How many ordered triples $(x,y,z)$ of positive integers satisfy $\text{lcm}(x,y) = 72, \text{lcm}(x,z) = 600$ and $\text{lcm}(y,z)=900$?

$\textbf{(A)}\ 15\qquad\textbf{(B)}\ 16\qquad\textbf{(C)}\ 24\qquad\textbf{(D)}\ 27\qquad\textbf{(E)}\ 64$

We prime factorize $72,600,$ and $900$. The prime factorizations are

$2^3\times 3^2$

, $2^3\times 3\times 5^2$

 and $2^2\times 3^2\times 5^2$, respectively.

Let $x=2^a\times 3^b\times 5^c$,

$y=2^d\times 3^e\times 5^f$ and

$z=2^g\times 3^h\times 5^i$. We know that

\[\max(a,d)=3\]

\[\max(b,e)=2\]

\[\max(a,g)=3\]

\[\max(b,h)=1\]

\[\max(c,i)=2\]

\[\max(d,g)=2\]

\[\max(e,h)=2\]

From here we can see a few things. Note that since the max of d and g is 2, a must equal 3. Because the max of b and h is 1, e must equal 2. From here, we only care about

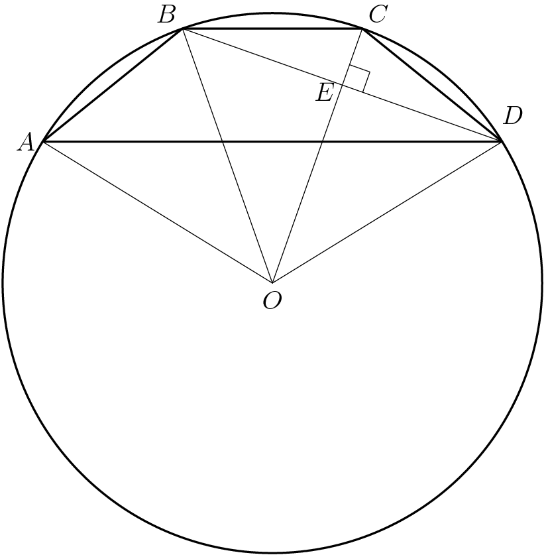
\[\max(b,h)=1\]

\[\max(d,g)=2\]

Running a casework on both of them, we have the first has 3, and second 5, so 3 \* 5 = 15.

A quadrilateral is inscribed in a circle of radius $200\sqrt{2}$. Three of the sides of this quadrilateral have length $200$. What is the length of the fourth side?

$\textbf{(A) }200\qquad \textbf{(B) }200\sqrt{2}\qquad\textbf{(C) }200\sqrt{3}\qquad\textbf{(D) }300\sqrt{2}\qquad\textbf{(E) } 500$



Let us divide everything by 200 and multiply that later, so the side lengths are all 1 and the radius √2.

Let BE = ED = x, so CE = √(12-x2) and OE = √(√22-x2). We also know CE + OE = √2.

We can then find x, then use Ptolemy’s theorem to solve for AD = 500.

The number $5^{867}$ is between $2^{2013}$ and $2^{2014}$. How many pairs of integers $(m,n)$ are there such that $1\leq m\leq 2012$ and\[5^n<2^m<2^{m+2}<5^{n+1}?\]$\textbf{(A) }278\qquad \textbf{(B) }279\qquad \textbf{(C) }280\qquad \textbf{(D) }281\qquad \textbf{(E) }282\qquad$

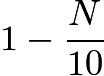
Between each consecutive power of 5, there are either 2 or 3 powers of two. This is because 22 = 4, and if a power of 2 is say 1 greater than a power of 5, then if we multiply that power of 2 by 22 it will still be less than the next power of 5, therefore there are 3 in this interval.

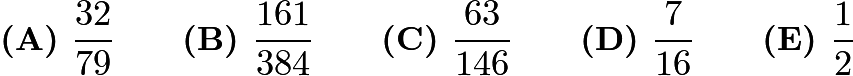
We know that up to 5867 there are 2013 powers of 2, so let x be the number of intervals with 2 powers of 2, and y with 3 powers of 2.

x + y = 867

2x + 3y = 2013.

Solve and y = 279

In a small pond there are eleven lily pads in a row labeled $0$ through $10$. A frog is sitting on pad $1$. When the frog is on pad $N$, $0<N<10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability . Each jump is independent of the previous jumps. If the frog reaches pad $0$ it will be eaten by a patiently waiting snake. If the frog reaches pad $10$ it will exit the pond, never to return. What is the probability that the frog will escape being eaten by the snake?



The probability is ½ at Lili pad 5. If we let Pk be the probability the frog will escape at pad k, then we obtain the following equations.

P1 = 9/10 P2

P2 = 1/5 P1 + 4/5 P3

P3 = 3/10 P2 + 7/10 P4

P4 = 2/5 P3 + 3/5 P5

We can then plug in P5  and solve P1 = 63/146