Forecasting Amtrak Ridership Using Time Series

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ADS-506 Applied Time Series Analysis

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Problem Statement

The Amtrak client would like to know the number of passengers for the next three months. Based on historical data, Amtrak would like to model the behavior of ridership from 1991 to 1999. This will allow Amtrak to understand how well the new high-speed rail service affected the northeast U.S. from 2000 to 2013. The ridership data is collected monthly, reporting the number of passengers. The task entails forecasting models to predict the number of passengers for the next three months in 2013. The forecasted number of passengers will determine the necessary adjustments to make improvements for the new service system.

Justification

The purpose of this project is to forecast Amtrak ridership. With this information, Amtrak can better plan how to allocate funds, mainly for changing the routes and equipment management. Deciding whether or not to introduce a new passenger train service or make changes to existing service comes down to one question: how many more people will ride the train after the changes are made? (Kenton, 2015). Being able to forecast which routes are close to their maximum ridership, Amtrak can prioritize these routes by adding rail cars and or more routes. This will increase profitability; using this information, routes and rail cars can be added to busy routes, increasing capacity for more paying customers who may have had to seek other means of transportation by mode or competitor. Amtrak can reallocate resources from slower routes to busier routes with this forecasting information, thereby maximizing their profits by either cutting down on routes or reallocating rail cars.

The outcome of this project will enable Amtrak to allocate vital resources for operations efficiently; Forecasting also helps plan maintenance during slower periods or seasons. Amtrak is also fine-tuning its analysis of the effects of a host of service changes on ridership, including on-time performance, station improvements, and changes in onboard amenities (Kenton, 2015).

Literature Review

On December 11, 2000, Amtrak introduced the U.S.'s only high-speed rail service nicknamed Acela, which is still the only high-speed rail service in the U.S. (Waite, 2021). Having made a significant investment in the service, Amtrak needs demand forecasts to maximize revenue.

Demand forecast involves using past historical data to predict or estimate future demand. In this case, the goal is to predict future demand for the next three months of service, using time series analysis to make the forecast.

The goal of this literature review is to find other research that has been done for forecasting ridership across different transportation modes.

Airline

Over the past several years, airline travel has increased in popularity in Indonesia. Forecasting has become crucial in planning for strategic growth to keep up with the demand. The Airline industry in Indonesia in 2016 reached the 2nd highest after China, and it's predicted to have a high passenger surge in 2037 (Ramadhani et al., 2020). Forecasting also affects fleet planning, route planning, and annual operation planning; poor forecasting will positively impact the company's revenue (Ramadhani et al., 2020).

Due to significant pattern components within the data, such as "trend, seasonality, cyclicity, and irregularity, time series was chosen as the forecasting method (Ramadhani et al., 2020). The time series model ARIMA was compared to the Neural network model to see which model would perform better; previous research has shown that neural networks had outperformed the ARIMA models. Monthly data from two of the most profitable routes over five years were used.

The metric for success was the model with the lowest minimum error; The results showed that the neural network model had the lowest minimum errors for both routes, the study has certain limitations, the parameters were manually combined using trial and error, and the data interval used was not focused on daily or weekly demand (Ramadhani et al., 2020).

Bus

Chinese urban public transportation has improved dramatically over the past decades; according to Ye et al. (2019), rapid economic development has played a role in this. Data from the China Academy of Transportation Science in 2017 shows that it has reached 55.9 million kilometers or four times its total highway mileage (Ye et al., 2019). Forecasting became critical after investigations on passenger demand showed that several popular routes were often in short supply; forecasting the demand will help the bus companies maximize the profit.

The main research objective was to predict the daily passenger volume in Jiaozuo city in China based on the ticket sales records; an experiment was performed to see if using just the weekday data would result in higher prediction accuracy (Ye et al., 2019).

The results showed that the data from the weekend was necessary for accuracy prediction; Ye et al., (2019) arrived at this conclusion after observing three months' worth of data. Using three months of data limited the study by not being long enough to analyze the impact of patterns such as trends and seasonality on the dataset (Ye et al., 2019).

Train

Guo et al., (2010) studied the ARIMA and Holt-Winters model to forecast China's monthly freight. Railway freight has shown volatility due to international fluctuations in the industrial product sector; most of the predictions are focused on the medium- and long-term forecast, and as such, the goal of this article is to build a model to forecast the short-term freight (Guo et al.,2010).

The data had 96 samples, and it showed seasonal fluctuations with a steady growth trend, which could also influence the short-term forecast results. Both models performed well in the short-term forecast. The Arima model performed better due to its lower minimum error and the estimates being closer to the actual real values than the Holt-Winters model (Guo et al.,2010).

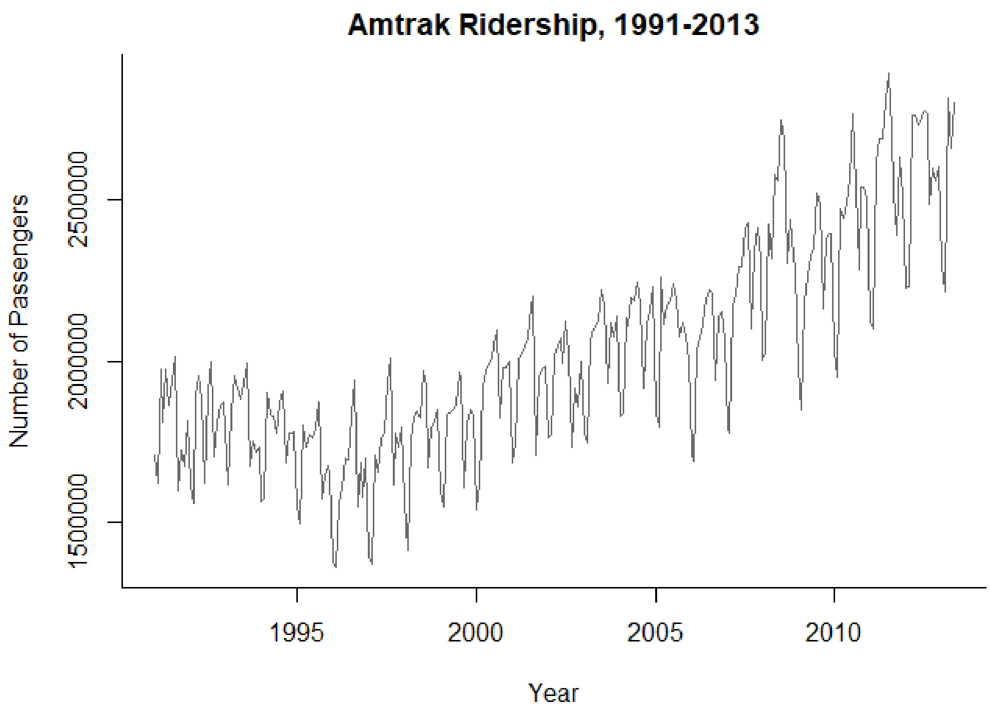
In conclusion, most of the existing research has been done using the ARIMA model, and it has made valuable predictions when it comes to forecasting. Trends, seasonality need to be taken into account when modeling.

Data Exploration

The data set was obtained from the Bureau of Transportation Statistics, which includes monthly Amtrak ridership data from January 1991 until May 2013. There are 269 rows with only two columns containing the timestamp of each month in that respective year and the total counts of ridership. A time-series trend was visible during the exploratory data analysis over the years. This series also shows signs of seasonality where this may be hypothesized as non-stationary.

Figure 1.

Amtrak Ridership (1991-2013).

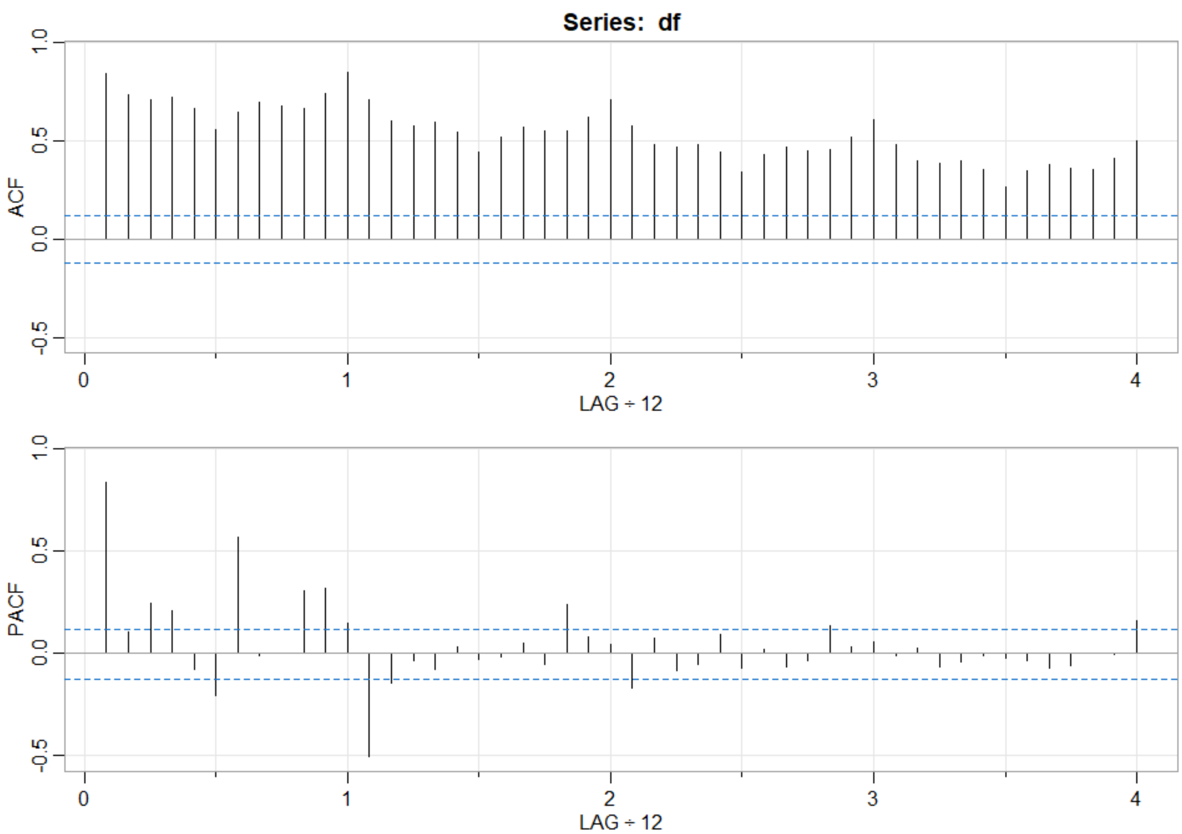


Note. This figure shows the time series plot of the ridership data Amtrak has collected monthly over the time periods of January 1991 to May 2013.

The next data exploration task is to explore the auto-correlation (ACF) and partial auto-correlation function (PACF) plots of this series. Figure 2 displays these plots below.

Figure 2.

ACF and PACF Plot



Note. This is the ACF and PACF plot of the original Amtrak ridership series.

The ACF plot shows clear trend and seasonality; high lag coefficients starting at lag 1, high lag coefficients are any lags above the significance range, which are the dotted blue lines, there is an oscillation in the series with a peak every twelve months at lags 1, 12, 24, 36, 48 indicate seasonality. The ACF plot is slowly decaying, indicating a trend; every twelve months, there is a slight decrease over the period. It is still above the significance range. The slow decay indicates that future values are highly affected by past values. The PACF plot has significant lags above the significance level that taper off, which will be helpful when picking an ARMA model.

Data Pre-processing

The first steps were renamed to the original columns of the data set to Dates and Number_of_Passengers. Then the data was converted to a time series object to allow for further time series analysis. The frequency of the time series object was set to 12 due to the observations being monthly each year.

Simple Smoothing Methods

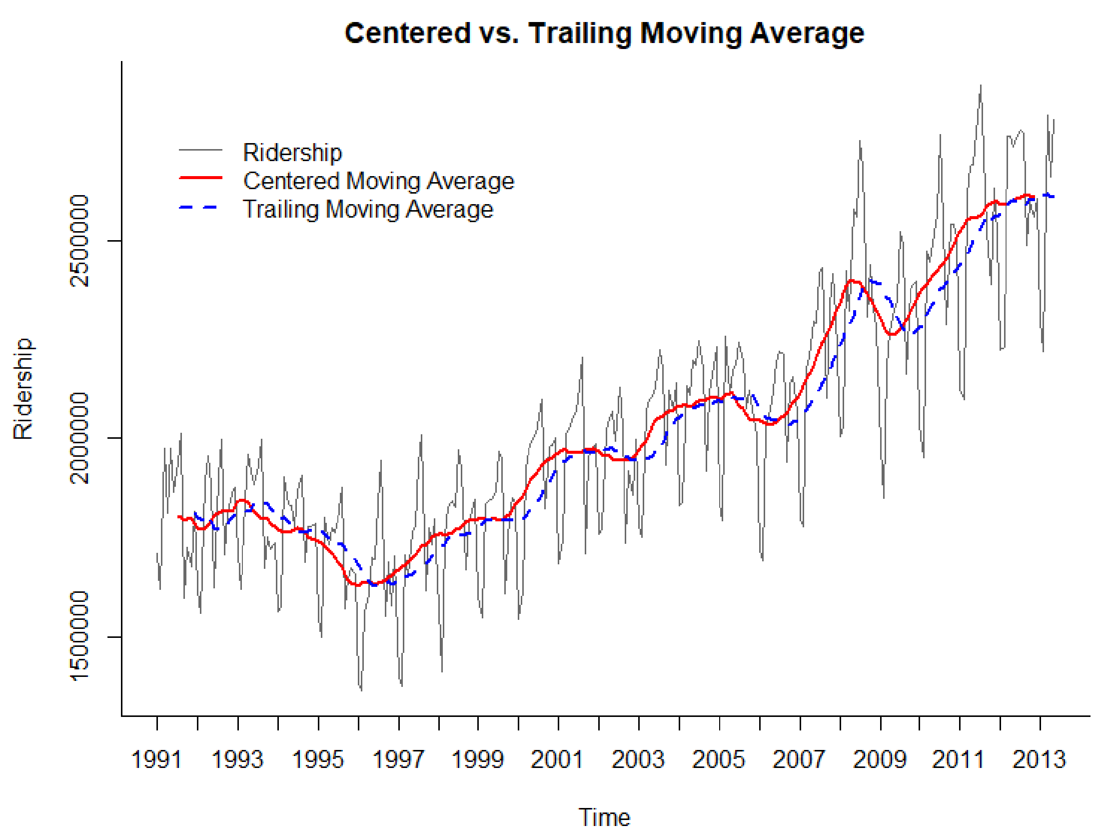
At the beginning of the data exploration, time components such as seasonality and trend are present over time. Thus, a good approach may be data-driven methods because it deals with data without a predetermined structure. An example of using a simple smoothing method is using the moving average. The moving average is a simple smoothing method containing average values across a time window, *w*, specified by the user. Moving average works by suppressing seasonality and noise in a dataset. However, there are two types of moving averages: the centered moving average and the trailing moving average.

Centered vs. Trailing Moving Average

The centered-moving average is useful for visualizing trends, and the trailing moving average is useful for forecasting. These two moving averages will be compared against each other in Figure 3.

Figure 3.

Centered vs. Trailing Moving Average



Note. Twelve was the selected time window, indicating the length of the seasonal cycle. This figure shows somewhat of a global U-shape, but the moving average looks to increase as the year progresses.

