Mid-Term Exam

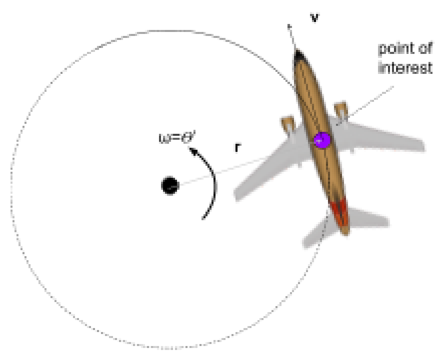
DUE *no later than MON 2025-11-03 @ 23:59*

(PDF submissions only; can be pictures/scans as needed)

1. Turning the airplane

While this class is about orbital dynamics, it is useful to relate the concepts with the (dynamic) behavior of things we may be familiar with. One such metaphor is to relate an aircraft that is turning.

In class and homeworks we discussed rotating bodies with respect to an inertial frame. We set up a problem where you position a “point of interest” in an object which is rotating about an inertial frame. This concept is illustrated in a “turning airplane” in the figure below:



Top View of turning airplane

The resulting acceleration in an inertial reference frame for that “point of interest” in the turning airplane is given by the “5-acceleration equation”:

$$\overline{a}_{in} = \overline{a}_0 + \overline{a}_{rel} + \overline{\alpha} \times \overline{r}_{rel} + \overline{\omega} \times \left(\overline{\omega} \times \overline{r}_{rel}\right) + 2\overline{\omega} \times \overline{v}_{rel}$$

where all variables are <u>vectors</u>.

Analyze the motion of the point of interest in the turning airplane drawing by taking the following steps:

- **a.** Identify in the figure the reference frame if you are standing in the ground. Is that an inertial reference frame? Why or why not?
- **b.** Identify in the figure the reference frame if you are standing inside the airplane. Is that an inertial reference frame? Why or why not?

c. Consider the case of the airplane rotating at a constant angular rate and constant radius of rotation.

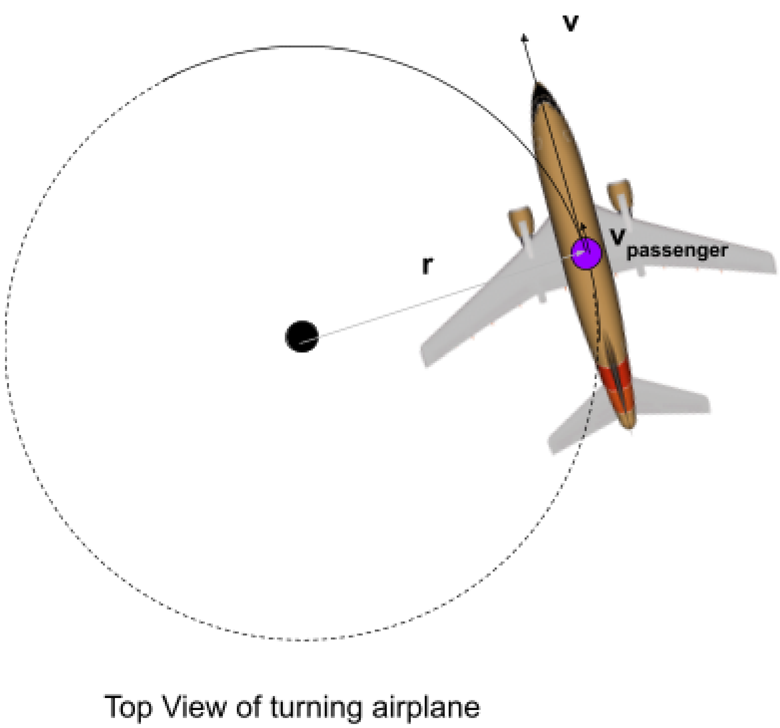
   i. Use the “5-acceleration equation” to determine the equation for the amount of “banking” (rotation about the roll axis) necessary so that a passenger seating at the c.g. does not feel any sideway forces (sideway acceleration with respect to the seat), as a function of the radius of rotation & the speed of the airplane. Include all equations necessary for an analytical solution.

