4340 Homework 4 (Due Tuesday, 2/16)

You only have to submit solutions to problems 1, 3, 4, 6 and 7, but please attempt all problems in preparation for the test next Thursday.

1. Two independent six-sided dice are rolled. Let X and Y denote, respectively, the outcomes of the first and second dice rolls. Let Ω denote the sample space containing all (x, y)-outcome pairs. Calculate:

   (a) $P(\min[X, Y] = 2)$

   (b) $P(\min[X, Y] \geq 5)$

   (c) $P(X = 2 | \min[X, Y] = 2)$

   (d) $P(X = 5 | \min[X, Y] \geq 5)$

   (Hint: it always helps to write out the elements of Ω that make up events.)

2. A 2010 study in the journal *Pediatric* found that 8% of children under the age of 18 in the US have at least one food allergy. Among those with food allergies, 39% have a history of severe reaction.

   (a) What is the probability that a child selected at random from this population has at least one food allergy and a history of severe reaction?

   (b) It was also reported that 30% of those with an allergy are, in fact, allergic to multiple foods. If a child is randomly selected from this population, what is the probability that he or she is allergic to multiple foods?

3. Application to national security: There has been a great deal of controversy over the last several years regarding what types of surveillance are appropriate to prevent terrorism. Suppose a particular surveillance system has a 99% chance of correctly identifying a future terrorist and a 99.9% chance of correctly identifying someone who is not a future terrorist. If there are 1000 future terrorists in a population of 300 million. One of these people is randomly selected and scrutinized by the system.

(a) If this person is identified as a future terrorist, what is the probability the he/she is indeed a future terrorist.

(b) If the person is not identified as a future terrorist, what is the probability the he/she is indeed a future terrorist.

(c) Comment on the use of this surveillance system. Also identify any flaws with screening for terrorists as described in this problem. Specifically, would one randomly select individuals from the population for screening? Why or why not?

4. A particular airline has 10am flights from Chicago to New York, Atlanta and Los Angeles. Let A denote the event that the flight to New York is full. Similarly, let B and C denote the flights to Atlanta and Los Angeles being full. Assume that the events A, B and C are independent with $P(A) = 0.9$, $P(B) = 0.7$ and $P(C) = 0.8$. Calculate

   (a) the probability that all three flights are full.

   (b) the probability that at least one flight is not full.

   (c) the probability that only the New York flight is full.

5. Suppose a randomly selected individual is equally likely to have been born on any of the 365 days of the year (we exclude the possibility of Feb. 29 birthdays to simplify this problem). If ten people are randomly selected, and these people are independent of one another, calculate

   (a) $P(\text{All 10 have different birthdays})$.

   (b) $P(\text{At least 2 have the same birthday})$.

6. Problem 19 on p. 108 (Sec. 3.2).

7. After a„ students have left the classroom, a statistics professor notices that three copies of the textbook were left behind in class. At the beginning of the next lecture, the professor distributes the three books back in random order to the three students (1, 2 and 3). One possible outcome is $(3, 2, 1)$, which means student 1 receives the book belonging to 3, student 2 receives her own book back and student 3 receives the book belonging to 1.

   (a) List all six ways in which the books can be distributed back to the students.

   (b) Let X denote the number of students who get their own books back. For outcome $(3, 2, 1)$, for example, 1 student gets their own book back. Write down the pmf of X.