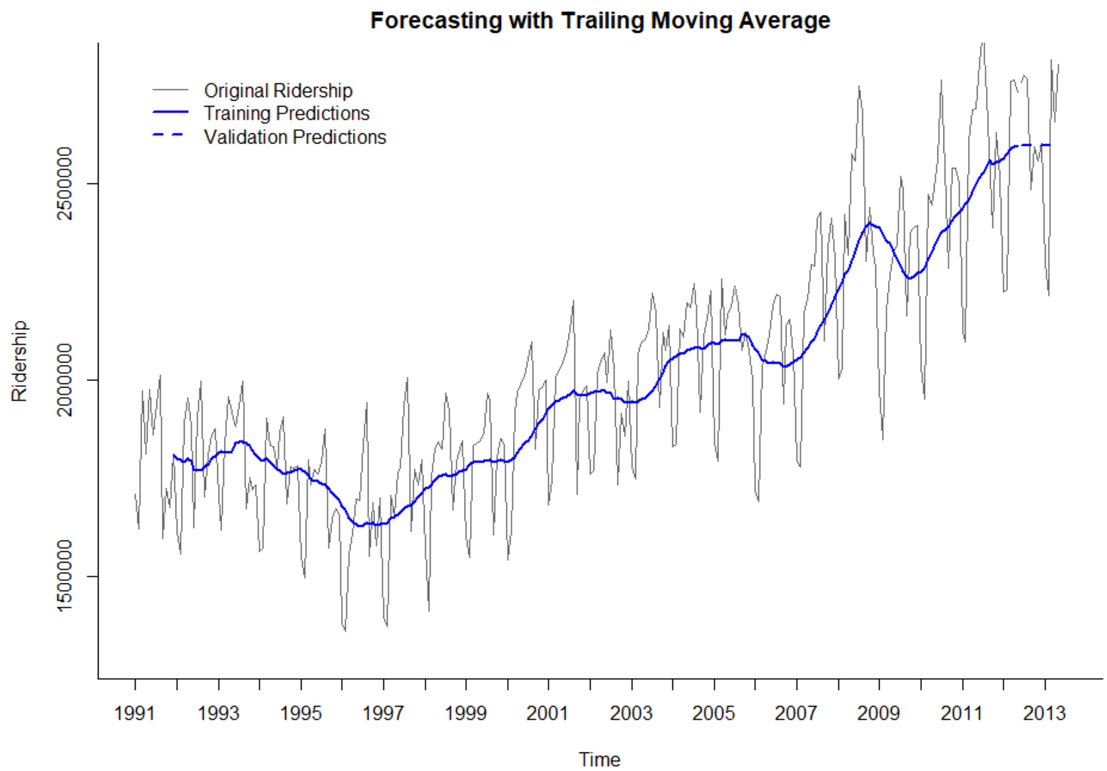
Since the centered-moving average uses past and future data, this is a limitation for making forecasts when future information is unavailable. Therefore, the better approach is to move forward with the trailing average for forecasting. This method selects the window that is placed over the most recent values.

Trailing Moving Average Forecasting

In Figure 4 below, the trailing moving average does not perform well with forecasting this time series data. The first thing to notice is that the forecasts for all the months in the validation period denoted in blue dashes are identical because this method is not roll-forward next month's forecasts. The trailing moving average forecaster is inadequate for the Amtrak monthly forecast task.

Figure 4.

Forecasting with a Trailing Moving Average



Note. The forecast is constant during the validation period, which means it does not capture the trend and seasonality of the data.

The reason why is because it does not capture the seasonality in the data. The forecaster predicted seasons with high ridership with lower ridership and seasons with low ridership with high ridership. This occurs because the moving average lags behind when forecasting a time series with a trend. Therefore, over-forecasting and under-forecasting in the presence of increasing and decreasing trends. Between the smoothing methods of moving averages, it should only be used for forecasting when a series lacks seasonality and trend, which is not valid for the Amtrak ridership data.

Alternative methods for Non-Stationary Series

Differencing is a popular method for removing trend or seasonality patterns by taking the difference between two values in a series. For example, the lag-1 difference (first-order difference) takes the difference between consecutive observations in time. On the other hand, differencing at lag-k takes the difference from every value from the k-periods back. However, before using differencing as a pre-processing step, a Dickey-Fuller test should be used to check if the assumptions for a series are non-stationary.

Dickey-Fuller Test

The Dickey-Fuller test checks if a time series data is stationary or not. However, it was evident during the data exploration that this series was not stationary. Figure 5 below shows the results of a Dickey-Fuller test.

Figure 5.

Results of the Dickey-Fuller Test

```
##
##  Augmented Dickey-Fuller Test
##
## data:  df
## Dickey-Fuller = -3.8062, Lag order = 6, p-value = 0.01915
## alternative hypothesis: stationary
```

Note. The results here indicate that a p-value was less than the chosen significance level at 0.05. This means that the Dickey-Fuller test rejects the null hypothesis stating that it is statistically significant as a stationary series instead of non-stationary.

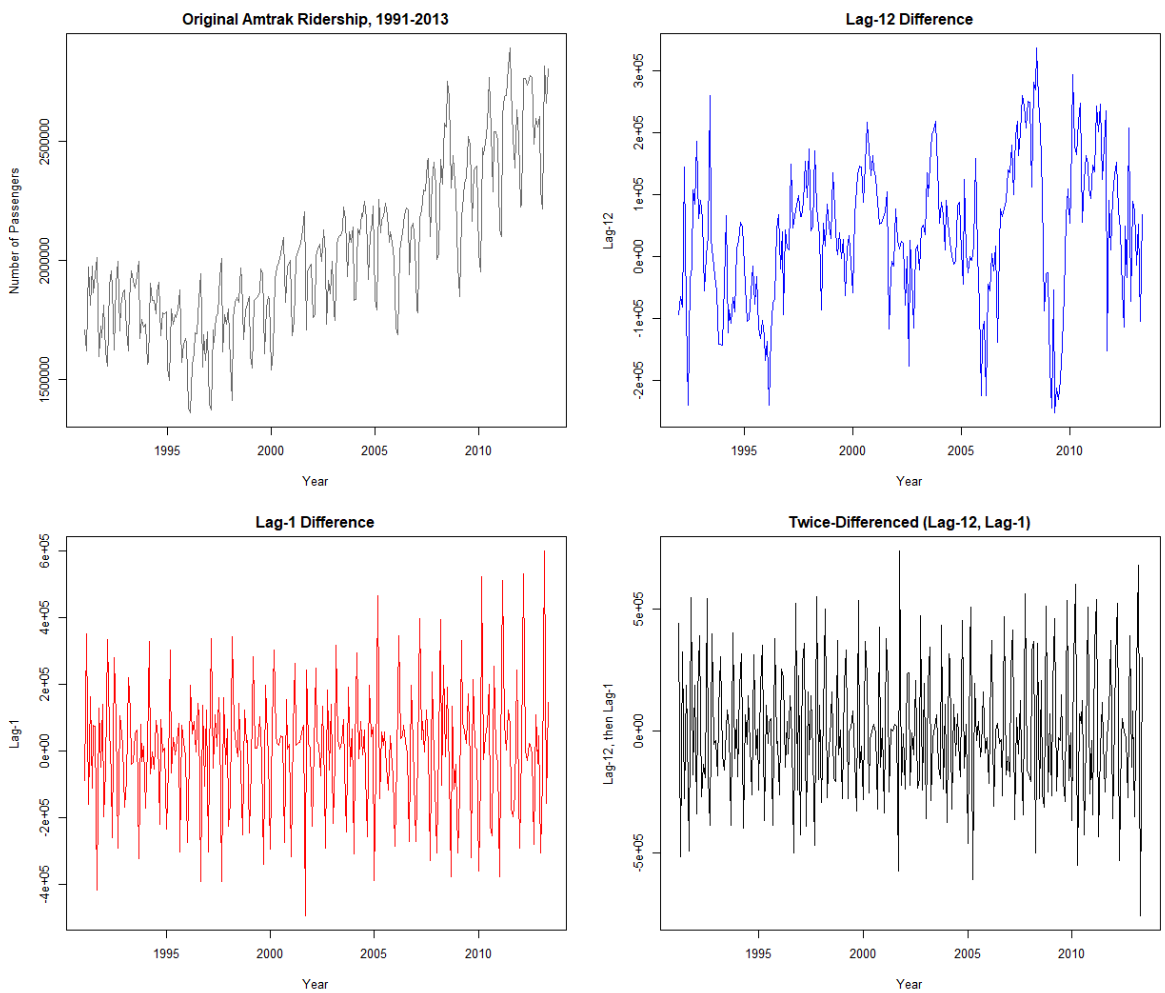
Although the dataset visually appears to be non-stationary, the Dickey-Fuller test rejects the null hypothesis in favor of the alternative hypothesis that the dataset is indeed stationary. However, these results are from a biased test with a type one error. A stationary time series has a constant mean, minimal seasonality effect, and no variance increase over time. Meanwhile, the Amtrak ridership data set has no constant mean with signs of seasonality and an increasing trend as the year progresses. Therefore, the next step is to continue with differencing for pre-processing the series.

Detrending and Dseasonalizing

There are different approaches to making a time series stationary, such as detrending and deseasonalizing. Detrending can be used by the lag-1 difference of a series. This would remove the somewhat U-shape of the Amtrak ridership series. Deasonalizing would remove the seasonality by using a lag-12 difference series. A k-period of 12 was used because of monthly data over the years. Double differencing can be applied to the series when both of these components exist. Figure 6 below shows the comparison between the original data, the lag-12 difference, the lag-1 difference, and, lastly, the double-differenced data.

Figure 6.

Comparisons between Different Differenced Data



Note. Since there are both seasonality and trend, the double differencing effect on the bottom right resulted in a series without trend or monthly seasonality.

The first plot on the top left shows the original Amtrak ridership with trend and seasonality. The lag-1 plot on the bottom left has been differenced by a month to remove the trend. In comparison, the lag-12 different plot on the top right was adjusted to remove monthly seasonality. Lastly, the bottom right panel has been double-differenced to remove both trend and monthly seasonality.

Data Splitting

Fixed-Partitioning

Forecasting with time-series data also requires the appropriate partitioning to evaluate the performance of the models. There will be a training set, a validation set, and a test set similar to cross-sectional. Although partitioning data into these sets are usually done randomly, this is not the case for a time series. The first step is to decide the different training and validation periods. Meaning, the time window will need to be specified before partitioning into different sets. The training set consists of the earlier periods, while the validation set is the periods the model has not yet been trained on. This will be the later period. For the test period, this partitioning set will be the forecasted months the client has requested. Thus, the data splitting strategy for the Amtrak ridership data will be a fixed-partitioning method. The training period will be from January 1991 to May 2012. The validation period will be from June 2012 to May 2013. While the test period Amtrak requested is from June 2013 to August 2013.

Model Strategies

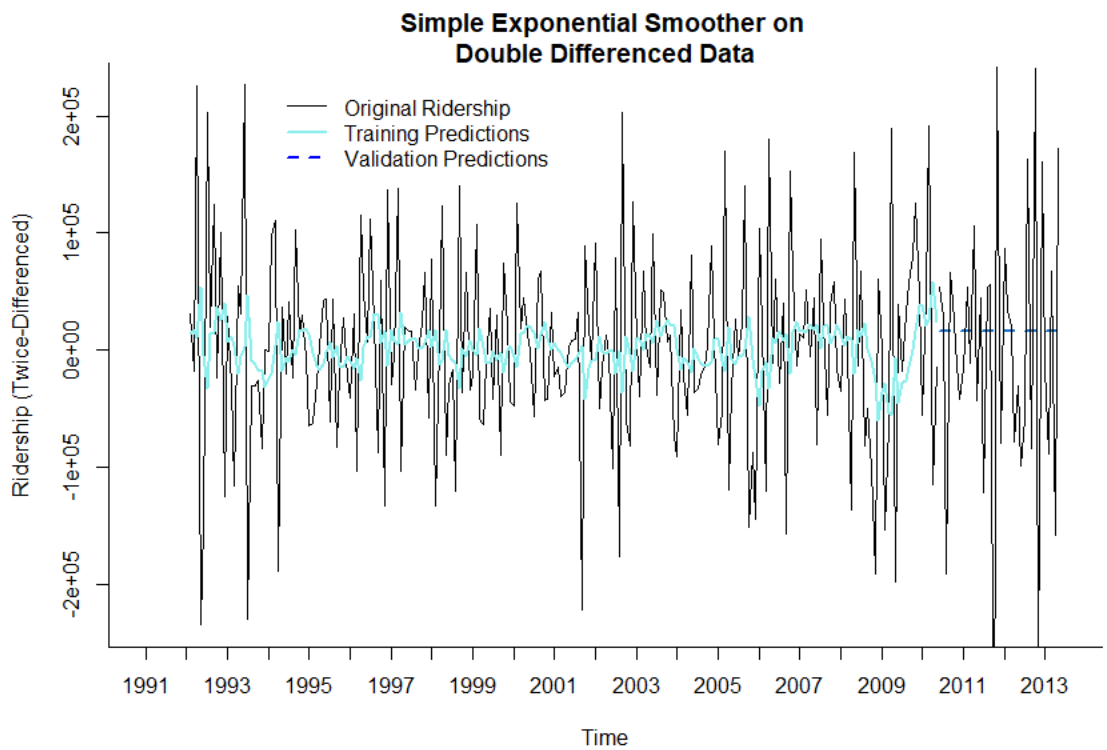
Simple Exponential Smoothing

Exponential smoothing works in a similar manner to forecasting with a moving average, although it differs by taking the weighted average of all past values of a series. The weights will decrease exponentially into the past, which is valuable as it gives more weight to recent data than past data. This is a popular method due to its low computational costs, easy automation, and good performance. Exponential smoothing works well with data without trend or seasonality, making it a logical model for the double-differenced series.

Figure 7 below shows the exponential smoother on the double-differenced data. This smoothing method is under the *ets* framework using the model called additive (A), no trend (N), and no seasonality (N), denoted as "ANN".

Figure 7.

Simple Exponential smoothing



Note. The forecasts for the validation set remained a singular value similar to the moving average forecast model.

Both moving average and exponential smoothing models were not able to have accurate forecasts, even though the pre-processing was already performed. Another solution is to use a more complex and sophisticated exponential smoothing that is able to model data with both trend and seasonality instead of removing them.

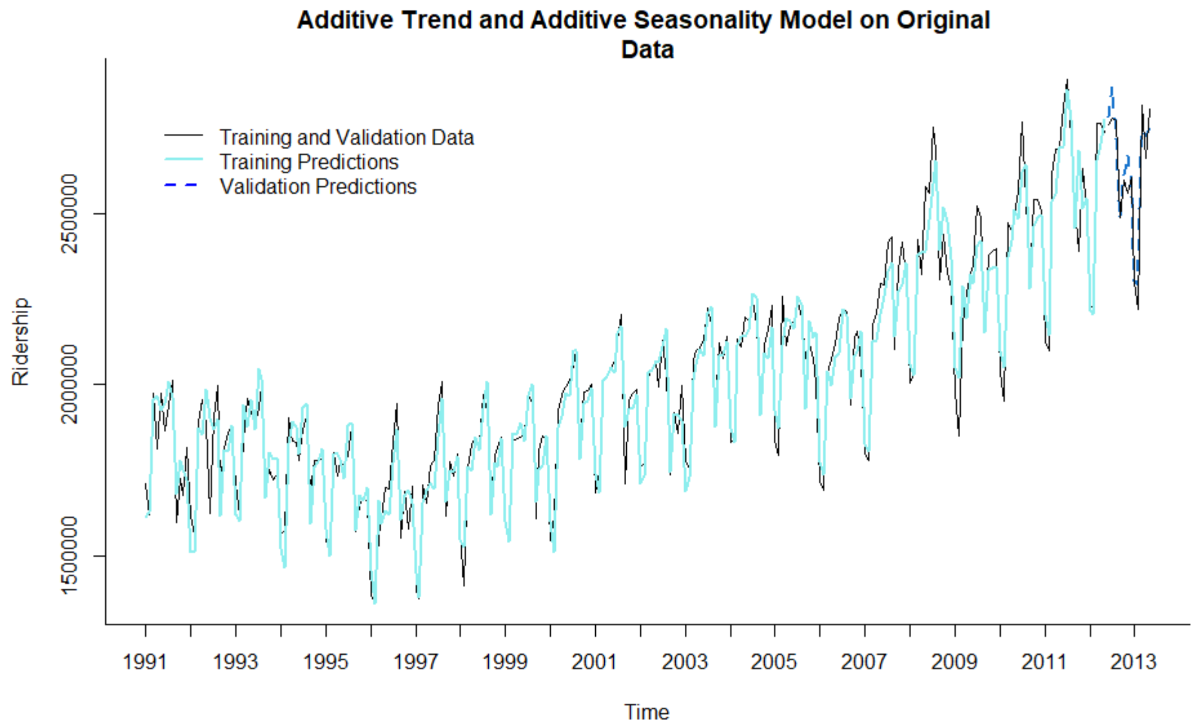
Models with Additive or Multiplicative Trends

Double exponential smoothing can be used on a series that contains an additive trend. This is also called Holt's linear trend model, where the local trend is estimated and is updated as more data comes in. Although additive trend models assume the level changes from one period to the next by a fixed amount, multiplicative trends assume it changes by a factor instead.

