

Bayesian Optimization

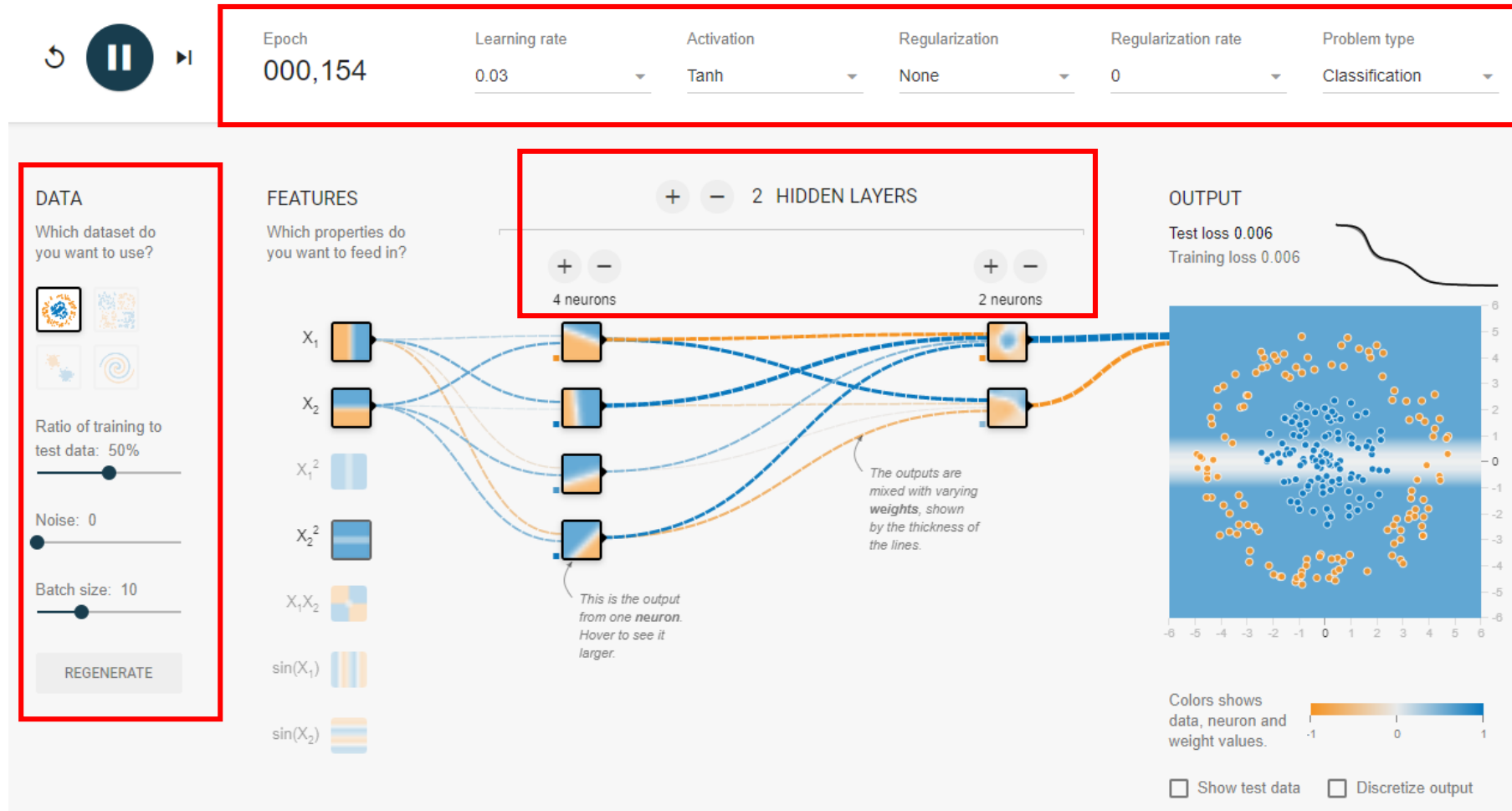
Feb.19.2020
Team .A

Kwangwoon University ML Study
Week #11

Machine Learning is
Extremely powerful

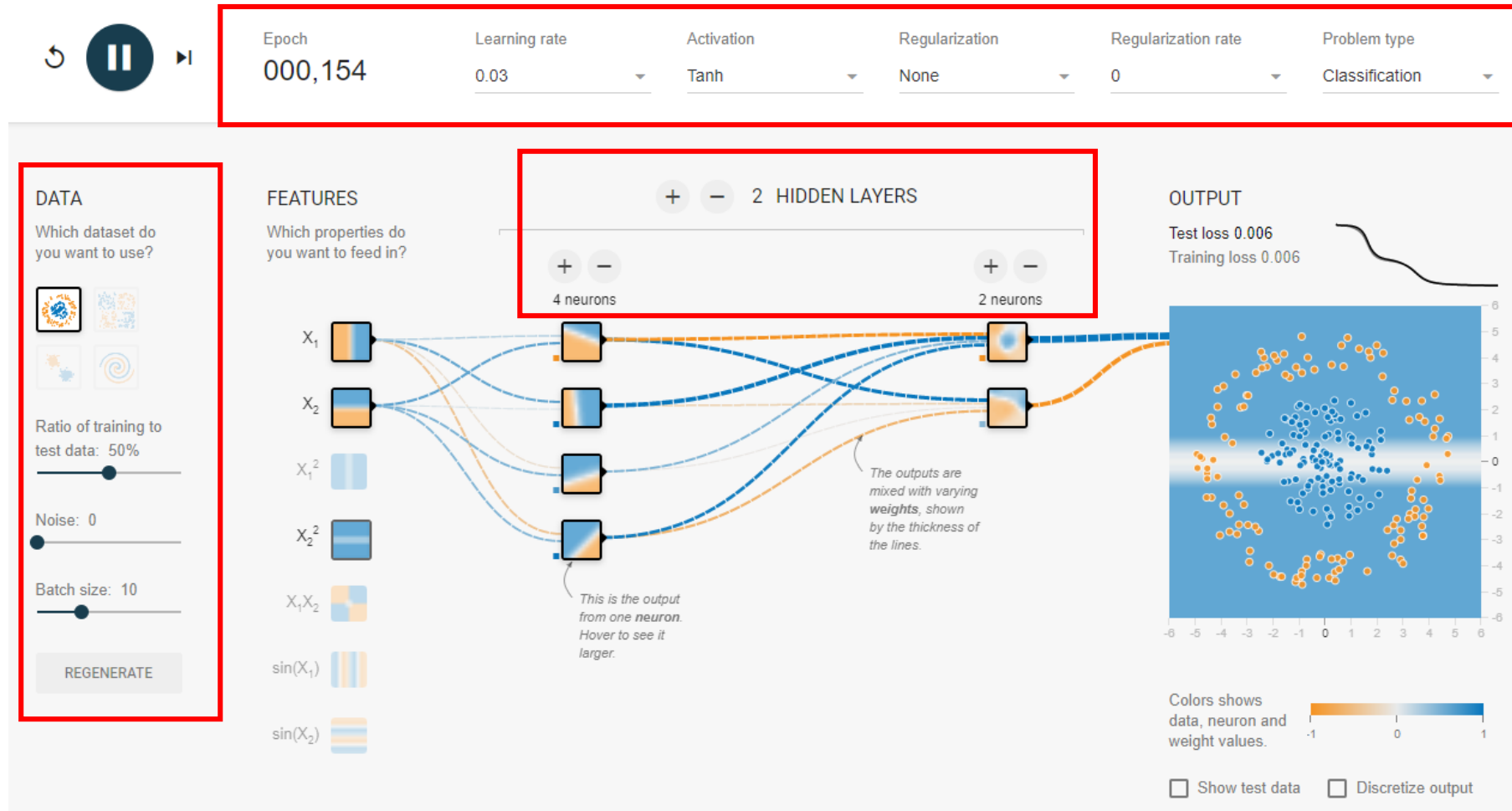
But Tuning Machine Learning system is
Extremely non-intuitive

