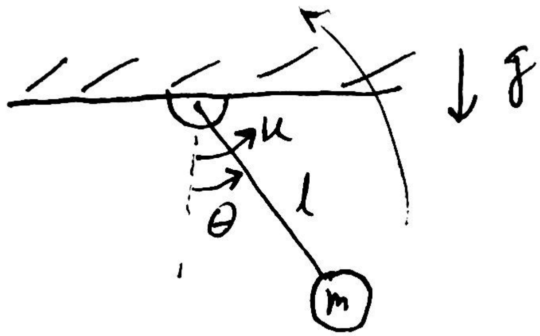
Swing up controller

Inverse pendulum Model



$$I\alpha = \sum \tau$$

$$ml^2\ddot{\theta} = u - mgl\sin\theta$$

$$ml^2\ddot{\theta} + mgl\sin\theta = u(t) \Rightarrow \ddot{\theta} = -\frac{g}{l}\sin\theta + \frac{u(t)}{ml^2}$$

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x} = \begin{matrix} \dot{\theta} \\ \ddot{\theta} \end{matrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin\theta_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u(t)$$

$$\boxed{\dot{x} = f(x) + Bu}$$

Energy Analysis

$$TE = \frac{1}{2}ml^2\dot{x}_2^2 + mgL(1 - \cos x_1)$$

$$\dot{E} = ml^2 x_2 \dot{x}_2 + mgL \sin x_1 \dot{x}_1$$

$$= ml^2\dot{\theta}\left[-\frac{g}{l}\sin\theta + \frac{u(t)}{ml^2}\right] + mgl\dot{\theta}\sin\theta$$

$$= \cancel{-mlg\dot{\theta}\sin\theta} + u(t)\dot{\theta} + \cancel{mlg\dot{\theta}\sin\theta}$$

$$= u(t)\dot{\theta}$$

The desired energy state is

$$E^d = mgL$$

so if we define

$$e_E(t) = E - E^d$$

then

$\dot{e}_E = u(t)\dot{\theta}$ and so we can use $u(t) = -k\dot{\theta}e$,

$\dot{e}_E = -k\dot{\theta}^2 e$ to drive the system to convergence except when $\dot{\theta} = 0$