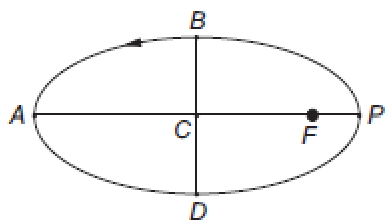
Homework 4

DUE: **WEDNESDAY** 2025-10-29 @ 23:59 on *Canvas*

(PDF submissions only; can be pictures/scans as needed)

NOTE: Answers to this homework will be published Thursday 2025-10-30 morning. The longest extension will be to Fri 2025-10-31 @ 23:59. All request for extensions MUST have attestation that the solutions will not be accessed or used in any.

1. Calculate the time required to fly from the given points in terms of the eccentricity e and the period T (only e & T in the final equations). The orbit is prograde (positive **h**, i.e. counter-clockwise in the diagram). Simplify your solutions as much as possible. B lies on the minor axis.



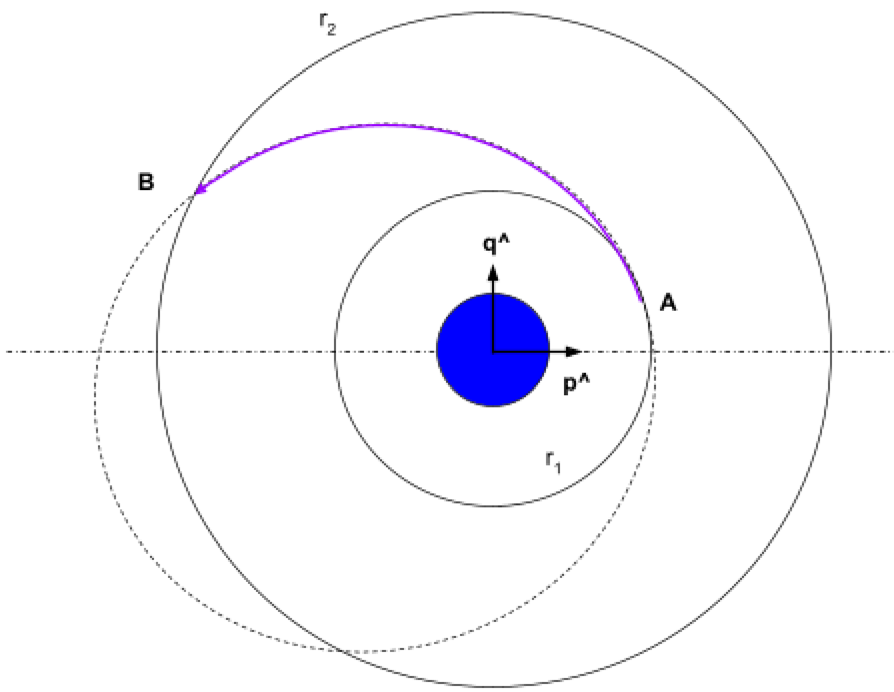
    a. P to B
    b. B to A
    c. A to D
    d. D to P
    e. P to A
    f. A to P
    g. P to D
    h. B to P
    i. B to D
    j. D to B
    k. D to A

2. A satellite around Earth wants to change orbit directly. It has been calculated that the velocities at points A and B on the orbits in the figure below are (in km/s):

   $v_{r1@A}$ = -3**p^** + 7**q^** (velocity in circle 1)
   $v_{AB@A}$ = -2**p^** + 10**q^** (velocity in the transfer ellipse periapsis)

   $v_{AB@B}$ = -6**p^** -3**q^** (velocity in the transfer ellipse at point B)
   $v_{r2@B}$ = -4**p^** - 4**q^** (velocity in circle 2)

   The *perifocal frame* is used to define the velocity vectors.



Not to scale.