Bayesian Optimization



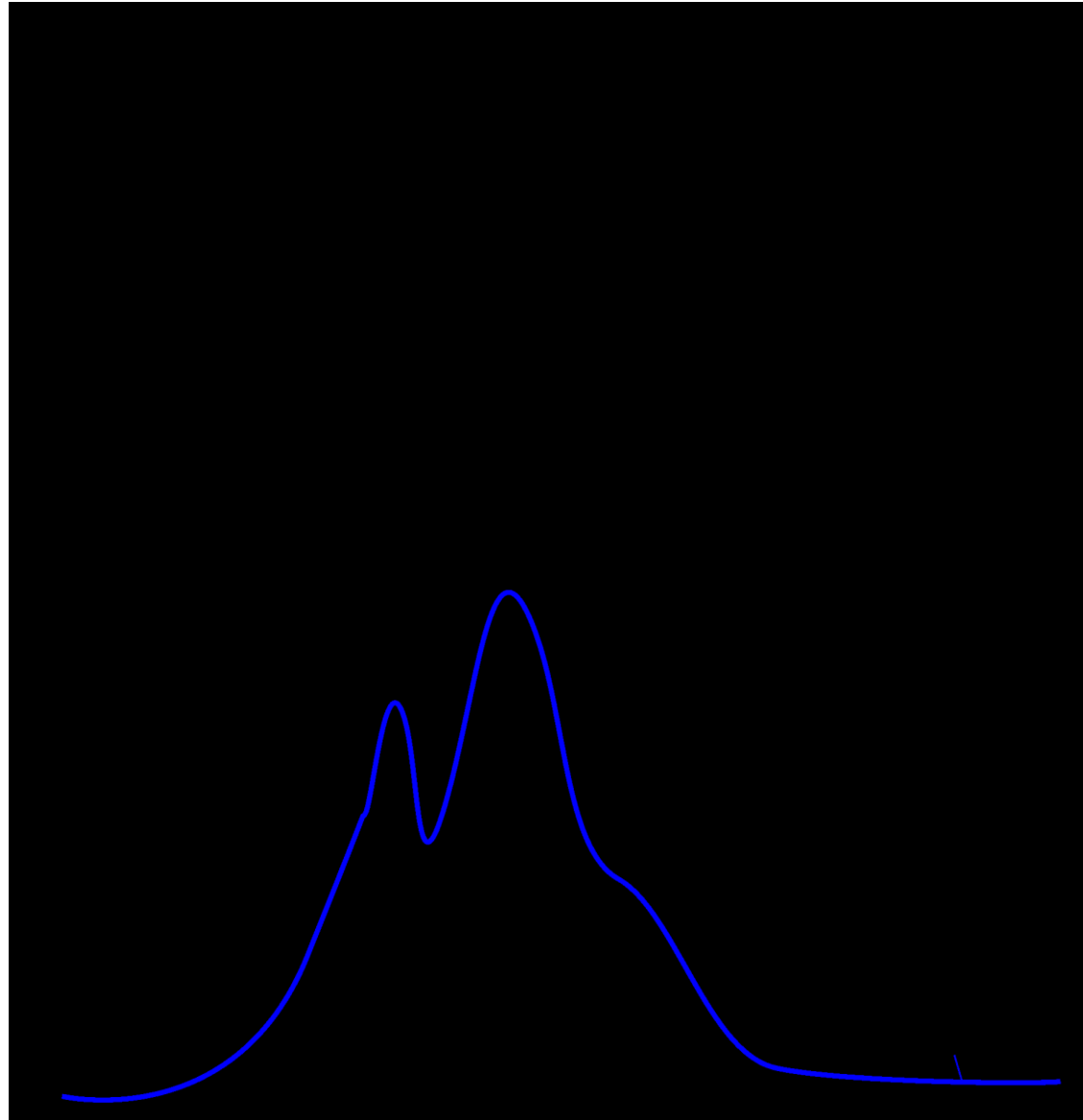
<https://playground.tensorflow.org/>

Bayesian Optimization



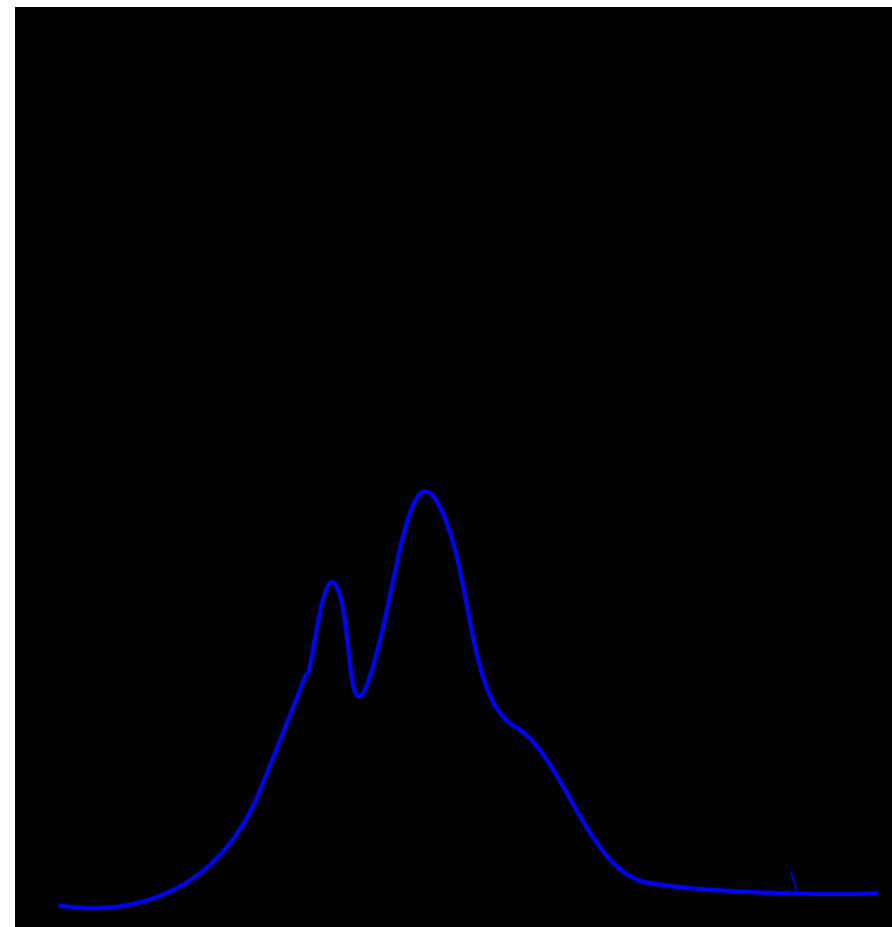
Many Tunable parameters in Machine learning

Bayesian Optimization



Manual Search

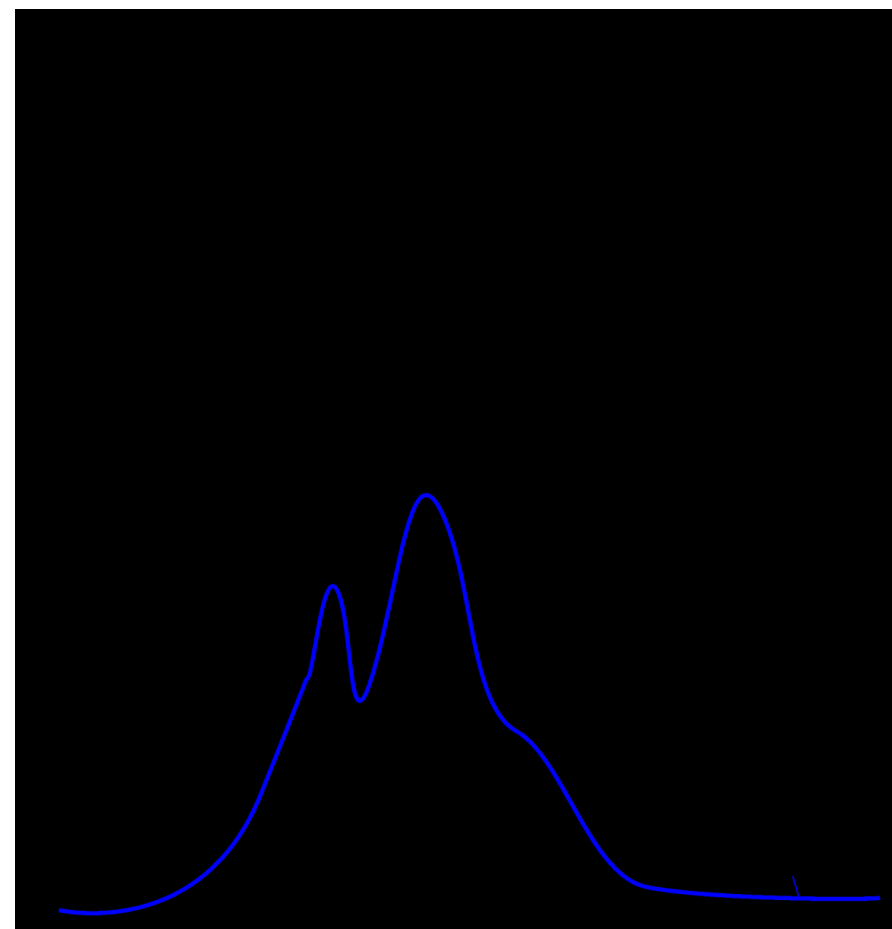
Select hyperparameter
manually
with using prior
knowledge
and intuition



