Beam up my quantum state, Scotty!
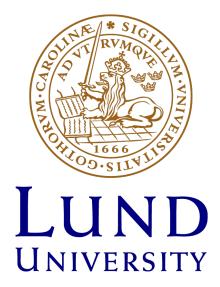## FYST85

Author

Max Eriksson & Lukas Nord

`maxerikss@gmail.com`

`Lukassigvard@gmail.com`

under the direction of

Peter Samuelsson

`peter.samuelsson@teorfys.lu.se`

Lund University

Department of Physics

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1   Introduction

> **Bullet points**
>
> - Quantum teleportation protocol
>
> - Why is it needed?
>
> - Areas of application, quantum communications, quantum computers
>
> - what is needed to realize it on a large scale, i.e. quantum repeaters, memory...
>
> - EPR-pairs and bell basis

1.1   Preliminaries

In quantum teleportation the sender and receiver are referred to as Alice and Bob, and are denoted A and B respectively. Sometimes a third party is relevant which will be called Charlie and be denoted C.

1.1.1   EPR-pairs and the Bell Basis

An Einstein-Podolsky-Rosen-pair (EPR-pair) is a maximally entangled state of two qubits [1] which can be written as

$$\left|\Phi^{\pm}\right\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \quad \text{and } \left|\Psi^{\pm}\right\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}. \tag{1}$$

When measuring a quantum state the basis of measurement is important as this determines the possible outcome states. Common basis used are the computational basis, consisting of $|0\rangle$ and $|1\rangle$, and the Bell basis consisting of the EPR-pairs, also known as Bell states, seen in Eq. (1). EPR-pairs and projective measurements in the Bell basis, henceforth called Bell measurements, play a crucial role in quantum teleportation protocols. [1]

1.2   Quantum Teleportation Protocol

Let Alice have a particle with a normalized state $|\phi\rangle = \alpha |0\rangle_\phi + \beta |1\rangle_\phi$, which is unknown to her, that she wants to send to Bob. Sending the particle itself is rarely possible since Alice does not necessarily know where Bob located. She also cannot measure the particle to get accurate information since the

particle is part of an unknown orthonormal set. To overcome these hurdles Alice can instead opt to send, or teleport, the state $|\phi\rangle$ to Bob. [2]

To realize this teleportation both a classical channel and a non-classical channel will be used. The non-classical channel is made of an EPR-pair, where one particle is with Alice and one with Bob. Let Alice and Bob share the EPR-pair

$$|\Psi^-\rangle_{AB} = \frac{|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B}{\sqrt{2}}. \tag{2}$$

The subscript denotes if Alice or Bob has the particle. Thus, the entire system is in the state

$$|\psi\rangle = |\phi\rangle |\Psi^-\rangle_{AB} \tag{3}$$
$$= \frac{\alpha}{\sqrt{2}}(|0\rangle_\phi |0\rangle_A |1\rangle_B - |0\rangle_\phi |1\rangle_A |0\rangle_B) + \frac{b}{\sqrt{2}}(|1\rangle_\phi |0\rangle_A |1\rangle_B - |1\rangle_\phi |1\rangle_A |0\rangle_B) \tag{4}$$

Rewriting the products $|x\rangle_\phi |x\rangle_A$, that is the part of the system that is with Alice, using the Bell basis the system can be written as

$$|\psi\rangle = \frac{1}{2} \left[|\Psi^-\rangle_{\phi A} (-\alpha |0\rangle_B - \beta |1\rangle_B) |\Psi^+\rangle_{\phi A} (-\alpha |0\rangle_B + \beta |1\rangle_B) \right.$$
$$\left. |\Phi^-\rangle_{\phi A} (\alpha |1\rangle_B + \beta |0\rangle_B) |\Phi^+\rangle_{\phi A} (\alpha |1\rangle_B - \beta |0\rangle_B) \right] \tag{5}$$

That is, if Alice performs a Bell measurement the system will collapse into one of these terms. [2]

Depending on the result of Alice's measurement Bob will have the following states

$$A : |\Psi^-\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B - \beta |1\rangle_B = -|\phi\rangle \tag{6}$$
$$A : |\Psi^+\rangle_{\phi A} \longrightarrow B : -\alpha |0\rangle_B + \beta |1\rangle_B = -Z |\phi\rangle \tag{7}$$
$$A : |\Phi^-\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B + \beta |0\rangle_B = X |\phi\rangle \tag{8}$$
$$A : |\Phi^+\rangle_{\phi A} \longrightarrow B : \alpha |1\rangle_B - \beta |0\rangle_B = -iY |\phi\rangle \tag{9}$$

$$\tag{10}$$

That is Bob will end up with a rotated version of $|\phi\rangle$. Thus, if Alice sends the result of her measurement classically to Bob he can perform the necessary rotation to obtain the state $|\phi\rangle$. The classical channel can

be a generic broadcast, i.e. a radio, which essentially means that Alice doesn't need to know where Bob is exactly. Note that the choice of $|\Psi^-\rangle$ as the shared EPR-pair was arbitrary. However, which state is shared must be known by both parties since that will change what gate Bob has to apply to obtain $|\phi\rangle$ [2]

2 Teleportation of Complex Quantum Systems

> **Bullet points**
>
> - What is a complex system?
>
> - How does the protocol differ from simple systems?
>
> - Why is it important to be able to teleport complex quantum systems?
>
> - Theoretical and experimental limits

3 Quantum Repeaters and Quantum Memory

Quantum internet [3]

> **Bullet points**
>
> - quantum repeater analogues to normal repeater?
>
> - How to realize quantum memory
>
> - why do we need quantum memory for quantum repeaters
>
> - how much does a quantum repeater reduce attenuation
>
> - how good are today's quantum repeaters?

4 Experimental Evidence

Experimental evidence for quantum teleportation in quantum communications.

> **Bullet points**
>
> - Quantum teleportation has experimental evidence
>
> - Experimental hurdles

4.1 Satellite Based

1400 km [4]

> **Bullet points**
>
> - Protocol used
>
> - distance
>
> - technical difficulties and innovations
>
> - what does this mean for quantum communications?

4.2 Fibre Network Based

100 km [5]. Metropolitan [6]

> **Bullet points**
>
> - Protocol used
>
> - distance
>
> - technical difficulties and innovations
>
> - what does this mean for quantum communications?

5 Summary & Conclusion

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