A further extension of the double exponential smoothing where the k-step-ahead forecasts also takes into consideration the seasonality of the current period. At the same time, the trend is considered from the additive and multiplicative seasonality. This is an adaptive method that allows the components (levels, trends, and seasonality) to change over time. Therefore, the Amtrak ridership series would not need the pre-processing performed previously. Figure 8 shows this model fitted to the training periods and made forecasts on the validation periods.

Figure 8.

Additive Trend and Additive Seasonality Model



Note. This approach uses the Holt-Winters method to build an additive trend and seasonality model with multiplicative error on the original data without a second-order difference.

This falls under the ETS framework using the M.A.A model. However, what if the choice was to use a multiplicative trend and seasonality model or try different combinations of these components.

Automated Model Selection for the Optimal Exponential Model

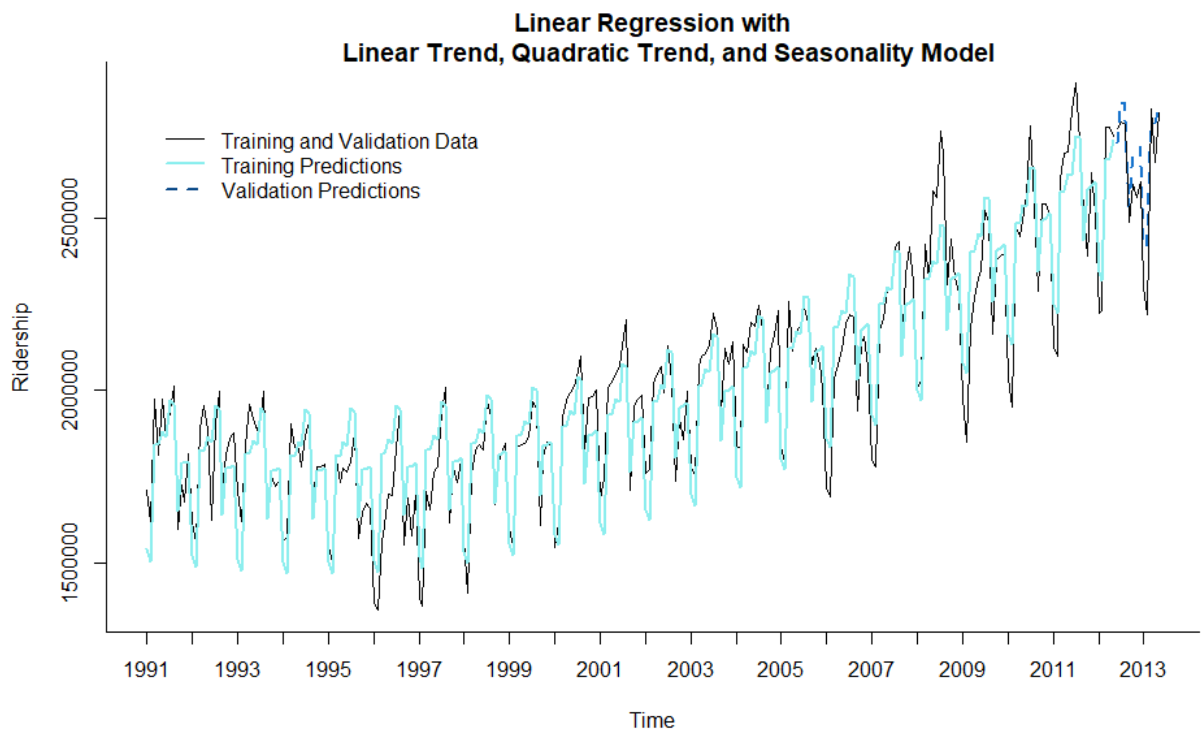
The optimal model chosen was a multiplicative error, additive trend, and multiplicative seasonality with an improved AIC score of 7155.16, which was lower than the previous MAA model with an AIC of 7181.09.

Regression-Based Models

There are different types of common trends and seasonality that can be modeled by regression-based models estimated during the training period and used to forecast future data. Such trends include linear, exponential, or polynomial, while the different seasonality is additive and multiplicative seasons. For example, a new model can be built with both linear and quadratic trends. Then a monthly seasonality can be added, which is a factor of twelve; however, it is redundant to use all twelve. In total, we will have thirteen predictors, with two for the linear and quadratic trends and the eleven dummy variables for each month, except January. Figure 9 shows what this model looks like.

Figure 9.

Regression Model with Linear and quadratic trends and Monthly Seasonality.



Note. This model uses a linear and quadratic trend along with the 11 monthly seasons as dummy variables, excluding January.

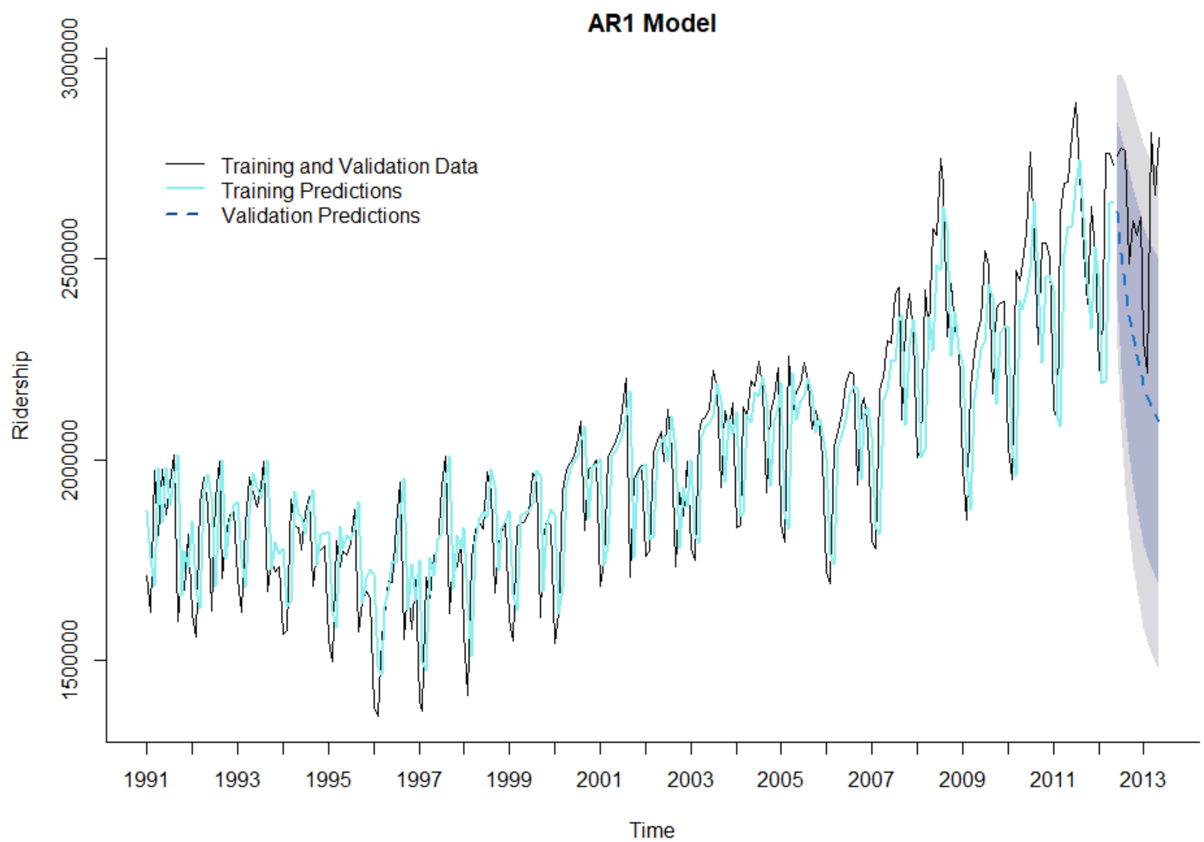
The AIC score has dropped significantly compared to the model with only a linear trend. Compared with the exponential smoothing model, this score has dropped drastically than before.

AR1 Model

ARIMA (autoregressive moving average) models are specified by the parameters (*p,d,q*), the A.R. parameter *p* refers to past values, the Integrated *d* parameter stands for differencing, differencing subtracts current and previous d values to stabilize the series, the M.A. *q* parameter represents a combination of the previous error terms. We choose the ARIMA model based on its ability to use past values to make future forecasts; the model used (*p,d,q*) of the order (1,0,0) as a base A.R. model, since the d differences the dataset the double differenced dataset was not used. The AR1 model had an AIC score of 6936.7, which is a slightly better AIC score than previous models; however, the validation predictions in Figure 10 below performed poorly in the forecast trailing off.

Figure 10.

AR1 Model



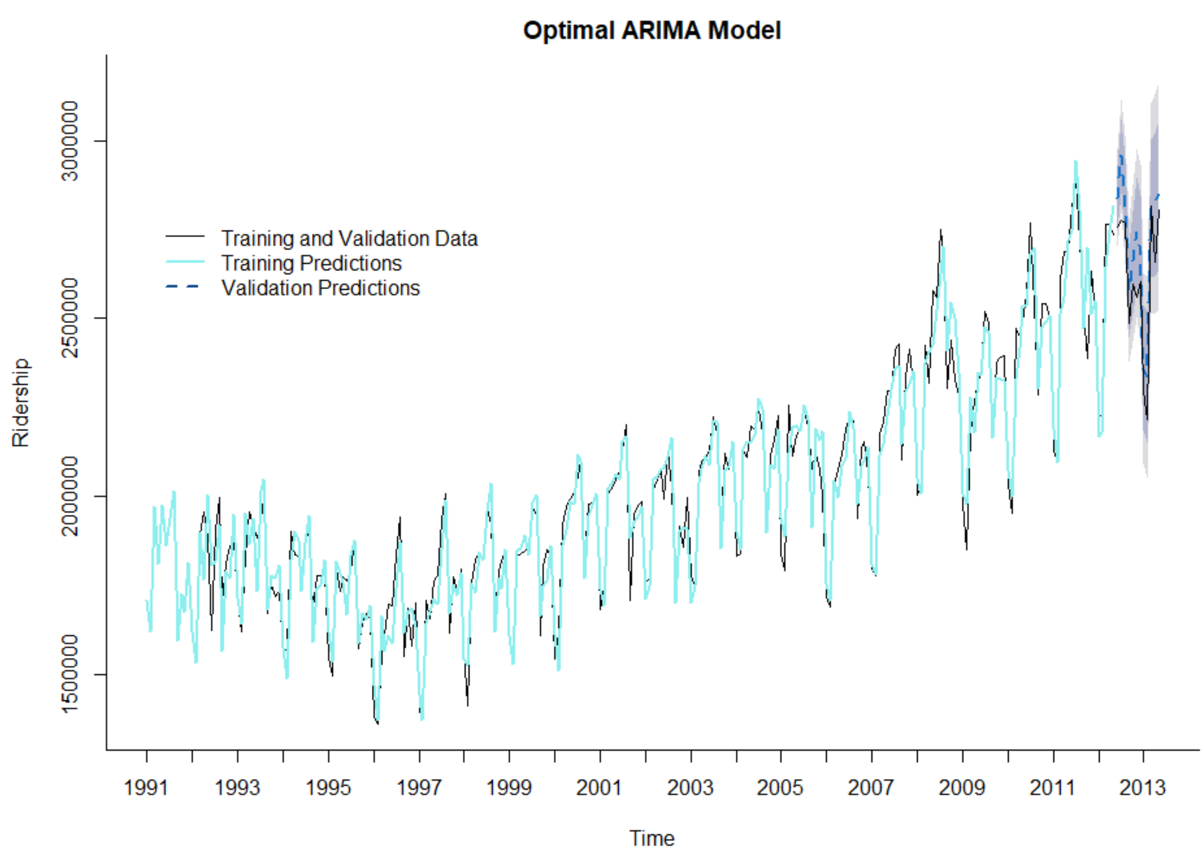
Note. These are the forecasts of an auto-regressive (AR1) model. Based on the validation data, this may indicate that the AR1 model may perform poorly on future data.

Auto Selected ARIMA Model

Based on the results of the AR1 model indicates the ARIMA model needs to be tuned to obtain optimal parameters , We implemented the Auto ARIMA function to obtain the parameters, Auto ARIMA runs many variations on the **(***p,d,q***)** to obtain the parameters, Figure 11 below shows the results of the Auto ARIMA the optimal parameter order were (2,1,1) , it generated a much better AIC score of 6152.79 which is much lower than the AR1 model which had an AIC score of 6936.7, it also has much better prediction forecast capturing the seasonality and trend of the data.

Figure 11.

Auto selected Optimal ARIMA Model



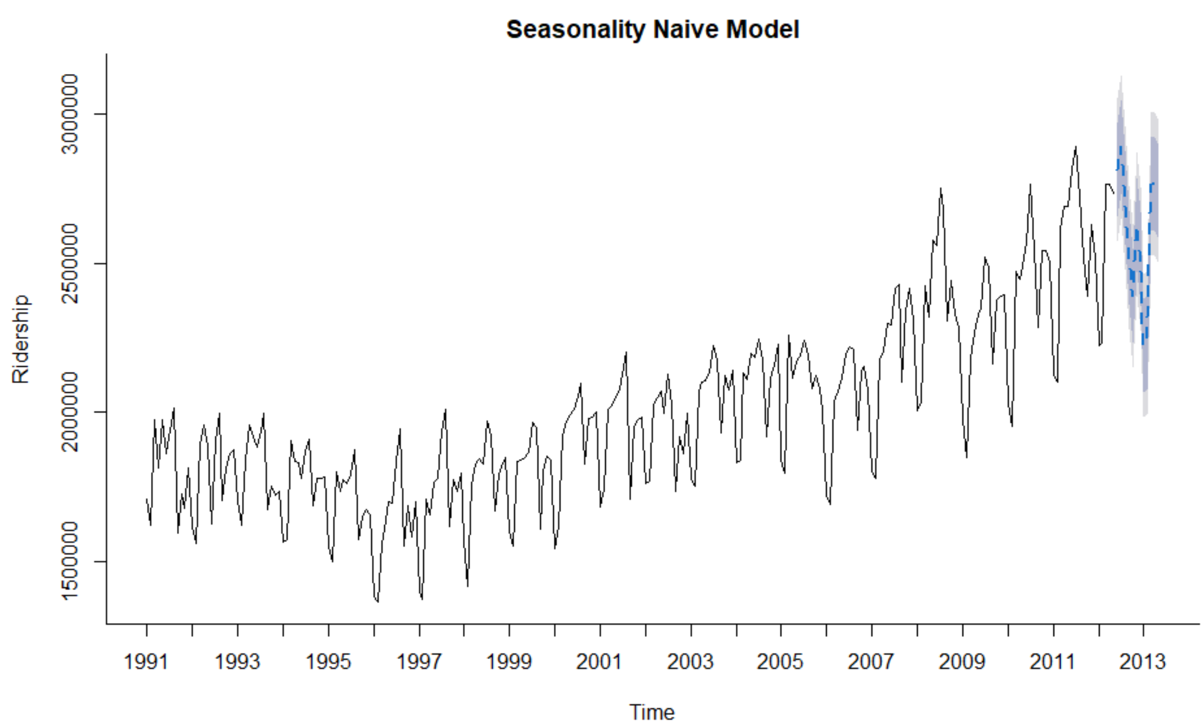
Note. These forecasts are from the auto selected parameters for an optimal ARIMA model.