Manual Search

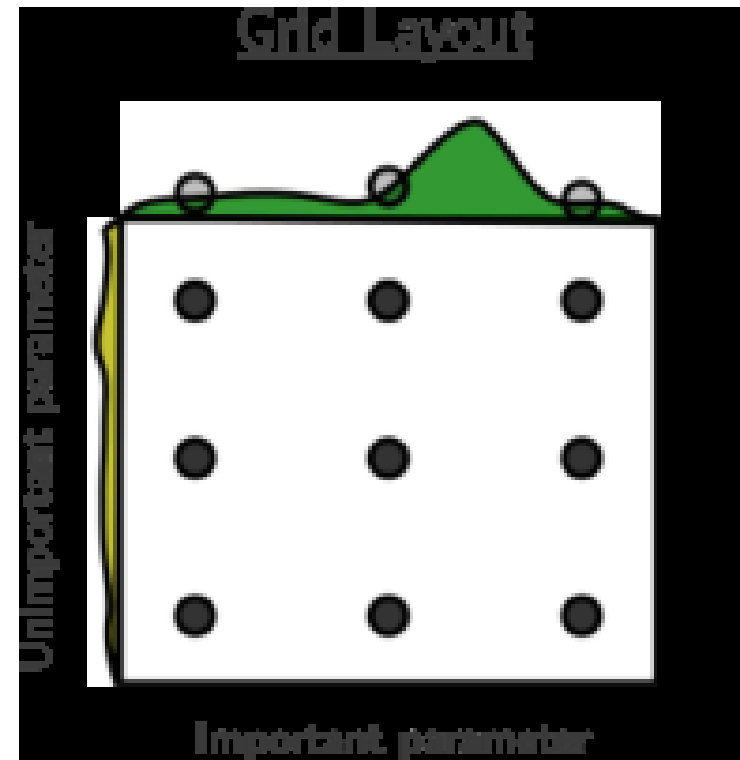
Problem

1. It doesn't guarantee that the solution is truly optimal.
2. As the number of hyperparameters increases, the problem becomes more complex



Grid Search

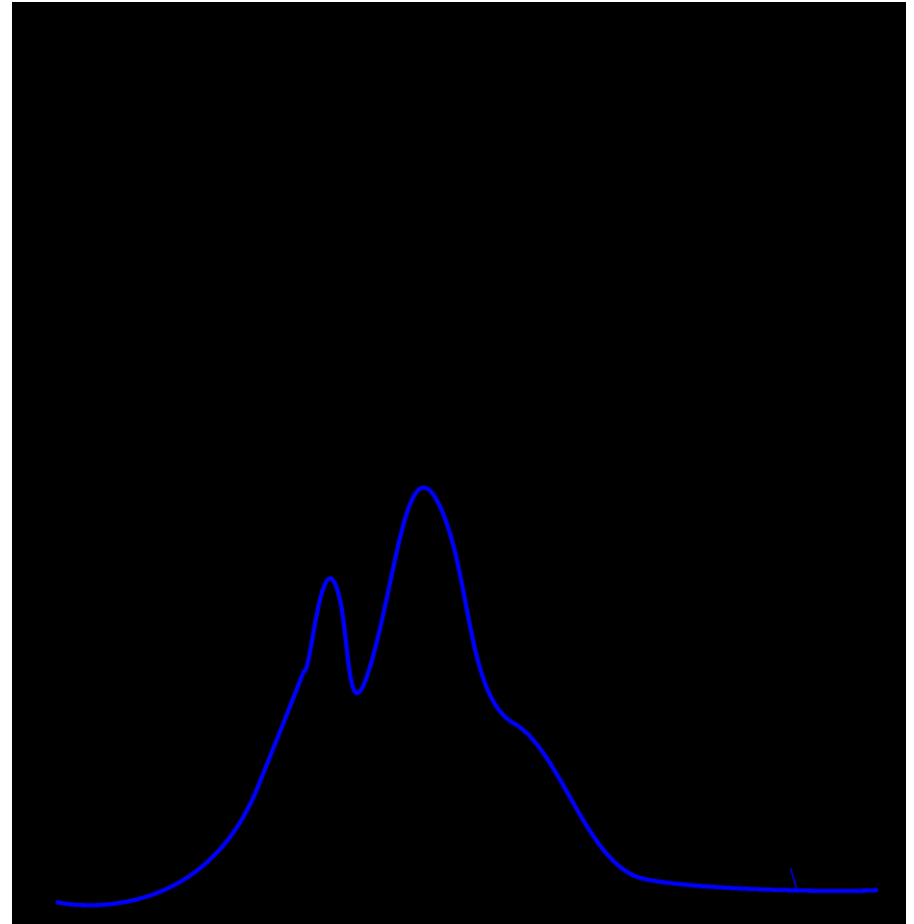
Select all hyperparameter within the certain section at regular intervals



Grid Search

Problem

1. As the number of hyperparameters increases, the problem becomes more complex
2. There is no exploitation

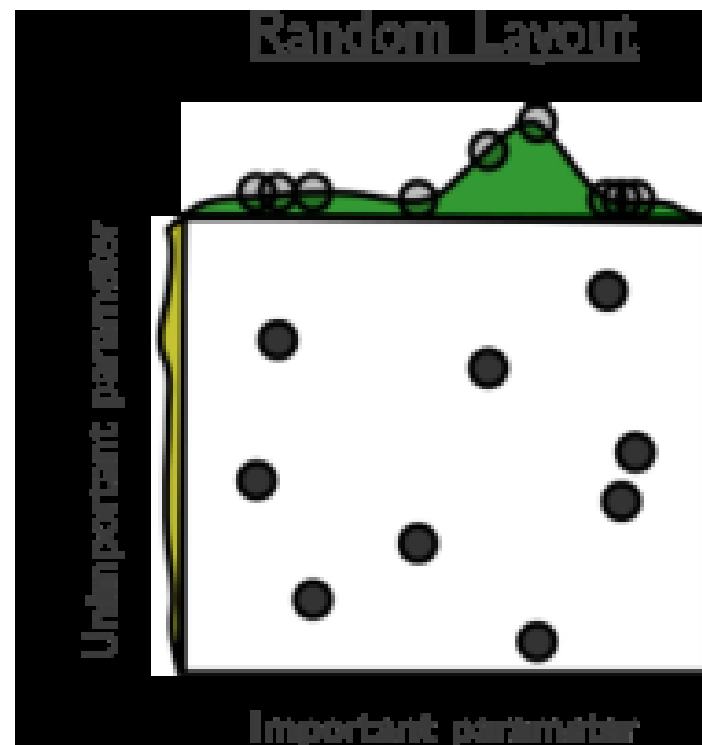


Random Search

Select hyperparameter
by random sampling

Advantage

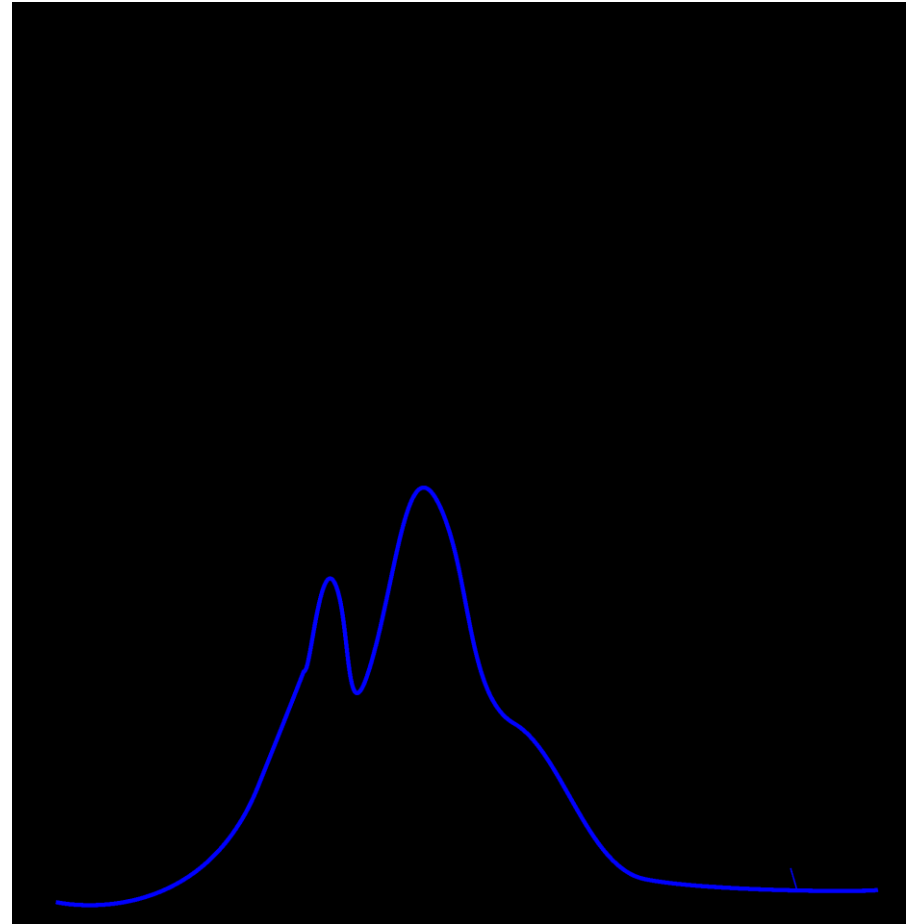
1. Reduce the number of iteration
2. The values located between grids can also be stochastically searched



