San Francisco Bay University

MATH201 - Calculus I

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Homework Assignment 2

Due Date: 11/17/2024

1.)

Plot each following group of functions in one graph respectively by Excel,  
covering the appropriate domain of x and y.  
a. 𝑦 = 𝑒^𝑥, 𝑦 = 𝑒^―𝑥, 𝑦 = 8^𝑥, 𝑦 = 8^―𝑥

Group A:

Functions:

- ( y = e^x )

- ( y = e^-x )

- ( y = 8x )

- ( y = 8^-x )

1. ( y = e^x ):

- Range: The range is ( y > 0 ). As ( x ) approaches negative infinity, ( y ) approaches 0, and as ( x ) increases, ( y ) increases exponentially.

- Behavior: The graph is increasing, starting very close to 0 on the left (as ( x to -infinity)) and rising steeply as ( x to infinity).

2. ( y = e^-x ):

- Range: The range is ( y > 0 ), just like ( y = e^x ). As ( x to infinity), ( y ) approaches 0, and as ( x to -infinity), ( y ) increases exponentially.

- Behavior: The graph is decreasing, starting high on the left (as ( x to -infinity)) and approaching 0 as ( x to infinity).

3. ( y = 8x ):

- Range: The range is all real numbers ( y in (-infinity, infinity) ).

- Behavior: This is a straight line with a slope of 8. The graph passes through the origin (0, 0), and for every 1-unit increase in ( x ), ( y ) increases by 8.

4. ( y = 8^-x ):

- Range: The range is ( y > 0 ). As ( x to infinity), ( y to 0 ), and as ( x to -infinity), ( y to infinity).

- Behavior: This is a decreasing exponential curve similar to ( y = e^-x ), but it decays more rapidly. As ( x ) increases, ( y ) approaches 0.

Graph Behavior Summary:

- ( y = e^x ) and ( y = e^-x ) are exponential curves, one increasing and one decreasing, both with ranges greater than zero.

- ( y = 8x ) is a straight line passing through the origin, with a positive slope.

- ( y = 8^-x ) is an exponential curve that decreases rapidly as ( x ) increases.

b. 𝑦 = 0.9^𝑥, 𝑦 = 0.6^𝑥, 𝑦 = 0.3^𝑥, 𝑦 = 0.1^𝑥

Group B:

Functions:

- ( y = 0.9x )

- ( y = 0.6x )

- ( y = 0.3x )

- ( y = 0.1x )

1. ( y = 0.9x ):

- Range: The range is all real numbers ( y in (-infinity, infinity) ).

- Behavior: This is a straight line with a slope of 0.9. It has a mild positive slope and passes through the origin.

2. ( y = 0.6x ):

- Range: The range is all real numbers ( y in (-infinity, infinity) ).

- Behavior: This is a straight line with a slope of 0.6, which is smaller than 0.9, so the line will be less steep. It also passes through the origin.

3. ( y = 0.3x ):

- Range: The range is all real numbers ( y in (-infinity, infinity) ).

- Behavior: This is a straight line with a slope of 0.3, which is even milder than 0.6 or 0.9. It passes through the origin as well.

4. ( y = 0.1x ):

- Range: The range is all real numbers ( y in (-infinity, infinity) ).

- Behavior: This is a straight line with the mildest slope of 0.1. It passes through the origin, and its slope is much gentler than the other lines.

Graph Behavior Summary:

- All four functions in this group are linear, with slopes of 0.9, 0.6, 0.3, and 0.1.

- The slope determines the steepness of the lines. As the slope decreases, the line becomes less steep, but all pass through the origin.

Conclusion:

- For Group A, you’ll see exponential growth and decay with ( y = e^x ) and ( y = e^-x ), and the linear functions ( y = 8x ) and ( y = 8^-x ) will show steep and rapid growth or decay.

- For Group B, the graphs are linear with different slopes. As the slope decreases, the line becomes less steep.

2.)

Given 𝑓(𝑥) = 10𝑥 , prove that 𝑓(𝑥 + ℎ) - 𝑓(𝑥)/ ℎ = 10^𝑥(10^ℎ - 1/ℎ ) and verify it by the plot  
in Excel.

Proof:

Given ( f(x) = 10^x ), we want to show that:

f(x + h) - f(x)/h = 10^x times 10^h – 1/h

We begin by evaluating ( f(x + h) ):

f(x + h) = 10^x + h = 10^x times 10^h

Now, subtract ( f(x) ) from ( f(x + h) ):

f(x + h) - f(x) = 10^x times 10^h - 10^x

Factoring out ( 10^x ), we get:

f(x + h) - f(x) = 10^x left( 10^h - 1 right)

Dividing the entire expression by ( h ), we obtain:

f(x + h) - f(x)/h = 10^x (10^h - 1)/h

Thus, we have shown that:

f(x + h) - f(x)/h = 10^x times 10^h – 1/h

3.)

