San Francisco Bay University

MATH201 - Calculus I

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Homework Assignment #5

Due Date: 11/30/2024

1.)

Average Tests = N \* (1 - q^x + 1 / x)

Where:

* q is the probability that any one person tests negative (q = 0.95).
* x is the group size.
* N is the total population size.

**Steps to Find the Optimal Group Size (x):**

1. **Understanding the Formula**:
   * The term q^x represents the probability that all individuals in a group of size x test negative. If this happens, only one test is required for the entire group.
   * The (1 / x) term accounts for the additional cost of testing individuals in groups where the pooled test is positive.
   * Balancing these two effects (pooling efficiency vs. individual testing) minimizes the number of tests.
2. **Optimization**:
   * To minimize the average tests, we vary x (the group size) over a range from 1 to 150.
   * We compute the average number of tests for each group size and find the x value where this number is minimized.
3. **Results**:
   * The optimal group size is approximately x = 5.03.
   * For x = 5.03, the average number of tests is minimized to around 42.62.
4. **Interpretation**:
   * Small groups (e.g., x = 1) require N = 100 individual tests, which is inefficient.
   * Very large groups (e.g., x = 100) often test positive, leading to many retests, which is also inefficient.
   * At x = 5.03, the balance between pooled tests and individual follow-ups is optimal, significantly reducing the number of tests required.

A graph with a blue line

Description automatically generated

**Summary:**

The optimal group size is approximately x = 5.03, resulting in an average of about 42.62 tests for a population of N = 100. This strikes the best balance between pooled and individual testing.

2.)

**Part a: Verifying Multiplicity**

1. **Root of Multiplicity 2:**
   * A root r of a function f(x) has multiplicity 2 if:
     + f(r) = 0
     + f'(r) = 0
     + f''(r) != 0
2. **Given Function**:  
   f1(x) = exp(2 \* sin(x)) - 2 \* x - 1
3. **Steps**:
   * Evaluate f1(0): Substitute x = 0 into the function: f1(0) = exp(0) - 2(0) - 1 = 0 Result: f1(0) = 0, so x = 0 is a root.
   * Evaluate f1'(x): Derivative of f1(x): f1'(x) = 2 \* exp(2 \* sin(x)) \* cos(x) - 2 Substitute x = 0:f1'(0) = 2 \* exp(0) \* cos(0) - 2 = 0 Result: f1'(0) = 0, so the root has at least multiplicity 2.
   * Evaluate f1''(x): Second derivative of f1(x): f1''(x) = 4 \* exp(2 \* sin(x)) \* cos(x)^2 - 2 \* exp(2 \* sin(x)) \* sin(x) Substitute x = 0: f1''(0) = 4 \* exp(0) \* cos(0)^2 - 0 = 4 Result: f1''(0) != 0, so the root's multiplicity is exactly 2.
4. **Conclusion**:  
   Since f1(0) = 0, f1'(0) = 0, and f1''(0) != 0, the root x = 0 has multiplicity 2.

**Part b: Newton's and Modified Newton's Methods for f1(x)**

**Newton's Method:**

1. Formula:  
   x\_{n+1} = x\_n - f(x\_n) / f'(x\_n)
2. Iteratively refines the guess x\_n by dividing the function value by the slope at x\_n.

**Modified Newton's Method:**

1. Formula (for multiplicity m = 2):  
   x\_{n+1} = x\_n - 2 \* f(x\_n) / f'(x\_n)
2. Scales the correction term to account for higher multiplicity, improving convergence.

**Steps:**

* Start with an initial guess x0 = 0.1.
* Perform 9 iterations of both methods using the respective formulas.
* Compare results to observe faster convergence of Modified Newton's Method due to its adjustment for multiplicity.

A graph of a method

Description automatically generated

Standard Newton's Method Results: [0.1, 0.05114870668095536, 0.02588832264635743, 0.01302627716676804, 0.0065341397288105375, 0.003272380449166415, 0.0016375254901840506, 0.0008190975191015522, 0.00040963257308487616, 0.00020483725493164077]

Modified Newton's Method Results: [0.1, 0.002297413361910708, 1.3172493128995634e-06, -1.5258027692339302e-11, -1.5258027692339302e-11, -1.5258027692339302e-11, -1.5258027692339302e-11, -1.5258027692339302e-11, -1.5258027692339302e-11, -1.5258027692339302e-11]

Final x9 for f1(x) using Standard Newton's Method: 0.00020483725493164077

Final x9 for f1(x) using Modified Newton's Method: -1.5258027692339302e-11

**Part c: Modified Newton's Method for f2(x)**

1. **Given Function**:  
   f2(x) = -8 \* x^2 / (3 \* x^2 + 1)
2. **Derivative**:  
   Manually derived: f2'(x) = [-16 \* x \* (3 \* x^2 + 1) + 48 \* x^3] / (3 \* x^2 + 1)^2
3. **Formula for Modified Newton's Method**:  
   x\_{n+1} = x\_n - 2 \* f2(x\_n) / f2'(x\_n)
4. **Handling Zero Derivatives**:
   * If f2'(x\_n) = 0, division by zero occurs. This is handled by stopping the iteration when f2'(x\_n) = 0.
5. **Steps**:
   * Start with an initial guess x0 = 0.15.
   * Perform 9 iterations of both Standard and Modified Newton's Methods.
   * Compare results, noting that Modified Newton's Method handles multiplicity better if the derivative does not become zero.

A graph with a line and a blue line

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Division by zero encountered at x = 0.0. Stopping iterations.

Standard Newton's Method Results for f2(x): [0.15, 0.06993749999999999, 0.03445562689489746, 0.017166455372294376, 0.008575639584402044, 0.0042868737928904894, 0.0021433187247807467, 0.0010716445933753101, 0.000535820450637098, 0.00026790999456461435]

Modified Newton's Method Results for f2(x): [0.15, -0.010125000000000023, 3.1139121093731736e-06, -9.058170337937488e-17, 0.0]

Final x9 for f2(x) using Modified Newton's Method: 0.0

Final x9 for f2(x) using Standard Newton's Method: 0.00026790999456461435