Random Search

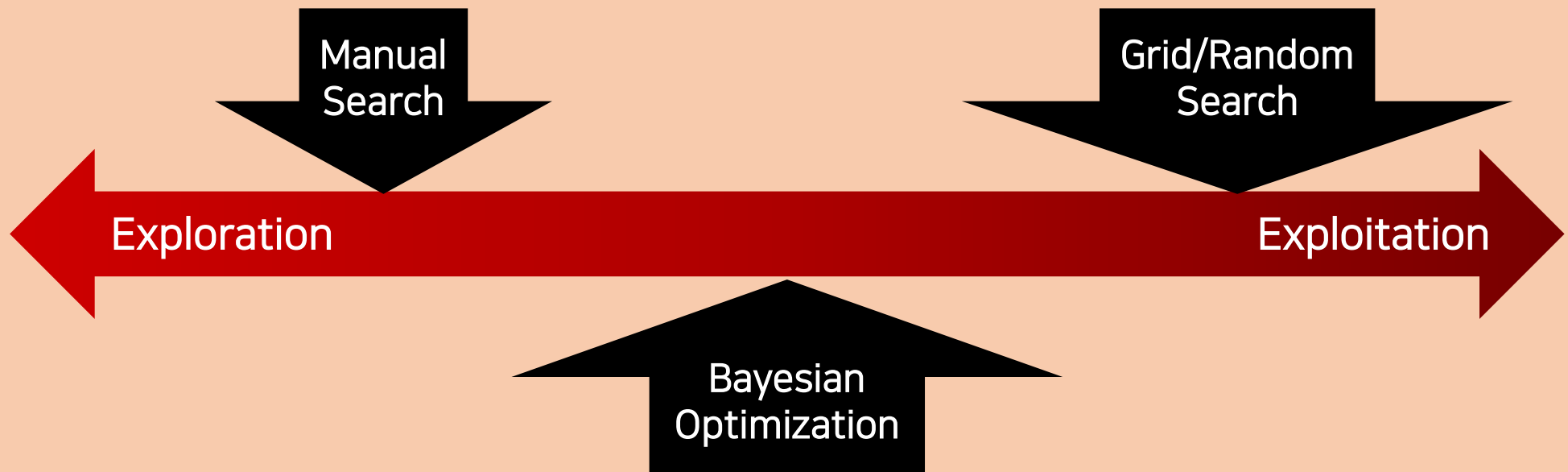
Problem

1. As the number of hyperparameters increases, the problem becomes more complex
2. There is no exploitation



Bayesian Optimization

Exploration-Exploitation Dilemma





Common Approach

1. Random search or grid search
2. Expert defined grid search near “good” points
3. Refine domain and repeat steps - “grad student descent”

Problem

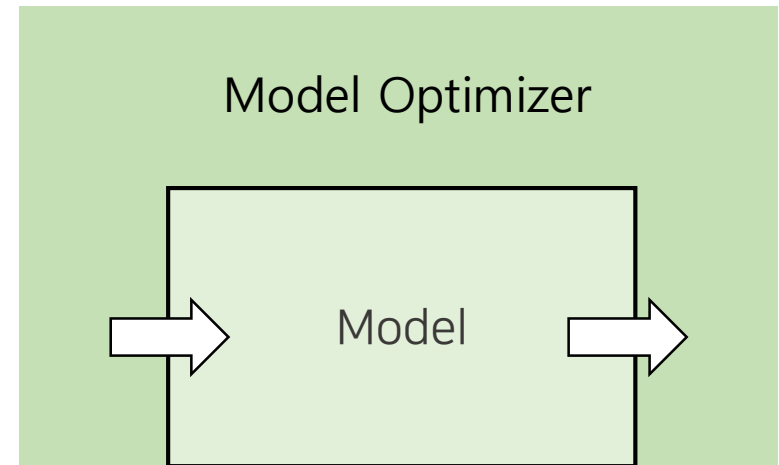
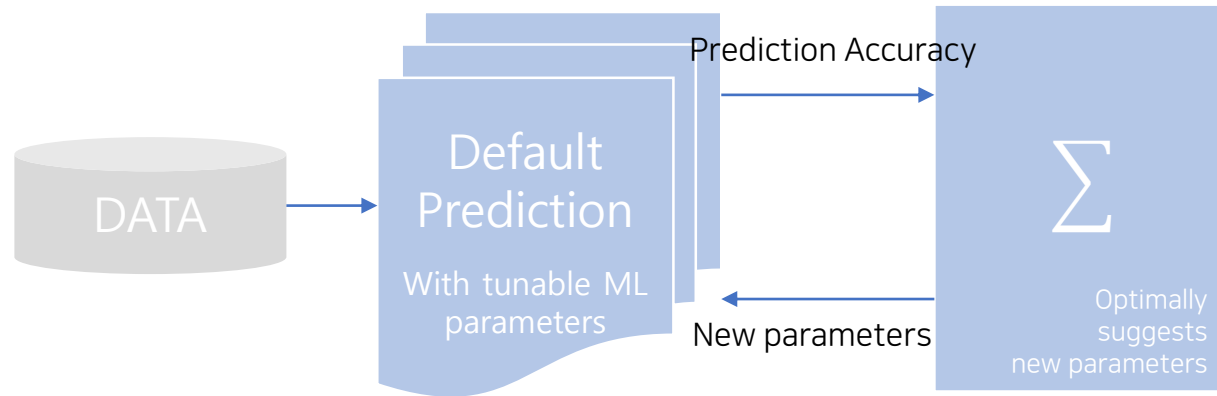
- Expert intensive
- Computationally intensive
- Finds potentially local optima
- Does not fully exploit useful information



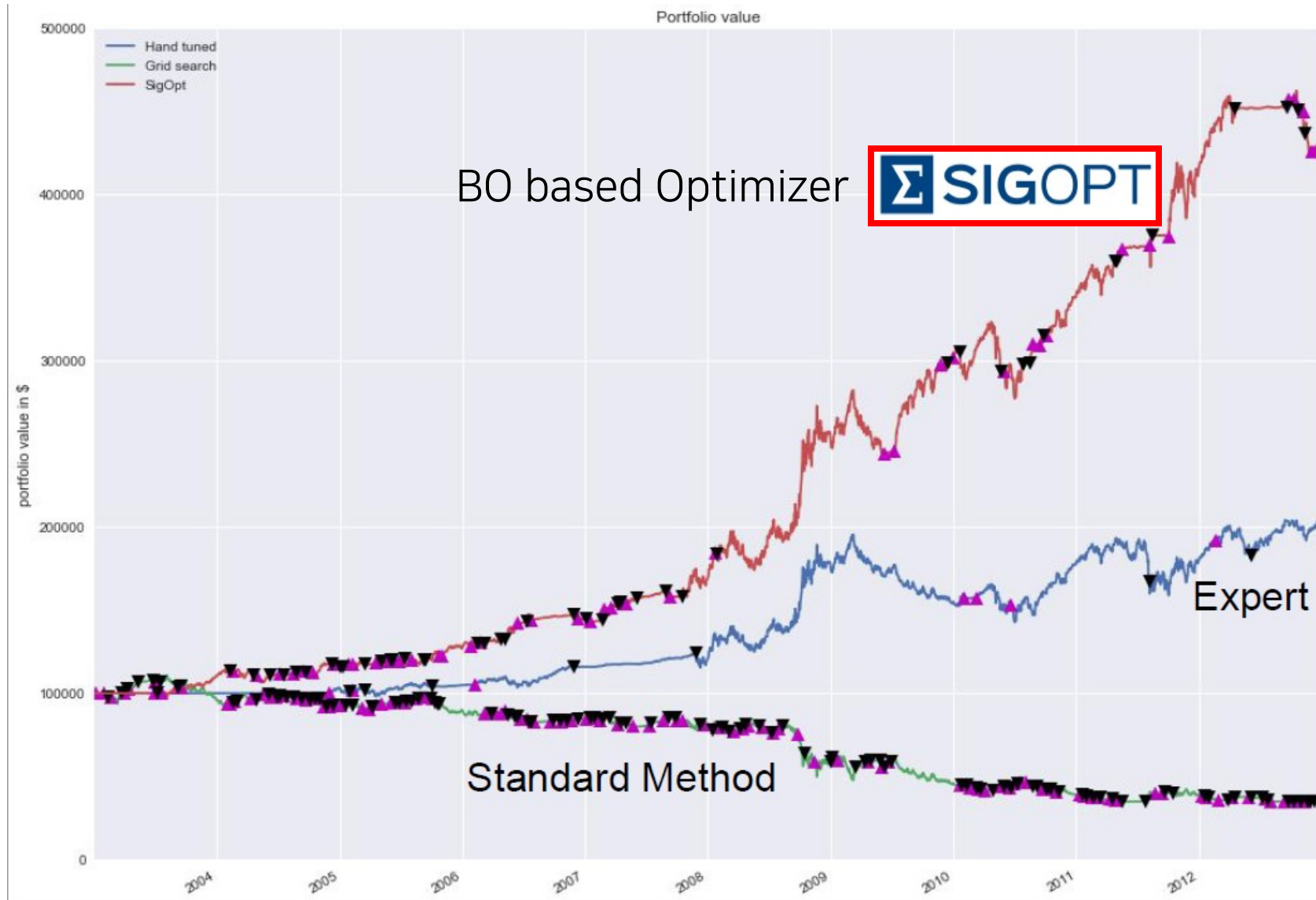
Common Approach

1. Optimize some Overall Evaluation Criterion (OEC)
 - Loss, Accuracy, Likelihood, Revenue
2. Given tunable parameters
 - Hyperparameters, feature parameters
3. In an efficient way
 - Sample function as few times as possible
 - Training on big data is expensive