Naive Forecast Models

Instead of sophisticated modeling, naive forecast models output the most recent values of the series. While a seasonal naive forecast model uses the recent value from the most identical season. For example, the forecast for August 2012 would be the value from August 2011 instead. Also, naive forecast models have great performance and are easy to understand and implement. Although this seasonality naive model in Figure 10 will be used as a baseline model to compare other models' predictive performances.

Figure 10.

Seasonal Naive Forecasting Model



Note. This seasonal naive model will only be used as a baseline model to compare performance with previous models.

Results and Final Model Selection

Based on the results, there are a handful of different performance metrics to evaluate across the models. The following models are competing against each other: the seasonal naive forecast model, the optimal exponential model, and the regression model with a monthly seasonality and linear and quadratic trends, and the ARIMA models. The selected performance metric is the root mean square error (RMSE). Lower values of the RMSE indicate a model with higher performance. A benefit of using this performance metric over the others is that the RMSE will be in the same units as the original data. Previously, the AIC scores were used for comparison for only a quick evaluation of the training set to see the complexity of each model. Since the task is forecasting future values, using the RMSE score from the test set is more appropriate. The following tables in Table 1 display the different performance metrics of the previously stated models.

Table 1.

Model Performance Results

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	6027	66103	52183	0.2028	2.624	0.5279
Test set	-21561	61488	49641	-0.824	1.866	0.5022

Table 3: Regression with Monthly Seasonality and Linear and Quadratic Trends Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	1.806e-12	92906	76160	-0.2116	3.915	0.7705
Test set	-72741	100661	87296	-2.964	3.486	0.8831

Table 4: AR1 Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	1492	173260	129640	-0.69	6.683	1.311
Test set	326770	388025	326770	12.16	12.16	3.306

Table 5: Optimal ARIMA Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	2825	67502	50835	0.06815	2.555	0.5143
Test set	-92458	107742	93867	-3.587	3.637	0.9496

Table 6: Seasonal Naive Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	38708	122794	98851	1.595	4.854	1
Test set	12386	91056	77756	0.4937	2.959	0.7866

Note. The primary performance metric that will be evaluated is the RMSE from the test set.

While the RMSE score from the seasonal naive model will serve as the baseline performance score to beat if there are models with a lower RMSE value, currently, the seasonal naive model has an RMSE score of 91,056 in the test set. For the optimal exponential model, the test set's RMSE score is 61,488, while the regression-based model has the highest RMSE score of 100,061. The AR1 model has a test set RMSE score of 388,025 and the optimal ARIMA model score of 107,742. Not only did the AR1 model fail to beat the baseline score (lower RMSE), but it also had the worst RMSE score of the five models. The final model selected to forecast the months of June to August of 2013 will be the optimal exponential model.

Conclusion

Findings

The final model was used to make forecasts for the month of June to August of 2013. June of 2013 is forecasted to have 2,793,575 ridership. Then July of 2013 will see an increase with a total of 2,900,579 ridership. However, the final model forecasted a decrease in ridership with a total of 2,807,599 ridership for the month of August. Table 2 below shows the point forecasts along with the 80% and 95% confidence intervals for the possible low and high forecasts range.

Table 2.

Final forecasts

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jun 2013	2793575	2523876	3063274	2381106	3206044
Jul 2013	2900579	2611706	3189452	2458786	3342372
Aug 2013	2807559	2519628	3095489	2367207	3247910

Note. This table displays the forecasting results using the final model selected.

Suggestions

In conclusion, the optimal exponential model was the best model with the lowest RMSE score. Table 2 shows the final forecasts of this model, where the points forecast shows the predictions for the upcoming months. These numbers capture the seasonality and trend in the data. The next steps would involve implementing the model and strategizing on the next business decisions. Since it is noted that the July forecast will be the highest, the recommendation will be to have more available workers and keep all maintenance up to date with Amtrak rail services.

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Appendix A - Monthly Amtrak Ridership Forecasting Final Project

Team 3: Jimmy Nguyen, Luke Awino

12/04/2021

Contents

Libraries

```r
library(astsa)
library(readr)
library(forecast)
library(zoo)
library(xts)
library(pander)
library(tidyverse)
library(tseries)
library(lubridate)

knitr::opts_chunk$set(warning = FALSE, message = FALSE)
```

Data Set

Code:

```r
# Load the data set from CSV file
df <- read_csv("../Data/Amtrak Ridership Data.csv")


# Rename columns
names(df)[1] <- 'Dates'
names(df[2]) <- 'Number_of_Passengers'


# First 12 months in 1991
head(df, n = 12) %>%
  pander(style = "grid", caption = "First 12 Months - 1991")
```

Table 1: First 12 Months - 1991

Dates	Number of Passengers
Jan-91	1708917
Feb-91	1620586
Mar-91	1972715
Apr-91	1811665
May-91	1974964
Jun-91	1862356
Jul-91	1939860
Aug-91	2013264
Sep-91	1595657
Oct-91	1724924
Nov-91	1675667
Dec-91	1813863

Data Exploration

Code:

```
# convert to time series object
df<- ts(data = df[,2], start = c(1991,1),
        end = c(2013,5), frequency = 12)

print("Starting Year and Month: ")
```

```
## [1] "Starting Year and Month: "
```

```
start(df)
```

```
## [1] 1991    1
```

```
print("Final Year and Month: ")
```

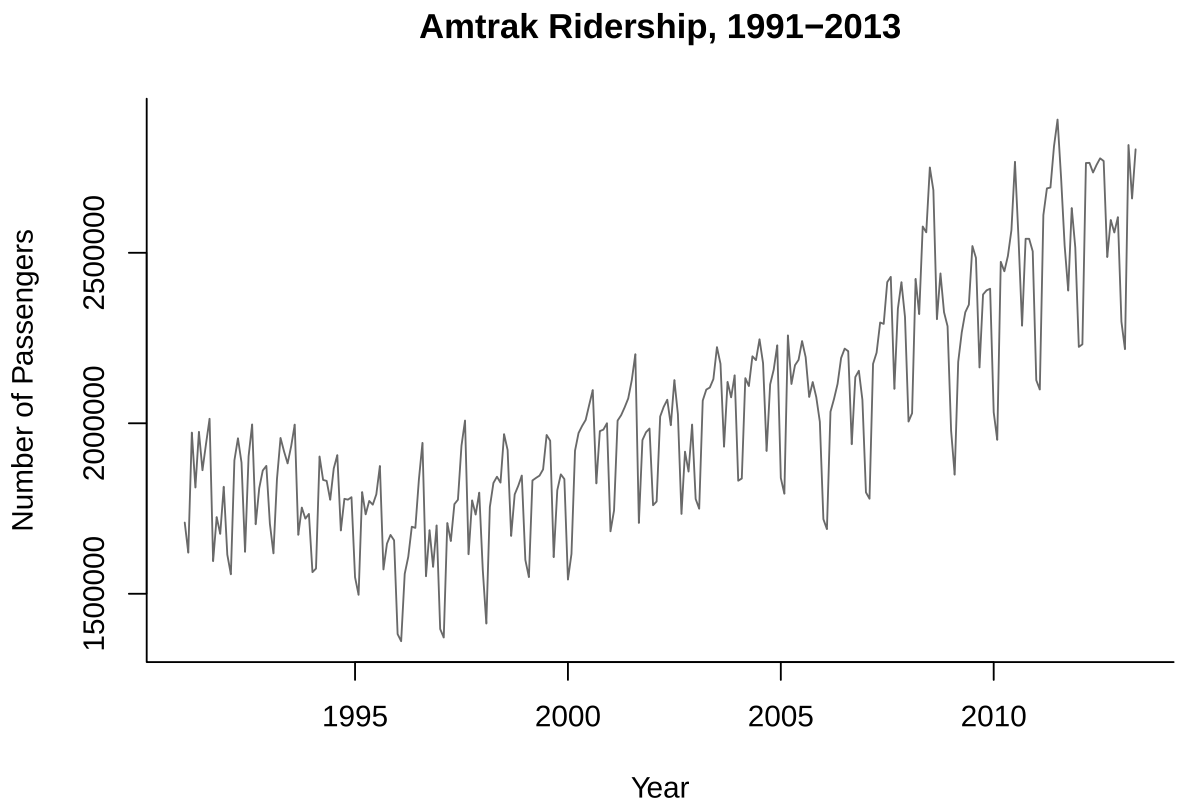
```
## [1] "Final Year and Month: "
```

```
end(df)
```

```
## [1] 2013    5
```

Code:

```
# Make a quick time-series plot
plot(df, xlab = "Year", ylab = "Number of Passengers",
     bty = "l", col = "grey41",
     main = "Amtrak Ridership, 1991-2013")
```



Analysis:

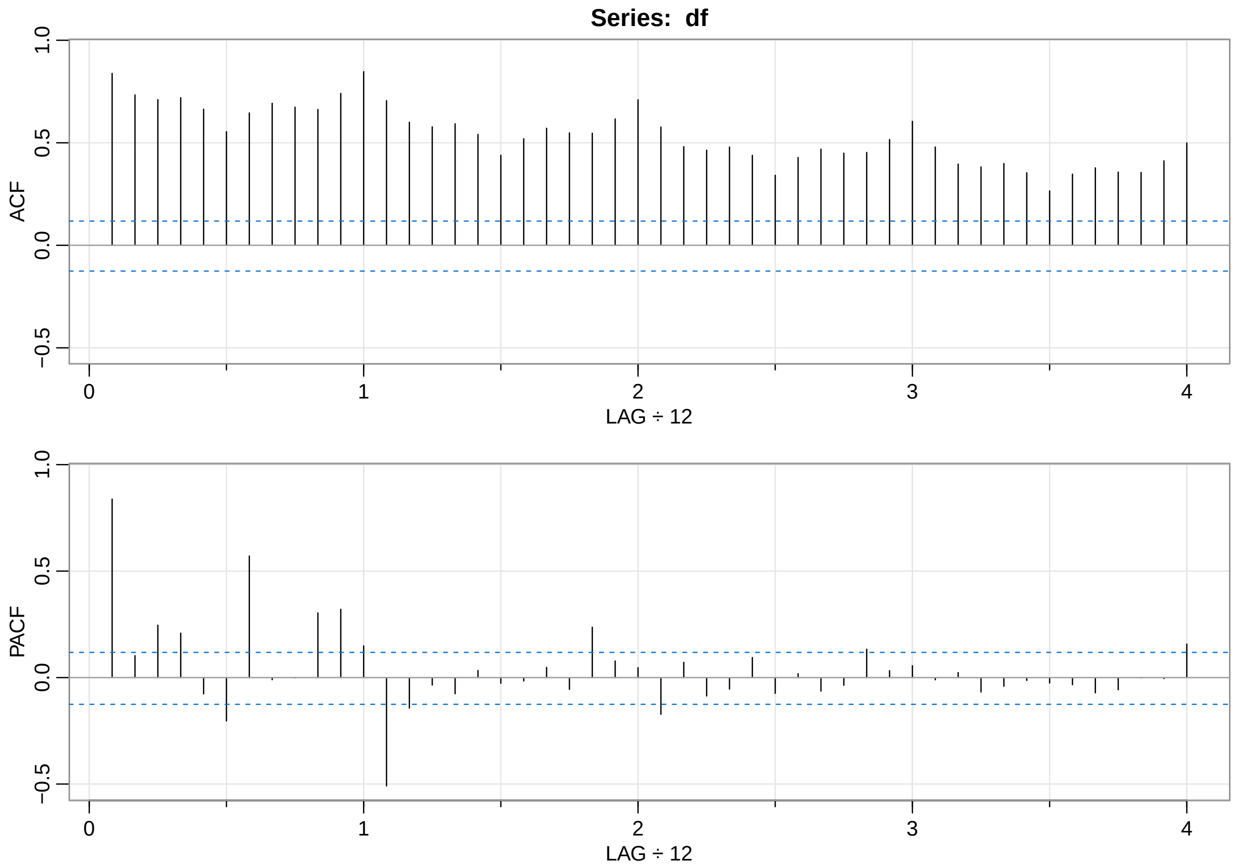
During exploratory data analysis, we were able to see that the trend of the data changed over time. It had signs of seasonality, showing it was not stationary.

ACF and PCF

Let's take a look at the auto-correlation and partial-correlation graphs for this time series data.

Code:

```
#Inspect ACF and PACF
acf_pcf <-acf2(df)
```




```
acf_pcf
```

```
##      [,1] [,2] [,3] [,4]  [,5]  [,6] [,7]  [,8] [,9] [,10] [,11] [,12] [,13]
## ACF  0.84 0.73 0.71 0.72  0.66  0.55 0.65  0.69 0.67  0.66  0.74  0.85  0.71
## PACF 0.84 0.10 0.25 0.21 -0.08 -0.20 0.57 -0.01 0.00  0.30  0.32  0.15 -0.51
##      [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25]
## ACF   0.60  0.58  0.59  0.54  0.44  0.52  0.57  0.55  0.55  0.62  0.71  0.58
## PACF -0.14 -0.04 -0.08  0.03 -0.03 -0.02  0.05 -0.06  0.24  0.08  0.05 -0.17
##      [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37]
## ACF   0.48  0.46  0.48  0.44  0.34  0.43  0.47  0.45  0.45  0.52  0.60  0.48
## PACF  0.07 -0.09 -0.05  0.09 -0.07  0.02 -0.06 -0.04  0.13  0.03  0.06 -0.01
##      [,38] [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48]
## ACF   0.40  0.38  0.40  0.35  0.27  0.35  0.38  0.36  0.36  0.41  0.50
## PACF  0.02 -0.07 -0.04 -0.01 -0.03 -0.03 -0.07 -0.06  0.00  0.00  0.16
```

Analysis:

The autocorrelation function shows high lag coefficients starting at lag 1, indicating seasonality.

Data Pre-processing

Smoothing Methods (Moving Average)

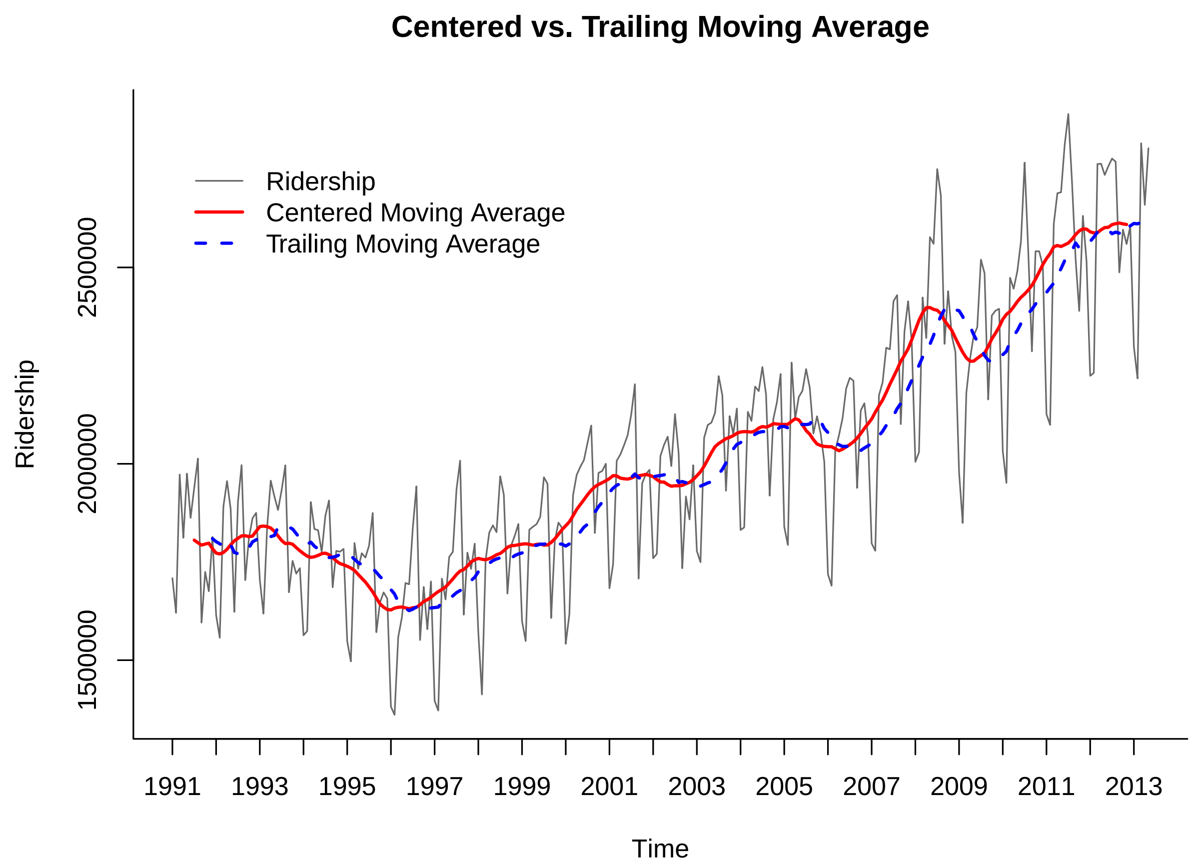
At the beginning of our data exploration, we were able to see there are components that change over time. Thus, we will need to look into data-driven methods because it deals data without a predetermined structure.

