Luke Gregory G. Pamaos

Calculus I Homework Assignment 2:

1. Plot each group of functions:

(a) 𝑦 = 𝑒^𝑥, 𝑦 = 𝑒^−𝑥, 𝑦 = 8^𝑥, 𝑦 = 8^−𝑥

- Answer: These functions show exponential growth (𝑦 = 𝑒^𝑥 and 𝑦 = 8^𝑥) and decay (𝑦 = 𝑒^−𝑥 and 𝑦 = 8^−𝑥).

- Explanation: 𝑒^𝑥 and 8^𝑥 increase exponentially as 𝑥 increases, while 𝑒^−𝑥 and 8^−𝑥 decrease exponentially. You should observe sharp growth for positive 𝑥 and rapid decay for negative 𝑥.

A graph of a function

Description automatically generated

(b) 𝑦 = 0.9^𝑥, 𝑦 = 0.6^𝑥, 𝑦 = 0.3^𝑥, 𝑦 = 0.1^𝑥

- Answer: These are exponential decay functions, where the base values are less than 1.

- Explanation: As 𝑥 increases, each function gets closer to zero. The smaller the base, the faster the function decays. This will be reflected in how quickly each curve approaches zero as 𝑥 increases.

A screenshot of a graph

Description automatically generated

2. Prove the Difference Quotient for 𝑓(𝑥) = 10^𝑥

- Answer: Using the difference quotient formula, you get 𝑓(𝑥 + ℎ) − 𝑓(𝑥) / ℎ = 10^𝑥(10^ℎ − 1) /

ℎ.

- Explanation: This shows the change in the function over an interval ℎ. As ℎ approaches zero, the result approaches the function's derivative, confirming the rate of change for exponential growth.

3. Compare 𝑓(𝑥) = 𝑥⁵ and 𝑔(𝑥) = 5𝑥

- Answer: 𝑓(𝑥) = 𝑥⁵ grows faster than 𝑔(𝑥) = 5𝑥 for large values of 𝑥.

- Explanation: While 𝑔(𝑥) is linear, 𝑓(𝑥) increases exponentially as 𝑥 becomes larger. Mathematically, for large 𝑥, powers of 𝑥 grow faster than linear functions.

4. Prove 𝑓(𝑥) = 1/𝑒^(1/𝑥) − 1/𝑒^(1/𝑥) is odd

- Answer: The function is odd because 𝑓(−𝑥) = −𝑓(𝑥).

- Explanation: A function is odd if it is symmetric about the origin, meaning 𝑓(𝑥) and 𝑓(−𝑥) are mirror reflections across the origin. This is verified by calculating 𝑓(−𝑥) and confirming it equals −𝑓(𝑥).

5. Parametrized Function Analysis:

(a) Effect of changing 𝑏 on 𝑓(𝑥) = 1/(1 + 𝑎𝑒^𝑏𝑥)

- Answer: As 𝑏 changes, the steepness of the curve changes.

- Explanation: Larger values of 𝑏 make the function steeper, while smaller values make the curve more gradual.

(b) Effect of changing 𝑎 on 𝑓(𝑥) = 1/(1 + 𝑎𝑒^𝑏𝑥)

- Answer: As 𝑎 changes, the height of the curve's plateau changes.

- Explanation: The parameter 𝑎 affects the horizontal asymptote. Increasing 𝑎 increases the maximum value the function can approach.

6. Inverse Function of 𝑔(𝑥) = 𝑥⁶ + 𝑥⁴

- Answer: The inverse function 𝑔⁻¹(𝑥) can be approximated numerically but doesn't have a simple closed form.

- Explanation: This is a complex function to invert algebraically. By plotting, you can compare 𝑦 = 𝑥 with 𝑦 = 𝑔(𝑥) and approximate 𝑔⁻¹(𝑥).

7. Capacitor Charging Function:

(a) Inverse of 𝑄(𝑡) = 𝑄₀(1 − 𝑒^−𝑡/𝑎)

- Answer: The inverse is 𝑡 = −𝑎 ln(1 − 𝑄/𝑄₀).

- Explanation: This inverse function tells us how long it takes to reach a certain charge, 𝑄, in terms of time 𝑡.

(b) Time to reach 90% of capacity with 𝑎 = 2

- Answer: It takes approximately 4.6 seconds to reach 90% capacity.

- Explanation: You can calculate this by solving 𝑄(𝑡) = 0.9𝑄₀ for 𝑡, resulting in 𝑡 = −𝑎 ln(1 − 0.9) = 4.6 seconds for 𝑎 = 2.