Beam up my quantum state, Scotty!

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December 16, 2024

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1 Introduction

Bullet points

- Quantum teleportation protocol
- Why is it needed?
- Areas of application, quantum communications, quantum computers
- what is needed to realize it on a large scale, i.e. quantum repeaters, memory...
- EPR-pairs and bell basis

1.1 Preliminaries

In quantum teleportation the sender and receiver are referred to as Alice and Bob, and are denoted A and B respectively. Sometimes a third party is relevant which will be called Charlie and be denoted C.

1.1.1 EPR-pairs and the Bell Basis

An Einstein-Podolsky-Rosen-pair (EPR-pair) is a maximally entangled state of two qubits [?] which can be written as

$$\left|\Phi^{\pm}\right\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \quad \text{and} \left|\Psi^{\pm}\right\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}. \tag{1}$$

When measuring a quantum state the basis of measurement is important as this determines the possible outcome states. Common basis used are the computational basis, consisting of $|0\rangle$ and $|1\rangle$, and the Bell basis consisting of the EPR-pairs, also known as Bell states, seen in Eq. (1). EPR-pairs and projective measurements in the Bell basis, henceforth called Bell measurements, play a crucial role in quantum teleportation protocols. [?]

1.2 Quantum Teleportation Protocol

Let Alice have a particle with a normalized state $|\phi\rangle = \alpha\,|0\rangle_{\phi} + \beta\,|1\rangle_{\phi}$, which is unknown to her, that she wants to send to Bob. Sending the particle itself is rarely possible since Alice does not necessarily know where Bob located. She also cannot measure the particle to get accurate information since the particle is part of an unknown orthonormal set. To overcome these hurdles Alice can instead opt to send, or teleport, the state $|\phi\rangle$ to Bob. [?]

To realize this teleportation both a classical channel and a non-classical channel will be used. The non-classical channel is made of an EPR-pair, where one particle is with Alice and one with Bob. Let Alice and Bob share the EPR-pair

$$\left|\Psi^-\right\rangle_{AB} = \frac{|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B}{\sqrt{2}}. \tag{2}$$

The subscript denotes if Alice or Bob has the particle. Thus, the entire system is in the state

$$|\psi\rangle = |\phi\rangle \left|\Psi^-\right\rangle_{AB} \tag{3}$$

$$= \frac{\alpha}{\sqrt{2}}(|0\rangle_\phi |0\rangle_A |1\rangle_B - |0\rangle_\phi |1\rangle_A |0\rangle_B) + \frac{b}{\sqrt{2}}(|1\rangle_\phi |0\rangle_A |1\rangle_B - |1\rangle_\phi |1\rangle_A |0\rangle_B) \tag{4}$$

Rewriting the products $|x\rangle_\phi |x\rangle_A$, that is the part of the system that is with Alice, using the Bell basis the system can be written as

$$|\psi\rangle = \frac{1}{2}\Big[\left|\Psi^-\right\rangle_{\phi A}(-\alpha|0\rangle_B - \beta|1\rangle_B)\left|\Psi^+\right\rangle_{\phi A}(-\alpha|0\rangle_B + \beta|1\rangle_B)$$

$$\left|\Phi^-\right\rangle_{\phi A}(\alpha|1\rangle_B + \beta|0\rangle_B)\left|\Phi^+\right\rangle_{\phi A}(\alpha|1\rangle_B - \beta|0\rangle_B)\Big] \tag{5}$$

That is, if Alice performs a Bell measurement the system will collapse into one of these terms. [?]

Depending on the result of Alice's measurement Bob will have the following states

$$A : \left|\Psi^-\right\rangle_{\phi A} \longrightarrow B : -\alpha|0\rangle_B - \beta|1\rangle_B = -|\phi\rangle \tag{6}$$

$$A : \left|\Psi^+\right\rangle_{\phi A} \longrightarrow B : -\alpha|0\rangle_B + \beta|1\rangle_B = -Z|\phi\rangle \tag{7}$$

$$A : \left|\Phi^-\right\rangle_{\phi A} \longrightarrow B : \alpha|1\rangle_B + \beta|0\rangle_B = X|\phi\rangle \tag{8}$$

$$A : \left|\Phi^+\right\rangle_{\phi A} \longrightarrow B : \alpha|1\rangle_B - \beta|0\rangle_B = -iY|\phi\rangle \tag{9}$$

$$\tag{10}$$

That is Bob will end up with a rotated version of $|\phi\rangle$. Thus, if Alice sends the result of her measurement classically to Bob he can perform the necessary rotation to obtain the state $|\phi\rangle$. The classical channel can be a generic broadcast, i.e. a radio, which essentially means that Alice doesn't need to know where Bob is exactly. Note that the choice of $|\Psi^-\rangle$ as the shared EPR-pair was arbitrary. However, which state is shared must be known by both parties since that will change what gate Bob has to apply to obtain $|\phi\rangle$. [?]

2 Teleportation of Complex Quantum Systems

Bullet points

- What is a complex system?
- How does the protocol differ from simple systems?
- Why is it important to be able to teleport complex quantum systems?
- Theoretical and experimental limits

3 Quantum Repeaters and Quantum Memory

Quantum internet [?]

Bullet points

- quantum repeater analogues to normal repeater?
- How to realize quantum memory
- why do we need quantum memory for quantum repeaters
- how much does a quantum repeater reduce attenuation
- how good are today's quantum repeaters?