Moving Average

- This method is a simple smoother, it contains the average values across a time window, w, specified by the user. Two types of moving averages:
- a centered-moving average: useful for visualizing trends since averaging can suppress seasonality and noise
- a trailing moving average: useful for forecasting

Code:

```
# Trailing Average
ma.trailing <- rollmean(df, k = 12, align = "right")

# Centered-Average
ma.centered <- ma(df, order = 12)

# Original Data
plot(df, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", col = 'grey41',
     main = "Centered vs. Trailing Moving Average")

# Labels
axis(1, at = seq(1991, 2013.50, 1), labels = format(seq(1991, 2013.50, 1)))

# Centered average lines
lines(ma.centered, lwd = 2, col = 'red')

# trailing moving average
lines(ma.trailing, lwd = 2, lty = 2, col ='blue')

# legend
legend(1991,2800000, c("Ridership","Centered Moving Average",
                       "Trailing Moving Average"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("grey41", 'red', 'blue'))
```



Analysis:

Since the goal is to suppress seasonality in the data to visualize the trend, we should choose the length of a seasonal cycle. *The Amtrak ridership data indicates a choice of* $w = 12$.

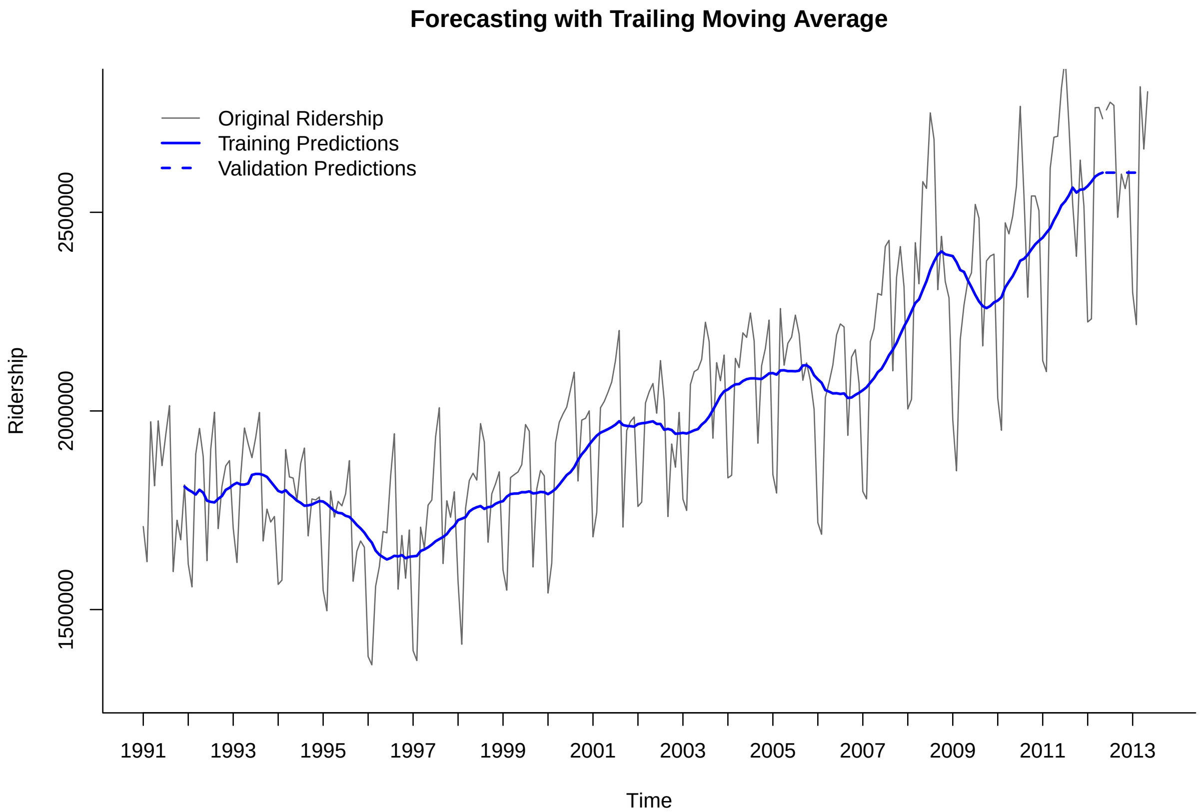
- This figure shows somewhat of a global U-shape, but the moving average looks to increase as the year passes.
- However, since centered moving averages uses data both in the past and future of a given time point, they cannot be used for forecasting because the future is typically unknown.

Trailing Moving Average

Therefore, trailing moving averages is the better approach here where the window of width is placed over the most recent available values.

```r
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))
```

```r
# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

# trailing moving average
ma.trailing <- rollmean(train.ts, k = 12, align = "right")

# last trailing moving average
last.ma <- tail(ma.trailing, 1)

# prediction by trailing moving average
ma.trailing.pred <- ts(rep(last.ma, nValid), start = c(1991, nTrain + 1),
end = c(1991, nTrain + nValid), freq = 12)

# plot training data
plot(train.ts, ylim = c(1300000, 2800000), ylab = "Ridership",
     xlab = "Time", bty="l", xaxt = "n", col = 'grey41',
xlim = c(1991,2013.50), main = "Forecasting with Trailing Moving Average")

# labels
axis(1, at = seq(1991, 2013.50, 1), labels = format(seq(1991, 2013.50, 1)))

# training model
lines(ma.trailing, lwd = 2, col = "blue")

# validation data
lines(valid.ts, col = 'grey41')

# predictions on validation
lines(ma.trailing.pred, lwd = 2, col = "blue", lty = 2)

# legend
legend(1991,2800000, c("Original Ridership","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("grey41", 'blue', 'blue'))
```



Analysis:

- The first thing to notice is that the forecasts for all the months in the validation period denoted in blue and blue dashes are identical because this method is not roll-forward next month forecasts. It is clear that the trailing moving average forecaster is inadequate for the Amtrak monthly forecast task. The reason why is because it does not capture the seasonality in the data. The forecaster predicted seasons with high ridership with lower ridership and seasons with low ridership with high ridership. This occurs because the moving average lags behind when forecasting a time series with a trend. Therefore, over-forecasting and under-forecasting in the presence of increasing and decreasing trends. So, between the smoothing methods of moving averages, it should only be use for forecasting when a series lack seasonality and trend, which is not true here for the Amtrak ridership data.
- However, there are other approaches for removing trends and seasonality, such as regression models or differencing.
- Then we can use the moving average to forecast a de-trended and de-seasonalized series.

Differencing

Differencing is a popular method for removing trend or seasonality patterns by taking the difference between two values in a series.

- For example, the lag-1 difference takes the difference between every two consecutive values $(y_t - y_{t-1})$.
- Meanwhile, differencing at lag-k means to subtract the value from k-periods back $(y_t - y_{t-k})$.

Dickey-Fuller Test

However, before using differencing as a pre-processing step, we should run a Dickey-Fuller test to see if differencing is actually needed. In other words, this test also check if the time series is stationary or not.

Code:

```
# Running the Dickey-Fuller Test on the original data without any pre-processing
adf.test(df)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  df
## Dickey-Fuller = -3.8062, Lag order = 6, p-value = 0.01915
## alternative hypothesis: stationary
```

Analysis:

This may be a biased Dickey-Fuller test where we have a type 1 error. There is definitely visible seasonality happening in the data.
Since the test rejects the null hypothesis that the series is non-stationary, in this case the series was actually non-stationary. Therefore, we will ignore that it ever happened because we will need to perform a second order difference due to a trend and seasonality pattern in the series.

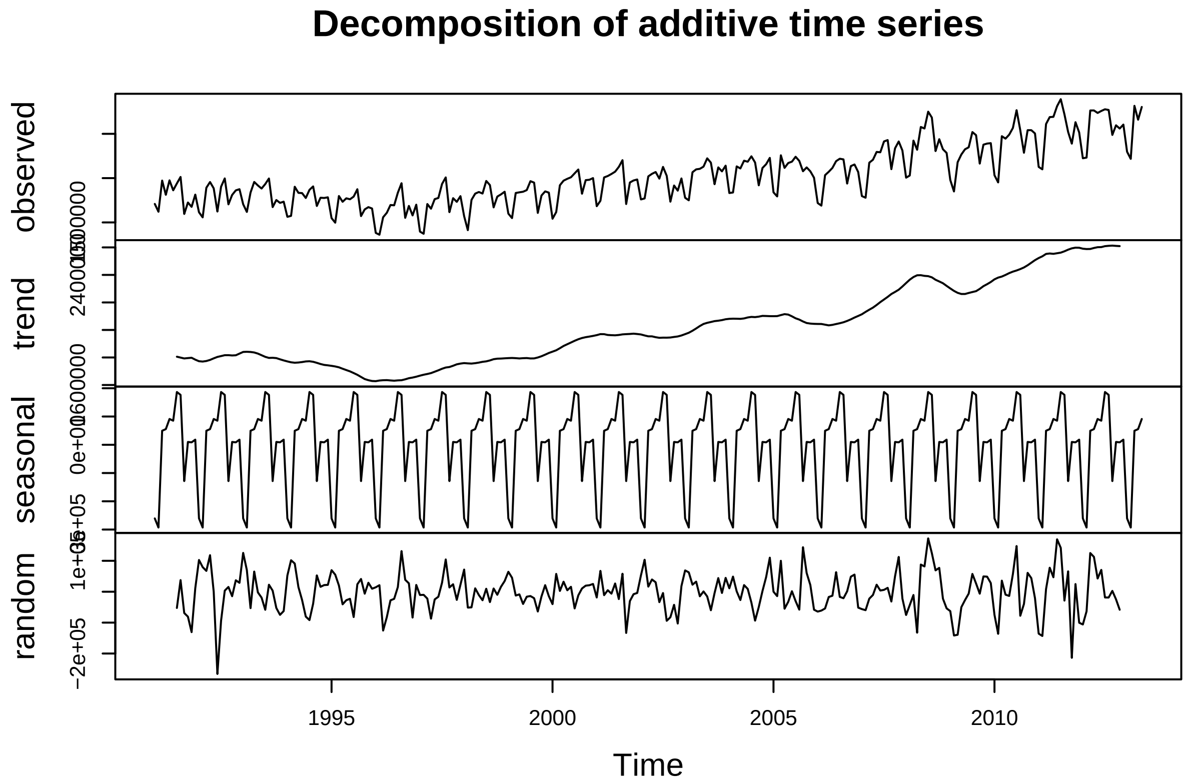
- Alternative approaches without differencing can be aggregating the monthly data into a coarser level such as yearly ridership as the total sum instead.

Other tests for Stationarity

Once again, we can visually inspect the data by decomposing it into different time series components, such as the seasonality and trend.

Code:

```
# decompose the components of the data set
d1 <-decompose(df, type = c("additive","multiplicative"))
plot(d1)
```



Analysis:

There is a strong seasonal component in the data set, there is a consistent upward trend in the dataset.

Detrending

Detrending can be used by the lag-1 difference of a series. This would remove the somewhat U-shape of Amtrak ridership series. An advantage of difference is there are no assumptions that the trend is global.

- For quadratic or exponential trends, one more step of lag-1 differencing must be applied to remove the trend.

Deseasonalizing

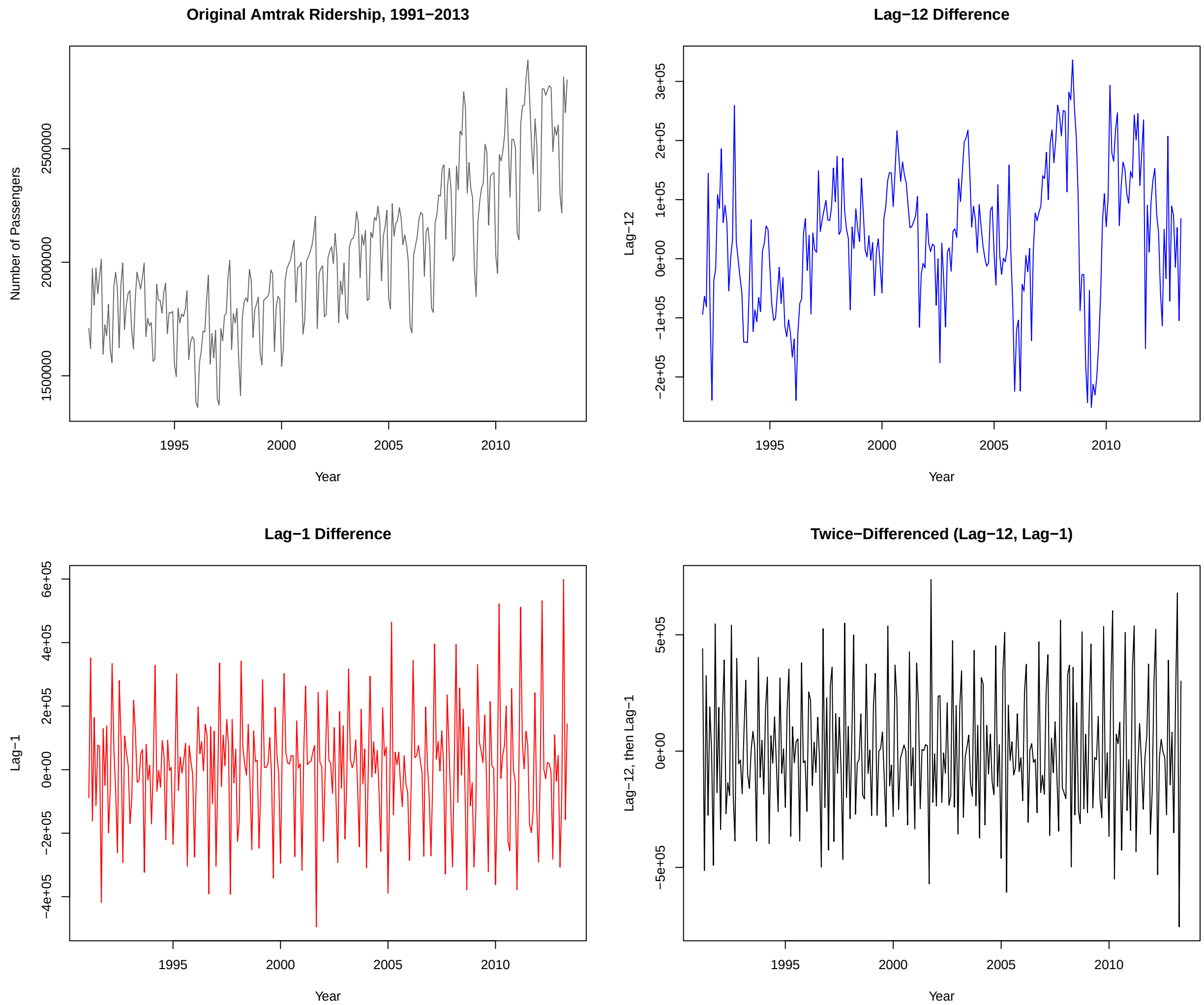
We can remove the seasonality of the Amtrak ridership data by using a lag-12 difference series. THis will remove the monthly pattern

Removing seasonality and trend

- When both of these components exist, we can apply differencing twice to the series.
- Since the Amtrak ridership data has both trend and seasonality, we will perform the doube differencing method in order to de-trend and deseasonalize it.

