Homework 1

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Exercise 2.1. Consider the low-speed flight of the Space Shuttle as it is nearing a landing. If the air pressure and temperature at the nose of the shuttle are 1.2 atm and 300 K, respectively, what are the density and specific volume?

Solution.

Given:

$$\text{P} = 1.2 \text{ atm}, \text{ T} = 300 \text{ K}$$

Find:

$$\rho, \nu$$

Properties:

$$P = \rho RT, \nu = \frac{1}{\rho}, R_{air} = 287\frac{\text{J}}{\text{kg K}},$$
$$1 \text{ atm} \approx 101325 \text{ Pa}$$

Analysis:

Start by converting to SI units: 1.2 atm = 121590 Pa. Consulting the ideal gas law, $P\nu = RT$ therefore

$$\nu = \frac{RT}{P} = \frac{(300\text{K})(287\frac{\text{J}}{\text{kg K}})}{(121590\text{Pa})} \approx 0.708 \text{ m}^3/\text{kg}$$

and finally

$$\rho = \frac{1}{\nu} = \frac{1}{0.708 \text{ m}^3/\text{kg}} \approx 1.412 \text{ kg/m}^3$$

□

Exercise 2.9. Consider a flat surface in an aerodynamic flow (say a flat sidewall of a wind tunnel). The dimensions of this surface are 3 ft in the flow direction (the x direction) and 1 ft perpendicular to the flow direction (the y direction). Assume that the pressure distribution (in pounds per square foot) is given by $p = 2116 - 10x$ and is independent of y. Assume also that the shear stress distribution (in pounds per square foot) is given by $\tau_w = \frac{90}{\sqrt{x+9}}$ and is independent of y as shown in the figure below. In these expressions, x is in feet, and $x = 0$ at the front of the surface. Calculate the magnitude and direction of the net aerodynamic force on the surface.

Solution.

Given:

$$\tau_w(x) = \frac{90}{\sqrt{x+9}}, P(x) = 2116 - 10x, L = 3\text{ft}, W = 1\text{ft}$$

Find:

aerodynamic force acting on the surface: magnitude & direction

Assumptions:

The surface is entirely flat & no forces act in the y direction

Properties:

$$F = \iint Pnda, A = L * W$$

Analysis:

Firstly, we can compute the force applied to the surface from both the pressure and shear stress on the surface. The pressure acts normal to the surface, therefore the force acting on the surface is also normal to the surface (-z direction). The shear stress acts in the positive x direction. The magnitude of force applied by the pressure is therefore:

$$F_p = \int_0^L \int_0^W P(x) \cdot da = \int_0^3 (2116 - 10x)\, dx = 6303 \text{ lbs}$$

its direction mirrors that of the pressure, being the -z direction. Next we can analyze the shear stress on the surface:

$$F_\tau = \int_0^L \int_0^W \tau(x) \cdot da = \int_0^3 \frac{90}{\sqrt{x+9}}\, dx = 83.54 \text{ lbs}$$

The direction of this force is in the same direction as the flow/shear stress.
Therefore the magnitude of the total aerodynamic force is given by:

$$||F_{aerodynamic}|| = \sqrt{F_\tau^2 + F_p^2} \approx 6304 \text{ lbs}$$

Breaking this into a vector

$$F_{aerodynamic} = \begin{bmatrix} 83.54 \\ 0 \\ -6303 \end{bmatrix}, \; \hat{\boldsymbol{e}} = \frac{F_{aerodynamic}}{||F_{aerodynamic}||} = \begin{bmatrix} 0.013 \\ 0 \\ -0.987 \end{bmatrix}$$

Results:

$$||F_{aerodynamic}|| \approx 6304 \text{ lbs } \& \; \hat{\boldsymbol{e}} = \begin{bmatrix} 0.013 \\ 0 \\ -0.987 \end{bmatrix}$$

Comments:

The pressure causes significantly more stress on the surface than the shear stress. I assume this is because we assume that the flow is only the x direction and that the surface only exists in the xy plane. Once we have a surface that is rotated out of this plane the flow exists in I expect to see a stronger shear force on the surface. □

Exercise 2.14. In a gas turbine jet engine, the pressure of the incoming air is increased by flowing through a compressor; the air then enters a combusto rthat looks vaguely like a long can (sometimes called the combustion can). Fuel is injected into the combustor and burns with the air, and then the burned fuel–air mixture exits the combustor at a higher temperature than the air coming into the combustor. (Gas turbine jet engines are discussed in Ch. 9). The pressure of the flow through the combustor remains relatively constant; that is, the combustion process is at constant pressure. Considerthe case where the gas pressure and temperature entering the combustor are $4 \cdot 10^6 \frac{\mathrm{N}}{\mathrm{m}^2}$ and 900 K, respectively, and the gas temperature exiting the combustor is 1500 K. Calculate the gas density at (a) the inlet to the combustor and (b) the exit of the combustor. Assume that the specific gas constant for the fuel–air mixture is the same as that for pure air.

Solution.

Given:

$$P = const = 4 \cdot 10^6 \frac{\mathrm{N}}{\mathrm{m}^2},\ T_{inlet} = 900\ \mathrm{K},\ T_{outlet} = 1500\ \mathrm{K},\ R = R_{air} = 287 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$$

Find:

$$\rho_{inlet}\ \&\ \rho_{outlet}$$

Properties:

$$P = \rho * RT$$

Analysis:

We can rewrite this equation $P = \rho * RT$ to be $\rho = \frac{P}{RT}$. We can then solve for the inlet and outlet density by simply plugging in values.

$$\rho_{inlet} = \frac{(4 \cdot 10^6\ \mathrm{Pa})}{(287\ \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}})(900\ \mathrm{K})} = 15.49\ \frac{\mathrm{kg}}{\mathrm{m}^3} \quad \& \quad \rho_{outlet} = \frac{(4 \cdot 10^6\ \mathrm{Pa})}{(287\ \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}})(1500\ \mathrm{K})} = 9.29\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

□