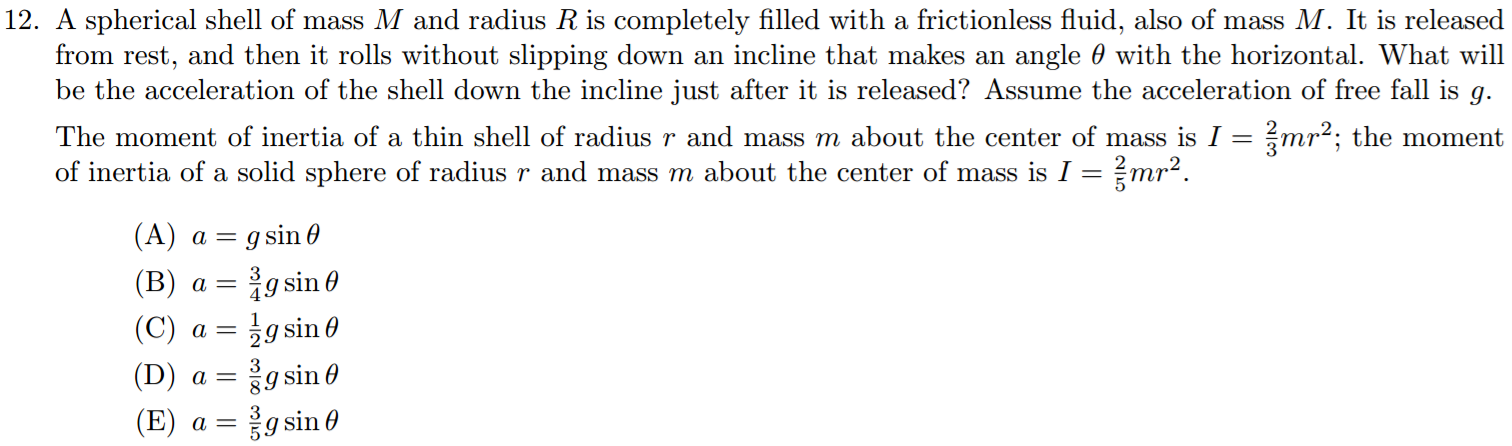
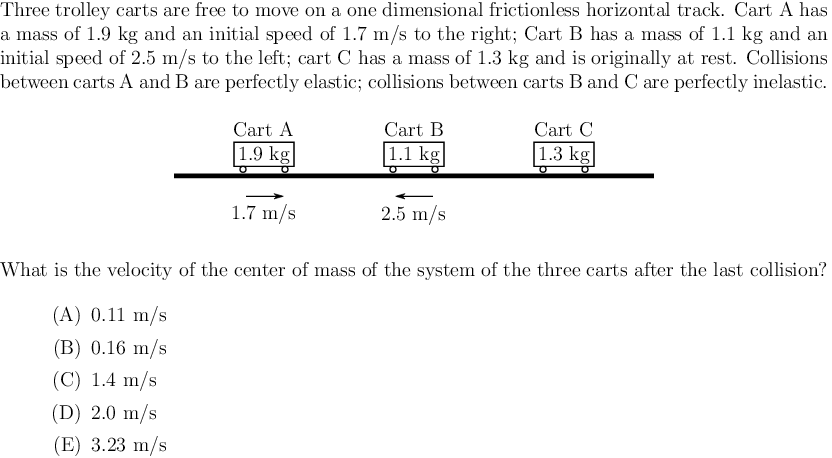
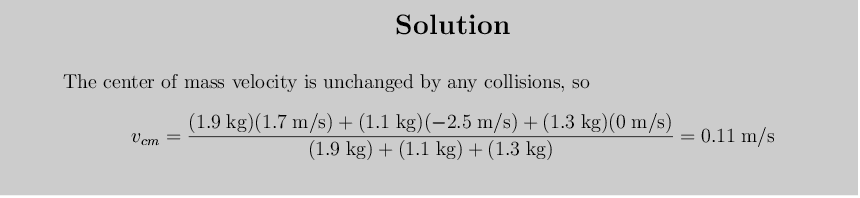


