Beam up my quantum state, Scotty!

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1 Introduction

1.1 Preliminaries

In quantum teleportation the sender and receiver are referred to as Alice and Bob, and are denoted A and B respectively. Sometimes a third party is relevant which will be called Charlie and be denoted C. [1]

1.1.1 EPR-pairs and the Bell Basis

An Einstein-Podolsky-Rosen-pair (EPR-pair) is a maximally entangled state of two qubits [1] which can be written as

$$\left|\Phi^{\pm}\right\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \quad \text{and} \left|\Psi^{\pm}\right\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}. \tag{1}$$

When measuring a quantum state, the basis of measurement is important as this determines the possible outcome states. Common basis used are the computational basis, consisting of $|0\rangle$ and $|1\rangle$, and the Bell basis consisting of the EPR-pairs, also known as Bell states, seen in Eq. (1). EPR-pairs and projective measurements in the Bell basis, henceforth called Bell measurements, play a crucial role in quantum teleportation protocols. [1]

1.2 Quantum Teleportation Protocol

Let Alice have a particle with a normalized state $|\phi\rangle = \alpha\,|0\rangle_{\phi} + \beta\,|1\rangle_{\phi}$ which is unknown to her, that she wants to send to Bob. Sending the particle itself is rarely possible since Alice does not necessarily know where Bob is located. She also cannot measure the particle to get accurate information since the particle is part of an unknown orthonormal set. To overcome these hurdles, Alice can instead opt to send, or teleport, the state $|\phi\rangle$ to Bob. [2]

To realize this teleportation both a classical channel and a non-classical channel will be used. The non-classical channel is made of an EPR-pair, where one particle is with Alice and one with Bob. Let Alice and Bob share the EPR-pair

$$\left|\Psi^{-}\right\rangle_{AB} = \frac{|0\rangle_A\,|1\rangle_B - |1\rangle_A\,|0\rangle_B}{\sqrt{2}}. \tag{2}$$

The subscript denotes if Alice or Bob has the particle. Thus, the entire system is in the state

$$\begin{aligned} |\psi\rangle &= |\phi\rangle \left|\Psi^-\right\rangle_{AB} && (3) \\ &= \frac{\alpha}{\sqrt{2}}(|0\rangle_\phi |0\rangle_A |1\rangle_B - |0\rangle_\phi |1\rangle_A |0\rangle_B) + \frac{\beta}{\sqrt{2}}(|1\rangle_\phi |0\rangle_A |1\rangle_B - |1\rangle_\phi |1\rangle_A |0\rangle_B) && (4) \end{aligned}$$

Rewriting the products $|x\rangle_\phi |x\rangle_A$, that is the Alice's system, using the Bell basis the system can be written as

$$\begin{aligned} |\psi\rangle = \frac{1}{2}\Big[&\left|\Psi^-\right\rangle_{\phi A}(-\alpha |0\rangle_B - \beta |1\rangle_B) + \left|\Psi^+\right\rangle_{\phi A}(-\alpha |0\rangle_B + \beta |1\rangle_B) \\ &+ \left|\Phi^-\right\rangle_{\phi A}(\alpha |1\rangle_B + \beta |0\rangle_B) + \left|\Phi^+\right\rangle_{\phi A}(\alpha |1\rangle_B - \beta |0\rangle_B)\Big] \qquad (5) \end{aligned}$$

That is, if Alice performs a Bell measurement the system will collapse into one of these terms. [2]

Depending on the result of Alice's measurement Bob will have the following states

$$\begin{aligned} A &: \left|\Psi^-\right\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B - \beta |1\rangle_B = -|\phi\rangle && (6) \\ A &: \left|\Psi^+\right\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B + \beta |1\rangle_B = -Z |\phi\rangle && (7) \\ A &: \left|\Phi^-\right\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B + \beta |0\rangle_B = X |\phi\rangle && (8) \\ A &: \left|\Phi^+\right\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B - \beta |0\rangle_B = -iY |\phi\rangle && (9) \\ & && (10) \end{aligned}$$

That is Bob will end up with a rotated version of $|\phi\rangle$. Thus, if Alice sends the result of her measurement classically to Bob he can perform the necessary rotation to obtain the state $|\phi\rangle$. The classical channel can be a generic broadcast, i.e. a radio, which essentially means that Alice doesn't need to know Bob's location. Note that the choice of $|\Psi^-\rangle$ as the shared EPR-pair was arbitrary. However, which state is shared must be known by both parties since that will change what gate Bob has to apply to obtain $|\phi\rangle$. [2]