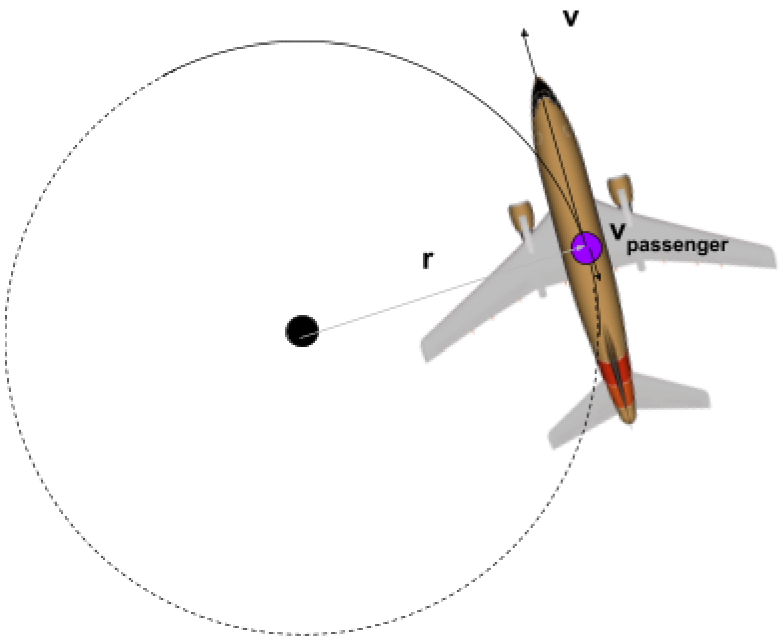
   ii. Draw a set of free body diagrams to help explain your answer.

   iii. How does this relate to orbits?

Parts d, e: Imagine (even if unrealistic) that the airplane does **not** bank at all, it remains flat. Find the resulting inertial acceleration (using the “5-acceleration equation”) for the passenger in the following scenarios.

Note: for the following answers the absolute magnitude (size / length) of the vectors is not important in the answer. However, the relative magnitudes, adding the vectors correctly, and the direction of the vectors must be in the correct directions.

<table>
<tr><td>Part</td><td>Scenario
The passenger is walking towards the front of the plane, at a $v_{passenger}$ speed, when the turn happens.</td></tr>
<tr><td>d</td><td></td></tr>
<tr><td colspan="2">What could this correspond to in orbital mechanics?</td></tr>
</table>

<table>
<tr><td>Part</td><td>Scenario
The passenger is walking toward the back of the plane, at a $v_{passenger}$ speed, when the turn happens.</td></tr>
<tr><td>e</td><td>

Top View of turning airplane</td></tr>
<tr><td colspan="2">Can this be the same orbital mechanics scenario as in part d? (why or why not)</td></tr>
</table>

2. Two Satellites deployed to Orbit

As you may know, a lot of *CubeSat* satellites are deployed from common launchers, with many satellites being deployed shortly after one another. Lets imagine a case of a couple of CubeSats deployed from the ISS (450km altitude, circular orbit) at *practically* the same time.

a. The CubeSat deployment happens from the arm of the ISS, which is only a few meters long. Given this, what is the approximate deployment altitude and radius?

b. Both CubeSat's must have a "smaller orbit" than ISS, for safety. In which direction must the deployment mechanism push the CubeSat's?

c. Will the CubeSat's and ISS ever meet again? Explain the assumptions (or non-assumptions) for your answer.

d. The desired orbit for Sat 1 has a maximum separation (in ideal orbital dynamics conditions) from the ISS's circular orbit of 50km. Find the parameters for the orbit of Sat 1:
   i. Use your existing functions in Matlab or Python to get all the other parameters. Submit a commented copy of your code.
      - e
      - h
      - r_p & r_a - periapsis / apoapsis radii, as applicable
      - a & b - semi- major & minor axes, as applicable
      - v_p & v_a - speed at periapsis / apoapsis
      - T - period
      - ε - specific energy

      Tip: first get the eccentricity e and either a or h
   ii. What is the Δv (magnitude) necessary to enter this orbit?
   iii. Use Matlab or Python to plot the resulting orbit with respect to the ISS orbit. Submit a commented copy of your code.

e. The objective for Sat 2 is for it to "meet" the ISS every 18 orbits:
   i. What is the difference in the periods between the ISS and Sat 2?
   ii. What is the Δv (magnitude) for Sat 2?