Code:

```r
par(mfrow=c(2,2))

# Original
plot(df, xlab = "Year", ylab = "Number of Passengers",
     main = "Original Amtrak Ridership, 1991-2013", col = 'grey41')

# lag-12 difference
plot(diff(df, lag = 12), xlab = "Year", ylab = "Lag-12",
     main = "Lag-12 Difference", col = 'blue')

# lag-1 difference
plot(diff(df), xlab = "Year", ylab = "Lag-1",
     main = "Lag-1 Difference", col = 'red')


# Double Differencing
plot(diff(diff(df, s = 12)), xlab = "Year", ylab = "Lag-12, then Lag-1",
     main = "Twice-Differenced (Lag-12, Lag-1)", col = 1)
```



Analysis:

- The lag-1 difference plot on the bottom left contains no visible trend compared to the original series above.
- If we are dealing with daily data ridership, then we could remove a seasonal pattern of lag-7 differences. However, since we are have monthly data, we are using a lag-12 difference series as shown in the top right figure where the monthly pattern is absent.
- Lastly, since there are both a seasonality and trend, the double differencing effect on the bottom right panel is a series without trend or monthly seasonality.

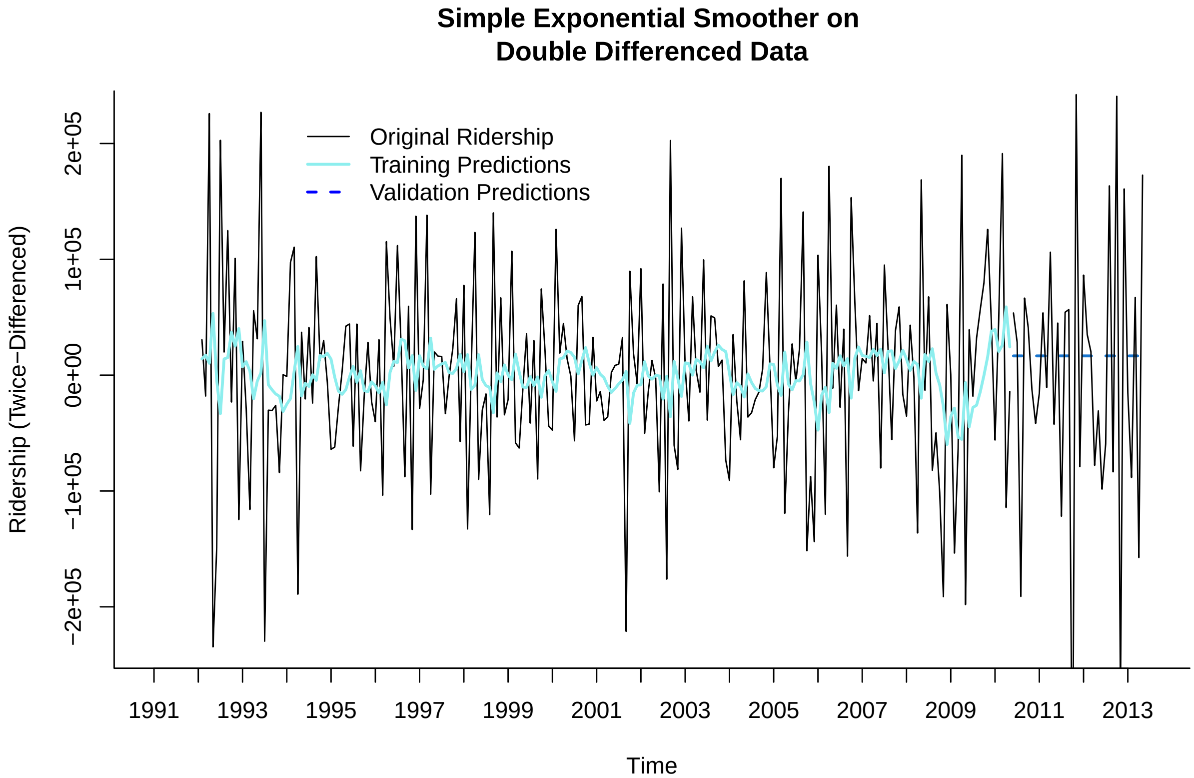
Data Modeling

Simple Exponential Smoothing

Exponential smoothing works very similar to forecasting with a moving average, except it takes the weighted average over all the past values of a series. By doing so, the weights will decrease exponentially into the past. This is valuable because we give weight to recent information more than the older information. This method is also very popular due to its low computation costs, easy automation, and good performance. However, it is important to note that using exponential smoothing for forecasting assumes no trend or seasonality in a series. So the idea is similar to before with moving averages, by first removing the trend and seasonality, then apply the exponential smoothing forecaster.

Code:

```
# remove trend and seasonality by doing double-differencing
diff_twice <- diff(diff(df, lag = 12), lag = 1)

# Number of validation data
nValid <- 36

# number of training data
nTrain <- length(diff_twice) - nValid

# specified time window for training data
train.ts <- window(diff_twice, start = c(1992, 2), end = c(1992, nTrain + 1))

# specified time window for validation data
valid.ts <- window(diff_twice, start = c(1992, nTrain + 2),
                   end = c(1992, nTrain + 1 + nValid))

# Additive, no trend, no seasonality model using a constant (learning rate) of 0.2
ses <- ets(train.ts, model = "ANN", alpha = 0.2)

# make predictions using model
ses.pred <- forecast(ses, h = nValid, level = 0)

# training data then validation forecasts
plot(ses.pred, ylab = "Ridership (Twice-Differenced)", xlab = "Time",
bty = "l", xaxt = "n", xlim = c(1991,2013.50), main = "Simple Exponential Smoother on
Double Differenced Data", flty = 2)

# labels
axis(1, at = seq(1991, 2013, 1), labels = format(seq(1991, 2013, 1)))

# training model - predictions
lines(ses.pred$fitted, lwd = 2, col = "darkslategray2")

# validaiton data
lines(valid.ts)

# legend
legend(1994,230000, c("Original Ridership","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'blue'))
```



Analysis:

For forecasting with simple exponential smoothing, the data trained on was the double-difference ridership data that contains no seasonality or trend. Then we fit the simple exponential smoothing model to the training data set with $\alpha = 0.2$ as default. This smoothing model is under the *ets* framework using the model with additive (A), no trend (N), and no seasonality (N), denoted as "ANN". The forecasts for the validation set for each month remained as the same value similar to the moving average forecast model. This would mean that the simple exponential forecaster or ANN model is also inadequate for the monthly forecasting task. The reason why is because it also does not capture the seasonality in the data.

- The simple exponential smoothing models did a poor job at forecasting this time series without trend or seasonality.
- Another solution is to use a more complex and sophisticated exponential smoothing that is able to model data with both trend and seasonality.

Additive Trends

Double exponential smoothing can be used on a series that contain an additive trend. This is also called the Holt's linear trend model. The local trend is estimated and is updated as more data comes in. The equation is specified by $F_{t+k} = L_t + kT_t$, where the k-step-ahead forecast is a combination of the level estimate at time t $L(t)$ and the trend estimate at time t (Tt). There is also two smoothing constants α and β which determine the rate of learning and can be constants between 0 and 1 set by the user (Higher values = faster learning). However, there are two types of errors in an exponential model:

- Additive error (additive trend): This is where errors are assumed to have a fixed magnitude, meaning the forecasts contain not only the level + trend but also an additional error. $y_{t+1} = L_t + T_t + e_t$
- Multiplicative error (additive trend): This is where the size of the error grows as the level of the series increase, or the error is a percentage increase in the current level plus trend. $y_{t+1} = (L_t + T_t) * (1 + e_t)$

Multiplicative trends

Although additive trend models assumes the level changes from one period to the next by a fixed amount, multiplicative trends assumes it changes by a factor instead. Thus, the formula is different specified by: $F_{t+k} = L_t * T_t^k$

- For the additive error, it will be $y_{t+1} = L_t * T_t + e_t$ for the multiplicative trend model instead.
- While the multiplicative error stays the same as $y_{t+1} = (L_t + T_t) * (1 + e_t)$

Exponential smoothing with both a trend and seasonality

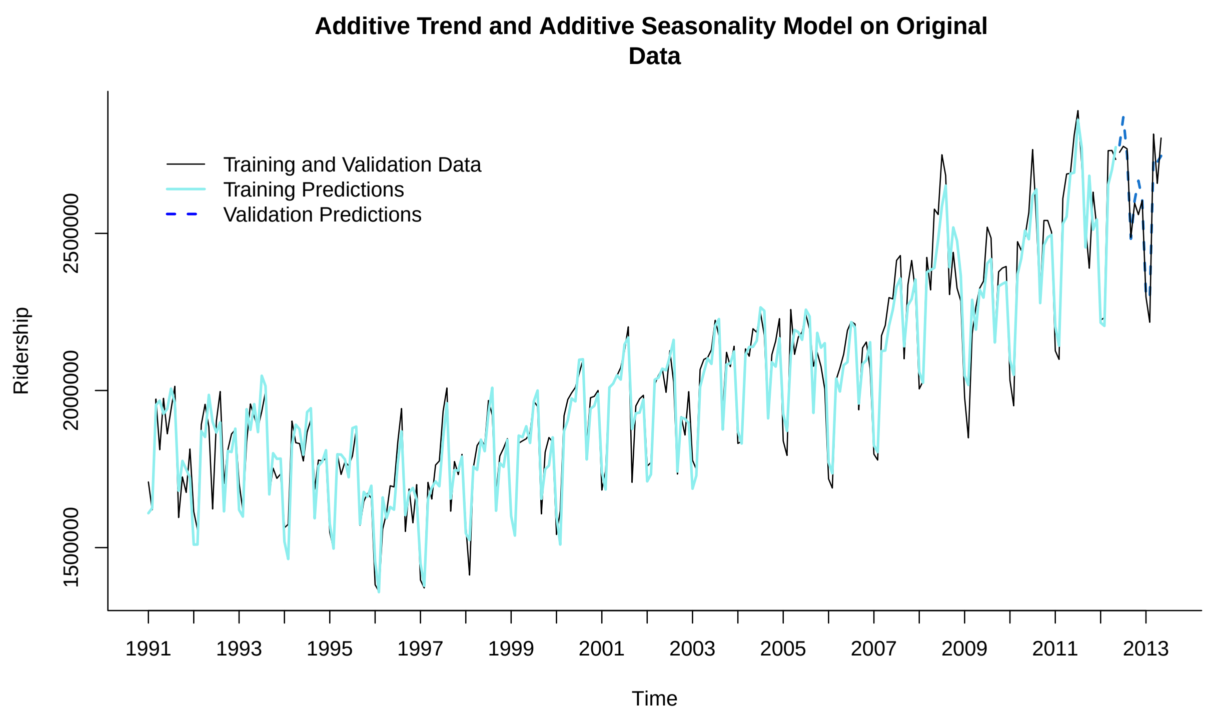
A further extension of the double exponential smoothing where the k-step-ahead forecasts also takes into consideration the seasonality of the current period. While the trend is considered from the additive and multiplicative seasonality. - This is specified as: $F_{t+k} = (L_t + kT_t)S_{t+k-M}$ where M denotes the number of seasons in a series (e.g., M = 12 for monthly seasonality). This is an adaptive method that allows the components (levels, trends, and seasonality) to change over time.

In this case we would make **June 2012 to May 2013** the validation period.

Code:

```
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

# multiplicative error with additive trend and additive seasonality model
maa <- ets(train.ts, model = "MAA")

# Forecasts
maa.pred <- forecast(maa, h = nValid, level = 0)
```

```
# training data and validation data - forecasts
plot(maa.pred, ylab = "Ridership", xlab = "Time",
main = "Additive Trend and Additive Seasonality Model on Original
Data", flty = 2, bty="l", xaxt = "n")

# labels
axis(1, at = seq(1991, 2013, 1), labels = format(seq(1991, 2013, 1)))

# training data - forecasts
lines(maa.pred$fitted, lwd = 2, col = "darkslategray2")

# validation data only
lines(valid.ts)

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                        "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'blue'))
```



```
## ETS(M,Ad,A)
##
## Call:
##  ets(y = train.ts, model = "MAA")
##
##   Smoothing parameters:
##     alpha = 0.5146
##     beta  = 1e-04
##     gamma = 0.2007
##     phi   = 0.8692
##
##   Initial states:
##     l = 1868559.4037
##     b = -3079.4402
##     s = 23126.2 11233.09 11079.87 -128642.4 177366.8 187591.1
##            81511.92 89042.49 49616.47 43289.88 -288877.1 -256338.3
##
##   sigma:  0.0357
##
##      AIC     AICc      BIC
## 7181.096 7183.970 7244.979
```

Analysis:

This approach uses Holt-Winter's method to build an additive trend and seasonality model with multiplicative error on the original data without a second order difference. This falls under the ets framework using the "MAA" model. However, what if the choice was to use an multiplicative trend and seasonality model, and so forth. There is an automated approach for model selection. In this case we would make **June 2012 to May 2013** the validation period.

Automated Exponential Model Sections

By leaving out the model parameter inside the ets function, this will fit several different models and choose the best one based on the lowest AIC score.

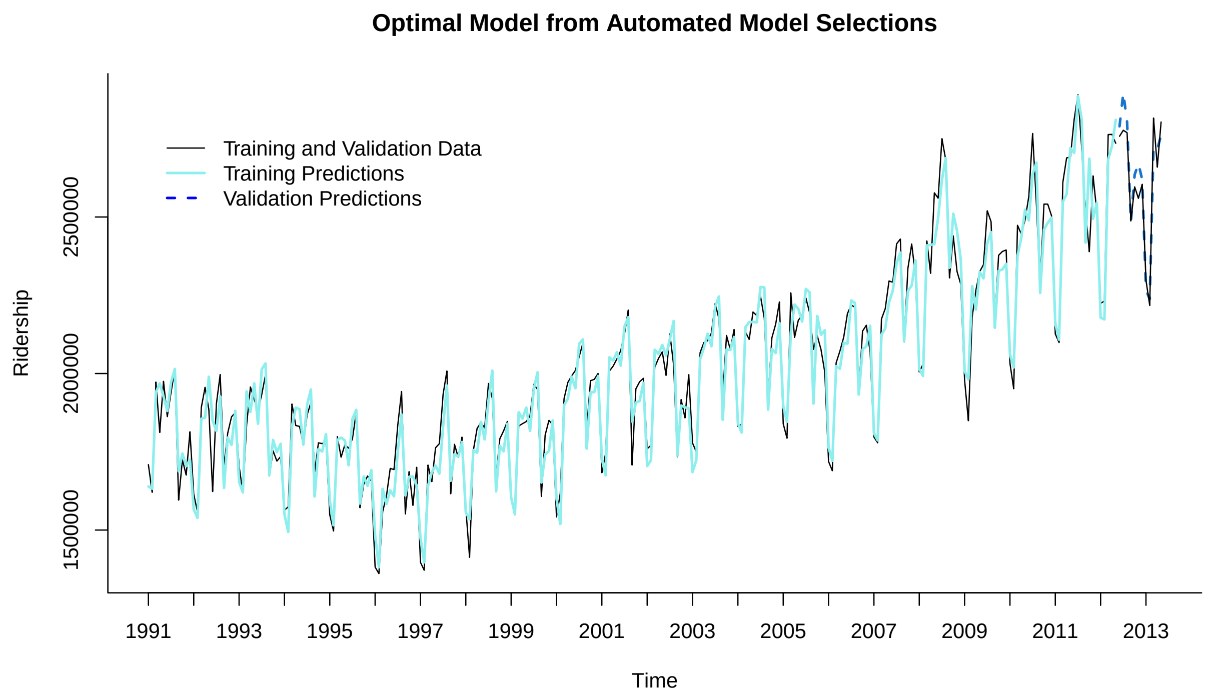
Code:

```r
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

#automated selection - optimal model
optimal <- ets(train.ts, restrict = FALSE, allow.multiplicative.trend = TRUE)
optimal
```

```
## ETS(M,Ad,M)
##
## Call:
##  ets(y = train.ts, restrict = FALSE, allow.multiplicative.trend = TRUE)
##
##   Smoothing parameters:
##     alpha = 0.5545
##     beta  = 6e-04
##     gamma = 0.1467
##     phi   = 0.98
##
##   Initial states:
##     l = 1860206.6141
##     b = -2853.0529
##     s = 1.0049 0.9822 0.9979 0.9348 1.1141 1.0792
##            1.0222 1.0606 1.0344 1.0276 0.8596 0.8826
##
##   sigma:  0.0339
##
##      AIC     AICc      BIC
## 7155.162 7158.036 7219.045
```

```r
# Optimal Forecasts
op.pred <- forecast(optimal, h = nValid, level = 0)

# training data - then validation forecasts
plot(op.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n",
main = "Optimal Model from Automated Model Selections", flty = 2)

# labels
```

```
axis(1, at = seq(1991, 2013, 1), labels = format(seq(1991, 2013, 1)))

# training data - forecasts
lines(op.pred$fitted, lwd = 2, col = "darkslategray2")

# validation data
lines(valid.ts)

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                        "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'blue'))
```



Analysis:

The optimal model chosen was a multiplicative error, additive trend, and multiplicative seasonality with an improved AIC score of 7155.16 which was lower than the previous ANN model with an AIC of 7181.09

Regression-Based Models

There are different types of common trends and seasonality that can be modeled by regression-based models estimated during the training period and used to forecast on future data. Such trends include linear, exponential, or polynomial, while the different seasonality are additive and multiplicative seasons.