1.3 Reasons and Applications

Quantum communications and quantum computing are two large fields being researched at the moment, and it is important to ask the question of why this effort is worth it. For one thing, in the age of information, sending and receiving data is increasingly more important as we rely more and more on technology [3]. A corollary of this is then that secure communication is of the utmost importance to keep our personal information secure. [4]

Quantum communication opens up possibilities of transmitting data unable to be compromised using entangled states [4]. Furthermore, quantum computing could, if developed sufficiently, apply Shor's algorithm to break the current encryption used by modern digital technology [1]. It could therefore prove useful, or even paramount, to have a quantum internet complementing the current digital one [3].

To realize a full scale quantum internet, or even larger quantum networks, technologies decreasing attenuation of the signal and storing information is essential [3]. That is, a successful quantum communication protocol will need to apply both quantum memory technologies and quantum repeater technologies. It is therefore not sufficient to just focus on quantum computers for calculation but also infrastructure and technologies to link multiple quantum computers together. Specifically one application could be secure cloud based computing [3] which could increase the accessibility to run quantum computation algorithms. Such algorithm's could solve some computational tasks exponentially faster, thus decreasing energy needed, and effects on the climate. Quantum simulation algorithms can also help in the quest of understanding molecular reaction, helping to create more efficient catalysts [5].

A recent paper [6] shows that quantum computers are scalable and are getting increasingly better regarding error-corrections and accuracy of measurements. This shows a great outlook for the field of quantum computers. The importance of being able to scale the network of quantum computers then becomes readily apparent.

2 Teleportation of Complex Quantum Systems

Bullet points

- Theoretical and experimental limits

2.1 Complex Quantum Systems

A complex quantum system is a system of particles where each particle has multiple degrees of freedom, in turn, one degree of freedom can have quantum numbers other than the basic $|1\rangle$ and $|0\rangle$. That is, to make quantum teleportation a practical technique for quantum communication it needs to be possible to teleport these high dimensional states, since we want to be able to teleport a complete quantum state for a given particle. For example, two of the ways a photon may be encoded in is polarization and orbital angular momentum. This also increases the information density of the particles used to transmit information. [4]

In contrast to the two-level encoding qubits used in teleportation of simple quantum systems, complex quantum system instead use qudits. Qudits are encoded in d dimensions, and systems with $d \geq 3$ are referred to as multilevel systems, that is, qudits are multilevel encoded. There are numerous advantages of qudits in many areas of quantum information, including quantum communication and quantum computation. There are also some challenges implementing teleportation for high dimensional systems. Notably these challenges include preparing entanglement with sufficient quality to perform teleportation and performing Bell measurements. It has also been theoretically proven that auxiliary particles are needed to perform Bell measurements when using linear optics. [4]

2.2 High-Dimensional Teleportation Protocol

For a 3-dimensional system, suppose Alice wants to teleport the normalized state $|\phi\rangle = \alpha |0\rangle_\phi + \beta |1\rangle_\phi + \gamma |2\rangle_\phi$ consisting of a single photon. As with simple systems, Alice and Bob need to share an entanglement source, for example

$$|\Psi\rangle_{AB} = \frac{|00\rangle_{AB} + |11\rangle_{AB} + |22\rangle_{AB}}{\sqrt{3}} \tag{11}$$

which is a 3-dimensional Bell state. The nine total 3-dimensional Bell states together create a basis of the Hilbert space. Then the entire system will be $|\phi\rangle |\Psi\rangle_{AB}$ and doing a similar, but more complicated, rewriting of the expression one can get to a point where there is equal probability to project the state into one of nine states when Alice performs her Bell measurement. Then, Alice classically transmits the result and Bob perform the necessary rotations to obtain $|\phi\rangle$. This can be extended to d-dimensions with d^2 Bell states in the basis. [7]

2.3 Experimental Realization

One experimentally proven Bell measurement scheme uses quantum Fourier transformations. The experiment teleported a 3-dimensional system using one auxiliary particle. However, it has been shown that for dimension d, $d-2$ auxiliary particles are needed. The fidelity of this experiment was $75(1)$ %. The system was encoded using path the path of the photon. That is, by using a series of mirrors and beam splitters, it is possible to generate superposition states. [7]

