \{v_2\} \to \{v_3\} \to \{v_1, v_2\} \to \{v_2, v_3\} \to \{v_1, v_3\} \to \{v_1, v_2, v_3\} \tag{4}$$

Each prefix forms a valid subcomplex: vertices appear before edges, and edges before the filled face (Fig. 5).

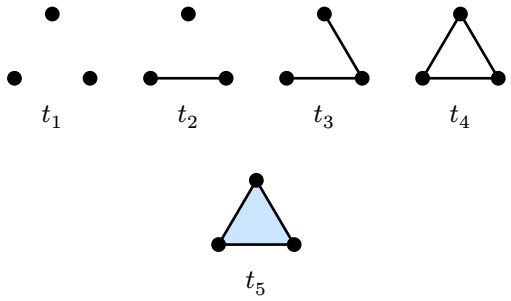


Fig. 5. A filtration building a 2-simplex. At t_1, only vertices exist. Edges are added at t_2-t_4. The filled 2-simplex appears at t_5, only after all its faces exist.

Rather than treating filtration as a global property, we compute it over the query-induced subcomplex. Closure-induced faces can be flagged for inference. The agent may then pose clarifying questions or bias search results to confirm missing relationships, or discount confidence in inferences that depend on closure-induced faces. The topology of the query-induced subcomplex thus serves not only as a retrieval mechanism but as an inference guide, directing attention toward gaps most relevant to the immediate task.

IV. Simplex Trees

The simplex tree, introduced by Boissonnat and Maria in 2014, provides an efficient data structure for representing abstract simplicial complexes of arbitrary dimension[5]. The simplex tree reconciles the need to explicitly store all faces of the complex with the desire for compact representation and efficient operations, making it particularly well-suited for database-backed memory systems.

For the simplicial complex $K = (V, S)$ of dimension k (that is, the dimension of the largest simplex in K), we label each vertice $v_i \in V$ a letter $l_i \in \{1, ..., |V|\}$ from the alphabet $1, ..., |V|$, where $1 \leq l_1 < ... < l_{|V|} \leq |V|$. Then, the simplex $s = \{v_1, ..., v_i\}$ can be represented as the word $[s] = [l_1, ..., l_j]$. This allows us to express every k-simplex $s \in S$ as a word of $(k+1)$ length with labels of ascending order. We start with an empty **trie**, and construct the tree as such (Fig. 6):

- Words are inserted from the root or node to the leaf of the tree, with the first letter as the root or node and the last letter as the leaf.
- If inserting a word $[s] = [l_1, ...l_j]$, and the longest prefix word already in the tree is $\{l_1, ..., l_i\}$, of which $i < j$, we append the rest of the word $[l_{i+1}, ..., l_j]$ to the l_i node.

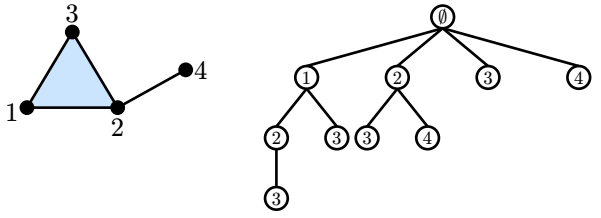


Fig. 6. A simplicial complex K with 2-simplex $\{1, 2, 3\}$ and edge $\{2, 4\}$ (left) and its simplex tree representation (right). Each path from root to node encodes a simplex: e.g., $\emptyset \to 1 \to 2 \to 3$ represents $\{1, 2, 3\}$.

