# Maximum Likelihood Estimation

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## Definitions

We denote the random variable $T$ as the number of vaccinated individuals from the 170 COVID cases.

$$T \sim Binom(n, \pi)$$

We can define $\pi = \text{P(Vaccine|COVID)} = \frac{\pi_1}{\pi_1+\pi_2}$ (as proved in lecture). Where $\pi_1$ is the proportion of vaccinated individuals who got COVID and $\pi_2$ is the proportion of unvaccinated individuals who got COVID. Moreover, we define the vaccine efficacy as $\psi = \frac{1-2\pi}{1-\pi}$.

## Log-Likelihood Calculation

We can first write the likelihood function of $\pi$:

$$L(\pi) = \binom{n}{t}\pi^t(1-\pi)^{n-t}$$

Then we write $\pi$ in the form $\pi = g(\psi)$:

$$\pi = \frac{1-\psi}{2-\psi}$$

We can then write the likelihood function for $\psi$.

$$L^*(\psi) = L(g(\psi)) = L(\frac{1-\psi}{2-\psi}) = \binom{n}{t} \cdot (\frac{1-\psi}{2-\psi})^t \cdot (1-(\frac{1-\psi}{2-\psi}))^{n-t}$$

Finally, we calculate the log-likelihood function for $\psi$:

$$\ell(\psi) = \ln(\binom{n}{t}) + t\ln(\frac{1-\psi}{2-\psi}) + (n-t)\cdot\ln(1-\frac{1-\psi}{2-\psi})$$

Now that we have calculated the log-likelihood function for $\psi$, we can use the Newton Raphson to estimate $\psi$.

## Newton Raphson Method

Observed Values:
$\pi_1 = 0.047$
$\pi_2 = 0.953$
$\pi = \frac{\pi_1}{\pi_1+\pi_2} = 0.047$
$\psi = \frac{1-2\pi}{1-\pi} = 0.9507$
$t = 8$
$n = 170$

```
loglik <- function(psi, T, n) {
  return(log(choose(n,T)) + T * log((1 - psi) /
          (2 - psi)) + (n - T) * log(1 - ((1 - psi) / (2 - psi))))
}

maxLik(logLik = loglik, start = 0.9507,
       method = "NR", tol = 1e-4,
       T = 8, n = 170)
```

```
## Maximum Likelihood estimation
## Newton-Raphson maximisation, 1 iterations
## Return code 2: successive function values within tolerance limit (tol)
## Log-Likelihood: -1.944994 (1 free parameter(s))
## Estimate(s): 0.9506174
```