Common Approach



Common Approach



Bayesian Optimization

Optimization

Find optimal solution x^* that maximize $f(x)$

Suggest Two essential element

Surrogate Model

$(x_1, f(x_1)), \dots, (x_t, f(x_t))$

Probabilistic Estimation of
Unknown Objective Function

Acquisition Function

Recommend x_{t+1}

Recommend candidate values to help
you find the optimal input value

Bayesian Optimization

Optimization

Find optimal solution x^* that maximize $f(x)$

Suggest Two essential element

알고리즘 Bayesian Optimization

- 1: **for** $t = 1, 2, \dots$ **do**
- 2: 기존 입력값-함숫값 점들의 모음 $(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_t, f(x_t))$ 에 대한 Surrogate Model의 확률적 추정 결과를 바탕으로, Acquisition Function을 최대화하는 다음 입력값 후보 x_{t+1} 을 선정한다.
- 3: 입력값 후보 x_{t+1} 에 대한 함숫값 $f(x_{t+1})$ 을 계산한다.
- 4: 기존 입력값-함숫값 점들의 모음에 $(x_{t+1}, f(x_{t+1}))$ 를 추가하고, Surrogate Model로 확률적 추정을 다시 수행한다.
- 5: **end for**

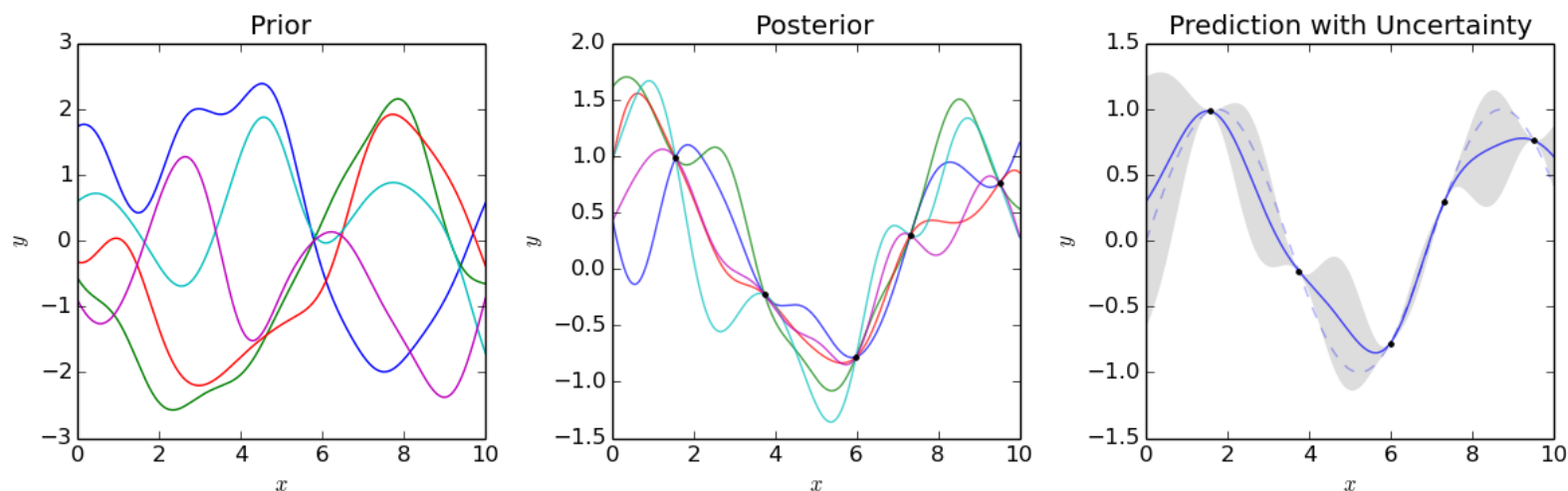
Pseudo-code for BO algorithm

Bayesian Optimization

Surrogate Model

Gaussian Processes(GP)

$$f(x) \sim GP(\mu(x), k(x, x'))$$



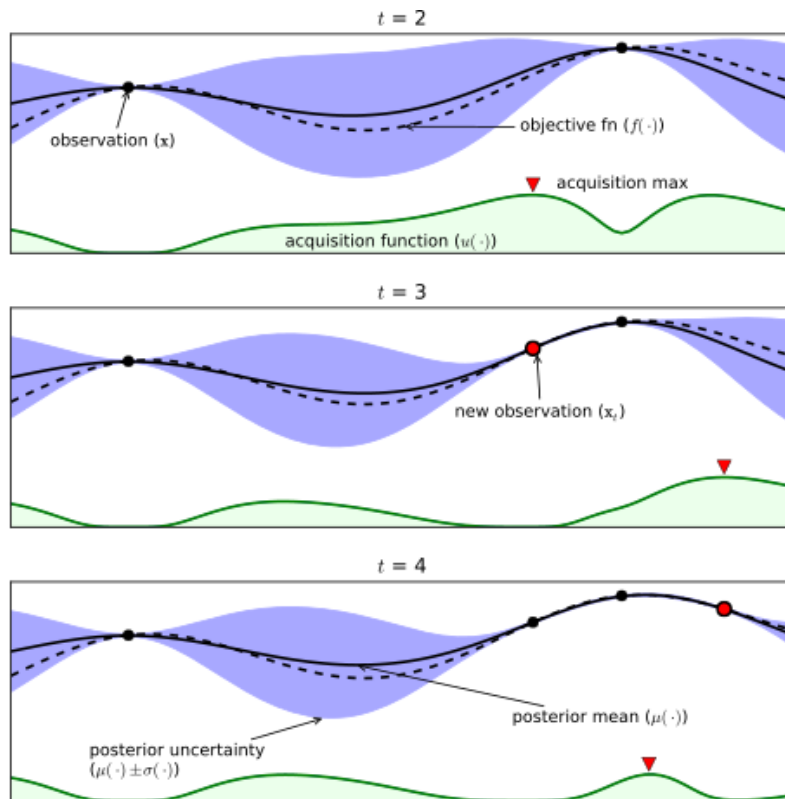
Gaussian process uses lazy learning and a measure of the similarity between points (the kernel function) to predict the value for an unseen point from training data

Gaussian processes can be seen as an infinite-dimensional generalization of multivariate normal distributions

Bayesian Optimization

Surrogate Model

Gaussian Processes(GP)