3.1 Entanglement swapping

Assuming that Alice and Bob are too far apart from each other to efficiently transport an entangled state between them, an option is entanglement swapping. Assume that Alice and Bob each share an entangled state with the third part, Charlie. These states would be $|\phi\rangle_{AC_1}$ and $|\phi\rangle_{BC_2}$. If Charlie now performs a simultaneus measurement on his two states, C_1 and C_2 Alice and Bobs qubits will end up in an entangled state. This is a way of propagating entaglement through a third part to achieve more reliable longer distance entaglement.

3.2 Quantum memories (p.34)??

3.3 Quantum repeater protocol

If long distance distribution of entanglement were to be achieved through optical fibers, one would experience an exponential loss of photons. To overcome this loss, quantum repeaters use heralded entanglement generation and entanglement swapping. Heralded entanglement generation involves establishing local Bell states between quantum memories and optical pulses at each party (e.g., Alice and a repeater node or two repeater nodes). The optical pulses are sent through fibers to a central station where a linear-optical Bell measurement is performed, projecting the pulses into an entangled Bell state with success probability

$$p_g(l) = e^{-l/L_{\text{att}}}/2, \tag{11}$$

where l is the fiber length and L_{att} is the attenuation length. Once neighboring parties share entanglement with a repeater node, entanglement swapping extends the entanglement between more distant parties. This is achieved via a Bell measurement on the quantum memories at the repeater node, succeeding with probability $p_s = 1/2$ in the ideal case. Without repeaters, directly linking Alice and Bob requires an average number of trials

$$\langle T_{\text{tot}}^{(0)} \rangle = p_g(L)^{-1} = 2e^{L/L_{\text{att}}}, \tag{12}$$

which grows exponentially with the distance L. With a single repeater node located midway, entanglement generation is performed in parallel for Alice-to-repeater and repeater-to-Bob links, each requiring

$$\langle T_g(L/2) \rangle = 2e^{L/(2L_{\text{att}})} \tag{13}$$

trials on average. Successful swapping at the repeater node then connects Alice and Bob, requiring

$$\langle T_{\text{tot}}^{(1)} \rangle \sim p_s^{-1} p_g(L/2)^{-1} = 2^2 e^{L/(2L_{\text{att}})} \tag{14}$$

trials, providing a square-root improvement compared to the direct link. This process generalizes with $N_{\text{QR}} = 2^n - 1$ equally spaced repeater nodes, where each step reduces the entanglement distance to $L/(N_{\text{QR}} + 1)$, achieving a total trial count of

$$\langle T_{\text{tot}}^{(N_{\text{QR}})} \rangle \sim 2^{1+\log_2(N_{\text{QR}}+1)} e^{L/((N_{\text{QR}}+1)L_{\text{att}})}, \tag{15}$$

exponentially improving efficiency. While this idealized protocol assumes perfect operations and quantum memories, practical implementations must account for memory errors, imperfect operations, and accumulated errors, mitigated by advanced error-suppression techniques. Thus, quantum repeaters enable scalable quantum communication by mitigating photon loss and other imperfections over long distances.

3.4 Experimental realizations???

4 Experimental Evidence

Experimental evidence for quantum teleportation in quantum communications.

Bullet points

- Quantum teleportation has experimental evidence
- Experimental hurdles

4.1 Satellite Based

1400 km [?]

Bullet points

- Protocol used
- distance
- technical difficulties and innovations
- what does this mean for quantum communications?

The qubit system used in this experiment was based on the polarization of a photon, represented as:

$$|\chi\rangle_1 = \alpha |H\rangle_1 + \beta |V\rangle_1 , \tag{16}$$

where α and β are complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$, and $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarization states, respectively.

At a ground station, two entangled photon pairs were prepared. One photon from each pair was transmitted to a satellite through a 130-mm-diameter telescope, designed with narrow beam divergence and high-precision tracking systems to counteract atmospheric turbulence. The entangled pair of photons is described by one of the Bell states:

$$\left|\psi^+\right\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2 |H\rangle_3 + |V\rangle_2 |V\rangle_3), \tag{17}$$

where photon 2 was retained at the ground station, and photon 3 was sent to the satellite.

A joint measurement, known as a Bell-state measurement, was performed on the photon to be teleported (photon 1) and photon 2 from the entangled pair. This measurement projected the two photons into one of the Bell states:

$$\left|\psi^\pm\right\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1 |H\rangle_2 \pm |V\rangle_1 |V\rangle_2). \tag{18}$$

The outcome of the measurement was then transmitted classically to the satellite, where it influenced the state of photon 3. If the measured state was $|\psi^+\rangle_{12}$, photon 3 adopted the original state of photon 1. If the measured state was $|\psi^-\rangle_{12}$, photon 3 carried the original state of photon 1, but with a π-phase shift.

This experiment demonstrated the successful teleportation of a photon's quantum state over distances of up to 1400 kilometers, from a ground station to a satellite in low-Earth orbit. The primary technical challenges included atmospheric turbulence, which caused beam wandering and broadening, resulting in significant signal losses. These challenges were addressed through the use of a narrow beam divergence, a high-precision telescope, and an advanced acquiring, pointing, and tracking (APT) system.

In the future, this method could be used to create a global quantum internet, connecting quantum computers across the globe and introducing secure cryptography protocols. This also raises the possibility of letting several quantum processors work together on a single problem, enhancing the efficacy of quantum computers.

4.2 Fibre Network Based

100 km [?]. Metropolitan [?]

Bullet points

- Protocol used
- distance
- technical difficulties and innovations
- what does this mean for quantum communications?

5 Summary & Conclusion