1) given prograde orbit   Schematic
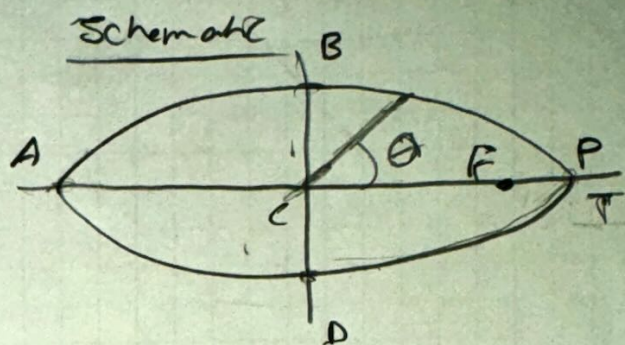

Find

Time to fly between
points on the orbit
in terms of T and e

Properties

$$\frac{2\pi \cdot t}{T} = E - e\sin E \qquad E = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}} \tan\left(\frac{\theta}{2}\right)\right]$$

$$t(\pi) = \frac{T}{2} \rightarrow \text{Half the time of flight will be half the period}$$

Analysis

The points of the ellipse are at points $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

Here we can pre compute the E values for given angles

$$E(0) = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}(0)\right] = 0 \qquad E\left(\frac{\pi}{2}\right) = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}(1)\right] = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]$$

$$E(\pi) = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\pi}{2}\right)\right] = \text{undefined} \quad E\left(\frac{3\pi}{2}\right) = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}(-1)\right] = -2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]$$

Knowing this will make it easier to solve the following
problems

a) $P \rightarrow B \quad \theta = \frac{\pi}{2} \rightarrow t\left(\frac{\pi}{2}\right)$

$$t = \left(\frac{T}{2\pi}\right)(E - e\sin E) = \frac{T}{2\pi}\left(2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)$$

b) $B \rightarrow A$ know $t(P \rightarrow B)$ and $t(P \rightarrow A) = t(\pi)$

$$t_{B \rightarrow A} = t(\pi) - t_{P \rightarrow B} = \frac{T}{2}\left(1 - \frac{1}{\pi}\left(2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$

c) $A \rightarrow D$

$$t_{A \rightarrow D} = t_{P \rightarrow D} - t_{P \rightarrow A} = \frac{T}{2}\left(\frac{1}{\pi}\left(-2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right) + e\sin\left(2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right)\right)\right) - 1\right)$$

d) $D \to P$

$$T_{D \to P} = T_{P \to A} - T_{P \to D} = T\left(1 - \frac{1}{2\pi}\left(-2Tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right) + e\sin\left(2Tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right)\right)\right)\right)$$

e) $P \to A \qquad \theta = \pi$

$$t_{P \to A}^{T} = \frac{T}{2}$$

f) $A \to P \qquad \theta = \pi$

$$t_{A \to P} = t_{P \to A} = \frac{T}{2}$$

g) $P \to D \quad \theta = \frac{3\pi}{2}$

$$t_{P \to D} = \frac{T}{2\pi}\left(-2Tan\left(\sqrt{\frac{1-e}{1+e}}\right) + e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)$$

h) $B \to P$

$$t_{B \to P} = t_{B \to B} - t_{P \to B} = T\left(1 - \frac{1}{2\pi}\left(2Tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right) - e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$

i) $B \to D$

$$t_{B \to D} = t_{B \to A} + t_{A \to D}$$

$$\to \frac{T}{2}\left(1 - \frac{1}{\pi}\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$

$$- \frac{T}{2}\left(1 + \frac{1}{\pi}\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$

$$\to \frac{T}{2}\left(-\frac{2}{\pi}\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$

$$t_{B \to D} \to \frac{T}{\pi}\left(-2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] + e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)$$

j) $t_{D \to B} = t_{D \to P} + t_{P \to B}$

$$t_{D \to B} = T\left(\frac{1}{\pi}\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right] - e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right) + T\right)$$

k) $T_{D \to A} = T_{D \to D} - T_{A \to D}$

$$t_{D \to A} = \frac{T}{2}\left(3 - \frac{1}{\pi}\left(-2Tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\right) + e\sin\left(2Tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\right]\right)\right)\right)$$