AP Calculus AB Cram Sheet

Definition of the Derivative Function:

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

Definition of Derivative at a Point:

$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ (note: the first definition results in a function, the second definition results in a number. Also note that the difference quotient, $\frac{f(a+h)-f(a)}{h}$, by itself, represents the average rate of change of f from x = a to x = a + h)

Interpretations of the Derivative: f ' (a) represents the instantaneous rate of change of f at x = a, the slope of the tangent line to the graph of f at x = a, and the slope of the curve at x = a.

Derivative Formulas: (note:a and k are constants)

$\frac{d}{dx}(k) = 0$

$\frac{d}{dx}(k \cdot f(x)) = k \cdot f'(x)$

$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} f'(x)$

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$

$\frac{d}{dx}\sin(f(x)) = \cos(f(x)) \cdot f'(x)$

$\frac{d}{dx}\cos(f(x)) = -\sin(f(x)) \cdot f'(x)$

$\frac{d}{dx}\tan(f(x)) = \sec^2(f(x)) \cdot f'(x)$

$\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)} \cdot f'(x)$

$\frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$

$\frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot \ln a \cdot f'(x)$

$\frac{d}{dx}\sin^{-1} f(x) = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$

$\frac{d}{dx}\cos^{-1} f(x) = -\frac{f'(x)}{\sqrt{1-(f(x))^2}}$

$\frac{d}{dx} \tan^{-1} f(x) = \frac{f'(x)}{1+(f(x))^2}$

$\frac{d}{dx} (f^{-1}(x))$ at $x = f(a)$ equals $\frac{1}{f'(x)}$ at $x = a$

L'Hopitals's Rule:

If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ and if $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists then

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

The same rule applies if you get an indeterminate form ($\frac{0}{0}$ or $\frac{\infty}{\infty}$) for $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ as well.

Slope; Critical Points: Any c in the domain of f such that either f ' (c) = 0 or f ' (c) is undefined is called a critical point or critical value of f.

Tangents and Normals
The equation of the tangent line to the curve y = f(x) at x = a is

y - f(a) = f ' (a) (x - a)

The tangent line to a graph can be used to approximate a function value at points very near the point of tangency. This is known as local linear approximations. Make sure you use ≈ instead of = when you approximate a function.

The equation of the line normal(perpendicular) to the curve y = f(x) at x = a is

y - f(a) = $-\frac{1}{f'(a)} (x - a)$

Increasing and Decreasing Functions A function y = f(x) is said to be increasing/decreasing on an interval if its derivative is positive/negative on the interval.

Maximum, Minimum, and Inflection Points
The curve y = f(x) has a local (relative) minimum at a point where x = c if the first derivative changes signs from negative to positive at c.

The curve y = f(x) has a local maximum at a point where x = c if the first deivative changes signs from positive to negative.

The curve y = f(x) is said to be concave upward on an interval if the second derivative is positive on that interval. Note that this would mean that the first derivative is increasing on that interval.

The curve y = f(x) is siad to be concave downward on an interval if the second derivative is negative on that interval. Note that this would mean that the first derivative is decreasing on that interval.

The point where the concavity of y = f(x) changes is called a point of inflection.

The curve y = f(x) has a global (absolute) minimum value at x = c on [a, b] if f(c) is less than all y values on the interval.

Similarly, y = f(x) has a global maximum value at x = c on [a, b] if f (c) is greater than all y values on the interval.

The global maximum or minimum value will occur at a critical point or one of the endpoints.

Related Rates: If several variables that are functions of time t are related by an equation (such as the Pythagorean Theorem or other formula), we can obtain a relation involving their (time)rates of change by differentiating with respect to t.

Approximating Areas: It is always possible to approximate the value of a definite integral, even when an integrand cannot be expressed in terms of elementary functions. If f is nonnegative on [a, b], we interpret $\int_a^b f(x)\,dx$ as the area bounded above by y = f(x), below by the x-axis, and vertically by the lines x = a and x = b. The value of the definite integral is then approximated by dividing the area into n strips, approximating the area of each strip by a rectangle or other geometric figure, then summing these approximations. For our discussion we will divide the interval from a to b into n strips of equal width, Δx. The four methods we learned this year are listed below.

Left sum: $\sum_{i=0}^{n-1} f(t_i)\,\Delta t = f(t_0)\,\Delta t + f(t_i)\,\Delta t + f(t_2)\,\Delta t + \cdots + f(t_{n-1})\,\Delta t$, using the value of f at the left endpoint of each subinterval.

