Beam up my quantum state, Scotty!
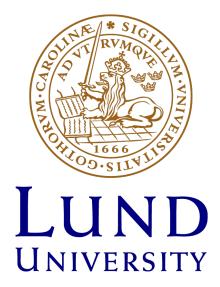
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Contents

1   Introduction

> **Bullet points**
>
> - Quantum teleportation protocol
>
> - Why is it needed?
>
> - Areas of application, quantum communications, quantum computers
>
> - what is needed to realize it on a large scale, i.e. quantum repeaters, memory...
>
> - EPR-pairs and bell basis

1.1   Preliminaries

In quantum teleportation the sender and receiver are referred to as Alice and Bob, and are denoted A and B respectively. Sometimes a third party is relevant which will be called Charlie and be denoted C.

1.1.1   EPR-pairs and the Bell Basis

An Einstein-Podolsky-Rosen-pair (EPR-pair) is a maximally entangled state of two qubits [**?**] which can be written as

$$\left|\Phi^{\pm}\right\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \quad \text{and} \left|\Psi^{\pm}\right\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}. \tag{1}$$

When measuring a quantum state the basis of measurement is important as this determines the possible outcome states. Common basis used are the computational basis, consisting of $|0\rangle$ and $|1\rangle$, and the Bell basis consisting of the EPR-pairs, also known as Bell states, seen in Eq. (1). EPR-pairs and projective measurements in the Bell basis, henceforth called Bell measurements, play a crucial role in quantum teleportation protocols. [**?**]

1.2   Quantum Teleportation Protocol

Let Alice have a particle with a normalized state $|\phi\rangle = \alpha |0\rangle_{\phi} + \beta |1\rangle_{\phi}$, which is unknown to her, that she wants to send to Bob. Sending the particle itself is rarely possible since Alice does not necessarily know where Bob located. She also cannot measure the particle to get accurate information since the

particle is part of an unknown orthonormal set. To overcome these hurdles Alice can instead opt to send, or teleport, the state $|\phi\rangle$ to Bob. [?]

To realize this teleportation both a classical channel and a non-classical channel will be used. The non-classical channel is made of an EPR-pair, where one particle is with Alice and one with Bob. Let Alice and Bob share the EPR-pair

$$\left|\Psi^-\right\rangle_{AB} = \frac{|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B}{\sqrt{2}}. \tag{2}$$

The subscript denotes if Alice or Bob has the particle. Thus, the entire system is in the state

$$|\psi\rangle = |\phi\rangle \left|\Psi^-\right\rangle_{AB} \tag{3}$$

$$= \frac{\alpha}{\sqrt{2}}(|0\rangle_\phi |0\rangle_A |1\rangle_B - |0\rangle_\phi |1\rangle_A |0\rangle_B) + \frac{b}{\sqrt{2}}(|1\rangle_\phi |0\rangle_A |1\rangle_B - |1\rangle_\phi |1\rangle_A |0\rangle_B) \tag{4}$$

Rewriting the products $|x\rangle_\phi |x\rangle_A$, that is the part of the system that is with Alice, using the Bell basis the system can be written as

$$|\psi\rangle = \frac{1}{2} \left[\left|\Psi^-\right\rangle_{\phi A} (-\alpha |0\rangle_B - \beta |1\rangle_B) \left|\Psi^+\right\rangle_{\phi A} (-\alpha |0\rangle_B + \beta |1\rangle_B) \right.$$

$$\left. \left|\Phi^-\right\rangle_{\phi A} (\alpha |1\rangle_B + \beta |0\rangle_B) \left|\Phi^+\right\rangle_{\phi A} (\alpha |1\rangle_B - \beta |0\rangle_B) \right] \tag{5}$$

That is, if Alice performs a Bell measurement the system will collapse into one of these terms. [?]

