Homework 5

DUE: **Wed** 2025-11-**12** @ 23:59 on *Canvas*

(PDF submissions only; can be pictures/scans as needed)

1. The *interplanetary transfer* trajectories we studied include the use of a *parking orbit* both at the departure and arrival stages. In this problem you will analyze the use of parking orbits on arrival.

   For this problem consider the following "steps" to arrive to Mars:
   1. Entrance to the Mars-centric parking orbit after arriving to apogee of the sun-centric elliptic trajectory (1 Δv)
   2. A Hohmann transfer (2 Δvs) from the parking orbit to *0 altitude*, but assuming no atmospheres, mountains, or rotation of the planet
   3. Slowing down to the planet rotation speed (1 Δv)

   Only consider the magnitude for all Δv's, you can ignore the direction.

   a. Develop a function in Matlab or Python which takes as input the altitude of the *parking orbit* and calculates the $|\Delta v|$ required to enter the parking orbit after arrival to Mars' SoI (i.e., the magnitude of the first Δv above).
      i. Submit a copy of your derivation and commented code.
      ii. Plot the function from the minimum to maximum useful altitudes
      iii. What *should* be the altitude of the parking orbit considering only this Δv? [Answer ~8800 km]

      (Tip: plot the function on a semi-log or log-log scale to visualize better.)

   b. Create a new formula that calculates the 2 Δv's required for the Homann transfer from the parking orbit to the surface (assuming no atmosphere, etc). **Add** that to the first Δv to calculate the **total** Δv to the surface.
      i. Submit a copy of your derivation and commented code.
      ii. Plot the function from the minimum to maximum useful altitudes.
      iii. What should be the altitude of the parking orbit considering these 3 Δvs? [Answer ~0 km]

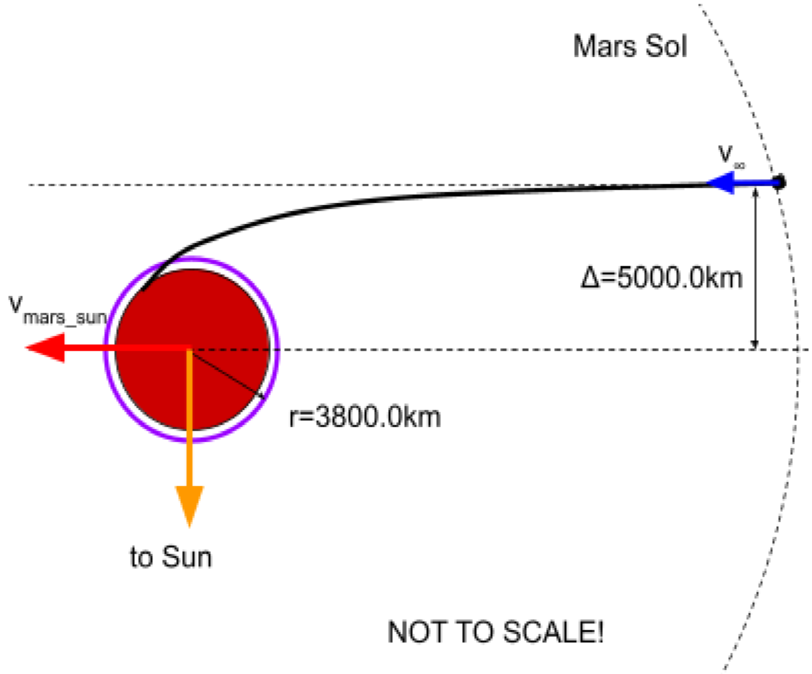
      (Tip: plot the function on a semi-log or log-log scale to visualize better.)

c. Calculate the Δv to slow down from “0-altitude orbit” to the actual surface speed of Mars and add it to the new **total** Δv which includes the 4 Δv’s.
    i. What is the total *minimum* Δv ? [Answer ~5.4380 km]

d. What would be your recommendation for a parking orbit?

e. What is the aiming radius (Δ=b) to get to the recommended parking orbit? [Answer: 7281.6 km]

2. The mission went wrong! After trying to arrive to Mars close to the surface (r_C = 3800), the satellite arrived *too* close to Mars. The satellite arrived with Δ=5000.0km on the shadow side at Sol:



Tip: don't forget to use μ for Mars!