black dotted line : Actual object function

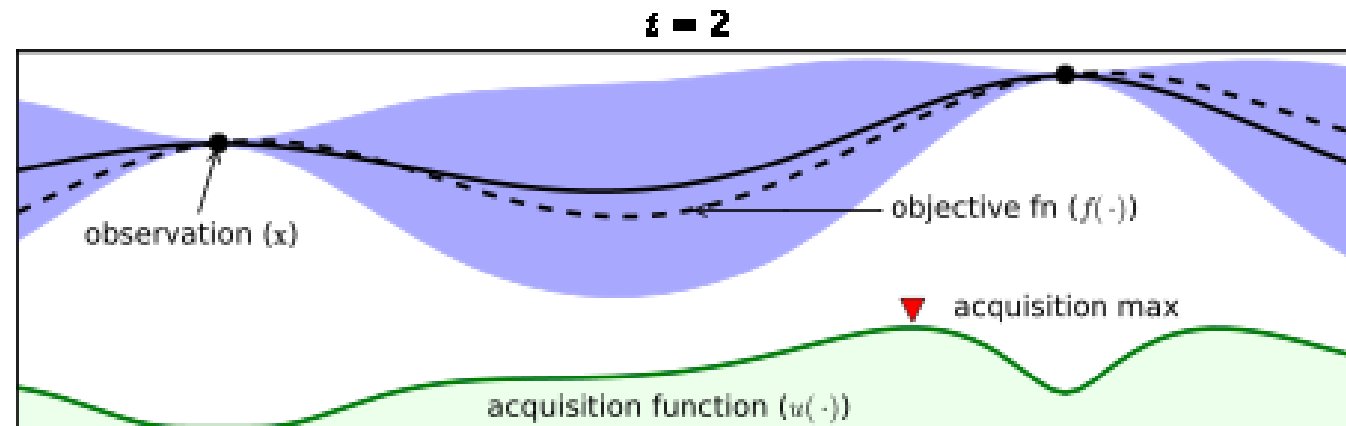
Black line : Estimated avg function

Blue space : Estimated Standard deviation

Black dots : searched input-function output point

Bayesian Optimization

Acquisition Function



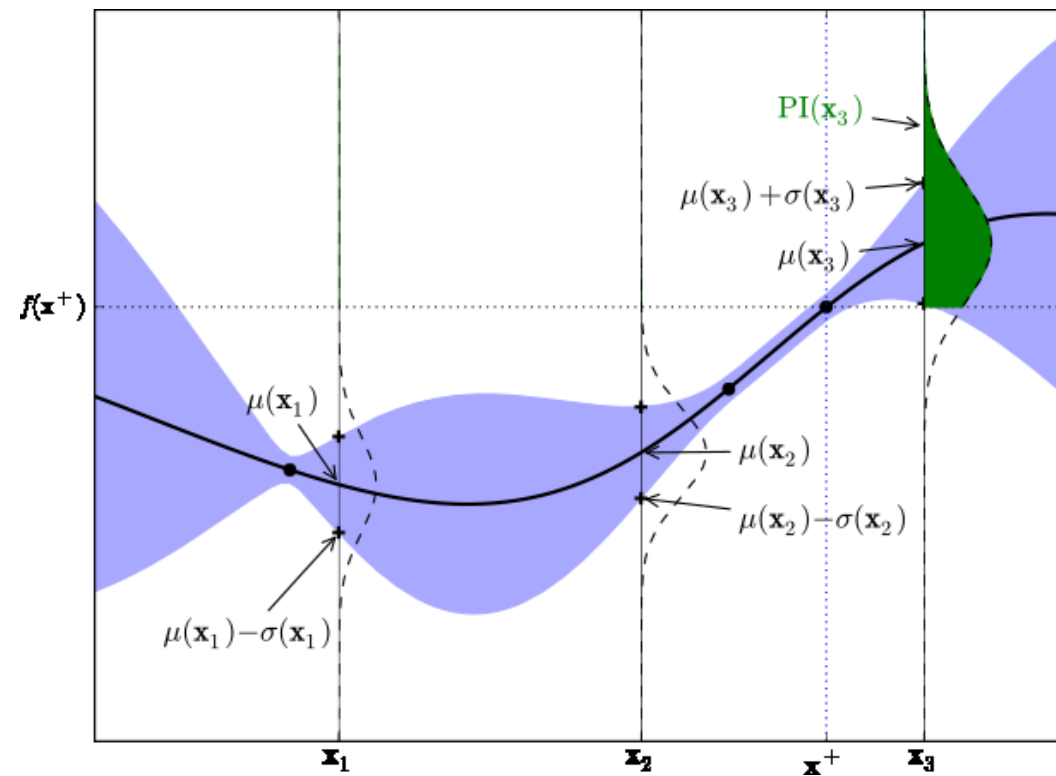
Trade-off

Exploitation vs Exploration

Bayesian Optimization

Acquisition Function

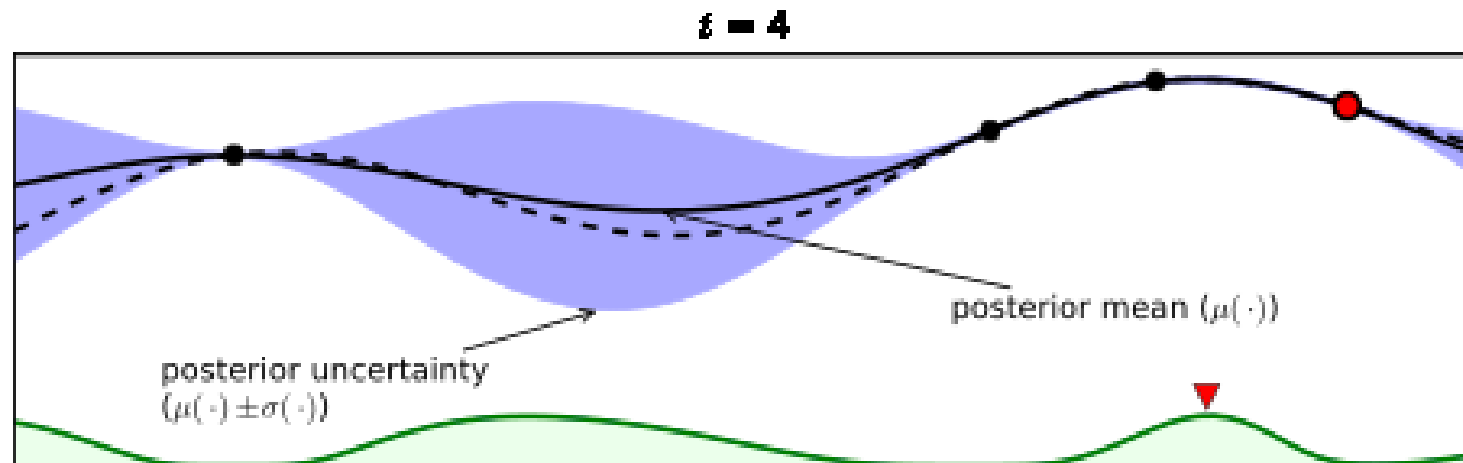
Expected Improvement (EI)



Bayesian Optimization

Acquisition Function

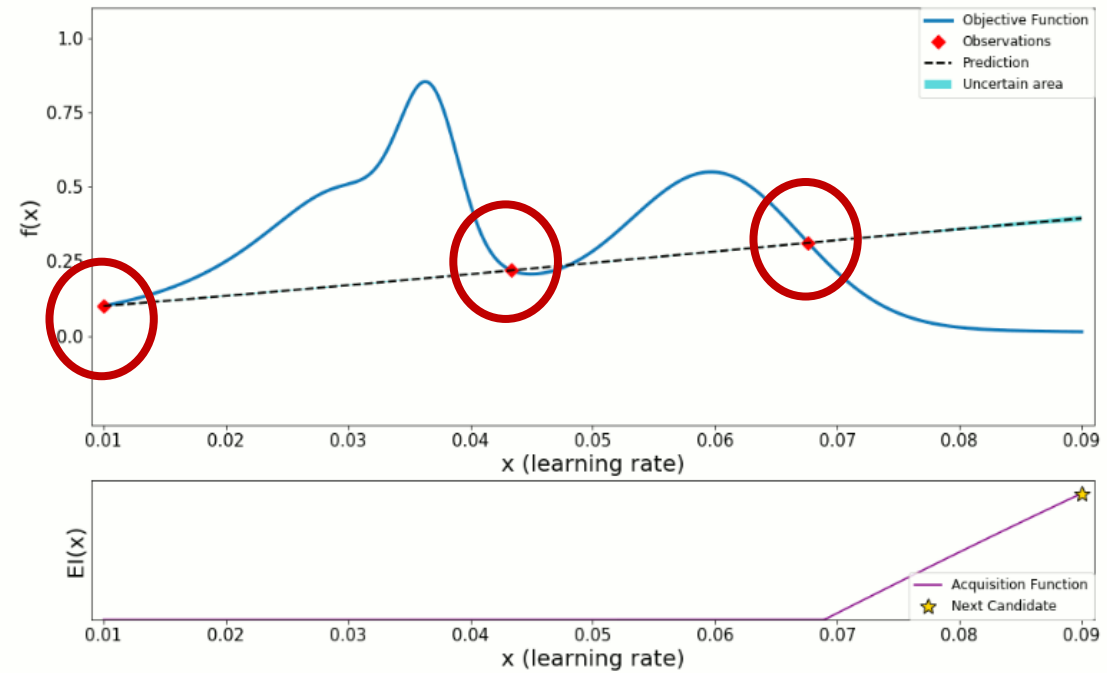
Expected Improvement (EI)



Define input, object function f and other setting values.

- Input: hyperparameters
- Objective function $f(x)$
- Others
 - Set lower/upper bound of x (a,b)
 - Determine N_{warmup} and N
 - N_{warmup} : the number of based points $(x_i, f(x_i))$
 - N : the number of training iteration

Warmup Phase



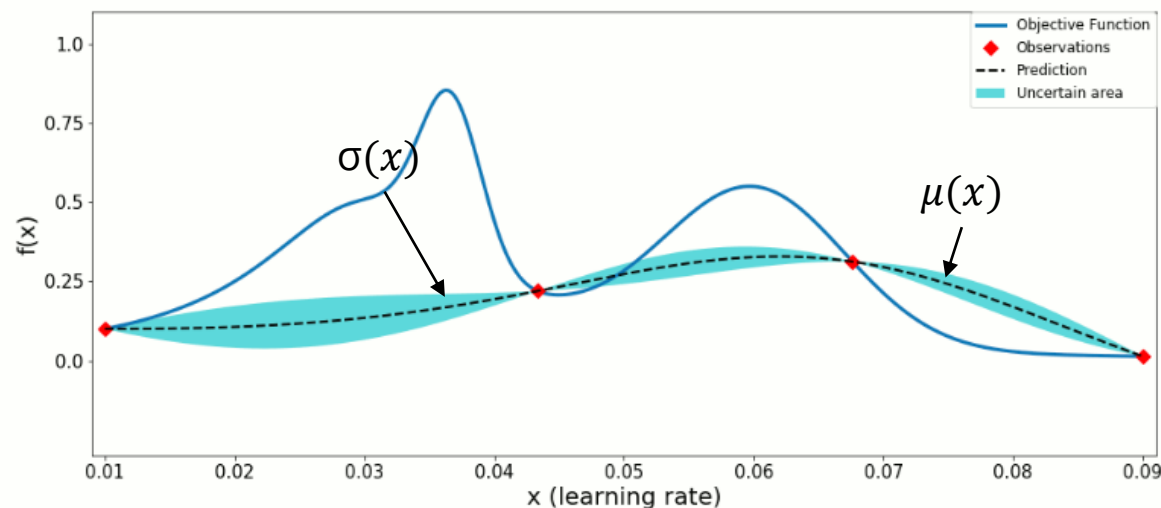
select x_i with random sampling and calculate $f(x_i)$ ($1 \leq i \leq N_{warmup}$)

