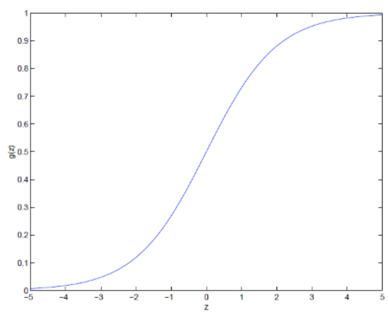
Hypothesis

$$h_\theta(x) = g(\theta^T x) = \frac{1}{1 + e^{-\theta^T x}}$$

where

$$g(z) = \frac{1}{1 + e^{-z}}$$

g(z) **is called the logistic function or the sigmoid function and looks like this:**



The derivative the sigmoid function is

$$\begin{aligned}
\frac{d}{dx}\sigma(x) &= \frac{d}{dx}\left[\frac{1}{1+e^{-x}}\right] \\
&= \frac{d}{dx}\left(1+e^{-x}\right)^{-1} \\
&= -(1+e^{-x})^{-2}(-e^{-x}) \\
&= \frac{e^{-x}}{(1+e^{-x})^2} \\
&= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} \\
&= \frac{1}{1+e^{-x}} \cdot \frac{(1+e^{-x})-1}{1+e^{-x}} \\
&= \frac{1}{1+e^{-x}} \cdot \left(\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right) \\
&= \frac{1}{1+e^{-x}} \cdot \left(1 - \frac{1}{1+e^{-x}}\right) \\
&= \sigma(x) \cdot (1 - \sigma(x))
\end{aligned}$$

Cost Function *J*

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))]$$

Gradient Descent

Remember that the general form of gradient descent is:

$$\begin{aligned}&Repeat\ \{\\&\theta_j := \theta_j - \alpha\frac{\partial}{\partial\theta_j}J(\theta)\\&\}\end{aligned}$$

We can work out the derivative part using calculus to get:

$$\begin{aligned}&Repeat\ \{\\&\theta_j := \theta_j - \frac{\alpha}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}\\&\}\end{aligned}$$

How do we obtain this?

In what follows, the superscript *(i)* denotes individual measurements or training examples.

$$\begin{aligned}
\frac{\partial J(\theta)}{\partial\theta_j} &= \frac{\partial}{\partial\theta_j}\frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\left(\log(h_\theta\left(x^{(i)}\right)\right) + (1-y^{(i)})\left(\log(1-h_\theta\left(x^{(i)}\right)\right)\right]\\
&\underset{\text{linearity}}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\frac{\partial}{\partial\theta_j}\log\left(h_\theta\left(x^{(i)}\right)\right) + (1-y^{(i)})\frac{\partial}{\partial\theta_j}\left(\log(1-h_\theta\left(x^{(i)}\right)\right)\right]\\
&\underset{\text{chain rule}}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\frac{\frac{\partial}{\partial\theta_j}(h_\theta\left(x^{(i)}\right)}{h_\theta\left(x^{(i)}\right)} + (1-y^{(i)})\frac{\frac{\partial}{\partial\theta_j}\left(1-h_\theta\left(x^{(i)}\right)\right)}{1-h_\theta\left(x^{(i)}\right)}\right]\\
&\underset{h_\theta(x)=\sigma(\theta^\top x)}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\frac{\frac{\partial}{\partial\theta_j}\sigma\left(\theta^\top x^{(i)}\right)}{h_\theta\left(x^{(i)}\right)} + (1-y^{(i)})\frac{\frac{\partial}{\partial\theta_j}\left(1-\sigma\left(\theta^\top x^{(i)}\right)\right)}{1-h_\theta\left(x^{(i)}\right)}\right]
\end{aligned}$$

$$\underset{\sigma'}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\frac{\sigma\left(\theta^\top x^{(i)}\right)\left(1-\sigma\left(\theta^\top x^{(i)}\right)\right)\frac{\partial}{\partial\theta_j}\left(\theta^\top x^{(i)}\right)}{h_\theta\left(x^{(i)}\right)} - (1-y^{(i)})\frac{\sigma\left(\theta^\top x^{(i)}\right)\left(1-\sigma\left(\theta^\top x^{(i)}\right)\right)\frac{\partial}{\partial\theta_j}\left(\theta^\top x^{(i)}\right)}{1-h_\theta\left(x^{(i)}\right)}\right]$$

$$\underset{\sigma(\theta^\top x)=h_\theta(x)}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\frac{h_\theta\left(x^{(i)}\right)\left(1-h_\theta\left(x^{(i)}\right)\right)\frac{\partial}{\partial\theta_j}\left(\theta^\top x^{(i)}\right)}{h_\theta\left(x^{(i)}\right)} - (1-y^{(i)})\frac{h_\theta\left(x^{(i)}\right)\left(1-h_\theta\left(x^{(i)}\right)\right)\frac{\partial}{\partial\theta_j}\left(\theta^\top x^{(i)}\right)}{1-h_\theta\left(x^{(i)}\right)}\right]$$

$$\underset{\frac{\partial}{\partial\theta_j}\left(\theta^\top x^{(i)}\right)=x_j^{(i)}}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}\left(1-h_\theta\left(x^{(i)}\right)\right)x_j^{(i)}-\left(1-y^{(i)}\right)h_\theta\left(x^{(i)}\right)x_j^{(i)}\right]$$

$$\underset{\text{distribute}}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}-y^{(i)}h_\theta\left(x^{(i)}\right)-h_\theta\left(x^{(i)}\right)+y^{(i)}h_\theta\left(x^{(i)}\right)\right]x_j^{(i)}$$

$$\underset{\text{cancel}}{=} \frac{-1}{m}\sum_{i=1}^{m}\left[y^{(i)}-h_\theta\left(x^{(i)}\right)\right]x_j^{(i)}$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta\left(x^{(i)}\right)-y^{(i)}\right]x_j^{(i)}$$