Right sum: $\sum_{i=1}^{n} f(t_i)\,\Delta t = f(t_1)\,\Delta t + f(t_2)\,\Delta t + f(t_3)\,\Delta t + \cdots + f(t_n)\,\Delta t$ using the value of f at the right endpoint of each subinterval.

Midpoint sum: $\sum_{i=0}^{n} f\left(\frac{t_i+t_{i+1}}{2}\right)\Delta t = f\left(\frac{t_0+t_1}{2}\right)\Delta t + f\left(\frac{t_1+t_2}{2}\right)\Delta t + \cdots + f\left(\frac{t_{n-1}+t_n}{2}\right)\Delta t$ using the value of f at the midpoint of each subinterval.

Trapezoidal Rule: $\frac{1}{2}\left(f(t_0)+f(t_1)\right)\Delta t + \frac{1}{2}\left(f(t_1)+f(t_2)\right)\Delta t + \cdots + \frac{1}{2}\left(f(t_{n-1})+f(t_n)\right)\Delta t$. Note that the trapezoidal approximation is the average of the left and right sum approximations.

Antiderivatives: The antiderivative or indefinite integral of a function f(x) is a function F(x) whose derivative is f(x). Since the derivative of a constant is zero, the antiderivative of f(x) is not unique; that is, if F(x) is an integral of f(x), then so is F(x) + C, where C is any constant. Remember when you are integrating a function, f(x), you are finding a family of functions F(x) +C whose derivatives are f(x).

Integration Formulas:

$\int k\,f(x)\,dx = k\int f(x)\,dx$

$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$

$\int \frac{1}{u}\,du = \ln|u| + C$

$\int \cos u\,du = \sin u + C$

$\int \sin u\,du = -\cos u + C$

$\int \tan u\, du = \ln|\sec u| + C$

$\int \sec^2 u\, du = \tan u + C$

$\int e^u\, du = e^u + C$

$\int a^u\, du = \frac{a^u}{\ln a} + C$

$\int \frac{1}{\sqrt{a^2-u^2}}\, du = \sin^{-1} \frac{u}{a} + C$

$\int \frac{1}{a^2+u^2}\, du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$

The Fundamental Theorems

The First Fundamental Theroem of Calculus states

If f is continuous on the closed interval [a, b] and F ' = f, then,

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

The Second Fundamental Theorem of Calculus States

If f is continuous on [a, b], then the function

$$F(x) = \int_a^x f(t)\, dt$$

has a derivative at every point in [a, b]. and

$$F'(x) = \frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$

Definite Integral Properties (in addition to the indefinite integral properties)

1. $\int_a^a f(x)\, dx = 0$

2. $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

3. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

Areas: If f(x) is positive for some values of x on [a, b] and negative for others, then

$$\int_a^b f(x)\, dx$$

represents the cumulative sum of the signed areas between the graph of y = f(x) and the x-axis (where the areas above the x-axis are counted positively and the areas below the x-axis are counted negatively)

Thus,

$$\int_a^b |f(x)|\,dx$$

represents the actual area **between** the curve and the x-axis.

The area between the graphs of f(x) and g(x) where f(x) ≥ g(x) on [a, b] is given by

$$\int_a^b [f(x) - g(x)]\,dx$$

Volumes:

The volume of a solid of revolution (consisting of disks) is given by

$$\text{Volume} = \pi \int_{\text{left endpoint}}^{\text{right endpoint}} (\text{radius})^2\,dx(\text{or dy})$$

The volume of a solid of revolution (consisting of washers) is given by

$$\text{Volume} = \pi \int_{\text{left endpoint}}^{\text{right endpoint}} [(\text{outside radius})^2 - (\text{inside radius })^2]\,dx(\text{or dy})$$

The volume of a solid of known cross-sectional areas is given by

$$\text{Volume} = \int_{\text{left endpoint}}^{\text{right endpoint}} (\text{cross sectional area})\,dx(\text{or dy})$$

Arc Length:

If the derivative of a function y = f(x) is continuous on the interval [a, b], then the length s of the arc of the curve of y = f(x) from the point where x = a to the point where x = b is given by

$$s = \int_a^b \sqrt{1 + \left(\frac{\text{dy}}{\text{dx}}\right)^2}\,dx \;\text{ or } \int_a^b \sqrt{1 + (f'(x))^2}\,dx$$