More specifically, one auxiliary photon was used to extend the Hilbert space to 4 dimensions. This was done to overcome the limits of linear optics concerning discrimination of Bell states for dimensions $d \geq 3$. The quantum Fourier transform was realized using a multiport beam splitter with 3-input-3-output all-to-all connected ports. This gives 9 different output positions for detectors, since one photon has 3-dimensional encoding, and depending on the state will end up in different places. Then, depending on the detector activation pattern, the state will be projected onto one of the possible Bell states. [7]

3 Quantum Repeaters and Quantum Memory

3.1 Entanglement swapping

Assuming that Alice and Bob are too far apart from each other to efficiently transport an entangled state between them, an option is entanglement swapping. Assume that Alice and Bob each share an entangled state with a third party, Charlie. These states would be $|\phi\rangle_{AC_1}$ and $|\phi\rangle_{BC_2}$. If Charlie now performs a simultaneous measurement on his two states, C_1 and C_2, Alice's and Bob's qubits will end up in an entangled state. This is a way of propagating entanglement through a third party in order to achieve more reliable long distance entanglement [3].

3.2 Quantum memory

For the entanglement swapping to be efficiently used in a repeater protocol, Charlie needs to store the quantum state sent by Alice, while waiting for Bob's state. This requires a way of storing the state without destroying the information it carries. Classically this is not a problem, since information can be duplicated exactly. In quantum information this is not possible due to the no-cloning theorem. A quantum state can not be completely duplicated, meaning that the qubit carrying the state must be stored itself. Such a memory has been realized in many different ways, including storing the state in trapped ions, with a memory time of 4 ms [8] and in quantum dots with a memory time of 3 μs [9].

3.3 Quantum repeater protocol

If long distance distribution of entanglement were to be achieved through optical fibers, one would experience an exponential loss of photons. To overcome this loss, quantum repeaters use heralded entanglement generation and entanglement swapping. Heralded entanglement generation involves establishing local Bell states between quantum memories and optical pulses at each party (e.g., Alice and a repeater node, or between two repeater nodes). Thus there needs to be a quantum memory unit at each party. The optical pulses are sent through fibers to a central station where a linear-optical Bell measurement is performed, projecting the pulses into an entangled Bell state with success probability

$$p_g(L) = e^{-L/L_{\text{att}}}/2, \tag{12}$$

where L is the fiber length and L_{att} is the attenuation length, defined as the distance over which the transmittance of the fiber has dropped to $1/e$ of its initial value [3]. Once neighboring parties share entanglement with a repeater node, entanglement swapping extends the entanglement between more distant parties. This is achieved via a Bell measurement on the quantum memories at the repeater node, succeeding with probability $p_s = 1/2$ in the ideal case. Without repeaters, directly linking Alice and Bob requires an average number of trials

$$\langle T_{\text{tot}}^{(0)} \rangle = p_g(L)^{-1} = 2e^{L/L_{\text{att}}}, \tag{13}$$

which grows exponentially with the distance L. With a single repeater node located midway, entanglement generation is performed in parallel for Alice-to-repeater and repeater-to-Bob links, each requiring

$$\langle T_g(L/2) \rangle = 2e^{L/(2L_{\text{att}})} \tag{14}$$

trials on average. Successful swapping at the repeater node then connects Alice and Bob, requiring

$$\langle T_{\text{tot}}^{(1)} \rangle \sim p_s^{-1} p_g(L/2)^{-1} = 2^2 e^{L/(2L_{\text{att}})} \tag{15}$$

trials, providing a square-root improvement compared to the direct link [3]. This process generalizes with the number of quantum repeaters $N_{\text{QR}} = 2^n - 1$ equally spaced repeater nodes, where each step reduces the entanglement distance to $L/(N_{\text{QR}} + 1)$, achieving a total trial count of

$$\langle T_{\text{tot}}^{(N_{\text{QR}})} \rangle \sim 2^{1+\log_2(N_{\text{QR}}+1)} e^{L/((N_{\text{QR}}+1)L_{\text{att}})}, \tag{16}$$

exponentially improving efficiency [3]. While this idealized protocol assumes perfect operations and quantum memories, practical implementations must account for memory errors, imperfect operations, and accumulated errors, mitigated by advanced error-suppression techniques. Thus, quantum repeaters enable scalable quantum communication by mitigating photon loss and other imperfections over long distances.