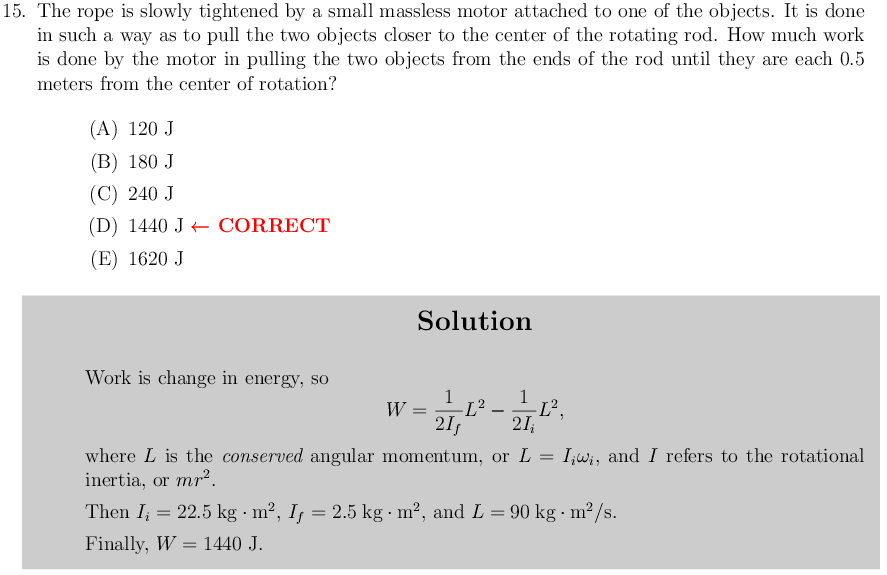
Using conservation of momentum. Since mass cancels, the new velocities are just ratios of the mass to the first velocity. mv0 = (am + bm + cm ….)v1

  
  
Torque acts as a friction here, countering the force of gravity of the sphere rolling down. Note that the water in the sphere does not rotate, so when considering the moment of inertia, we use the inertia of the empty shell. So the force of the sphere rolling down, 2MA is the result of the frictional torque, 2/3 MA(FR = Ia, FR = 2/3 MR^2 \* A/R) subtracted from the force of gravity, 2Mgsin@. Since the ball needs to roll, what causes it to roll? What causes the angular acceleration? That is the friction with the ground that is translated only to torque.

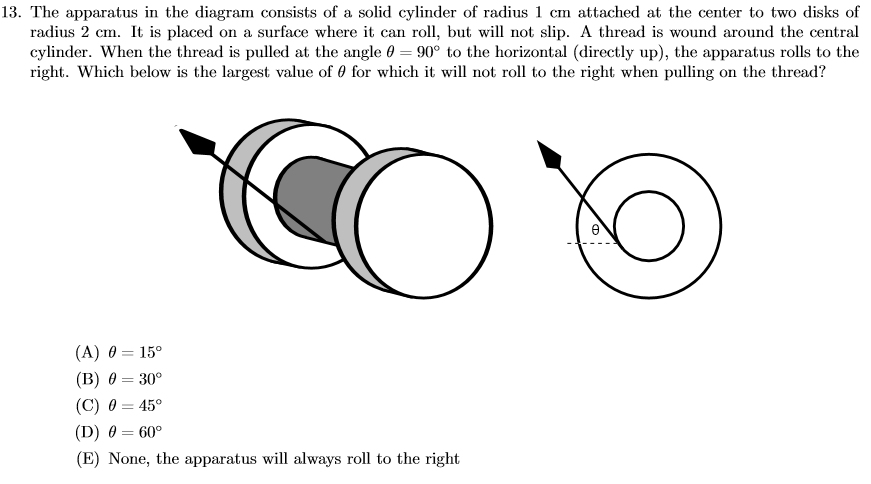




If momentum is conserved, then the velocity of the center of mass never changes. More on this in CCof Physics.



We cannot use ½ Iw^2 here. The conserved angular momentum must be used instead.   
  