In the original Boissonnat and Maria paper, the simplex tree was implemented as an in-memory structure. While in-memory databases could host these structures directly, they are unsuitable for persistent agent memory: RAM costs more per gigabyte than SSD storage, and requires continuous power to retain state. In-memory stores excel as caches, not as primary memory layers that must persist across sessions and scale with conversation history. We therefore implement the simplex tree using disk-backed persistence. Translating the requirements of the in-memory tree required some unique considerations:

1) An in-memory simplex tree uses red-black trees or hash tables for the trie structure, with the top nodes being an array. However, red-black trees are not used as database indexes due to being optimised for in-memory operations rather than disk-based operations[6]. Hash indexes are thus used in place, preserving the original time complexity of operations in the paper. A breakdown of the time complexity of operations in the persistent agent memory is given in Table I.

TABLE I
Time complexity of simplex tree operations.

Operation	**Purpose**	**Complexity**
Insert simplex	Record observed co-occurrence from extraction	$O(j)$
Membership check	Verify whether a face exists during gap detection	$O(j)$
Cofaces of vertex set	Find all simplices containing query-matched vertices	$O\left(kT^{>0}_{\text{last}(v)}\right)$
Enumerate theoretical faces	Compute expected faces for filtration comparison	$O(2^j)$
Delete simplex	Memory management, forgetting	$O(j)$

Here, j is the simplex dimension, k is the dimension of the simplicial complex, and $T^{>0}_{\text{last}(s)}$ is the total number of nodes in the simplex tree storing $\text{last}(s)$, the last letter of the simplex s, at a tree depth greater than 0. In practice, this value is bounded by $C_{\{\text{last}(s)\}}$, the number of cofaces containing the last letter of the simplex.

2) In the in-memory implementation, each sibling node has a pointer to its parent node, and for all nodes in the same tree depth of the same letter label l_i, they are connected in a circular linked list. Both cases are achievable in the relational model by using foreign-key primary-key relations, which can be shown in greater detail in the next section.

V. Database Design

We translate simplex trees to persistent database schemas with three core tables, adapting standard approaches to modelling graphs in relational models[7]. The full schema definition is provided in the Appendix.

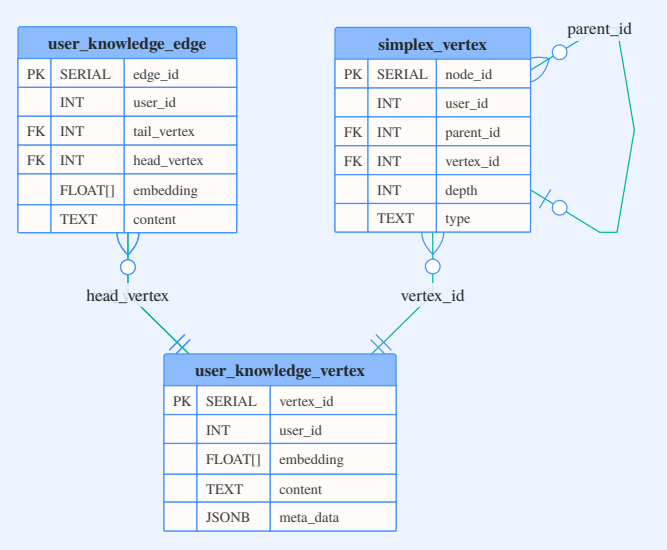


Fig. 7. Entity-relationship diagram for the persistence layer. The `simplex_vertex` table implements the trie structure with a self-referential foreign key for parent traversal. Both `user_knowledge_edge` and `simplex_vertex` reference vertices in `user_knowledge_vertex`.

There are several fields of interests here. The `meta_data` JSONB field encodes multi-dimensional context. An example body for what meta_data could look like is shown below.

```
{
  "first_created":
"2025-01-15T09:23:41Z",
  "last_updated": "2025-01-18T14:02:17Z",
  "frequency": 7,
  "confidence": 0.85,
  "latitude": 55.6116,
  "longitude": -4.4958,
  "session_id": "conv_a1b2c3d4"
}
```

Each key in the JSON body can be used for **witness complex** construction. That is, given the set of witnesses (W, d) and the set of landmarks (W, d) in metric space, a witness $w \in W$ witnesses a simplex $s \subseteq L$ if and only if for all $x \in s$ and for all $y \in L \setminus s$, we have $d(w, x) \leq d(w, y)$. The `type` TEXT field identifies which `meta_data` dimension generated the witness complex. Finally, we have the `depth` INT field, which records the depth of the simplex vertice in the simplex tree. This allows for efficient coface calculations without dynamic depth computation.