4 Experimental Evidence

Quantum teleportation has been implemented in many experiments, each with different protocols to address challenges in fidelity, distance and scalability. Experimental realization is necessary for validating theoretical models and advancing quantum communication technologies. This section examines two key implementations, satellite-based and fiber network-based teleportation, that demonstrate solutions to distance-dependent losses and environmental decoherence, contributing to the development of robust quantum networks.

4.1 Satellite Based

The qubit system used in this experiment was based on the polarization of a photon, represented as:

$$|\chi\rangle_1 = \alpha |H\rangle_1 + \beta |V\rangle_1 , \tag{17}$$

where α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$, and $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarization states, respectively [10].

At a ground station, two entangled photon pairs were prepared. One photon from each pair was transmitted to a satellite through a 130-mm-diameter telescope, designed with narrow beam divergence and high-precision tracking systems to counteract atmospheric turbulence. The entangled pair of photons is described by one of the Bell states:

$$\left|\psi^+\right\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2 |H\rangle_3 + |V\rangle_2 |V\rangle_3), \tag{18}$$

where photon 2 was retained at the ground station, and photon 3 was sent to the satellite [10].

A Bell-state measurement was performed on the photon to be teleported (photon 1) and the ground station photon (photon 2) from the entangled pair. This measurement projected the two photons into one of the Bell states:

$$\left|\psi^\pm\right\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1 |H\rangle_2 \pm |V\rangle_1 |V\rangle_2). \tag{19}$$

The outcome of the measurement was then transmitted classically to the satellite, where the relevant rotations can be performed on photon 3 to obtain $|\chi\rangle_1$. If the measured state was $|\psi^+\rangle_{12}$, photon 3 adopted the original state of photon 1. If the measured state was $|\psi^-\rangle_{12}$, photon 3 carried the original state of photon 1, but with a π-phase shift.

This experiment demonstrated the successful teleportation of a photon's quantum state over distances of up to 1400 kilometers with a fidelity of 80(1) %, from a ground station to a satellite in low-Earth orbit. The primary technical challenges included atmospheric turbulence, which caused beam wandering and broadening, resulting in significant signal losses. These challenges were addressed through the use of a narrow beam divergence, a high-precision telescope, and an advanced acquiring, pointing, and tracking (APT) system [10].

In the future, this method could be used to create a global quantum internet, connecting quantum computers across the globe and introducing secure cryptography protocols. This also raises the possibility of letting several quantum processors work together on a single problem, enhancing the efficacy of quantum computers.

4.2 Fibre Network Based

When it comes to fiber-based quantum teleportation, experiments have achieved notable advancements in efficiency and distance. Using time-bin encoded qubits, which are resilient to fiber transmission noise, researchers demonstrated teleportation over 100 kilometers of optical fiber. This was made possible through the use of high-detection-efficiency superconducting nanowire single-photon detectors (SNSPDs). These detectors made precise multi-photon coincidence measurements possible. These are crucial for the reliably performing Bell-state projections. The experiment achieved an average teleportation fidelity of 83.7(20) %, surpassing the classical limit [11].

Teleportation across a metropolitan fiber network showed the potential of quantum communications in urban settings. This experiment utilized time-bin entangled photons at telecommunication wavelengths, transmitted through the Calgary fiber network over a combined distance of 8.2 kilometers. The protocol required rigorous stabilization of timing and polarization to ensure indistinguishability between photons arriving at the Bell-state measurement station. The results demonstrated that teleportation in real-world fiber networks is possible and it achieved an average fidelity of 80(2) % for single-photon states [12].

Across these experiments, significant technical challenges were addressed to achieve successful teleportation over large distances. Fiber-based teleportation experiments had to overcome loss and noise over long distances, forcing development of efficient photon detectors like SNSPDs. In metropolitan networks, maintaining the indistinguishability of photons despite environmental fluctuations required active stabilization of polarization and timing using feedback systems.[12]

These advancements mark critical milestones toward the development of a global quantum internet. The ability to teleport quantum states over unprecedented distances enables secure quantum key distribution (QKD), enabling unbreakable encryption protocols. Moreover, the integration of teleportation with quantum repeater technologies promises scalable quantum networks capable of connecting quantum computers across continents. These networks hold the potential to revolutionize secure communication, distributed quantum computing, and fundamental tests of quantum mechanics, laying the groundwork for a new era in quantum information.[12]

5 Summary & Conclusion

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