Limiti notevoli

$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$

$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^x = e^a$

$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^{nx} = e^{na}$

$\lim_{x \to -\infty} \left(1 - \frac{1}{x}\right)^x = \frac{1}{e}$

$\lim_{x \to 0} (1 + ax)^{\frac{1}{x}} = e^a$

$\lim_{x \to 0} \frac{ln(1+x)}{x} = 1$

$\lim_{x \to 0} \frac{a^x - 1}{x} = ln(a)$

$\lim_{x \to 0} \frac{(1+x)^a - 1}{x} = a$

$\lim_{x \to 0} \frac{(1+x)^a - 1}{ax} = 1$

$\lim_{x \to 0} \frac{sin(x)}{x} = 1$

$\lim_{x \to 0} \frac{sin(ax)}{bx} = \frac{a}{b}$

$\lim_{x \to 0} \frac{tan(x)}{x} = 1$

$\lim_{x \to 0} \frac{tan(ax)}{bx} = \frac{a}{b}$

$\lim_{x \to 0} \frac{1 - cos(x)}{x} = 0$

$\lim_{x \to 0} \frac{1 - cos(x)}{x^2} = \frac{1}{2}$

$\lim_{x \to 0} \frac{arcsin(x)}{x} = 1$

$\lim_{x \to 0} \frac{arcsin(ax)}{bx} = \frac{a}{b}$

$\lim_{x \to 0} \frac{arctan(x)}{x} = 1$

$\lim_{x \to 0} \frac{arctan(ax)}{bx} = \frac{a}{b}$

$\lim_{x \to 0} \frac{x - sin(x)}{x^3} = \frac{1}{6}$

$\lim_{x \to 0} \frac{x - arctan(x)}{x^3} = \frac{1}{3}$

Derivate

D: costante k $\to 0$

D: $x^n \to nx^{n-1}$ — D: $[f(x)]^m \to m[f(x)]^{m-1} \cdot f'(x)$

D: $\sqrt{x} \to \frac{1}{2\sqrt{x}}$ — D: $\sqrt{f(x)} \to \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$

D: $\sqrt[n]{x^m} \to \frac{1}{n\sqrt[n]{x^{n-m}}}$ — D: $\sqrt[n]{[f(x)]^m} \to \frac{m}{n\sqrt[n]{[f(x)]^{n-m}}} \cdot f'(x)$

D: $sin(x) \to cos(x)$ — D: $sin(f(x)) \to cos(f(x)) \cdot f'(x)$

D: $cos(x) \to -sin(x)$ — D: $cos(f(x)) \to -sin(f(x)) \cdot f'(x)$

D: $tan(x) \to \frac{1}{cos^2(x)}$ — D: $tan(f(x)) \to \frac{1}{cos^2(f(x))} \cdot f'(x)$

D: $cotan(x) \to -\frac{1}{sin^2(x)}$ — D: $cotan(f(x)) \to -\frac{1}{sin^2(f(x))} \cdot f'(x)$

D: $arcsin(x) \to \frac{1}{\sqrt{1-x^2}}$ — D: $arcsin(f(x)) \to \frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x)$

D: $arccos(x) \to -\frac{1}{\sqrt{1-x^2}}$ — D: $arccos(f(x)) \to -\frac{1}{\sqrt{1-[f(x)]^2}} \cdot f'(x)$

D: $arctan(x) \to \frac{1}{1+x^2}$ — D: $arctan(f(x)) \to \frac{1}{1+[f(x)]^2} \cdot f'(x)$

D: $arccotan(x) \to -\frac{1}{1+x^2}$ — D: $arccotan(f(x)) \to -\frac{1}{1+[f(x)]^2} \cdot f'(x)$

D: $a^x \to a^x ln(a)$ — D: $a^{f(x)} \to a^{f(x)} \cdot f'(x) \cdot log_a(x)$

D: $e^x \to e^x$ — D: $e^{f(x)} \to e^{f(x)} \cdot f'(x)$

D: $ln(x) \to \frac{1}{x}$ — D: $ln(f(x)) \to \frac{1}{f(x)} \cdot f'(x)$

D: $log_a(x) \to \frac{1}{x} log_a(e)$ — D: $log_a(f(x)) \to \frac{1}{f(x)} \cdot f'(x) \cdot log_a(e)$

Derivate di funzioni

D: $[f(x)]^{g(x)} \to [f(x)]^{g(x)} \cdot [g'(x) ln(f(x)) + g(x) \frac{f'(x)}{f(x)}]$

D: $[f(g(x))] \to f'[g(x)] \cdot g'(x)$

D: $[f(x) \cdot g(x)] \to f'(x) \cdot g(x) + f(x) \cdot g'(x)$

D: $[\frac{f(x)}{g(x)}] \to \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Integrali immediati