VI. Knowledge Extraction and Retrieval

When a user queries an agent or enters a search query, the knowledge extraction process transforms raw user interactions into simplicial structure through three stages.

1) **Entity resolution**, where mentions of entities are identified within user-provided content, linking surface forms to canonical representations in `user_knowledge_vertex`. New entities are inserted with computed embeddings; existing entities may have their metadata updated to reflect the new observation.
2) **Structured extraction** identifies relational patterns among resolved entities, populating `user_knowledge_edge` with labelled relationships where applicable. This stage also determines which entities co-occur within the witnessing context.
3) **Simplex construction** invokes `insert-simplex` for the observed co-occurrence set. The type field is set according to which metadata dimension generated the witness relationship.

In contrast, when user information or memories are required to augment a query result, knowledge retrieval goes through several steps:

1) Vertex matching performs approximate nearest-neighbour search over entity embeddings in `user_knowledge_vertex`, returning vertices whose semantic representations lie within a similarity threshold of the query embedding. These matched vertices form the query set Q.
2) For each vertex in Q, `locate-cofaces` is invoked. All simplices that contain at least one query-matched vertex is collected. The union of these coface sets forms the query-induced subcomplex K_Q.
3) Filtration comparison examines K_Q against the theoretical filtration implied by the matched vertices. For a set of n matched vertices, the complete simplex on those vertices would contain $2^n - 1$ faces. The algorithm enumerates these theoretical faces and checks membership in K_Q via `search-simplex`. Faces present in the theoretical complex but absent from K_Q represent topological holes: that is, gaps in knowledge.
4) Gap-driven inference surfaces these holes as implicit queries. If vertices representing "user's sister" and "Edinburgh" both match, but the 1-simplex connecting them is absent, the system may infer that the user's sister's location is unknown and prompt accordingly. This mechanism operationalises the homological perspective: holes in the simplicial complex correspond to gaps in relational knowledge. A reference implementation of these operations is provided in Section VII.B.

VII. Appendix

A. Database Schema Definition

```
CREATE TABLE user_knowledge_vertex (
    vertex_id   SERIAL PRIMARY KEY,
    user_id     INT NOT NULL,
    embedding   FLOAT[] NOT NULL,
    content     TEXT NOT NULL,
    meta_data   JSONB DEFAULT '{}'
);

CREATE TABLE user_knowledge_edge (
    edge_id     SERIAL PRIMARY KEY,
    user_id     INT NOT NULL,
    tail_vertex INT REFERENCES
user_knowledge_vertex(vertex_id),
    head_vertex INT REFERENCES
user_knowledge_vertex(vertex_id),
    embedding   FLOAT[],
    content     TEXT,
    meta_data   JSONB DEFAULT '{}'
);

CREATE TABLE simplex_vertex (
    node_id     SERIAL PRIMARY KEY,
    user_id     INT NOT NULL,
    parent_id   INT REFERENCES
simplex_vertex(node_id),
    vertex_id   INT REFERENCES
user_knowledge_vertex(vertex_id),
    depth       INT NOT NULL,
    type        TEXT,
    meta_data   JSONB DEFAULT '{}'
);

-- Unique constraint handling NULL
parent_id for root nodes
CREATE UNIQUE INDEX idx_simplex_unique
    ON simplex_vertex (user_id,
parent_id, vertex_id)
    WHERE parent_id IS NOT NULL;
CREATE UNIQUE INDEX
idx_simplex_root_unique
    ON simplex_vertex (user_id,
vertex_id)
    WHERE parent_id IS NULL;

-- Child lookup (hash for O(1) expected)
CREATE INDEX idx_children
    ON simplex_vertex USING HASH
(parent_id);

-- L_j(ℓ) lists: find all nodes
referencing a vertex at given depth
CREATE INDEX idx_vertex_depth
    ON simplex_vertex (user_id,
vertex_id, depth);

-- Parent traversal
CREATE INDEX idx_parent
    ON simplex_vertex (parent_id);
```