   Submit commented copy of any code you use.

f. For this final 3 satellite system: after the ISS completes one full rotation, what will be the *difference* in true anomaly between:
   i. ISS and Sat 1
   ii. ISS and Sat 2
   iii. Sat 1 and Sat 2

Use Matlab (or Python) to get the answer. Submit a commented copy of your code.

3. Observing the Poles

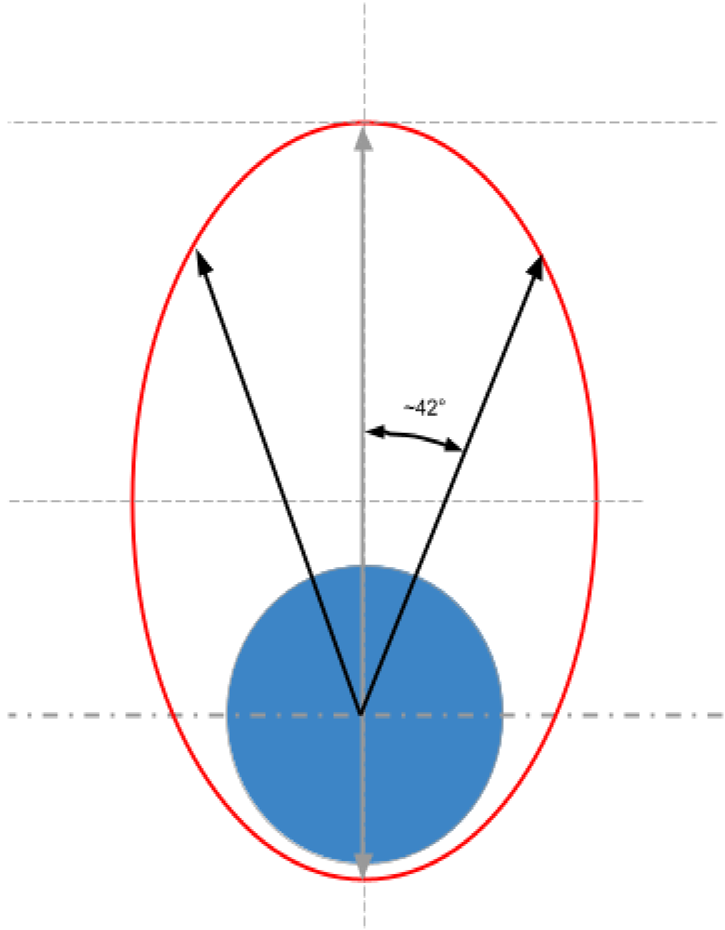
In order to observe the North and South poles we cannot use a "Geostationary orbit", because the Earth's rotation is perpendicular to the rotation of the satellite. The next best thing is to use *High Eccentricity Orbits*. Let's imagine that NASA wants satellites that observe the North or South poles. Assume they can be placed exactly perpendicular to the equator.

a. Find as much information as can you find about the orbit if NASA desires the satellites to have a 48 hour period. Submit commented code that you use (you are welcome to reuse any previous code used for homework).

b. In addition NASA tells you the maximum *altitude* for the sensor to work correctly is 90,000km. Calculate all the parameters of the orbit (using any previous code you have, submit with comments):
- r_p, r_a, a, b, h, v_p, v_a, ε.

c. For how much time will a satellite be on its "side" of the equator? (i.e., on the North or the South side)

d. The sensors ended up not being so good, and can only be used when the satellite is within ±42° of the pole:



What is the “useful” time for this satellite for the given location? Show your work (code recommended, commented).

e. In order to get into the highly elliptical orbit the satellite is first placed into a circular Earth orbit at 38000km altitude.

i. Sketch the 2 options for Hohmann Transfer orbits using the diagram below (or an equivalent version of your own)
ii. What are the origin & destination of the 2 Hohmann Transfer orbits?
iii. Determine the orbital parameters (same list as part b) of the 2 Hohmann Transfer orbits options.
iv. Calculate the <u>total</u> Δv required to enter the final orbit via the 2 Hohmann transfer options.
v. Recommend a transfer option based on those results.

You may re-use any code previously written. Submit a commented copy of your code with results.