Homework 1[1]

DUE: MON 2025-10-06 @ 23:59 on *Canvas*

(PDF submissions only; can be pictures/scans as needed)

1. Vector Algebra
    a. In Matlab or Python create **your own** functions to get numerical solutions to:
        i. Vector Dot Product
        ii. Vector Cross Product

    Comment your functions.
    Copy the complete commented code as your answer.

    b. Use your functions from (a) to make **your own** functions for the triple products:
        i. Scalar Triple Product (**A** . **(B** x **C**))
        ii. Vector Triple Product ((**A** x **(B** x **C**))

    Comment your functions.
    Copy the complete commented code as your answer.

    Consider the vectors:
    A = 6**i** + 18**j** + 5**k**
    B = -2**i** - 5**j** - 8**k**
    C = 2**i** + 1**j** + 12**k**

    c. Use your functions to determine how aligned is vector **A** with the plane created by **B** and **C**, comment on the result
    d. Use the functions to determine how aligned is vector **C** to the plane formed by **A** and **B**, comment on the result
    e. Use the functions to determine how aligned is vector **B** to the plane formed by **A** and **C**, comment on the result

[1] Note, parts marked with {L3} mean the material is introduced during Lecture 3.

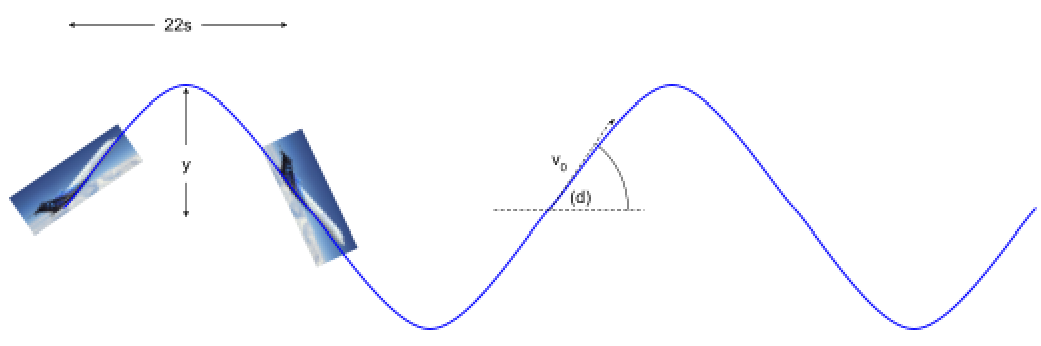
2. Reduced Gravity Airplanes (RGAs) fly a parabolic trajectory in order to simulate microgravity. RGAs provide approximately 22s of microgravity time. Curtis Examples 1.7 & 1.8 shows some important dynamics of the airplane. Lets study a few things about these vehicles (*assuming you can ignore drag completely*) - provide all your answers in SI (metric) units:
    a. If instead of following a parabolic trajectory, this was to be done solely by a vertical drop (like a drop tower) in the direction of **g** (with no lateral speed), what initial height would be needed to achieve 22s of free-fall (microgravity) time? *[Answer 2373 m]*

    b. At what speed would the vehicle be going after 22s? (assuming it started with no speed) [*Answer: 215.75 m/s*]

    c. A Boeing 727 (ie, "G-Force 1" which currently flies both NASA and commercial Zero-g flights) has a mass of approximately 80,000kg and thrust of 14,000lbf per engine (with 3 engines). The airplane flies between 30,000-40,000ft altitude normally (more than 4x the vertical drop). Explain with very basic physics why "G-Force 1" does not "simply" fall straight down (in "reverse") and then use its engines to stop itself?
    (Do not over think this! Remember: assume no drag.)

    d. The plane has a lift-to-thrust (i.e. lift-to-drag) ratio of 12. What is the maximum angle that the plane can achieve with respect to gravity? [*Answer: 69.5°*]

    e. The plane actually flies a "sinusoidal parabola". Using the basic equations of motion (shown in Curtis Example 1) and assuming the parabola starts at the angle from (d): determine the initial speed v_0 needed to achieve 22s of microgravity and the expected change in altitude (y). *[Answer: 1.1516m/s, 593.32 m]*



    f. {L3} Can the airplane be used as an inertial frame of reference for experiments inside it? Why? What does Curtis Example 1.8 have to do with this question?

3. Orbital Speeds
    a. A satellite of mass *m* is in a circular orbit around the Earth, whose mass is *M*. The orbital radius from the center of the earth is *r*. Use **Newton's Second Law of Motion** and of **Gravitational Attraction**, together with Curtis [1.25], to calculate the speed *v* of the satellite in terms of *M*, *r*, and the gravitational constant *G*.

    b. Enter the resulting equation into Matlab or Python. Provide a commented copy of your function.

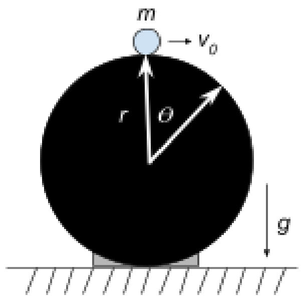
    c. Use it to calculate the speed for the following altitudes:
        i. LEO (like ISS or CubeSats, 460km altitude)
        ii. MEO (like GPS and other positioning systems, 20,200km) *[Answer 3.873 km/s]*
        iii. GEO (like communications satellites, 35,780km) *[Answer 3.075 km/s]*

    Present the results in a clean table (do not just copy Matlab or Python output, format it so that it is easy to read).

    *NOTE: altitude = distance from **surface** to the satellite - you **must** use the full radius from the center of the Earth to the satellite!*

    d. bonus: Astronaut patches say they flew at "Mach 25", why?

4. {L3} Ball *m* in the figure below is a placed on top of the **fixed** cylinder with radius **R**. The only force acting on the full system is that due to **gravity** (this is **not** orbital dynamics, it is a problem set in Earth's gravity; ignore friction, air drag, etc.). Ball *m* is released at time 0 with a velocity v_0.



   a. Before any math:
      i. Sketch at what point you think the ball *m* will stop making contact with the cylinder.
      ii. Sketch a free body diagram of all the physical forces acting on the ball when $\theta>0$ (but before contact is lost).
      iii. To remain moving in a circle (the shape of the cylinder) what do the forces in the free body diagram must balance out?

The following steps guide you through the necessary equations to understand the behavior of the ball:

   b. Define the potential energy V_0 when $\theta=0$?

   c. What is the kinetic energy T_0 when $\theta=0$?

   d. What is the kinetic energy $T(\theta)$ [i.e., as a function of θ and only θ, everything else must be a constant], as the ball *m* moves around the cylinder when $\theta>0$?

   e. What is the centripetal acceleration $a_c(\theta)$ as the ball *m* moves around the cylinder?

Using the resulting equation with Matlab or Python, determine numerically (providing a copy of your commented code):

   f. if $v_0 \simeq 0$ what is the departure angle? *[Answer 48.19°]*

   g. if you want the departure angle to be 45°, what should be v_0 as a function of r? [Answer: $1.0908\sqrt{r}$]