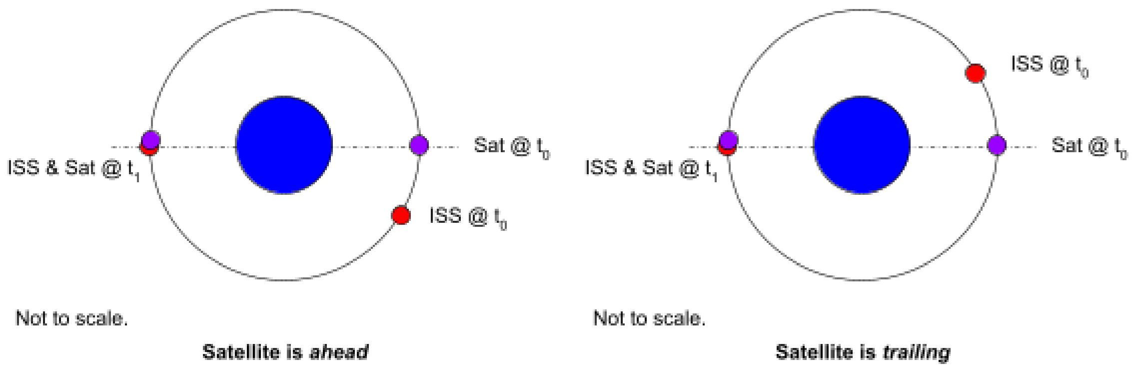
   a. Calculate the **total** Δv (<u>magnitude</u>) for a transfer to orbit r_2 by means of orbit r_{AB}. Provide a commented copy of your code, partial results of the intermediate calculations at A and at B, and the final answer for the total Δv required. [*Answer: Δv = 5.3983 km/s*].

   b. Point A becomes the periapsis of the elliptical *transfer orbit*. If the true anomaly of point B in the transfer orbit (Tip: *changes perifocal frame!*) is θ = 120°, calculate the time it takes to do the transfer [Answer: *59.204 minutes*]

c. The satellite looses communications for 10 hours after firing at B, at what true anomaly will it be if it stayed in the elliptical orbit? (Note: if its < 240° in the transfer orbit frame, it can get back into the circle at that point!) [*Answer: -171°*]

3. A mission to the International Space Station (450km *altitude*) reaches the orbit (*assume a circular orbit*), but the timing was wrong, leaving the spacecraft too far away from the ISS.

   The following drawings define “satellite is ahead” and “satellite is trailing” for this problem:



   Calculate (any way you want) the Δv and time required to complete the rendezvous with the ISS via:

   a. A maneuver of approximately 1 ISS orbit if the spacecraft is *ahead* 14° [*Partial Answer:* $\Delta v_{tot} = 0.19072 km/s$]
   Provide relevant intermediate answers and the final solution.

   Note: this would be an “emergency” to get there within ~90 minutes! The “shortest” time to ISS has in fact been about 2 orbits (~3 hours).

   b. A maneuver of 2 days (closest to, but not more than that) if the spacecraft is *trailing* 14° [*Partial Answer:* $\Delta v_{tot} = 0.0066124 km/s$]
   Provide relevant intermediate answers and the final solution.
   Note: this is similar to what real missions to ISS usually do.

   Tip: if you write a function that performs a phasing maneuver, you can re-use it later. Extra tip: this is hard maneuver to code.

4. A rocket failed to put a satellite into MEO circular orbit (22,000km *altitude*). Instead it ended in an elliptical orbit with a perigee *altitude* of 1650km and an e= 0.58.
    a. Use Matlab or Python to plot the intended and failed orbits; provide a commented copy of your code and an image of the output.

    b. Calculate the **Δv** (vector) for a **single** maneuver to change the elliptical orbit into the circular. [*Partial answer: |**Δv**| = 1.5824 km/s*]

    c. Draw the "**Δv** = **v2** - **v1**" triangle in the figure from part (a) (you can do it by "hand", in a drawing program, annotating a picture, or you can program it into Matlab or Python, any method *you* do is acceptable), include:
        i. the magnitude **and** direction of the **Δv**, **v1**, and **v2** [*Partial answer: Δv = 1.5824 km/s, v1=3.7484km/s, v2=2.6909km/s* magnitudes]
        ii. the change in the flight-path angle $\Delta\gamma$ [*Answer: -21.362°*]

Another way to change orbit is to use a Hohmann transfer.

    d. Draw a *sketch* (by hand or a drawing program) of your initial guess for the two Hohmann Transfer options to go from the bad elliptical orbit to the intended circular orbit, include the *direction* of the required **Δv**'s.

    *Attention: this is **not** a transfer between two circular orbits, it is a transfer between an elliptical and a circular orbit.*

    e. Calculate the required Δv's for the two transfers. [*Answer: Δv1 = 1.3182km/s, Δv2 = 1.2796km/s*]
    Tip: you can make a generic function to calculate the Δv *for a Hohmann transfer between any two (co-axial) orbits.*