B. Python Implementation

The full implementation is provided on the following page.

```
from typing import Optional
import asyncpg

class SimplexTree:
  """Simplex tree operations for user knowledge."""

  def __init__(self, pool: asyncpg.Pool, user_id: int):
    self.pool = pool
    self.user_id = user_id

  async def search_simplex(self, vertex_ids: list[int]) -> Optional[int]:
    """Verify whether a simplex exists. Complexity: O(j)"""
    if not vertex_ids:
      return None
    vertex_ids = sorted(vertex_ids)
    async with self.pool.acquire() as conn:
      current_parent = None
      for vertex_id in vertex_ids:
        row = await conn.fetchrow(
          """
          SELECT node_id FROM simplex_vertex
          WHERE user_id = $1
            AND parent_id IS NOT DISTINCT FROM $2
            AND vertex_id = $3
          """,
          self.user_id, current_parent, vertex_id
        )
        if row is None:
          return None
        current_parent = row['node_id']
      return current_parent

  async def insert_simplex(
    self, vertex_ids: list[int], simplex_type: str, meta_data: dict
  ) -> int:
    """Insert a simplex. Complexity: O(j)"""
    if not vertex_ids:
      raise ValueError("Cannot insert empty simplex")
    vertex_ids = sorted(vertex_ids)
    async with self.pool.acquire() as conn:
      async with conn.transaction():
        current_parent, current_depth = None, 0
        for vertex_id in vertex_ids:
          row = await conn.fetchrow(
            """
            SELECT node_id FROM simplex_vertex
            WHERE user_id = $1
              AND parent_id IS NOT DISTINCT FROM $2
              AND vertex_id = $3
            """,
            self.user_id, current_parent, vertex_id
          )
          if row is not None:
            current_parent = row['node_id']
          else:
            row = await conn.fetchrow(
              """
              INSERT INTO simplex_vertex
                (user_id, parent_id, vertex_id, depth, type, meta_data)
              VALUES ($1, $2, $3, $4, $5, $6)
              RETURNING node_id
              """,
              self.user_id, current_parent, vertex_id,
```

```
            current_depth + 1, simplex_type, meta_data
          )
          current_parent = row['node_id']
        current_depth += 1
      return current_parent

async def locate_cofaces(self, vertex_ids: list[int]) -> list[list[int]]:
  """Find all simplices containing vertex set. Complexity: O(k T)"""
  if not vertex_ids:
    return []
  vertex_ids = sorted(vertex_ids)
  last_vertex, min_depth = vertex_ids[-1], len(vertex_ids)
  async with self.pool.acquire() as conn:
    candidates = await conn.fetch(
      """
      SELECT node_id, depth FROM simplex_vertex
      WHERE user_id = $1 AND vertex_id = $2 AND depth >= $3
      """,
      self.user_id, last_vertex, min_depth
    )
    cofaces = []
    for candidate in candidates:
      path = await self._collect_path(conn, candidate['node_id'])
      if self._is_subsequence(vertex_ids, path):
        cofaces.append(path)
        cofaces.extend(
          await self._collect_subtree(conn, candidate['node_id'], path)
        )
    return cofaces

async def _collect_path(self, conn, node_id: int) -> list[int]:
  """Traverse upward from node to root."""
  vertices, current_id = [], node_id
  while current_id is not None:
    row = await conn.fetchrow(
      "SELECT vertex_id, parent_id FROM simplex_vertex WHERE node_id = $1",
      current_id
    )
    if row['vertex_id'] is not None:
      vertices.append(row['vertex_id'])
    current_id = row['parent_id']