$\int x^k = \frac{x^{k+1}}{k+1}$ — $\int \frac{1}{sin^2(x)} = -cotan(x)$

$\int e^x = e^x$ — $\int \frac{1}{\sqrt{1-x^2}} = arcsin(x)$

$\int \frac{1}{x} = ln|x|$ — $\int \frac{1}{\sqrt{1-x^2}} = arctan(x)$

$\int cos(x) = sin(x)$ — $\int \frac{1}{\sqrt{1+x^2}} = ln(x + \sqrt{1 + x^2})$

$\int sin(x) = -cos(x)$ — $\int \frac{1}{\sqrt{x^2-1}} = ln(x + \sqrt{x^2 - 1})$

$\int \frac{1}{cos^2(x)} = tan(x)$ — $\int \frac{1}{1-x^2} = \frac{1}{2} ln(\frac{1+x}{1-x})$

Integrali con la prima regola di sostituzione

$\int f(x)^k \cdot f'(x) = \frac{1}{a+1} [f(x)]^{a+1}$

$\int \frac{f'(x)}{f(x)} = ln|f(x)|$

$\int e^{f(x)} \cdot f'(x) = e^{f(x)}$

$\int cos(f(x)) \cdot f'(x) = sin(f(x))$

$\int sin(f(x)) \cdot f'(x) = -cos(f(x))$

$\int \frac{f'(x)}{cos^2(f(x))} = tan(f(x))$

$\int \frac{f'(x)}{\sqrt{1-f(x)^2}} = arcsin(f(x))$

$\int \frac{f'(x)}{1+f(x)^2} = arctan(f(x))$

$\int \frac{f'(x)}{\sqrt{1+f(x)^2}} = ln(f(x) + \sqrt{1 + f(x)^2})$

$\int \frac{f'(x)}{\sqrt{f(x)^2-1}} = ln(f(x) + \sqrt{f(x)^2 - 1})$

$\int \frac{f'(x)}{1-f(x)^2} = \frac{1}{2} ln(\frac{1+f(x)}{1-f(x)})$

Formula di Taylor

$e^x = 1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + o(x^n)$

$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^n)$

$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^n)$

$tan(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{7}{315}x^7 + \frac{62}{2835}x^9 + o(x^n)$

$ln(1 + x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} + ... + (-1)^{n+1} \frac{x^n}{n!} + o(x^n)$

$\frac{1}{1+x} = 1 - x + x^2 + ... + (-1)^n \cdot x^n + o(x^n)$

$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3}{8}x^2 + ... + (-1)^n \cdot \frac{(2n-1)!!}{(2n)!!} \cdot x^n + o(x^n)$

$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5}{128}x^4 + \frac{7}{256}x^5 + o(x^n)$

$\frac{1}{1+x^2} = 1 - x^2 + x^4 + ... + (-1)^n \cdot x^{2n} + o(x^n)$

$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2} + \frac{3}{8}x^4 + ... + \frac{(2n-1)!!}{(2n)!!} \cdot x^{2n} + o(x^n)$

$\frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \frac{3}{8}x^2 + ... + \frac{(2n-1)!!}{(2n)!!} \cdot x^n + o(x^n)$

$arcsin(x) = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + o(x^n)$

$arccos(x) = \frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3}{40}x^5 - \frac{5}{112}x^7 - \frac{35}{1152}x^9 + o(x^n)$

$arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + o(x^n)$

$sinh(x) = x + \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^7}{5040} + \frac{x^9}{362880} + o(x^n)$

$arccos(x) = 1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \frac{x^8}{40320} + o(x^n)$

$(1 + x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + ... + (\alpha\ n)x^n + o(x^n)$

Integrali impropri notevoli

$\int_0^\alpha \frac{1}{x^p} dx$, converge se p<1, diverge se p>=1 — $\int_\alpha^{+\infty} \frac{1}{x^p} dx$, converge se p>1, diverge se p<=1

$\int_0^\alpha \frac{1}{x^a |ln^b(x)|} dx$, $\int_\alpha^{+\infty} \frac{1}{x^a |ln^b(x)|} dx$, converge se (a>1) o se (a=1 e b>1), diverge se (a<1) o se (a=1 e b<=1).

$\int_1^a \frac{1}{ln^p(x)} dx$, converge se p<1, diverge se p>=1.

Proprietà dei numeri complessi

$z = a + bi$

$w = c + di$

$q = a - bi$

$z \cdot w = ac - bd + (ad + bc)i$

$\frac{1}{z} = \frac{1}{z} \cdot \frac{q}{q} = \frac{q}{|z|^2}$

$z \cdot q = a^2 + b^2 = |z|^2$

$|z| = \sqrt{a^2 + b^2}$

$|z \cdot w| = |z| \cdot |w|$

Scala degli infiniti (k->costante)

x^x	+grande
$x!$	
k^x	
x^k	
x	
$\sqrt[k]{x}$	
$log(x)$	-grande

Equazioni differenziali

1° ordine formula classica

$y' + a(x)y = b(x) \rightarrow y = e^{-\int a(x)} \cdot \left[\int b(x) \cdot e^{\int a(x)} + C\right]$

1° ordine con variabili separabili

$y'(x) = f(t) \cdot g(x) \rightarrow \int \frac{1}{g(x)} dx = \int f(t)dt$

2° ordine omogeneo

$ay'' + by' + cy = 0$, da risolvere per $\lambda_{1,2}$ $\qquad \lambda_1 = \alpha + i\beta, \lambda_2 = \alpha - i\beta$