Depending on the result of Alice's measurement Bob will have the following states

$$A : \left|\Psi^-\right\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B - \beta |1\rangle_B = -|\phi\rangle \tag{6}$$

$$A : \left|\Psi^+\right\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B + \beta |1\rangle_B = -Z |\phi\rangle \tag{7}$$

$$A : \left|\Phi^-\right\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B + \beta |0\rangle_B = X |\phi\rangle \tag{8}$$

$$A : \left|\Phi^+\right\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B - \beta |0\rangle_B = -iY |\phi\rangle \tag{9}$$

$$\tag{10}$$

That is Bob will end up with a rotated version of $|\phi\rangle$. Thus, if Alice sends the result of her measurement classically to Bob he can perform the necessary rotation to obtain the state $|\phi\rangle$. The classical channel can

be a generic broadcast, i.e. a radio, which essentially means that Alice doesn't need to know where Bob is exactly. Note that the choice of $|\Psi^-\rangle$ as the shared EPR-pair was arbitrary. However, which state is shared must be known by both parties since that will change what gate Bob has to apply to obtain $|\phi\rangle$. [?]

2   Teleportation of Complex Quantum Systems

> **Bullet points**
>
> - What is a complex system?
>
> - How does the protocol differ from simple systems?
>
> - Why is it important to be able to teleport complex quantum systems?
>
> - Theoretical and experimental limits

3   Quantum Repeaters and Quantum Memory

Quantum internet [?]

> **Bullet points**
>
> - quantum repeater analogues to normal repeater?
>
> - How to realize quantum memory
>
> - why do we need quantum memory for quantum repeaters
>
> - how much does a quantum repeater reduce attenuation
>
> - how good are today's quantum repeaters?

4   Experimental Evidence

Experimental evidence for quantum teleportation in quantum communications.

> **Bullet points**
>
> - Quantum teleportation has experimental evidence
>
> - Experimental hurdles

4.1   Satellite Based

1400 km [?]

> **Bullet points**
>
> - Protocol used
>
> - distance
>
> - technical difficulties and innovations
>
> - what does this mean for quantum communications?

The qubit system used in this experiment is based on the polarization of a photon with a state represented as $|\chi\rangle_1 = \alpha\,|H\rangle_1 + \beta\,|V\rangle_1$, with complex numbers satisfying $|\alpha|^2 + |\beta|^2 = 1$ and $|V\rangle$ and $|H\rangle$ represents vertical and horizontal polarization respectively. At a ground station, two entangled photon pairs were prepared and one photon from each pair was transmitted to a Satellite through a 130-mm diameter telescope with narrow beam divergence and high-precision tracking systmes to counteract the atmospheric turbulence. The entangled pair of photons can be written as one of the Bell states,

$$\left|\psi^+\right\rangle_{23} = (|H\rangle_2\,|H\rangle_3 + |V\rangle_2\,|V\rangle_3)/\sqrt{2}, \tag{11}$$

where photon 2 is the photon kept at the ground station and photon 3 was sent to the Satellite.

A joint measurement was one on the photon that was to be teleported and one of the entangled photons. This projects these photons into one of the Bell states. The result from this measurment was then classically sent to the Satellite. This measurement forces the entangled pair into a new state. If this state is $|\psi^+\rangle_{12} = (|H\rangle_1\,|H\rangle_2 + |V\rangle_1\,|V\rangle_2)/\sqrt{2}$ the photon sent to the Satellite (photon 3) carries the state that photon 1 was in originally and if the measured state is $|\psi^-\rangle_{12} = (|H\rangle_1\,|H\rangle_2 - |V\rangle_1\,|V\rangle_2)/\sqrt{2}$,

the state of photon 3 is equivalent of the state of photon 1 shifted π.

In this paper it was shown that a photon's quantum state could be teleported over a distance up to 1400 kilometers, from the ground station to the Satellite in low-Earth orbit. The biggest technical difficulties with this approach is the atmospheric turbulence causing the beam to wander and broaden, leading to significant signal losses. This was dealt with by using a narrow beam divergence from a high precision telescope and an advanced acquiring, pointing and tracking system.

In the future, this method could be used to create a global quantum internet, connecting quantum computers across the globe and introducing secure cryptography protocols. This also raises the possibility of letting several quantum processors work together on a single problem, enhancing the efficacy of quantum computers.

4.2   Fibre Network Based

100 km [?]. Metropolitan [?]

> **Bullet points**
> - Protocol used
> - distance
> - technical difficulties and innovations
> - what does this mean for quantum communications?

5   Summary & Conclusion