   



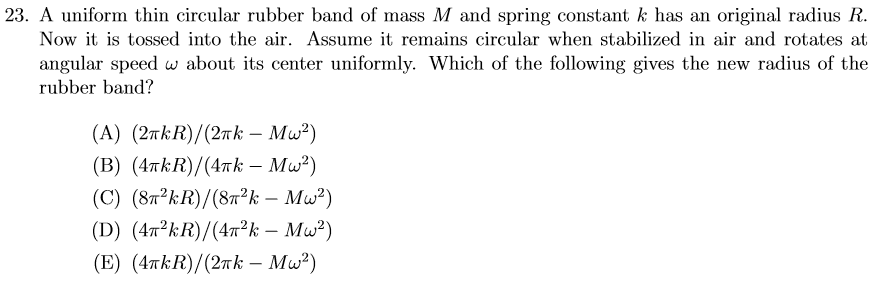
Torques and forced need to be balanced. As the apparatus clearly does not rotate and does not slide in any direction, we must balance both the two torques (from friction and the applied force) and the two forces (same).

For the two torques, we have from friction .2f and from applied force .1F. Since these two act in opposite directions, they need to be equal : .2f = F

The two horizontal forces also cancel, this is from the applied force’s horizontal component Fcos0 and the force of friction f.

Fcos0 = f;

Solving for 0 we get 0 = 60.



If we look at this, there is only one force acting upon the rubber band as it rises and that is the spring tension force.

To properly analyze this force, we use energy.

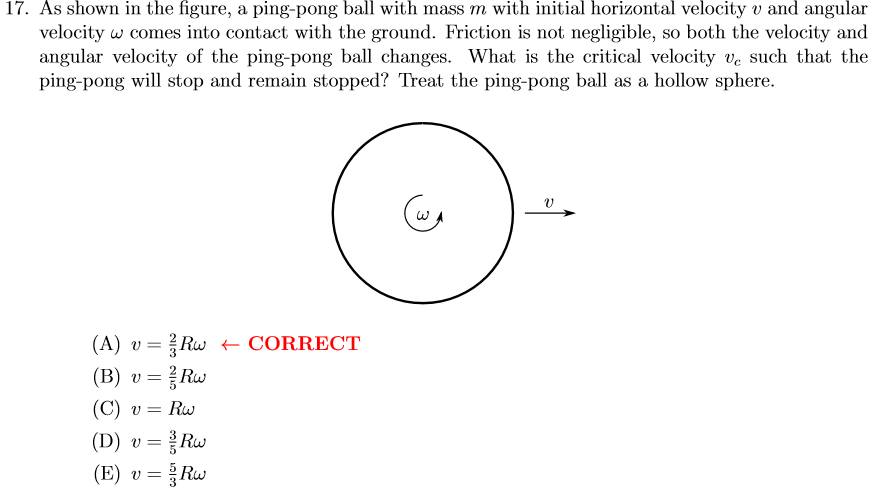
-dU/dx = F

-d(1/2k(2pir – 2piR))/d(2pir – 2piR) = F, where r is the new radius.

However, this gives us the force along the circumference of the circle, not towards the center (which is what centripetal force is directed as). To mitigate this, we simply pull out a 2pi from the bottom to change to radial spring force, and the energy remains unchanged (as energy is an existence).

From this, we get that 2piF = mrw^2, where F = 2kpi(r – R).

And there is our solution D.