  return list(reversed(vertices))

async def _collect_subtree(
  self, conn, root_id: int, root_verts: list[int]
) -> list[list[int]]:
  """Collect all simplices in subtree."""
  children = await conn.fetch(
    "SELECT node_id, vertex_id FROM simplex_vertex WHERE parent_id = $1",
    root_id
  )
  results = []
  for child in children:
    child_verts = root_verts + [child['vertex_id']]
    results.append(child_verts)
    results.extend(
      await self._collect_subtree(conn, child['node_id'], child_verts)
    )
  return results

def _is_subsequence(self, needle: list[int], haystack: list[int]) -> bool:
  it = iter(haystack)
  return all(v in it for v in needle)
```

```
@staticmethod
def enumerate_theoretical_faces(vertex_ids: list[int]) -> list[list[int]]:
  """Generate all non-empty subsets. Complexity: O(2^j)"""
  vertex_ids = sorted(vertex_ids)
  n = len(vertex_ids)
  return [
    [vertex_ids[i] for i in range(n) if mask & (1 << i)]
    for mask in range(1, 2 ** n)
  ]

async def remove_simplex(
  self, vertex_ids: list[int], remove_cofaces: bool = True
) -> bool:
  """Remove a simplex. Complexity: O(j) + O(k T) if removing cofaces"""
  node_id = await self.search_simplex(vertex_ids)
  if node_id is None:
    return False
  async with self.pool.acquire() as conn:
    async with conn.transaction():
      if remove_cofaces:
        await conn.execute(
          """
          WITH RECURSIVE descendants AS (
            SELECT node_id FROM simplex_vertex WHERE parent_id = $1
            UNION ALL
            SELECT sv.node_id FROM simplex_vertex sv
            JOIN descendants d ON sv.parent_id = d.node_id
          )
          DELETE FROM simplex_vertex
          WHERE node_id IN (SELECT node_id FROM descendants)
          """,
          node_id
        )
      else:
        has_children = await conn.fetchval(
          "SELECT EXISTS(SELECT 1 FROM simplex_vertex WHERE parent_id=$1)",
          node_id
        )
        if has_children:
          raise ValueError("Simplex has cofaces")
        await conn.execute(
          "DELETE FROM simplex_vertex WHERE node_id = $1", node_id
        )
  return True

async def construct_from_witness(
  self,
  conn,
  entity_ids: list[int],
  witness_type: str,
  threshold: dict
) -> list[int]:
  """
  Build simplices from co-occurring entities under witness conditions.
  Queries meta_data to group entities by witness criteria.
  Returns node_ids of inserted simplices.
  """
  # Fetch meta_data for all candidate entities
  rows = await conn.fetch(
    """
    SELECT vertex_id, meta_data FROM user_knowledge_vertex
    WHERE user_id = $1 AND vertex_id = ANY($2)
```

```
  """,
  self.user_id, entity_ids
)
meta_by_id = {r['vertex_id']: r['meta_data'] for r in rows}

# Group entities by witness criterion
groups: dict[str, list[int]] = {}
for vid, meta in meta_by_id.items():
  if witness_type == 'session':
    key = meta.get('session_id')
  elif witness_type == 'temporal':
    ts = meta.get('timestamp')
    if ts and threshold.get('window'):
      key = ts // threshold['window']
    else:
      key = None
  elif witness_type == 'spatial':
    loc = meta.get('location')
    key = loc if loc else None
  else:
    key = None
  if key is not None:
    groups.setdefault(str(key), []).append(vid)

# Insert simplex for each co-occurrence group with >= 2 entities
inserted = []
for group_key, verts in groups.items():
  if len(verts) >= 2:
    node_id = await self.insert_simplex(
      verts, witness_type, {'witness_key': group_key}
    )
    inserted.append(node_id)
return inserted
```

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