Homework 1

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1. Because $f(x)$ is a linear function, Let $f(x)=ax+b$ .Then

$$g(x)\equiv y_k = f\left(\sum_j w_{k_j} f\left(\sum_i w_{ji}x_i + w_{j0}\right) + w_{k0}\right.$$

$$= f\left(\sum_j w_{kj}\left[a\left(\sum_j w_j x_j + w_{j0}\right)+b\right]+w_{k0}\right.$$

$$= f\left(\sum_j w_{kj}\left[\sum_i a w_j x_i + a w_{j0} + b\right]+w_{k0}\right)$$

$$= a\left(\sum_j\sum_i a w_{kj} w_{ji} x_i + \sum_j a w_{kj} w_{j0} + \sum_j b w_{kj} + \sum_j w_{kj}\cdot w_{k0}\right)+b$$

$$= \sum_j\sum_i a^2 w_{kj} w_{j+xi} + \sum_j a^2 w_j w_j + \sum_j ab w_{kj} + \sum_j a w_{kj} w_{k0} + ab$$

You can see that $g(x)$ is still a linear function.

2.1

$$y = \mathrm{s}(W^T X) = \frac{1}{1+e^{-W^T X}}$$

$$E = 0.5\times(g-y)^2 = 0.5\left(g-\frac{1}{1+e^{-W^T X}}\right)^2$$

$$\frac{\partial E}{\partial w} = \left(g-\frac{1}{1+e^{-W^T X}}\right)\cdot\left(1+e^{-W^T X}\right)^{-2}\cdot e^{-W^T X}\cdot(-X)$$

$$= -\left(g-\frac{1}{1+e^{-W^T X}}\right)\cdot\frac{e^{-W^T X}\cdot X}{\left(1+e^{-W^T X}\right)^2}$$

$$W = W - \lambda\frac{\partial E}{\partial W}$$

$$= W + \lambda\left(g-\frac{1}{1+e^{-W^T X}}\right)\cdot\frac{e^{-W^T X}\cdot X}{\left(1+e^{-W^T X}\right)^2}$$

2.2

$$W = (0.5, 1.1)^T$$

$$X = (1, 2, 0.5)^T$$

$$W = W - \lambda \frac{\partial E}{\partial W}$$

$$= \begin{pmatrix} 0.5 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{10} \times \left(1 - \frac{1}{1+e^{-3}}\right) \cdot \frac{e^{-3}}{\left(1+e^{-3}\right)^2} \cdot \begin{pmatrix} 1 \\ 2 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5 \\ 1 \\ 1 \end{pmatrix} + \frac{e^{-6}}{10 \times \left(1+e^{-3}\right)^3} \cdot \begin{pmatrix} 1 \\ 2 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 0.50021425 \\ 1.0004285 \\ 1.00010713 \end{pmatrix}$$