AA 311 | HW8 | 12/1

Given

Sea level $V_\infty = 100\ m/s$ $S = 1.5\ m^2$

$C = 0.45\ m$ $l_t = 1\ m$ $S_t = 0.4\ m^2$

$a_t = \frac{\partial C_{L_t}}{\partial \alpha} = 0.12$ $i_t = 2^\circ$ $\varepsilon_0 = 0$

$\frac{\partial \varepsilon}{\partial \alpha} = 0.42$ $\alpha_a = 5^\circ$ $L = 4134\ N$

Find

M_{cg}

Properties

$\varepsilon = \varepsilon_0 + \frac{\partial \varepsilon}{\partial \alpha}\alpha_a$ $V_H = \frac{l_t S_t}{c S}$ $C_{m,cg_t} = -V_H C_{L,t}$

$C_{m_{cg}} = C_{m_{cg},wB} + C_{m_{cg},t}$ $q_\infty = \frac{1}{2}V_\infty^2 \rho_\infty$ $L = C_L q_\infty S$ $\rho_\infty = 1.225\ \frac{kg}{m^3}$

$\alpha_t = \alpha_{wb} - i_t - \varepsilon$

$C_{m_{cg}} = C_{m_{ac,wB}} + C_L\left(h - h_{ac} - V_H \frac{a_t}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right] + V_H a_t (i_t + \varepsilon_0)$

Analysis

$\varepsilon = (0) + (0.42)(5) = 2.1^\circ$

$V_H = \frac{(1m)(0.4m^2)}{(0.45m)(1.5m^2)} = 0.59$

$\alpha_T = \alpha_{wb} - i_t - \varepsilon = 0.9^\circ$

$C_{L_t} = a_T \alpha_T = (0.12)(0.9^\circ) = 0.108$ $q_\infty = (1.225\frac{kg}{m^3})(100\ m)^2 = 6125.0\ Pa$

$C_L = \frac{4134\ N}{(6125.0 Pa)(1.5 m^2)} = 0.45$ $a = \frac{0.45}{5^\circ} = 0.090$

from 7.2 $\rightarrow$ $C_{m_{ac,wB}} = -0.003$ $h - h_{ac} = 0.02$

$C_{m_{cg}} = -0.003 + 0.45\left(0.02 - 0.59\left(1 - 0.42\right)\cdot \frac{0.12}{0.090}\right) + 0.59(0.12)(2+0)$

$= -0.038$

$M_{cg} = C_{m_{cg}} \cdot S \cdot C \cdot q_\infty = (-0.038)(1.5m^2)(0.45m)(6125\ Pa) = \boxed{-239.8\ Nm}$

AA 311

7.6)

Given

Sea level $V_\infty = 100\ m/s$ $S = 1.5\ m^2$

$C = 0.45\ m$ $l_t = 1\ m$ $S_t = 0.4\ m^2$

$a_t = \frac{dC_{L_t}}{d\alpha} = 0.12$ $i_t = 2^\circ$ $\varepsilon_0 = 0$

$\frac{\partial \varepsilon}{\partial \alpha} = 0.42$ $\alpha_a = 5^\circ$ $L = 4134\ N$ $h = 0.26$

Find

Static margin, neutral point

Properties

$$h_n = h_{ac,wb} + V_H \frac{a_t}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$h_n - h \rightarrow$ static margin

Analysis

From previous question

$V_H = 0.59$ $a = 0.090$ $h - h_{ac} = 0.02$

$$h_{ac} = 0.26 - 0.2 = 0.24$$

$$h_n = (0.24) + 0.59 \cdot \frac{0.12}{0.09}\left(1 - 0.42\right) = \boxed{0.696}$$

Static Margin $= 0.696 - 0.26 = \boxed{0.436}$

AA 311

given 7.8

quantities from 7.4, 7.2

$$\frac{\partial C_{he}}{\partial \alpha_T} = -0.007 \qquad \frac{\partial C_{he}}{\partial \delta_e} = -0.012$$

$$\frac{\partial C_{Lt}}{\partial \delta_e} = 0.04$$

Find

Stick free static stability

Properties

$$F = 1 - \frac{1}{a_t}\frac{\partial C_{Lt}}{\partial \delta_e} \cdot \frac{\partial C_{He}/\partial \alpha_T}{\partial C_{He}/\partial \delta_e} \qquad h_n' = h_{ac,wB} + F V_H \frac{a_T}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

Analysis

$$\frac{\partial C_{mcg}}{\partial \alpha} = -a\left(h_n - h\right)$$

from prev

$V_H = 0.59$ $a_T = 0.12$ $\frac{\partial \varepsilon}{\partial \alpha} = 0.42$ $h_{ac,wB} = 0.24$

$h_n = 0.696$ $h = 0.26$

$$F = 1 - \frac{1}{0.12}\left(0.04 \cdot \frac{(-0.007)}{(-0.012)}\right) = 0.806$$

$$h_n' = 0.24 + 0.806 \cdot 0.59\left(\frac{0.12}{0.09}\right)\left(1 - 0.42\right) = 0.609$$

Stick-fixed Stability

$$\frac{\partial C_{mcg}}{\partial \alpha} = -a(h_n - h) = -0.0393$$

Stick-Free Stability

$$\frac{\partial C'_{mcg}}{\partial \alpha} = -a(h'_n - h) = -0.0314$$

Stick fixed has a more negative $\frac{\partial C_m}{\partial \alpha}$

Therefore it is more longitudally Stable

Additionally $\frac{h'_n - h}{h_n - h} = 80.04\%$

Therefore the static margin decreased by 20%