Homework 4

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November 14, 2019

(a)

$$J^{(t)}(\theta) = -\sum_{j=1}^{|V|} y_j^{(t)} \log \hat{y}_j^{(t)} = -\log \hat{y}_j^{(t)}, \boldsymbol{x}^{(t+1)} = \boldsymbol{v}_j$$

$$PP^{(t)}(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = \frac{1}{\sum_{j=1}^{|V|} y_j^{(t)} \cdot \hat{y}_j^{(t)}} = \frac{1}{\hat{y}_j^{(t)}}, \boldsymbol{x}^{(t+1)} = \boldsymbol{v}_j$$

$$PP^{(t)}(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = 2^{J^{(t)}(\theta)}$$

$$\mathbb{E}(PP^{(t)}(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)})) = \mathbb{E}\left(\frac{1}{\hat{y}_j^{(t)}}\right) = \frac{1}{\mathbb{E}(\hat{y}_j^{(t)})} = |V|$$

$$\mathbb{E}(J^{(t)}(\theta)) = \log |V| \approx 13.29$$

(b)

Let

$$\boldsymbol{g}^{(t)} = \boldsymbol{h}^{(t-1)}\boldsymbol{H} + \boldsymbol{e}^{(t)}\boldsymbol{I} + \boldsymbol{b}_1$$

$$\boldsymbol{\theta}^{(t)} = \boldsymbol{h}^{(t)}\boldsymbol{U} + \boldsymbol{b}_2$$

Then

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{\theta}^{(t)}} = \hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)}$$

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{g}^{(t)}} = \frac{\partial J^{(t)}}{\partial \boldsymbol{\theta}^{(t)}} \frac{\partial \boldsymbol{\theta}^{(t)}}{\partial \boldsymbol{h}^{(t)}} \frac{\partial \boldsymbol{h}^{(t)}}{\partial \boldsymbol{g}^{(t)}} = (\hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)})\boldsymbol{U}^T\boldsymbol{h}^{(t)}(1 - \boldsymbol{h}^{(t)})$$

Therefore

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{b}_2} = \frac{\partial J^{(t)}}{\partial \boldsymbol{\theta}^{(t)}} \frac{\partial \boldsymbol{\theta}^{(t)}}{\partial \boldsymbol{b}_2} = \hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)}$$

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{L}_{x^{(t)}}} = \frac{\partial J^{(t)}}{\partial \boldsymbol{g}^{(t)}} \frac{\partial \boldsymbol{g}^{(t)}}{\partial \boldsymbol{e}^{(t)}} \frac{\partial \boldsymbol{e}^{(t)}}{\partial \boldsymbol{L}_{x^{(t)}}} = (\hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)})\boldsymbol{U}^T\boldsymbol{h}^{(t)}(1 - \boldsymbol{h}^{(t)})\boldsymbol{I}^T$$

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{I}} = \frac{\partial J^{(t)}}{\partial \boldsymbol{g}^{(t)}} \frac{\partial \boldsymbol{g}^{(t)}}{\partial \boldsymbol{I}} = \boldsymbol{e}^{(t)^T}(\hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)})\boldsymbol{U}^T\boldsymbol{h}^{(t)}(1 - \boldsymbol{h}^{(t)})$$

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{H}} = \frac{\partial J^{(t)}}{\partial \boldsymbol{g}^{(t)}} \frac{\partial \boldsymbol{g}^{(t)}}{\partial H} = \boldsymbol{h}^{(t-1)^T}(\hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)})\boldsymbol{U}^T\boldsymbol{h}^{(t)}(1 - \boldsymbol{h}^{(t)})$$

$$\frac{\partial J^{(t)}}{\partial \boldsymbol{h}^{(t-1)}} = \frac{\partial J^{(t)}}{\partial \boldsymbol{g}^{(t)}} \frac{\partial \boldsymbol{g}^{(t)}}{\partial \boldsymbol{h}^{(t-1)}} = (\hat{\boldsymbol{y}}^{(t)} - \boldsymbol{y}^{(t)})\boldsymbol{U}^T\boldsymbol{h}^{(t)}(1 - \boldsymbol{h}^{(t)})\boldsymbol{H}^T$$