Bayesian Optimization

$(x_1, f(x_1)), \dots, (x_i, f(x_i))$

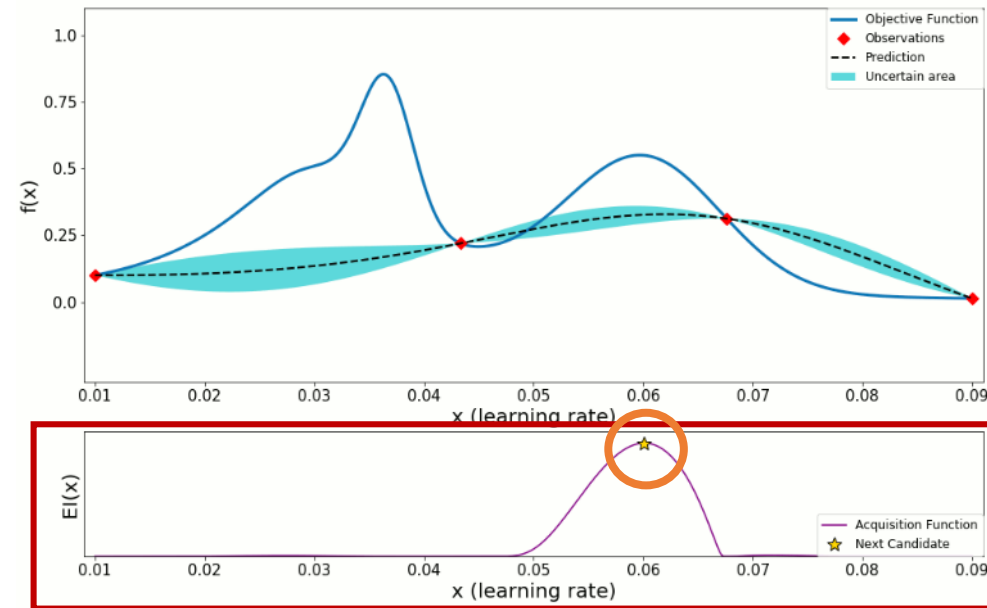


$\sigma(x), \mu(x)$



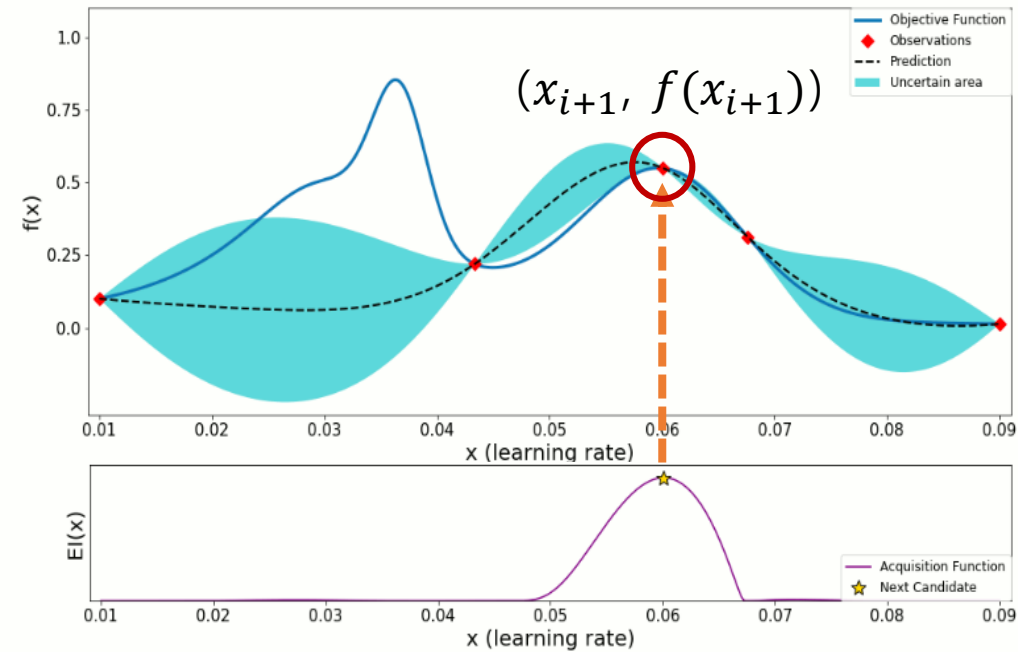
Execute probabilistic modeling and estimation under the gaussian process model, conditioned on all the previous observations of $(x_i, f(x_i))$

Bayesian Optimization



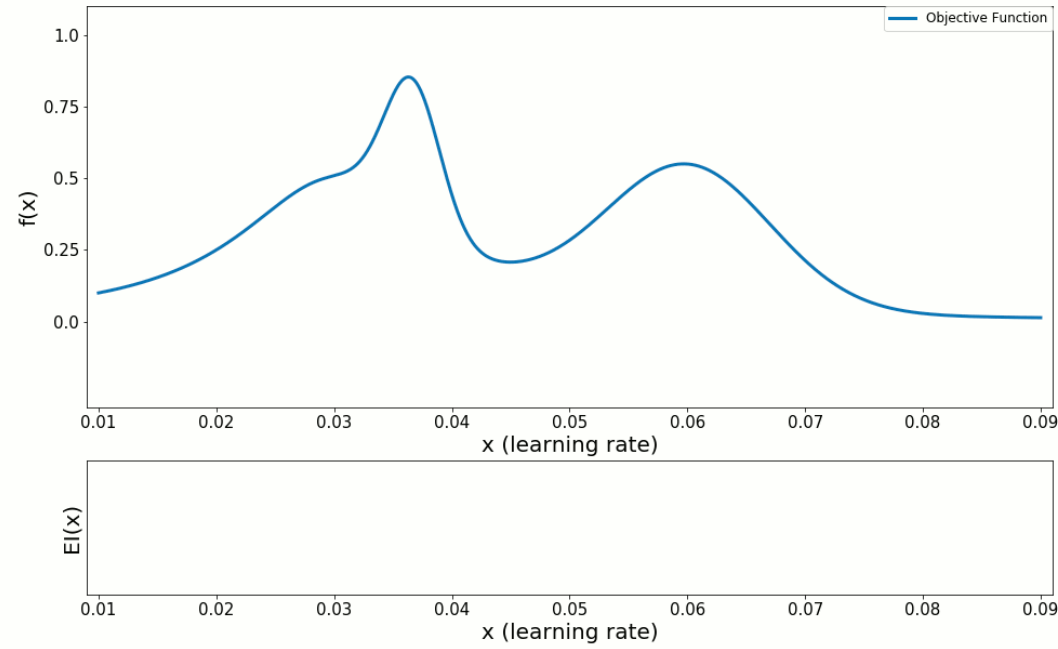
Calculate Expected Improvement function based on $\sigma(\mathbf{x})$, $\boldsymbol{\mu}(\mathbf{x})$, and select $x_{i+1} = \arg EI(x)$

Bayesian Optimization



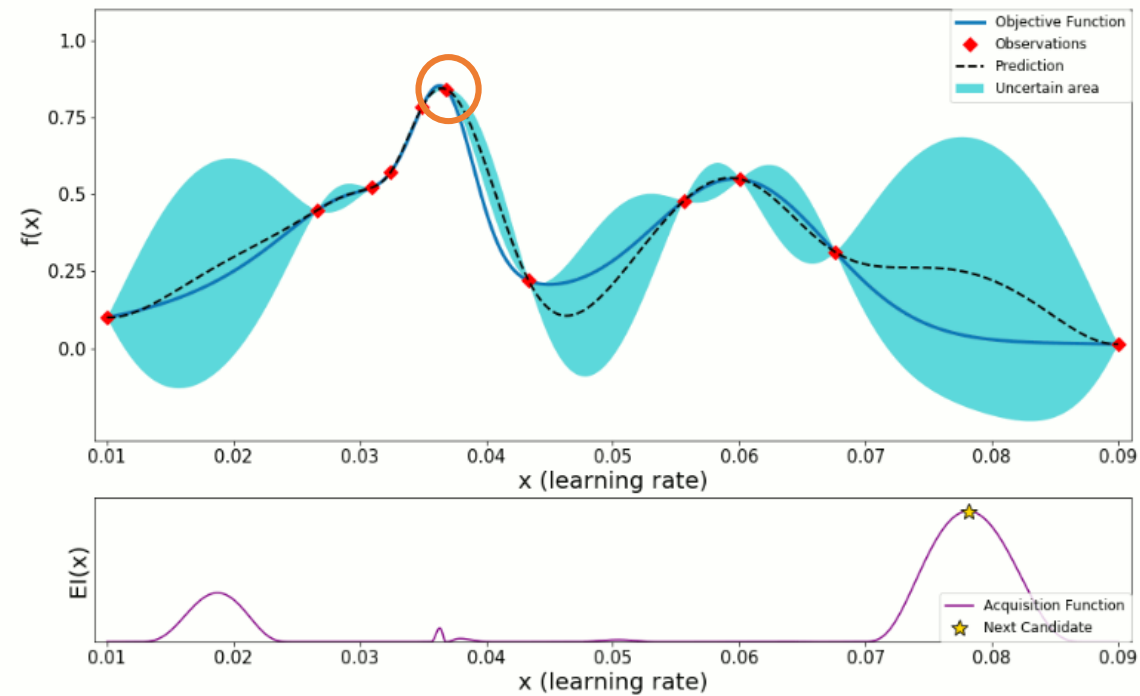
Train DL model with x_{i+1} , and
the performance of the result is considered $f(x_{i+1})$

