Homework 2

DUE: Mon 2025-10-13 @ 23:59 on *Canvas*

(PDF submissions only; can be pictures/scans as needed)

1. At a given time a 6kg CubeSat has the position and velocity:
   r = 7000**i** + 0**j** + 0**k** [km]
   v = 0**i** + 8**j** + 0**k** [km/s]

   a. Write a Matlab or Python function, ***which uses the functions you wrote before***, to calculate the specific angular momentum of a satellite given the **r** and **v** vectors. Provide a commented copy of your code.

   b. What is the angular momentum **H** of the satellite? [*Partial answer |H| = 336,000 kg km^2/s*]
   Info: this would be a somewhat realistic H for a cubesat in LEO.

   c. At another time a measurement comes in that says it is at:
   r = 3500**i** + 6062**j** + 0**k** [km]
   v = -6.928**i** + 4.000**j** + 0**k** [km/s]
   Is it possible that is the same satellite? Why or why not?

   d. A third measurement comes in that says it is now at:
   r = 0**i** + 7000**j** + 0**k** [km]
   v = 8**i** + 0**j** + 0**k** [km/s]
   Is it possible that is the same satellite? Why or why not?

   e. Give an example of a new valid measurement of position and velocity for the same satellite, and show that it is valid.

*Tip: this problem is all about (specific) angular momentum where **h** = **r** x **v***

2. The traditional orbital dynamics equations we will use assume that the inertial axis is the center of the larger body (e.g., the planet or the sun, depending on the problem). However, the CG <u>does</u> move, it “wobbles”. For our purposes, define the “wobble” as the offset of the CG from the center of the larger body as presented in Figure 1.

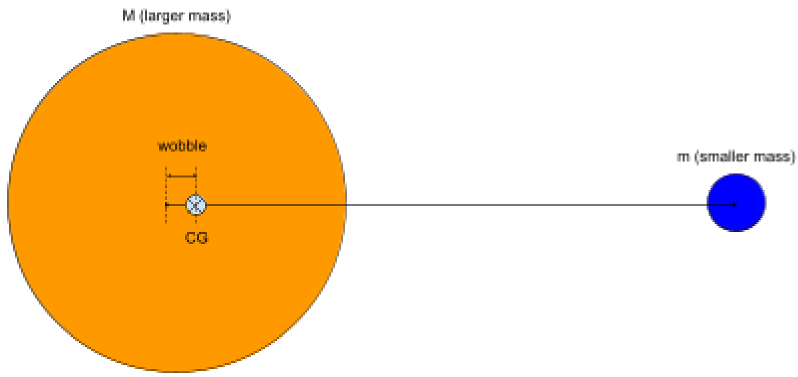

Figure 1 - Definition of “wobble” of the larger body in a 2-body problem.

   a. Write a Matlab (or Python) function which determines the “wobble” when given the two masses and the separation between them. Provide a commented copy of your code.

   Quantitatively, using data from Curtis Appendix A or online, calculate the wobble both as a % of the separation and in absolute kilometers (assume the *semimajor axis* in the appendix is the radius of circular orbits).

   b. Provide a table which uses that function to show the wobble between:
      i. Earth and the Sun
      ii. Jupiter and the Sun
      iii. The moon and Earth
      iv. A cubesat (~3kg) in LEO (~450km altitude) and Earth
      v. The ISS (~510,000kg, 400km altitude) and Earth

   Comment on the observed results.

3. Kepler's equation is about the *position* (**r**) of a satellite. When we discussed the *state* we said that we care about position **and** velocity.

    a. Determine the equation for the *speed* in terms of h, e, θ, and μ (scalars) only. Simplify as much as you can. Because Kepler's Equation does not include time you can<u>not</u> differentiate with respect to time directly. Instead, use the concept discussed about tangential and radial velocities:

    $$\mathbf{v} = v_{\perp}\mathbf{u}_{\perp} + v_r\mathbf{u}_r$$

    Both the tangential ($v_{\perp}$) and radial (v_r) speeds were found in the process to derive Kepler's equation (Section 2.4).

    b. Write a Matlab or Python function that outputs the scalar state [r v] of a satellite given h, e, θ, μ. Provide a commented copy of your code.

    c. Use the function to obtain the output at several interesting combinations of e (eccentricity) and θ (true anomaly):

e	**θ**
0	0, π/2, -π/2, π
0.5	0, π/2, -π/2, π
1	0, π/2, -π/2, π
3	0, π/2, -π/2, π

    Use h = 1, μ=1 for this exercise.

    Comment on your findings.

    Tip: use loops in your code to simplify it - this exercise is about using programing efficiently so that you can then have good comments on the results - the coding should not be the interesting part, the comments on your results are most important!

4. Future spacecraft (like the HSL HuskySat-3) will orbit the moon on the NASA Lunar Gateway[1] with a periapsis of 3000km and a period of 7 days.

   a. Determine a & e with the information given. [*Answers: a=35684km; e=0.91593*]

   b. Write a Matlab or Python function that can calculate all the parameters of an elliptical orbit (r_p, r_a, v_p, v_a, b, h, T, ε) given a & e (and μ). Provide a commented copy of your code.
   Tip: put this function in your 310 "library" so you can use it later!

   c. Calculate the parameters using your function and provide a clear listing of the outputs (identify each variable clearly and provide units in either comments or the output). [*Partial answer: r_a = 68,639 km*]

   d. A spacecraft at the Lunar Gateway periapsis (3000km) wants to enter a new elliptical orbit with a minimum *altitude* of 50 km *above the moon surface*, while keeping (the new) apoapsis at 3000km. What should the *new* velocity be at 3000km for it to be in this new orbit? [*Answer: 1.1048 km/s*]

[1] This is **not** a precise description of the Lunar Gateway orbit - that orbit is a complicated 3-body problem called "near-rectilinear halo orbit" (NRHO) - approximated in here as fully planar elliptical orbit.