- Linear trends = a series is increasing/decreasing linearly over time
- Quadratic/polynomials functions = These are used to capture the more complex trends.

Linear Trend

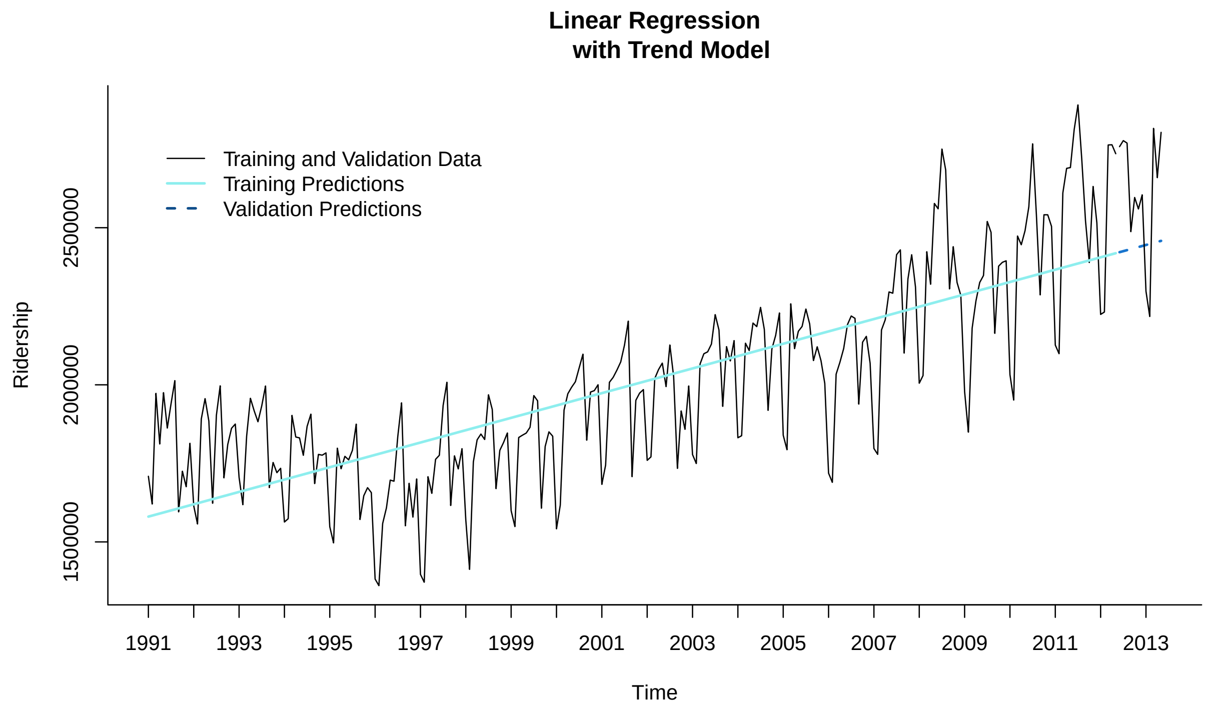
We can start with creating a linear regression model using a predictor index by time and the output, y, which is the number of ridership each month. Keep in mind that a linear regression model will capture the global linear trend in a time series.

However, before we move on we will need to partition the series into training and validation periods. In this case we would make **June 2012 to May 2013** the validation period. This will allow us to keep 12 months in the validation set to provide monthly forecasts for the following year and evaluate each forecasts individually from their respective years.

Code:

```
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

# trend of data
trend <- time(df)

# linear regression model
train.lm <- tslm(train.ts ~ trend)


# validation forecasts
train.lm.pred <- forecast(train.lm, h = nValid, level = 0)

# Plot training data along with validation forecasts
plot(train.lm.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", main = "Linear Regression
     with Trend Model", flty = 2)

# labels
axis(1, at = seq(1991, 2013.9, 1), labels = format(seq(1991, 2013.9, 1)))

# validation data
lines(valid.ts)
```

```r
# training model forecasts
lines(train.lm.pred$fitted, lwd = 2, col = "darkslategray2")

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'dodgerblue4'))
```




```r
summary(train.lm)
```

```
##
## Call:
## tslm(formula = train.ts ~ trend)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -483625 -116651   24726  119568  504688
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1577216.9    24624.3   64.05   <2e-16 ***
## trend          3274.7      165.5   19.79   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 196800 on 255 degrees of freedom
```

```
## Multiple R-squared:  0.6057, Adjusted R-squared:  0.6041
## F-statistic: 391.6 on 1 and 255 DF,  p-value: < 2.2e-16
```

```r
print(paste0("AIC: ", round(AIC(train.lm),2)))
```

```
## [1] "AIC: 6998.97"
```

Analysis:

This linear trend does not fit the global trend at all. The global trend in fact is not linear. This is why we will consider other models for this series. Also the reason why this linear trend model performed poorly is because we did not model seasonality.

Model with Trend and Seasonality

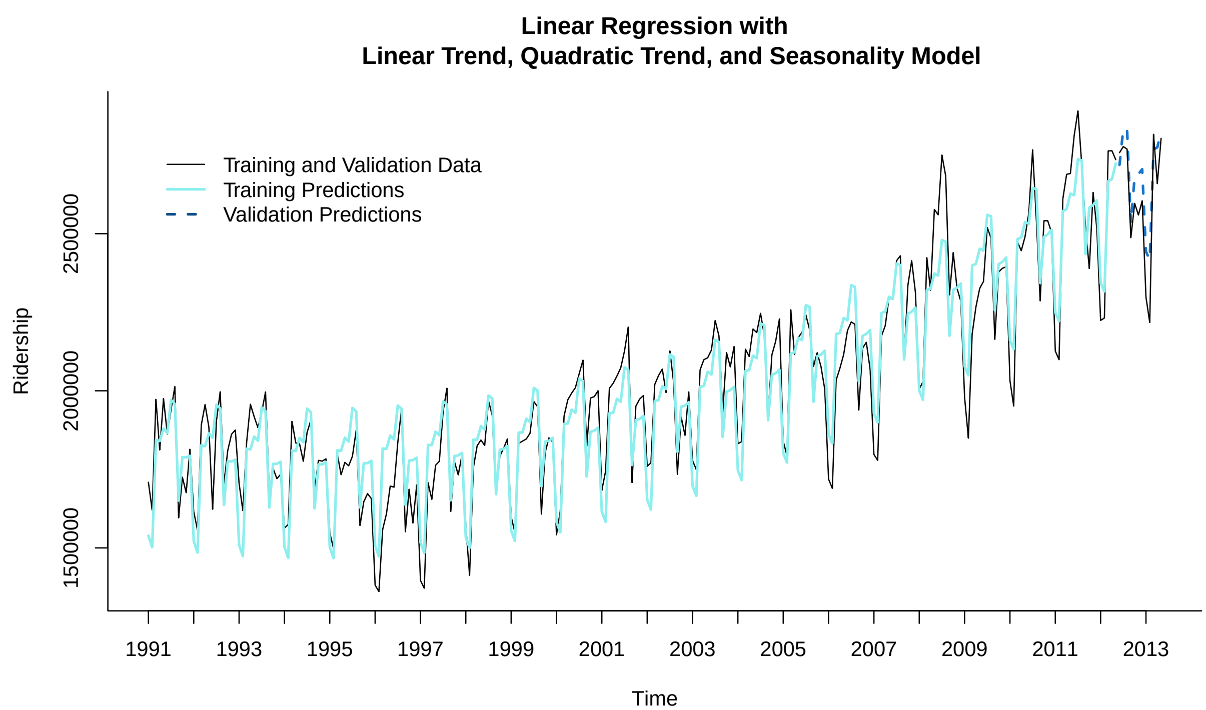
We can model a series with both trend and seasonality by adding predictors of both types. For example, we can build a new model with both linear and quadratic trend. Then we can add a monthly seasonality which was a factor of 12, however it is redundant to use all 12. In this case, except season 1 which was January. In total, we will have 13 predictors, with $tandt^2$ for the trends and the 11 dummy variables for months.

Code:

```r
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

# trend of data
trend <- time(df)

# seasonality
season <- factor(cycle(df))

# linear model with linear trend, quadratic trend and seasonality
model.lm <- tslm(train.ts ~ trend + I(trend^2) +  season)
model.lm
```

```
##
## Call:
## tslm(formula = train.ts ~ trend + I(trend^2) + season)
##
## Coefficients:
## (Intercept)        trend   I(trend^2)      season2      season3      season4
##  1540670.38     -1732.47        19.39    -35131.64    307758.22    305931.76
##     season5      season6      season7      season8      season9     season10
##   347646.93    334705.28    441363.24    430168.00    123687.04    263579.06
##    season11     season12
##   263809.12    270626.31
```

```r
# validation forecasts
model.lm.pred <- forecast(model.lm, h = nValid, level = 0)

# Plot training data along with validation forecasts
plot(model.lm.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", flty = 2,
     main = "Linear Regression with
     Linear Trend, Quadratic Trend, and Seasonality Model")

# labels
axis(1, at = seq(1991, 2013.9, 1), labels = format(seq(1991, 2013.9, 1)))

# validation data
lines(valid.ts)

# training model forecasts
lines(model.lm.pred$fitted, lwd = 2, col = "darkslategray2")

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'dodgerblue4'))
```




```r
# summary of model
summary(model.lm)
```

```
##
```

```
## Call:
## tslm(formula = train.ts ~ trend + I(trend^2) + season)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -257653  -64600    1511   67372  270695
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.541e+06  2.630e+04  58.575  < 2e-16 ***
## trend       -1.732e+03  3.229e+02  -5.365 1.89e-07 ***
## I(trend^2)   1.939e+01  1.212e+00  15.991  < 2e-16 ***
## season2     -3.513e+04  2.881e+04  -1.220    0.224
## season3      3.078e+05  2.881e+04  10.683  < 2e-16 ***
## season4      3.059e+05  2.881e+04  10.619  < 2e-16 ***
## season5      3.476e+05  2.881e+04  12.067  < 2e-16 ***
## season6      3.347e+05  2.916e+04  11.480  < 2e-16 ***
## season7      4.414e+05  2.916e+04  15.138  < 2e-16 ***
## season8      4.302e+05  2.916e+04  14.754  < 2e-16 ***
## season9      1.237e+05  2.916e+04   4.242 3.15e-05 ***
## season10     2.636e+05  2.916e+04   9.040  < 2e-16 ***
## season11     2.638e+05  2.916e+04   9.048  < 2e-16 ***
## season12     2.706e+05  2.916e+04   9.281  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 95550 on 243 degrees of freedom
## Multiple R-squared:  0.9114, Adjusted R-squared:  0.9067
## F-statistic: 192.4 on 13 and 243 DF,  p-value: < 2.2e-16
```

```
print(paste0("AIC: ", round(AIC(model.lm),2)))
```

```
## [1] "AIC: 6639.16"
```

```
accuracy(model.lm.pred, valid.ts)
```

```
##                         ME      RMSE      MAE        MPE     MAPE      MASE
## Training set  1.806244e-12  92906.36 76159.61 -0.2116348 3.915444 0.7704515
## Test set     -7.274081e+04 100660.82 87295.84 -2.9639564 3.485683 0.8831086
##                    ACF1 Theil's U
## Training set  0.6770800        NA
## Test set     -0.1252874  0.424056
```

Analysis:

The AIC score has dropped significantly compared to the model with only linear trend. In comparison with the exponential smoothing model, this score has dropped drastically, making the linear, quadratic, and monthly seasonality model the best.

AR1 Model

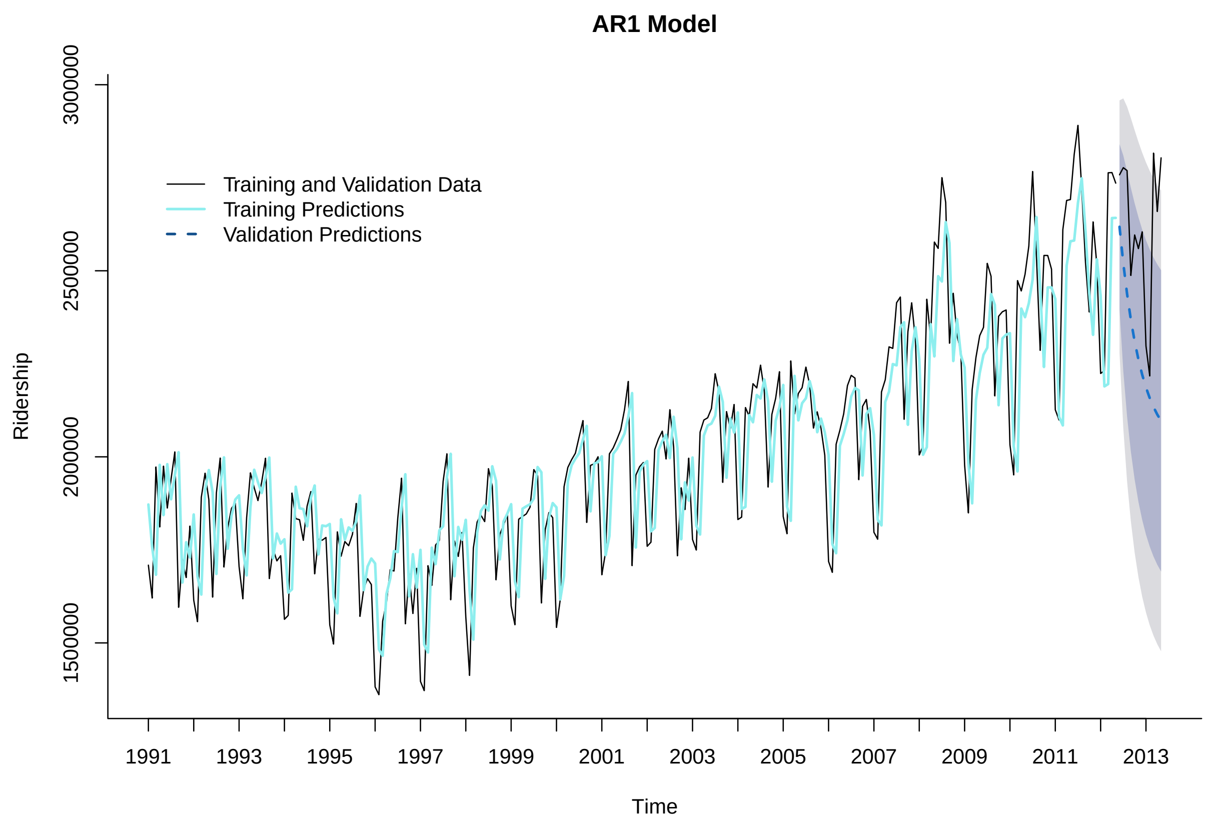
The following is an Auto-regression model

