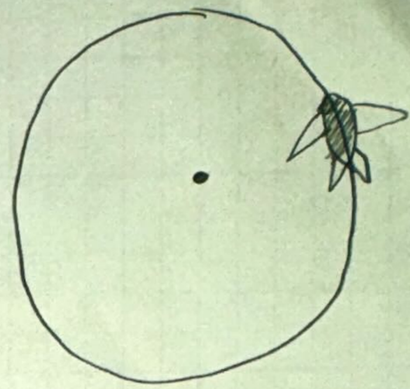
AA 310 Midterm | Luke Verlangieri | 11/2/2025

1) Turning The Airplane



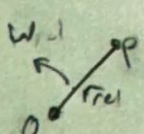
a) standing on the ground is an $\boxed{\text{inertial}}$ reference frame. This is because all motion of the plane is taken relative to the "stationary" earth (we can consider the surface of the earth to be an inertial reference frame because every object close to the surface of the earth undergoes the same forces from rotation and gravity). All behavior of the plane follows newtonian laws, which defines this as inertial.

b) The reference frame of someone standing on the plane is $\boxed{\text{non-inertial}}$. This is because the rotation of the plane in a circle is a centripetal accelleration and the reason the people in the plane rotate with the plane is because the seats they are strapped into apply a real force to keep everyone with the movement of the plane.

If you threw a ball up in the air on a plane while it was turning it would move in the direction opposite of the rotation of the plane. To an observer on the plane the ball would experience an acceleration with no real forces acting on it which disobeys newtons laws. Therefore this is a non-inertial frame of reference.

c) $\omega = \text{Const} \rightarrow \alpha = 0 \quad r = \text{Const}$

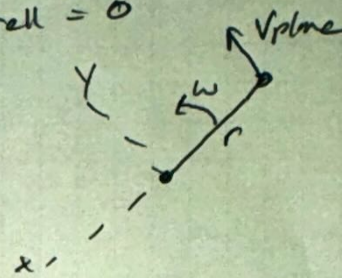
$$\bar{a}_P = \cancel{\bar{a}_0}^{\,0} + \bar{a}_{rel} + \cancel{\dot{\bar{\omega}} \times \bar{r}_{rel}}^{\,0} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{rel}) + \cancel{2\bar{\omega} \times \bar{v}_{rel}}^{\,0}$$



Taking the earth to be the inertial reference frame in this case. The origin is not moving so $a_0 = 0$, $\omega =$ const so $\alpha = 0$. This equation is used for a rotating coordinate axis with rotation speed and accells of ω and α respectively. In this situation because $r_{rel} =$ Const, $V_{rel} = 0$, since the objects position does not change relative to point O.

C) Cont

Similarly, because V_{rel} is constant $a_{rel} = 0$



$\bar{a}_{plane} = \bar{\omega} \times (\bar{\omega} \times \bar{r}_{rel})$

for counter clockwise rotation

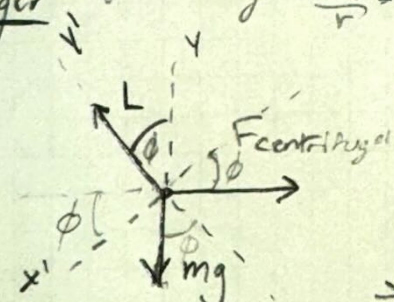
$\omega > 0$ (+k direction) → r in the $-\hat{i}$

Therefore the people in the plane experience the Centripetal acceleration of

$$\boxed{\bar{a}_{plane} = \omega^2 r \hat{i} \rightarrow \|a_{plane}\| = \frac{V_{plane}^2}{r} = a_c}$$

In the perspective of the observers, on the plane a "Centrifugal" force is trying to fling them out equivalent to the their mass times the centripetal acceleration of the plane. This is a ficticious force, but it must be accounted for because the frame is non-inertial

Passenger $\rightarrow F_{centrifugal} = \frac{mV^2}{r} = ma_c$



In order for the passengers to experience no sideways forces, the sum of forces in the x' direction on the passenger must be zero.

$$\rightarrow \Sigma F_{x'} = mg\sin\phi - \frac{m V_{plane}^2}{r}\cos\phi = 0$$

where y' and x' are the rolled axis of the plane. Lift will always be along the y' direction.

$$mg\sin\phi = \frac{mV_{plane}^2}{r}\cos\phi$$

$$g\tan\phi = \frac{V_{plane}^2}{r}$$

$$\phi = \tan^{-1}\left(\frac{V_{plane}^2}{rg}\right)$$

Note: This is in the observer on earths inertial frame of reference, because the passenger is in a non inertial frame, it experiences a centrifugal force. From someone observing this on the ground There is no centrifugal force, just the centripetal acceleration.

c) cont.