a. What is v_∞ on arrival from Earth via a Hohmann transfer? (assume circular orbits for the planets) [Answer: -2.6483 km/s]

b. What is the periapsis (r_p) of the arrival hyperbolic trajectory (with respect to Mars)? [Answer 1785.4km]

c. At what θ (wrt to the arrival hyperbola) would it reach the target radius of 3800km? [*Answer: -86.583°*]

d. What will be the corresponding flight path angle, γ, at that point? [*Answer:* -50.141°]

e. What will be the speed of the spacecraft when it reaches the target radius of r_C=3800km? (i.e, *before* the Δv, along the hyperbola) [Answer: 5.4371km/s]

f. What would be the required Δv for insertion into circular orbit? [*Answer: 4.1756km/s*]

g. Draw the "**Δv** triangle" (**Δv** = $\mathbf{v_2}$ **-** $\mathbf{v_1}$) for this problem, scale the velocities to each other (no need to draw the orbital trajectory itself, just the triangle; you can draw it by hand or code it, but be careful with the scales of the vectors!).

h. **Bonus**[1] (+2) : After the Δv, did the velocity increase or decrease with respect to:
   i. Mars
   ii. the sun

   Show your work to determine that numerically (you can do with code or by hand, but show your work).

[1] 1 point for Mars 1 point for sun, all or nothing each.

3. The Mission Project (to Mars) only considers the required Δv (fuel use) for the mission, but it ignores time. This question asks you to complement what you learn in the mission project, with thinking about the amount of time it would take to complete the mission. For this example we will only care about the "inter-planetary" times:
    - Time to takeoff from Earth and be in parking orbit before departure : ignored
    - Time to travel Earth to Mars
    - Time to orbit Mars in parking orbit / land : ignored
    - Wait in Mars until the planets align again
    - Time to takeoff from Mars and be in parking orbit before departure : ignored
    - Time to travel to Mars from Earth
    - Time to orbit Earth in parking orbit / land : ignored

    a. What is the flight time, in days, from Earth to Mars? (if you created a Hohmann Transfer function in Matlab/Python for HW4, you can use that function) [*Answer: 258.79 days*]

    b. Where does Mars need to be with respect to Earth, so that the satellite reaches Mars at the end of the Hohmann transfer?
    [*Answer: +44.329°* from Hohmann periapsis]

    c. To return to Earth with a Hohmann transfer, the opposite will need to be true: Where does Earth need to be with respect to Mars, so that the satellite reaches Earth?
    [*Answer: -75.097°* from Hohmann apoapsis]

    d. What is the "wait time" (in days and/or Earth years) from arrival to the next departures for a Hohmann transfer? [Answer: 454.64 days]

    e. What is the *minimum* **total** round trip time to Mars for this *ideal* Hohmann transfer?

4. Jupiter is so big it always makes sense to use it. Consider a mission to Saturn that wants to use Jupiter for a "sling shot". The management wants to minimize fuel, so its willing to wait to get to Jupiter via a Hohmann transfer. (Assume circular co-planar planetary orbits.)

   a. For the arrival to Jupiter:
      i. On what side does the satellite need to arrive: when Jupiter is trailing or ahead of the satellite?
      ii. Draw 2 sketches (by hand OK, similar to Curtis Fig 8.18) for the two cases of arrival:
         1. Sun side
         2. Shade side

      Include in your drawings:
      - Jupiter
      - the probe
      - the direction to the sun
      - $v_{Jupiter\_Sun}$
      - $v_{sat\_sun\_in}$
      - $v_{\infty in}$
      - $v_{\infty out}$
      - Δv_{∞}
      - $v_{sat\_sun\_out}$
      - Δv_{sun}

   b. Draw (by hand, drawing program, or any way you want) 2 sketches the expected trajectories from Earth to Jupiter and from Jupiter to Saturn of the Sun and shadow side arrivals.

   c. Determine the magnitude of v_{∞}. Show your work and/or Matlab/Python commented code. (Tip: you should be able to use previous functions from this HW with different input parameters!) [*Answer: -5.6441 km/s*]

   d. The required exit velocity and flight path angle for the sun-side arrival are:

      v_{sat_saturn} = 14.410 km/s

      γ_{sat_saturn} = -23.044 deg

      What is the required aiming radius (Δ=b) for arrival to Jupiter? [Answer 3,838,500km]

Useful data for this problem:

	Orbital radius [km][2]	μ [km^3/s^2]	Planet Radius [km]
Earth	1.496×10^8	3.986×10^5	6378
Jupiter	7.786×10^8	1.26686×10^8	71490
Saturn	1.433×10^9	n/a	n/a
Sun	n/a	1.327×10^{11}	696000

Note: ***it is your responsibility to confirm these data with Curtis Appendix A.***

[2] Curtis Appendix A: Semi-Major axis