Bayesian Optimization



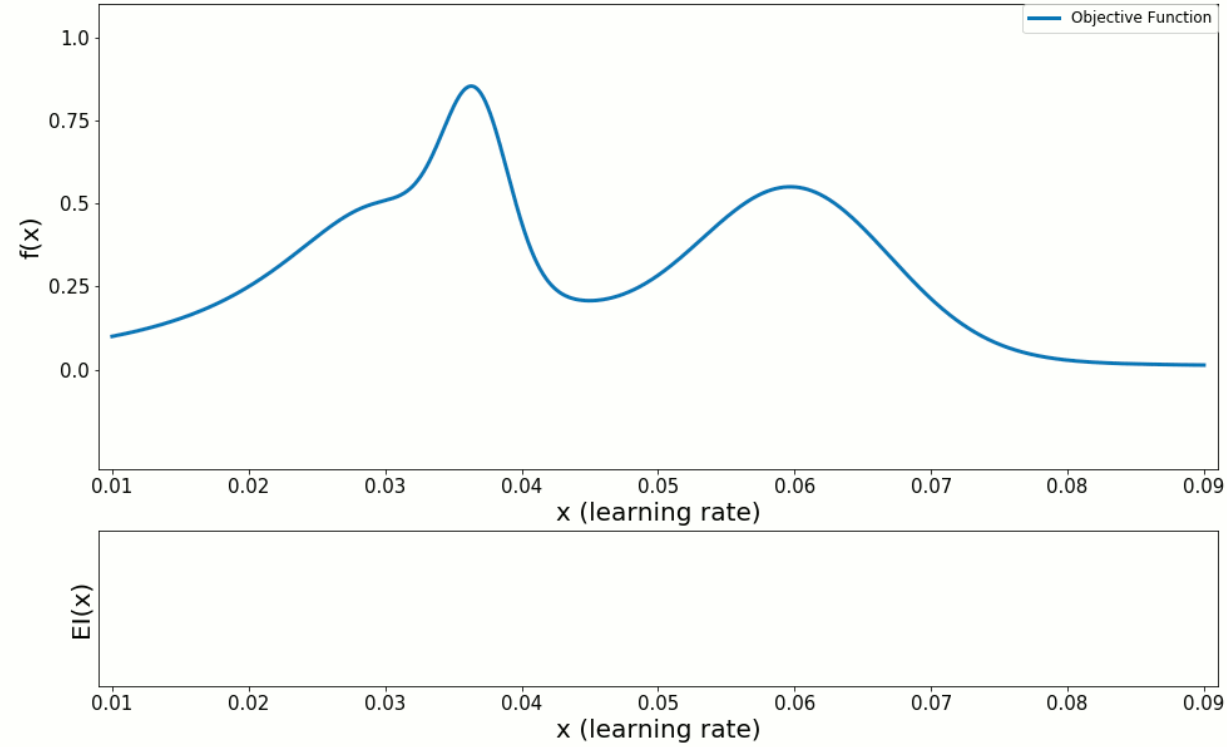
Repeat Upper process until the number of iteration is equal to N

Bayesian Optimization



Select x^* that maximize $\mu(x)$ based on objective function

Bayesian Optimization



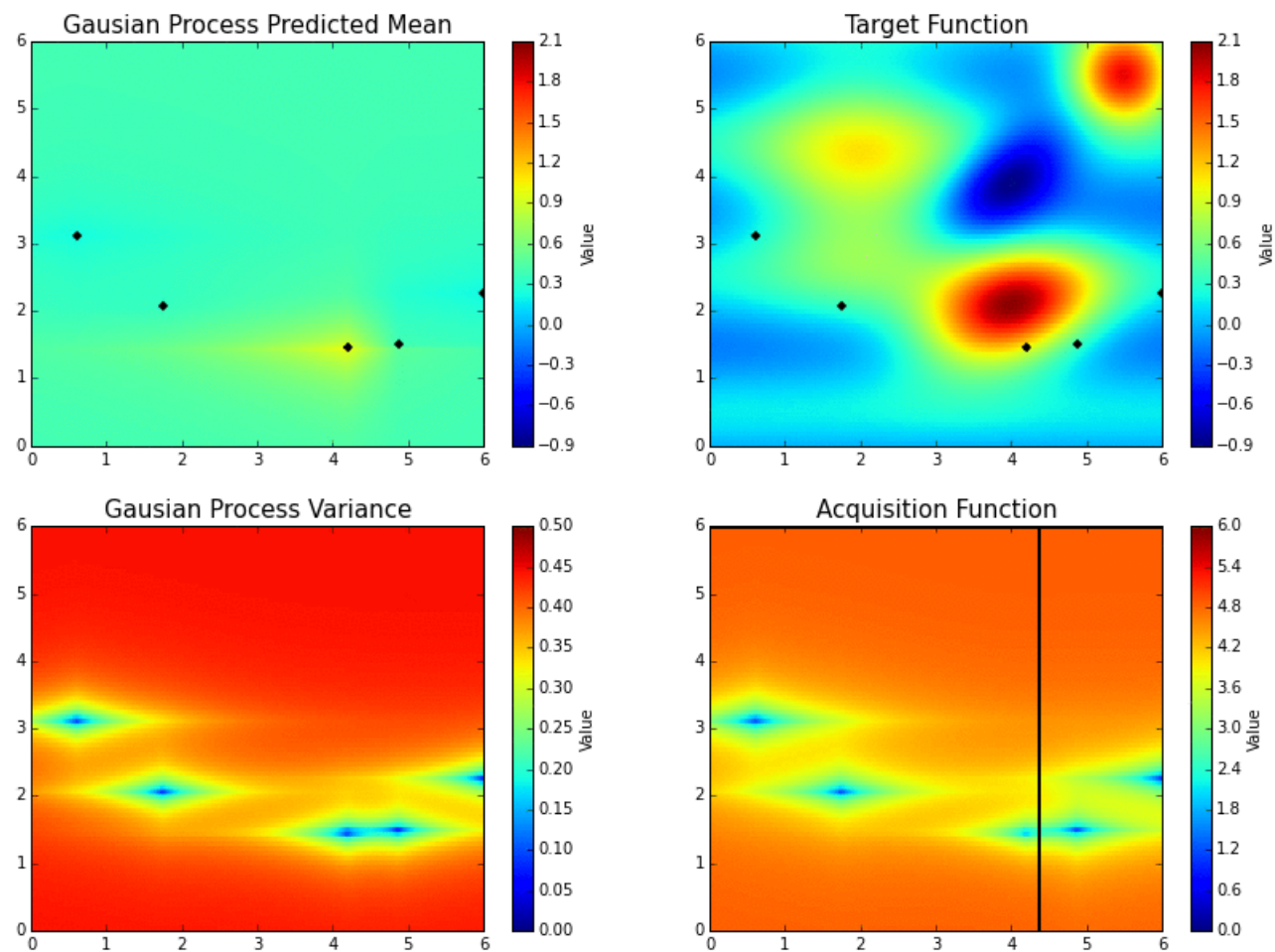
Bayesian Optimization

| | EGO | RBF | DIRECT | GPGO 1-Step | | GPGO 2-Step |
|------|--------------|--------------|--------------|--------------|--------------|--------------|
| | | | | Non-Periodic | Periodic | Non-Periodic |
| Br | 0.943 | 0.960 | 0.958 | 0.980 | — | — |
| C6 | 0.962 | 0.962 | 0.940 | 0.890 | — | 0.967 |
| G-P | 0.783 | 0.815 | 0.989 | 0.804 | — | 0.989 |
| H3 | 0.970 | 0.867 | 0.868 | 0.980 | — | — |
| H6 | 0.837 | 0.701 | 0.689 | 0.999 | — | — |
| Sh5 | 0.218 | 0.092 | 0.090 | 0.485 | — | — |
| Sh7 | 0.159 | 0.102 | 0.099 | 0.650 | — | — |
| Sh10 | 0.135 | 0.100 | 0.100 | 0.591 | — | — |
| GK2 | 0.571 | 0.567 | 0.538 | 0.643 | — | — |
| GK3 | 0.519 | 0.207 | 0.368 | 0.532 | — | — |
| Shu | 0.492 | 0.383 | 0.396 | 0.437 | 0.348 | 0.348 |
| G2 | 0.979 | 1.000 | 0.981 | 1.000 | 1.000 | — |
| G5 | 1.000 | 0.998 | 0.908 | 0.925 | 0.957 | — |
| A2 | 0.347 | 0.703 | 0.675 | 0.606 | 0.612 | 0.781 |
| A5 | 0.192 | 0.381 | 0.295 | 0.089 | 0.161 | — |
| R | 0.652 | 0.647 | 0.776 | 0.675 | 0.933 | — |
| mean | 0.610 | 0.593 | 0.604 | 0.705 | — | — |

BO works better than others in practice

Bayesian Optimization

Bayesian Optimization in Action



S M B O

Sequential Model-Based Optimization



Common Approach

1. Build Gaussian Process (GP) with points
2. Optimize the fit of the GP (covariance
3. Find the point(s) of highest Expected Improvement within parameter domain
4. Return optimal next best point(s) to sample