Code:

```r
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

#create basic AR model
ar1 = arima(train.ts, order=c(1,0,0))
ar1
```

```
##
## Call:
## arima(x = train.ts, order = c(1, 0, 0))
##
## Coefficients:
##          ar1   intercept
##       0.8388  2008415.12
## s.e.  0.0346    65740.01
##
## sigma^2 estimated as 3.002e+10:  log likelihood = -3465.35,  aic = 6936.7
```

```r
#forecast ar1 model
ar1.pred <- forecast(ar1, h = nValid)

# Plot training data along with validation forecasts
plot(ar1.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", flty = 2,
     main = "AR1 Model")

# labels
axis(1, at = seq(1991, 2013.9, 1), labels = format(seq(1991, 2013.9, 1)))

# validation data
lines(valid.ts)

# training model forecasts
lines(ar1.pred$fitted, lwd = 2, col = "darkslategray2")

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'dodgerblue4'))
```




```r
#summary
summary(ar1)
```

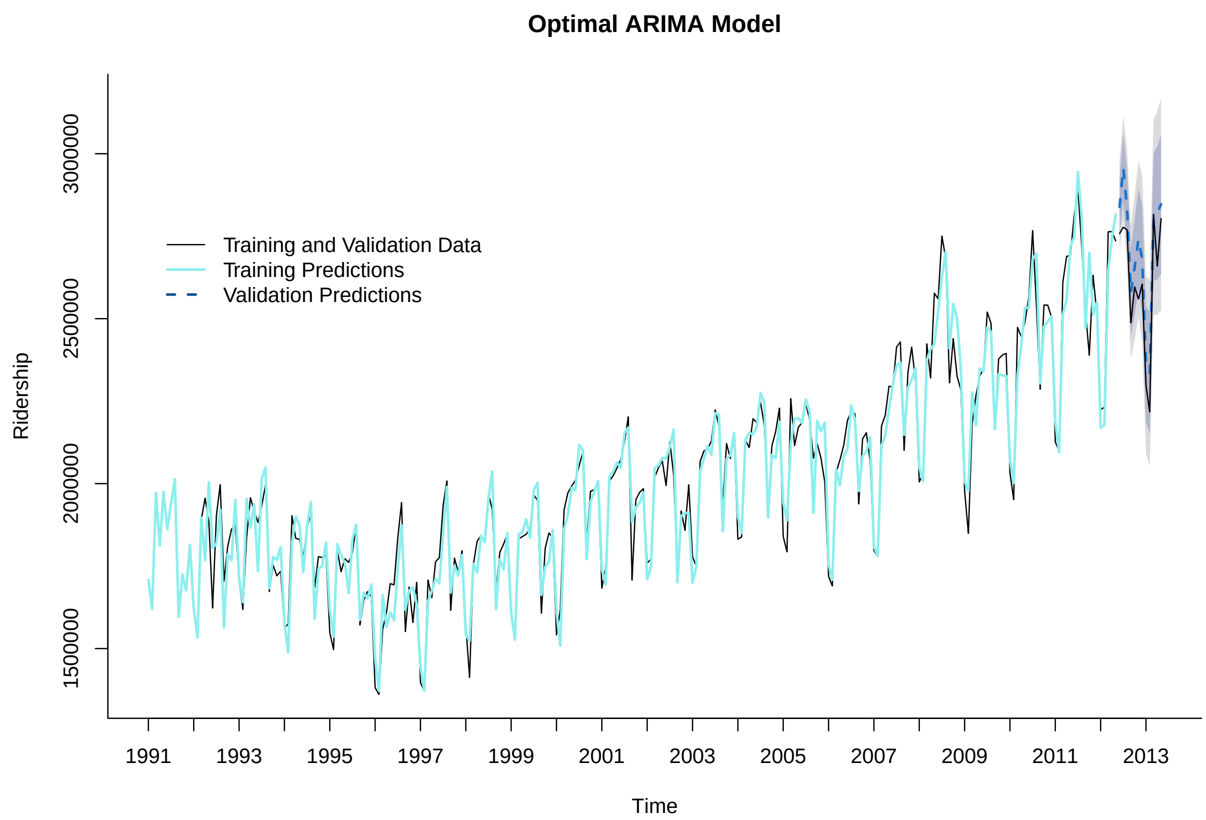
```
##
## Call:
## arima(x = train.ts, order = c(1, 0, 0))
##
## Coefficients:
##          ar1   intercept
##       0.8388  2008415.12
## s.e.  0.0346    65740.01
##
## sigma^2 estimated as 3.002e+10:  log likelihood = -3465.35,  aic = 6936.7
##
## Training set error measures:
##                    ME     RMSE      MAE        MPE     MAPE      MASE
## Training set 1492.036 173259.5 129640.3 -0.6899507 6.683191 0.9819261
##                    ACF1
## Training set -0.0747508
```

Autoselection for Optimal ARIMA Parameters

We used the auto-arima function to get the best parameters for an optimal ARIMA model

Code:

```
#auto arima
# Validation Data
nValid <- 12

# number of training data
nTrain <- length(df) - nValid

# time window for training data
train.ts <- window(df, start = c(1991, 1), end = c(1991, nTrain))

# time window for validation data
valid.ts <- window(df, start = c(1991, nTrain + 1), end = c(1991, nTrain + nValid))

# autoselection for optimal arima parameters
autoarima <- auto.arima(train.ts)

#forecast ar1 model
autoarima.pred <- forecast(autoarima, h = nValid)

# Plot training data along with validation forecasts
plot(autoarima.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", flty = 2,
     main = "Optimal ARIMA Model")

# labels
axis(1, at = seq(1991, 2013.9, 1), labels = format(seq(1991, 2013.9, 1)))

# validation data
lines(valid.ts)

# training model forecasts
lines(autoarima.pred$fitted, lwd = 2, col = "darkslategray2")

# legend
legend(1991,2800000, c("Training and Validation Data","Training Predictions",
                       "Validation Predictions"), lty=c(1,1,2),
lwd=c(1,2,2), bty = "n", col = c("black", 'darkslategray2', 'dodgerblue4'))
```




```r
#summary
summary(autoarima)
```

```
## Series: train.ts
## ARIMA(2,1,1)(2,1,1)[12]
##
## Coefficients:
##           ar1      ar2     ma1    sar1    sar2     sma1
##       -0.5596  -0.2037  0.1352  0.0763  0.0017  -0.7054
## s.e.   0.4954   0.1855  0.5050  0.1184  0.0952   0.0976
##
## sigma^2 estimated as 4.92e+09:  log likelihood=-3069.4
## AIC=6152.79   AICc=6153.27   BIC=6177.27
##
## Training set error measures:
##                    ME     RMSE      MAE        MPE     MAPE      MASE
## Training set 2825.372 67501.85 50835.07 0.06815184 2.555057 0.5142615
##                      ACF1
## Training set -0.005063544
```

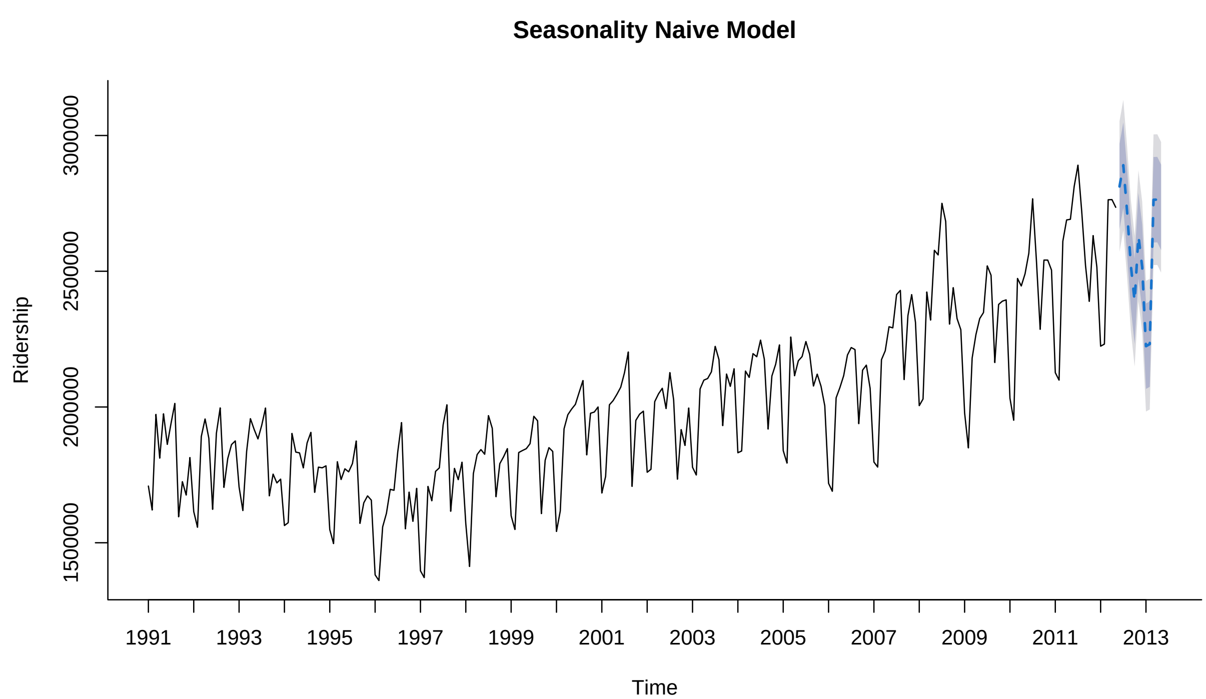
Naive Forecasts

Instead of sophisticated modeling, naive forecasts are basically the most recent values of the ridership data. While a seasonal naive forecast is from the recent value from the most identical season. For example, if we are forecasting August 2012, the forecasts for this month would be the value from August 2011 instead.

Ironically, naive forecasts are actually good at achieving great performance even though its easy to understand and deploy. Also, this naive model should be used as a baseline to compare other model's predictive performance against.

Code:

```
# Seasonality naive forecasts
snaive.pred <- snaive(train.ts, h = nValid)

# Plot training data along with validation forecasts
plot(snaive.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", flty = 2,
     main = "Seasonality Naive Model")

# labels
axis(1, at = seq(1991, 2013.9, 1), labels = format(seq(1991, 2013.9, 1)))
```



Results and Model Selection

Code:

Table 2: Optimal Exponential Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	6027	66103	52183	0.2028	2.624	0.5279
Test set	-21561	61488	49641	-0.824	1.866	0.5022

Table 3: Regression with Monthly Seasonality and Linear and Quadratic Trends Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	1.806e-12	92906	76160	-0.2116	3.915	0.7705
Test set	-72741	100661	87296	-2.964	3.486	0.8831

Table 4: AR1 Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	1492	173260	129640	-0.69	6.683	1.311
Test set	326770	388025	326770	12.16	12.16	3.306

Table 5: Optimal ARIMA Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	2825	67502	50835	0.06815	2.555	0.5143
Test set	-92458	107742	93867	-3.587	3.637	0.9496

Table 6: Seasonal Naive Model

	ME	RMSE	MAE	MPE	MAPE	MASE
Training set	38708	122794	98851	1.595	4.854	1
Test set	12386	91056	77756	0.4937	2.959	0.7866

Analysis:

The key performance metric is RMSE in the test set. Since this metric is on the same scale as the observed data values. For the seasonal naive model, its RMSE score will serve as the baseline performance to beat, meaning if any model has a lower RMSE value in the test set.

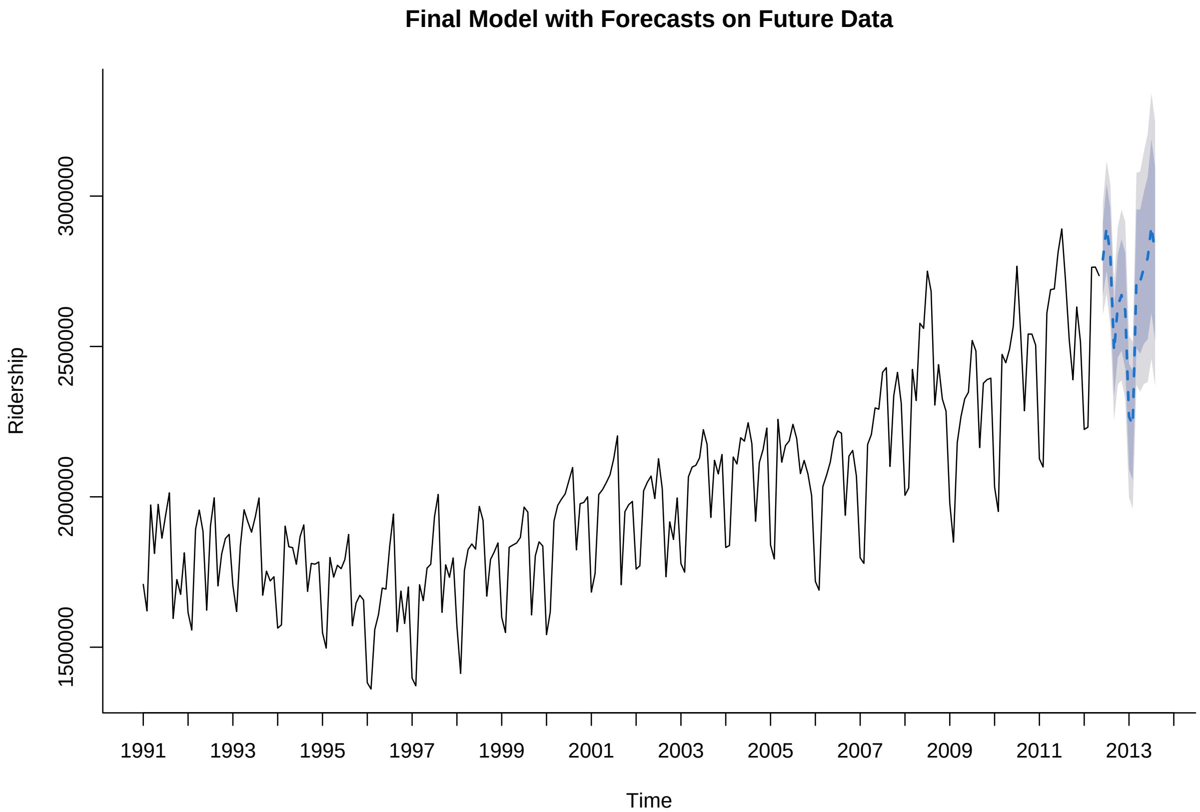
- Seasonal Naive Model - test set's RMSE: 91,056
- Optimal exponential model - test set's RMSE score: 61,488.
- Regression-based model - test set's RMSE score: 100,061
- AR1 Model - test's RMSE score: 388,025
- Optimal ARIMA Model - test's RMSE score: 107,742

The final model selected to forecast the months of June to August of 2013 will be the optimal exponential model with the lowest RMSE score of 61,488 on the test set.

Conclusion

Code:

```
# the final model selected was the optimal exponential model
final_model <- optimal

#forecasts for June to August 2013 (3 months ahead)
final.pred <- forecast(final_model, h = nValid + 3)

# plot of predictions
plot(final.pred, ylab = "Ridership", xlab = "Time",
     bty = "l", xaxt = "n", flty = 2,
     main = "Final Model with Forecasts on Future Data")

# labels
axis(1, at = seq(1991, 2014, 1), labels = format(seq(1991, 2014, 1)))
```



June to August 2013 Forecasts:

Table 7: June to August 2013 - Ridership Forecasts

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jun 2013	2793575	2523876	3063274	2381106	3206044
Jul 2013	2900579	2611706	3189452	2458786	3342372
Aug 2013	2807559	2519628	3095489	2367207	3247910