1°caso: Δ>0, $y = C_1 \cdot e^{\lambda_1 t} + C_2 \cdot e^{\lambda_2 t}$ 2°caso: Δ=0, $y = C_1 \cdot e^{\lambda_1 t} + C_2 \cdot t \cdot e^{\lambda_1 t}$ 3°caso: Δ<0, $y = C_1 \cdot e^{\alpha t} \cdot sin(\beta t) + C_2 \cdot e^{\alpha t} \cdot cos(\beta t)$

2° ordine non omogeneo

$ay'' + by' + cy = f(x)$, si risolve il sistema come se fosse omogenea, si ricava $yp(x)$ e lo si deriva per ricavare le varie costanti, rispettando la formula $y = yo + yp$ si mette tutto a sistema e si trovano le costanti tramite Cauchy.

Proprietà delle potenze

$a^{n+m} = a^n \cdot a^m$

$\left(a^n\right)^m = a^{n \cdot m}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Proprietà dei logaritmi

$ln(x \cdot y) = ln(x) + ln(y)$

$ln(\frac{x}{y}) = ln(x) - ln(y)$

$ln(x)^y = y \cdot ln(x)$

$e^{ln(x)} = x$

Forme di indeterminazione

$\left[\frac{0}{0}\right], \left[\frac{\infty}{\infty}\right], [0 \cdot \infty], \left[1^\infty\right], [\infty - \infty], \left[0^0\right], \left[\infty^0\right]$

Relazioni trigonometriche

$sin^2(x) + cos^2(x) = 1$

$sin^2(x) = 1 - cos^2(x)$

$cos^2(x) = 1 - sin^2(x)$

$cosh^2(x) - sinh^2(x) = 1$

$cosh(x) = \frac{e^x + e^{-x}}{2}$

$sinh(x) = \frac{e^x - e^{-x}}{2}$

Prodotti notevoli

$\left(a^2 - b^2\right) = (a + b) \cdot (a - b)$

$\left(a^3 - b^3\right) = (a - b) \cdot \left(a^2 + ab + b^2\right)$

$\left(a^4 - b^4\right) = \left(a^2 + b^2\right) \cdot \left(a^2 - b^2\right)$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Formule di duplicazione

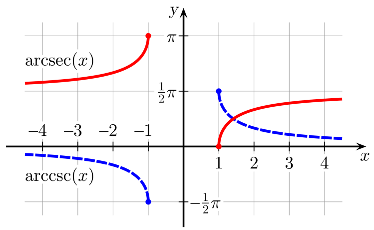
$sin(2x) = 2sin(x)cos(x)$

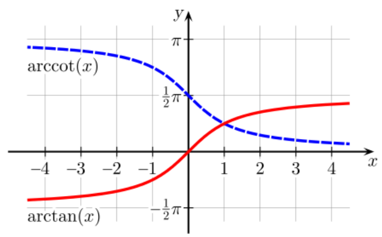
$cos(2x) = cos^2(x) - sin^2(x) = 1 - 2sin^2(x) = 2cos^2(x) - 1$

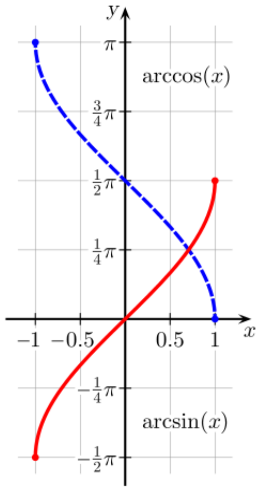
$sin^2(x) = \frac{1 - cos(2x)}{2}$

$cos^2(x) = \frac{1 + cos(2x)}{2}$

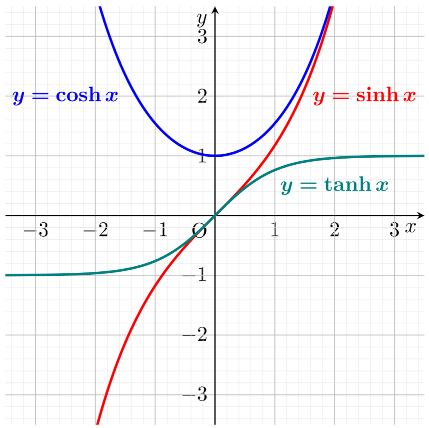
$tan(2x) = \frac{2\,tan(x)}{1 - tan^2(x)}$







gradi	sin	cos	tan	cotan
0°	0	1	0	± ∞
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
90°	1	0	± ∞	0
180°	0	-1	0	± ∞



Funzione	Dominio	Codominio	Monotonia
Sin	-∞ : +∞	-1 : +1	oscillante
Cos	-∞ : +∞	-1 : +1	oscillante
Tan	-∞ : +∞	-∞ : +∞	crescente
Cotan	-∞ : +∞	-∞ : +∞	decrescente
Arcsin	-1 : +1	-π/2 : +π/2	crescente
Arccos	-1 : +1	0 : +π	decrescente
Arctan	-∞ : +∞	-π/2 : +π/2	crescente