This relates to orbits because in orbital mechanics we consider the inside of an orbiting object to be inertial, while in this case of the plane thats not entirely inertial, the people inside do not feel the centrifugal force away from the center. In an orbit an astronaught does not feel like its turning toward the focus, but they are in the global frame. But because gravity is the only force, in space. The whole frame inside the satellite is inertial.

d)

$$\bar{a}_{person} = \cancel{\bar{a}_o}^{0} + \cancel{a_{rel}}^{0} + \cancel{\alpha \times r_{rel}}^{0} + \omega \times (\omega \times r_{rel}) + 2\omega \times V_{rel} \; \hat{\kappa} \times$$

$\boxed{\bar{a}_{person} = \omega^2 r \,\hat{\imath} + 2\omega V_{pas} \hat{\imath}}$ → both terms in Centripital direction

In this case, relative to the center of rotation the passenger is moving at a faster tangential velocity than the plane. This means that the person on the plane requires a greater centripital acceleration than the plane to stay in a perfect circle. This corresponds to the person accelerating outward relative to the plane. $\|a_{plane}\| < \|a_{person}\|$

In orbital mechanics this relates to a $+\Delta V$. In that case, speeding up means you need more acceleration towards the planet to maintain your orbit. Because the planet can't "just" "give" more gravity, your orbit gets bigger to a range that balances your new speed to keep you in a stable orbit.

e)

$$\bar{a}_{person} = \cancel{\bar{a}_o}^{0} + \cancel{a_{rel}}^{0} + \cancel{\alpha \times r_{rel}}^{0} + \omega \times (\omega \times r_{rel}) + 2\omega \times V_{rel}$$

$\boxed{\bar{a}_{person} = \omega^2 r \;\hat{\imath} - 2\omega V_{pas} \hat{\imath}}$

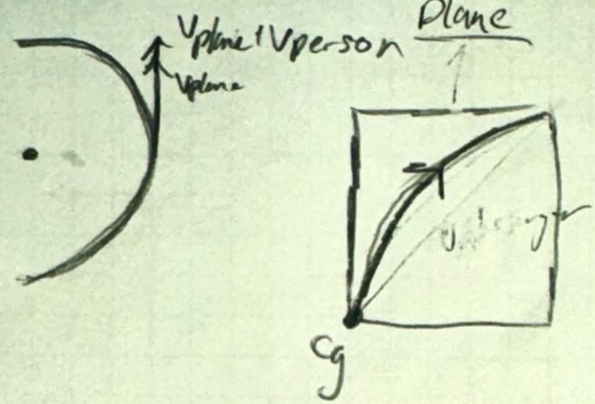
Inversely in this case, the relative centripital acceleration to keep the person in a circle is less than that of the plane to stay in a perfect circle. This corresponds to the person accelerating inward relative to the plane. $\|a_{plane}\| \geq \|a_{person}\|$

In orbital mechanics, this relates to a $-\Delta V$. Because the magnitude of acceleration required to keep the object in the current orbit is greater than it needs to be, you get pulled closer to the planet, in other words your orbit shrinks and you speed up until the orbit is stable.

Note: Look at the attached diagrams

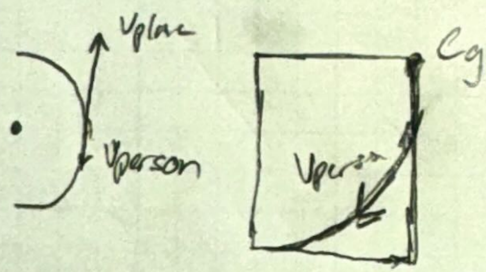
diagrams for parts D and E

D|



Becase the passenger is moving faster than the plane, and the passenger starts at the cg. relative to the cg of the plane the passenger moves forward ~~parallell at a~~ constant velocity. but also accellerates outward radially with the magnitude $2\omega V_{passenger}$.

E|



Becawse the person is moving slower than the plane, the passenger both has a velocity relative to the plane, and an ~~r~~ acceleration radially inward of $2\omega V_{passenger}$