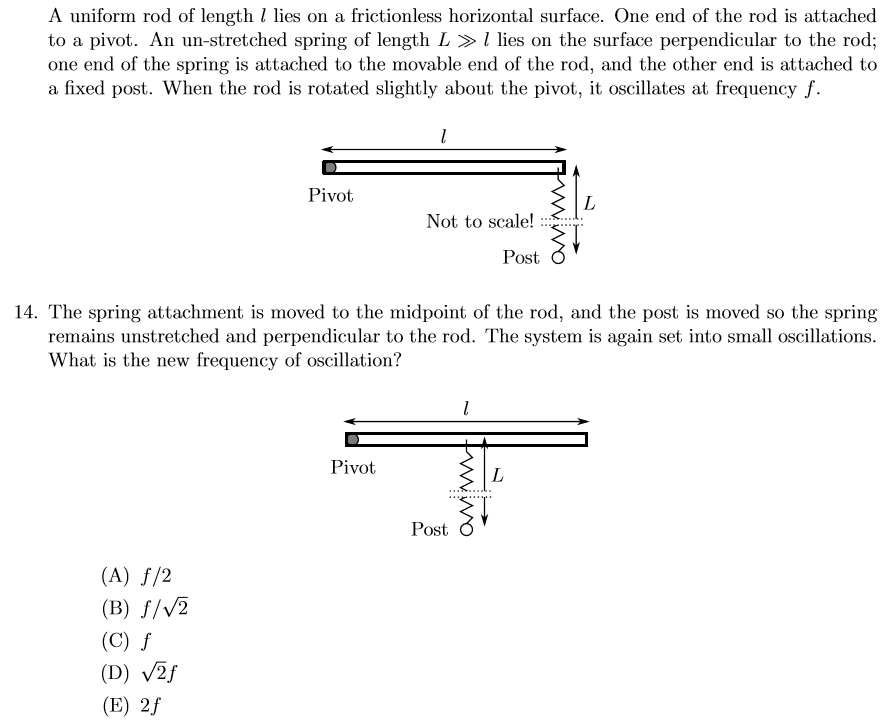


When the ball hits the ground, two things occur, the momentum changing to 0 and the angular momentum doing the same.

Starts with momentum mv and moment of angular momentum Iw which is 2/3mr^2w.

So the change in momentum from frictional force F = mv, and change in angular momentum from torque is Fr = 2/3mr^2w.

Therefore 2/3mrw = mv, and v = 2/3rw



We want to find a general formula for the frequency of this rod at any point. We will use energy to solve this problem.

Let’s say that the maximum stretch of the spring is a distance x. So at the maximum point, we have

1/2kx^2 as the spring potential energy. Now once it returns to equilibrium, we only have rotational velocity, so

1/2kx^2 = 1/2Iw^2 (that is the capital letter i for moment of inertia)

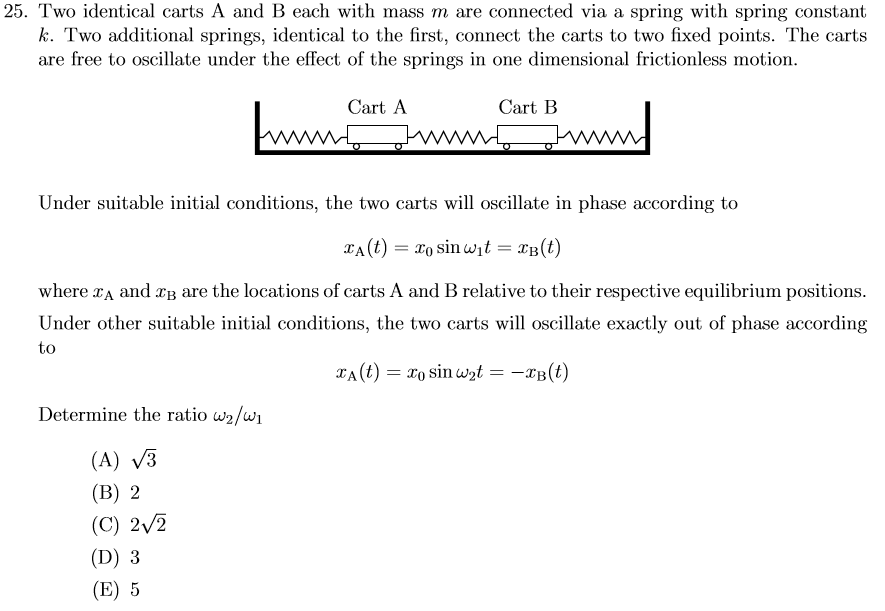
So we know that w = sqrt(k/l)x

From here, we want to find the frequency. Note that frequency of a spring can be analogized to the frequency of a point moving at a constant velocity around a circle and its projection to the diameter. This is the frequency of oscillation with which we wish to observe, with frequency w/2pi where w is the rotational velocity of the point moving about the circle, in other words the rate of oscillation for the spring.

