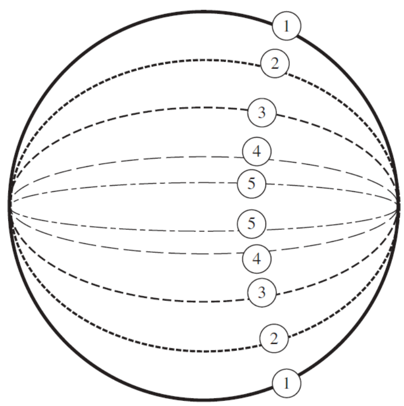
Homework 3

DUE: Mon 2025-10-20 @ 23:59 on *Canvas*

(PDF submissions only; can be pictures/scans as needed)

1. The caption in *Curtis Figure 2.20* states that the *periods* and *energies* of the 5 shown orbits are the same.



   a. Determine approximately the eccentricity of the orbits using a and b (the semi-major and semi-minor apses), assuming b = <fraction>*a. List the 5 eccentricities on a table.

   b. Using Matlab or Python, redraw the figure, showing all 5 orbits in a single plot, but with the center on the focus of the orbit, rather than the geometric center. Provide your commented code and the plot.

   c. Compare the two figures, what are the most drastic changes when looking at it from the “orbit” rather than the geometry perspective?

   d. Determine the functions to compare the velocities at apoapsis and periapsis of each of the orbits with that of the first (circular) orbit:

      $v_p = f(e)*v_1$

      $v_a = f(e)*v_1$

      Use Matlab or Python to plot v_p/v_1 and v_a/v_1 for the range e=0:0.9 in .125 increments. (You may assume r=1, μ=1.) Provide your commented code and the plots. [*Partial Answer @ e=0.125 v_p=1.1339v_1, v_a=0.8819v_1*]

e. Compare and comment on the behavior of r_p and r_a (from part b) with the v_p and v_a (from part d). What does it have to do with conservation of energy?

2. Escaping Earth!
   a. Determine an equation to calculate the change in speed (v_{escape} - v_{circle}) you would need to escape from Earth if you start in a circular orbit, with the orbit's altitude as a variable. Write a Matlab or Python function for it. Provide your commented code.

   b. Use your function to create a table to escape from:
      i. LEO circular @ 450km *altitude* (low Earth orbit; e.g. the ISS) [*Answer: change in v=3.1653km/s*]
      ii. MEO circular @ 22,200km *altitude* (middle Earth orbit; e.g. GPS)
      iii. GEO circular @ 35,700km *altitude* (geostationary orbit; e.g. communications satellite)

   c. Assuming the mass of the Earth could stay constant but that Earth could shrink in radius (increase in density), what would the radius of Earth need to be so that the escape velocity is the speed of light? (Optional: what do we usually call this?) [*Answer: 8.8726mm*]

   d. For the LEO (450km) case: if your satellite does not have enough fuel and only ½ the Δv needed is created (i.e. $v_{new} = v_{circle} + \Delta v/2$), what is the resulting orbit of the satellite? Use functions you have done before to show all the known information about the orbit. [*Partial answer: new r_a = 18,326km*]

3. On 2025-Oct-01 the asteroid "2025 TF" passed as close as 428km from the surface of Earth at 20.88km/s. Calculate:

   a. the eccentricity; what type of orbit is it? [Answer: e=6.442]

   b. Write a function that will give you all the information (e, r_p, r_a, a, b (= Δ), h, θ_∞, β, δ, v_∞, v_{esc}, v_p, ε) about a hyperbolic trajectory given e & r_p as inputs. Provide the numerical outputs of your function for 2025 TF.
   Tip: make this part of your library

   c. Sketch the orbit (or better yet, plot it with Matlab or Python - be careful not to plot beyond the asymptote!), and annotate the unique elements of a hyperbola: r_p, a, θ_∞, δ, Δ (=b), v_∞, v_p.

d. Imagine that Dr Evil wanted to make 2025 TF crash into the moon; the Dr determined that requires the new required true anomaly of the asymptote to be $\theta_\infty = 138°$. What is the new speed at periapsis (v_p) that 2025 TF needs to have to end up in that trajectory to the moon? Explain why the change in velocity (slower or faster) makes sense to end up from the old asymptote to the new one.

e. If we assume that the moon is "infinitely" far away from Earth for the purposes of 2025TF, at what speed would 2025TF reach the moon?

4. Imagine an elliptical orbit (e ~ 0.5, visually elliptical) that is perfectly parallel with Earth's equator. Draw the 2D "top view" orbit and the *perifocal frames* if their perigee (periapsis around Earth) is at the longitude of the places below[1]; keep the Earth aligned so that 0° longitude is always pointing "right" (like +X in a "normal" cartesian axis):
   a. 0° England/Africa
   b. 90° India
   c. 135° Australia
   d. -120° Seattle

[1] This is **very** simplified and the degrees are definitely approximations (Except Greenwich is 0)! In the 2nd half of the term will learn all the details about satellites orbiting Earth, including accounting for Earth's rotation about itself and about the sun. All these are ignored for this problem.