

a)

$$V_{\text{sum}} = V_0 \left[\sin[(\bar{\omega} - \Delta\omega/2)t] + \sin[(\bar{\omega} + \Delta\omega/2)t] \right]$$

$$\text{Using } \sin(a \pm b) = \sin(a)\cos(b) \pm \sin(b)\cos(a)$$

$$V_{\text{sum}} = V_0 \left[\left(\sin(\bar{\omega}t)\cos\left(\frac{\Delta\omega t}{2}\right) - \sin\left(\frac{\Delta\omega t}{2}\right)\cos(\bar{\omega}t) \right) + \left(\sin(\bar{\omega}t)\cos\left(\frac{\Delta\omega t}{2}\right) + \sin\left(\frac{\Delta\omega t}{2}\right)\cos(\bar{\omega}t) \right) \right]$$

$$\rightarrow V_{\text{sum}} = 2V_0 \sin(\bar{\omega}t) \cos\left(\frac{\Delta\omega t}{2}\right)$$

b) From the problem statement:

$\Delta f = f_2 - f_1$, because a negative or positive Δf results in the frequencies being out of phase. There will be the same amount of constructive or destructive interference for the same $|\Delta f|$. We also saw this in lab when $f = 58 \text{ Hz}$ and $f = 62 \text{ Hz}$ had the same beat frequencies when paired with $f = 60 \text{ Hz}$.

Therefore $f_{\text{beat}} = |f_2 - f_1|$. We also learned this in Phys 123 for waves.

c) A low pass filter with cut off frequency

at f_c [nothing is observed] on the other

end of the filter because neither frequency (f_1, f_2) is lower than the cutoff frequency. This low pass filter. The filter doesn't care about the observed beat frequency because the beat frequency is not representative of the frequencies or the signals inside of the beat.

Overall the voltage will hover around 0, with of course some added noise.