However, the w in our previous equation is not that same w as the one we want to solve for here. So we want to find a relation between these two. Looking at it, the w in our equation follows the path of a larger circle, of radius l (lowercase L), and the w of the spring a circle of radius x. So we have that the ratio between the two w’s is l/x.

So we have w(spring) = l/x w(rod)

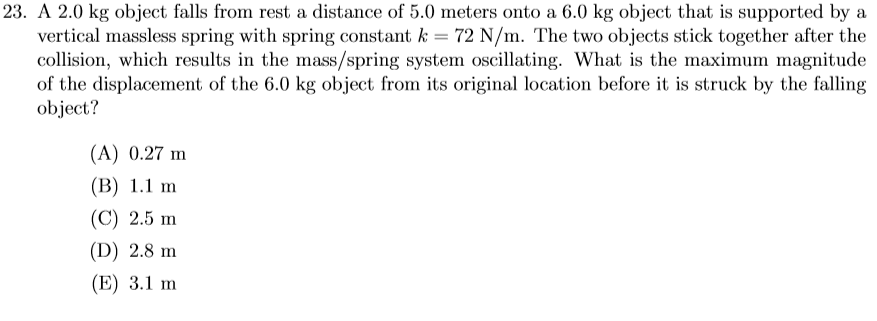
Plugging this in, we have that f = l/x w(spring) / 2pi which would equal l \* sqrt(k/i). So the frequency is completely dependent on l, the distance the spring is from the pivot.



We know the normal equation of resonance is of the form x = Asin(wt – o) + B where w is the frequency. So the question is asking for a ratio of frequencies.

For the first phase, where they are moving in motion as 1, all the springs are compressed the same length at the same time, so it is equivalent to one spring of spring constant k, so sqrt(k/m).

For the second phase, the two outer spring are compressed the same amount with spring constant k, but the middle spring is compressed and stretched by both. Note the center of the middle spring does not move, for it is the same thing as a spring of half the length and constant 2k acting on a single cart. So for one cart, we have a total spring constant of 3k, so sqrt(3k/m).



First part is normal, we have the potential energy = kinetic energy of the falling object right before it hits the 6 kg, and we find it is traveling 10 m/s. The collision is inelastic, using conservation of momentum we find the 8 kg together mass moves at 2.5 m/s at the moment of impact. Therefore, by 1/2mv^2, both of them together have kinetic energy of 25 J.

So the spring is compressed, changing the gravitational potential energy and the spring potential energy. Note however, that the spring is already compressed with the 6 kg block resting on it. We solve this with mg = kx and find it is compressed a distance of 5/6 m.

So our change in spring potential energy, given it is compressed a distance x, is

½k(x + 5/6)^2 – ½k(5/6)^2 = ½kx^2 + 5/6kx

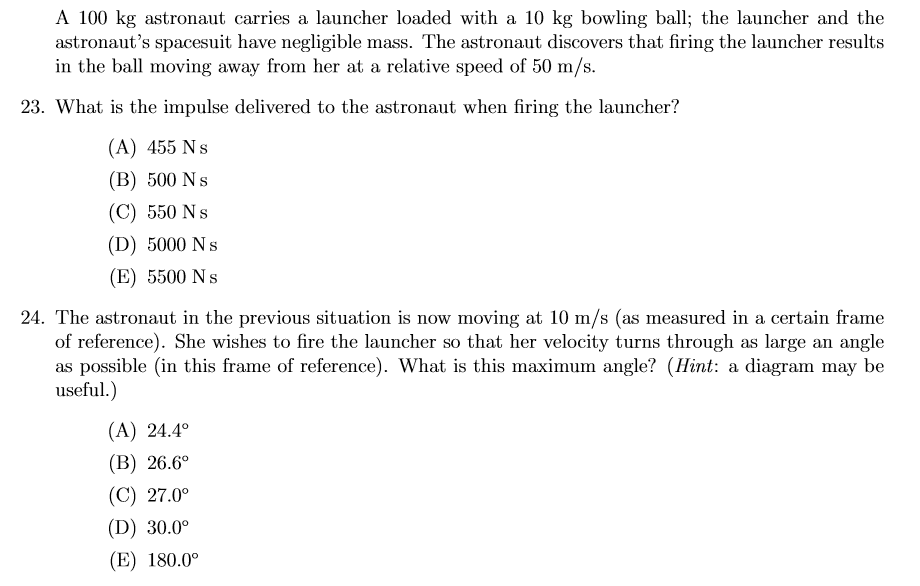
We know k = 72, so we have this equal to 36x^2 +60x

