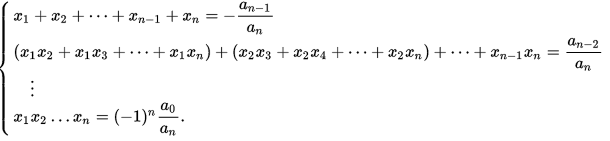
Vieta's formulas

### Basic formulas[[edit](https://en.wikipedia.org/w/index.php?title=Vieta%27s_formulas&action=edit&section=2)]

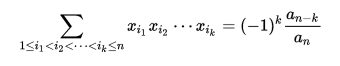
Any general polynomial of degree *n*

{\displaystyle P(x)=a\_{n}x^{n}+a\_{n-1}x^{n-1}+\cdots +a\_{1}x+a\_{0}\,}P(x) = anxn + an-1xn-1 + ….. + a1x + a0

(with the coefficients being real or complex numbers and *an* ≠ 0) is known by the [fundamental theorem of algebra](https://en.wikipedia.org/wiki/Fundamental_theorem_of_algebra) to have *n* (not necessarily distinct) complex roots *x*1, *x*2, ..., *xn*. Vieta's formulas relate the polynomial's coefficients { *ak* } to signed sums and products of its roots { *xi* } as follows:

{\displaystyle {\begin{cases}x\_{1}+x\_{2}+\dots +x\_{n-1}+x\_{n}=-{\dfrac {a\_{n-1}}{a\_{n}}}\\(x\_{1}x\_{2}+x\_{1}x\_{3}+\cdots +x\_{1}x\_{n})+(x\_{2}x\_{3}+x\_{2}x\_{4}+\cdots +x\_{2}x\_{n})+\cdots +x\_{n-1}x\_{n}={\dfrac {a\_{n-2}}{a\_{n}}}\\{}\quad \vdots \\x\_{1}x\_{2}\dots x\_{n}=(-1)^{n}{\dfrac {a\_{0}}{a\_{n}}}.\end{cases}}}

Equivalently stated, the (*n* − *k*)th coefficient *an*−*k* is related to a signed sum of all possible subproducts of roots, taken *k*-at-a-time:

{\displaystyle \sum \_{1\leq i\_{1}<i\_{2}<\cdots <i\_{k}\leq n}x\_{i\_{1}}x\_{i\_{2}}\cdots x\_{i\_{k}}=(-1)^{k}{\frac {a\_{n-k}}{a\_{n}}}}

for *k* = 1, 2, ..., *n* (where we wrote the indices *ik* in increasing order to ensure each subproduct of roots is used exactly once).

The left hand sides of Vieta's formulas are the [**elementary symmetric functions**](https://en.wikipedia.org/wiki/Elementary_symmetric_polynomial) of the roots.

# Rational root theorem

In [algebra](https://en.wikipedia.org/wiki/Algebra), the **rational root theorem** (or **rational root test**, **rational zero theorem**, **rational zero test** or ***p*/*q* theorem**) states a constraint on [rational](https://en.wikipedia.org/wiki/Rational_number) [solutions](https://en.wikipedia.org/wiki/Equation_solving) of a [polynomial equation](https://en.wikipedia.org/wiki/Polynomial_equation)

{\displaystyle P(x)=a\_{n}x^{n}+a\_{n-1}x^{n-1}+\cdots +a\_{1}x+a\_{0}\,}0 = anxn + an-1xn-1 + ….. + a1x + a0{\displaystyle a\_{n}x^{n}+a\_{n-1}x^{n-1}+\cdots +a\_{0}=0\,\!

with [integer](https://en.wikipedia.org/wiki/Integer) coefficients. These solutions are the possible [roots](https://en.wikipedia.org/wiki/Root_of_a_polynomial) (equivalently, zeroes) of the [polynomial](https://en.wikipedia.org/wiki/Polynomial) of the equation.

If *a*0 and *an* are nonzero, then each [rational](https://en.wikipedia.org/wiki/Rational_number) solution *x*, when written as a fraction *x* = *p*/*q* in lowest terms (i.e., the [greatest common divisor](https://en.wikipedia.org/wiki/Greatest_common_divisor) of *p* and *q* is 1), satisfies

* *p* is an integer [factor](https://en.wikipedia.org/wiki/Divisor) of the [constant term](https://en.wikipedia.org/wiki/Constant_term) *a*0, and
* *q* is an integer factor of the leading [coefficient](https://en.wikipedia.org/wiki/Coefficient) *an*.

The rational root theorem is a special case (for a single linear factor) of [Gauss's lemma](https://en.wikipedia.org/wiki/Gauss%27s_lemma_(polynomial)) on the factorization of polynomials. The **integral root theorem** is a special case of the rational root theorem if the leading coefficient *an* = 1.

# Irrational root theorem

The irrational conjugate roots theorem says:

Let p(x) be any polynomial with rational coefficients. If

a + b\*sqrt(c) is a root of p(x), where sqrt(c) is irrational and

a and b are rational, then another root is a - b\*sqrt(c).

Also if a + bi is a root, then a – bi will also be a root.