Compare the functions 𝑓(𝑥) = 𝑥5 and 𝑔(𝑥) = 5𝑥 by plotting curve in Excel and  
which function grows more rapidly when x is large? And prove it mathematically

To prove which function grows more rapidly as ( x ) becomes large, we can consider the limit of the ratio of the two functions:

lim\_x to infinity f(x/)g(x) = lim\_x to infinity x^5/5x

Simplifying this:

x^5/5x = x^4/5

Now, as ( x ) grows larger, ( x^4 ) increases without bound, meaning:

lim\_x to infinity x^4/5 = infinity

Thus, ( f(x) = x^5 ) grows much faster than ( g(x) = 5x ) as ( x ) becomes large. The ratio tends to infinity, indicating that ( f(x) ) eventually grows far more rapidly than ( g(x) ) for large ( x ).

Conclusion:

- Graphically: By plotting ( f(x) = x^5 ) and ( g(x) = 5x ), you'll clearly see that ( f(x) ) grows much faster for larger values of ( x ).

- Mathematically: We proved that ( f(x) = x^5 ) grows faster than ( g(x) = 5x ) as ( x to infinity ) since the limit of their ratio tends to infinity.

4.)  
Plot the function 𝑓(𝑥) = 1 𝑒1/𝑥  
1 𝑒1/𝑥 in Excel. And then prove that 𝑓(𝑥) is an odd function.

To verify that the function ( f(x) = {1}{e^{{1}{x}}} ) is odd, we need to show that ( f(-x) = -f(x) ). This means that for every value of ( x ), the function evaluated at ( -x ) should be equal to the negative of the function evaluated at ( x ).

First, calculate ( f(-x) ):

f(-x) = {1}{e^{{1}{-x}}} = {1}{e^{-{1}{x}}}

Now simplify:

f(-x) = {1}{e^{-{1}{x}}} = {e^{{1}{x}}}{1} = -f(x)

Since ( f(-x) = -f(x) ), this confirms that the function is indeed odd.

5. Parametrized Function Analysis:

(a) Effect of changing 𝑏 on 𝑓(𝑥) = 1/(1 + 𝑎𝑒^𝑏𝑥)

- Answer: As 𝑏 changes, the steepness of the curve changes.

- Explanation: Larger values of 𝑏 make the function steeper, while smaller values make the curve more gradual.

(b) Effect of changing 𝑎 on 𝑓(𝑥) = 1/(1 + 𝑎𝑒^𝑏𝑥)

- Answer: As 𝑎 changes, the height of the curve's plateau changes.

- Explanation: The parameter 𝑎 affects the horizontal asymptote. Increasing 𝑎 increases the maximum value the function can approach.

6. Inverse Function of 𝑔(𝑥) = 𝑥⁶ + 𝑥⁴

- Answer: The inverse function 𝑔⁻¹(𝑥) can be approximated numerically but doesn't have a simple closed form.

- Explanation: This is a complex function to invert algebraically. By plotting, you can compare 𝑦 = 𝑥 with 𝑦 = 𝑔(𝑥) and approximate 𝑔⁻¹(𝑥).

7. Capacitor Charging Function:

(a) Inverse of 𝑄(𝑡) = 𝑄₀(1 − 𝑒^−𝑡/𝑎)

The function describing the electric charge in the capacitor over time is:

Q(t) = Q\_0 left( 1 - e^{-t/a} right) ]

Where:

- ( Q\_0 ) is the maximum charge capacity.

- ( a ) is a constant related to the rate of charging.

- ( t ) is the time in seconds.

To isolate the exponential term, we start with:

{Q(t)}{Q\_0} = 1 - e^{-t/a} ]

Rearranging this:

e^{-t/a} = 1 - {Q(t)}{Q\_0} ]

Next, we take the natural logarithm (ln) of both sides:

ln left( e^{-t/a} right) = ln left( 1 - {Q(t)}{Q\_0} right) ]

Simplifying the left-hand side:

-{t}{a} = ln left( 1 - {Q(t)}{Q\_0} right) ]

Solving for ( t ):

t = -a ln left( 1 - {Q(t)}{Q\_0} right) ]

Thus, the inverse function is:

t = -a ln left( 1 - {Q(t)}{Q\_0} right) = Q^{-1}(t) ]

This inverse function ( Q^{-1}(t) ) gives the time required to reach a certain charge ( Q(t) ) in the capacitor. It shows how long it takes for the charge to reach a specified level based on the exponential decay characteristic of the charging process.

(b) Time to reach 90% of capacity with 𝑎 = 2

- Answer: It takes approximately 4.6 seconds to reach 90% capacity.

- Explanation: You can calculate this by solving 𝑄(𝑡) = 0.9𝑄₀ for 𝑡, resulting in 𝑡 = −𝑎 ln(1 − 0.9) = 4.6 seconds for 𝑎 = 2.