So our energy equation is

25 = 36x^2 + 60x – 80x (from potential energy),

36x^2 – 20x -25 = 0

Solving for x gives us around 1.15, so 1.1 is our answer.

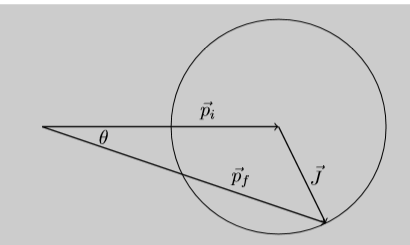


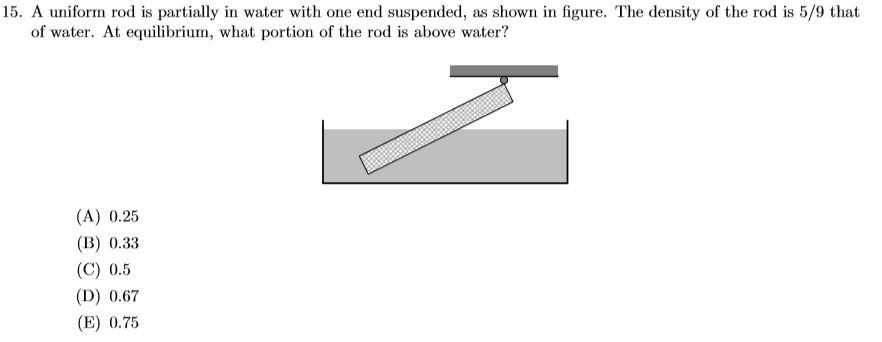
Impulse is just the change in momentum. Since the astronaut starts with 0 momentum, the answer is simply her final momentum.

We have 100x + 10(x – 50) = 0, so x = 500/110

So the momentum of the astronaut is 500/110 \* 100 = 455.

For the second part, we look at her velocity as a vector with length 10. The impulse has a magnitude of 4.55 for the velocity change. So that means her final velocity will be a vector addition of these two. But since the ball is fired in any direction, the second 4.55 vector is effectively confined within a circle like so.

So The largest theta angle that can be obtained is when p\_f is tangent to the circle. Our answer is 27.



This problem is a balance of torques problem. We have on torque on the entire rod is gravity pulling it down, so let us say the density of the rod is p\_r, and the volume is V, and the length is L. The weight of the rod is then Vgp\_r, and the torque is Vgp\_r \* ½L, since the center of mass is ½L away from the pivot point.

We have another torque, which is the buoyant force from the water. Let’s say that x is the fraction of the length submerged. Therefore, the Vx of the volume is submerged. That means, the mass of the water displaced is Vxp\_w.

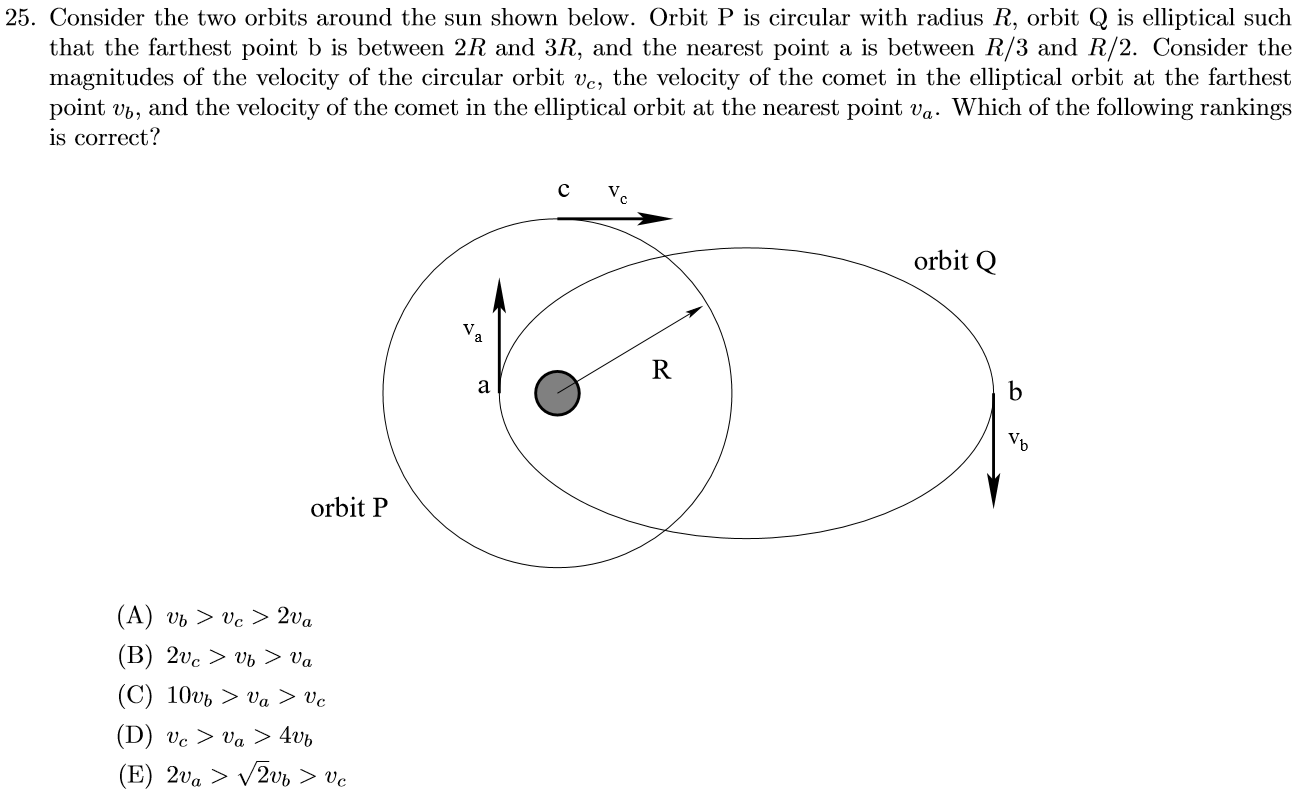
The distance from the center of the submerged portion to the pivot point would be (1-x + x/2)L.

Therefore, the torque of the buoyant force would be Vgxp\_w( -x + x/2)L.

So the two torques need to balance, so

Vgp\_r \* ½L = Vgxp\_w(1-x + x/2)L

Solving for x, and using the fact p\_r/p\_w = 5/9, we get x = 1/3, so 1-x or 2/3 is above water.



We use the vis-viva equation, v^2 = GM(2/r – 1/a) where G is the gravitational constant, m is the mass of the central object, r is the distance from the satellite to the central mass, and a is the semi-major axis of the orbit path.

Here, we only need to consider one of the cases (2R, 3R, R/3, R/2). We will use 2r as the longer distance, and r/2 as the shorter.

For v\_c, we have v\_c = sqrt(GM(2/r – 1/r)) = sqrt(GM \* 1/r).

For v\_a, we have v\_a = sqrt(GM(2/(r/2) – 1/(5/4r)) = sqrt(GM(16/5r)).

For v\_b, we have v\_b = sqrt(GM(2/2r) – 1/(5/4r)) = sqrt(GM(1/5R)).

Only answer C is true.