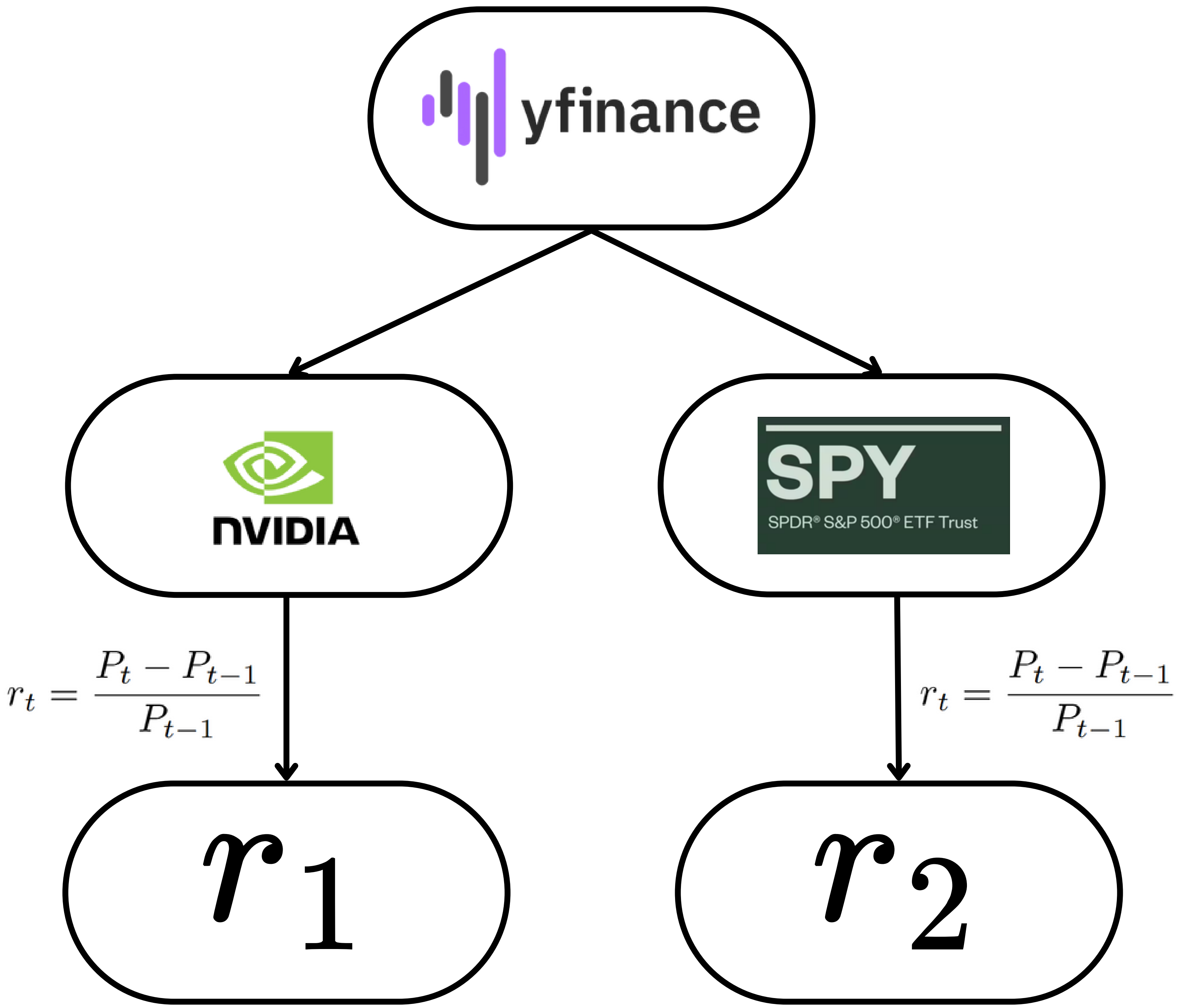
Stress Testing a Markowitz Portfolio

AI & Finance project

Fidanza Riccardo
Loda Enrico
Panariello Luca

Data Collection and Preparation



Dataset creation

Our model leverages 22 key features to make predictions:

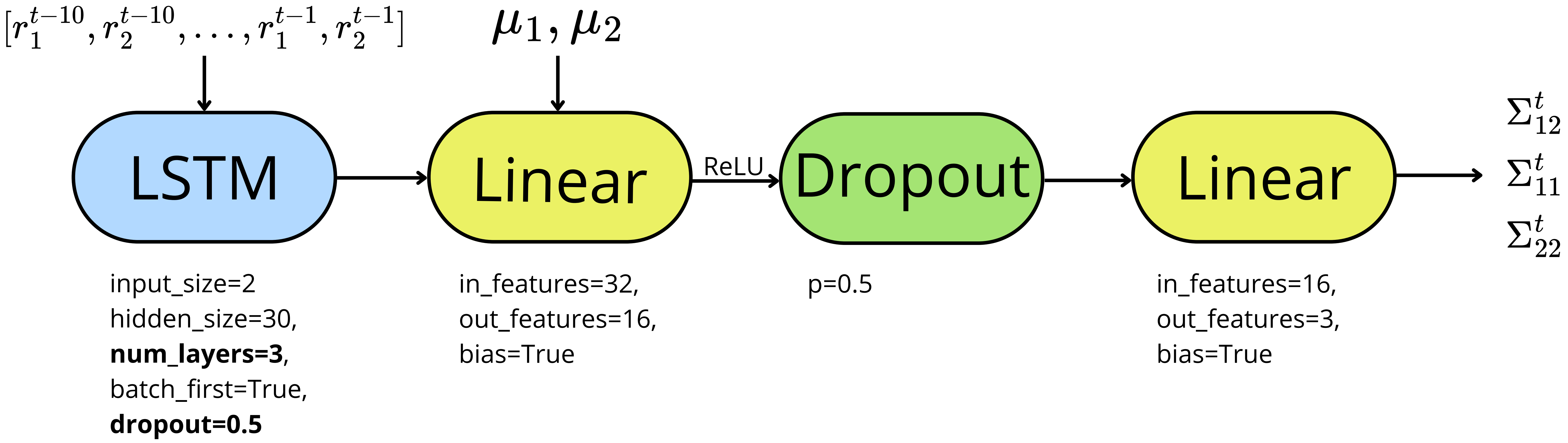
- Lagged Returns: $[r_1^{t-1}, \ldots, r_1^{t-10}], [r_2^{t-1}, \ldots, r_2^{t-10}]$
- Recent Means: μ_1, μ_2

The target features are the values of the covariance matrix between today's returns for both assets.

Training set: 2010/01/01 - 2023/12/31
Test set: 2024/01/01 - 2025/05/30

Markowitz Portfolio Construction

Model implementation



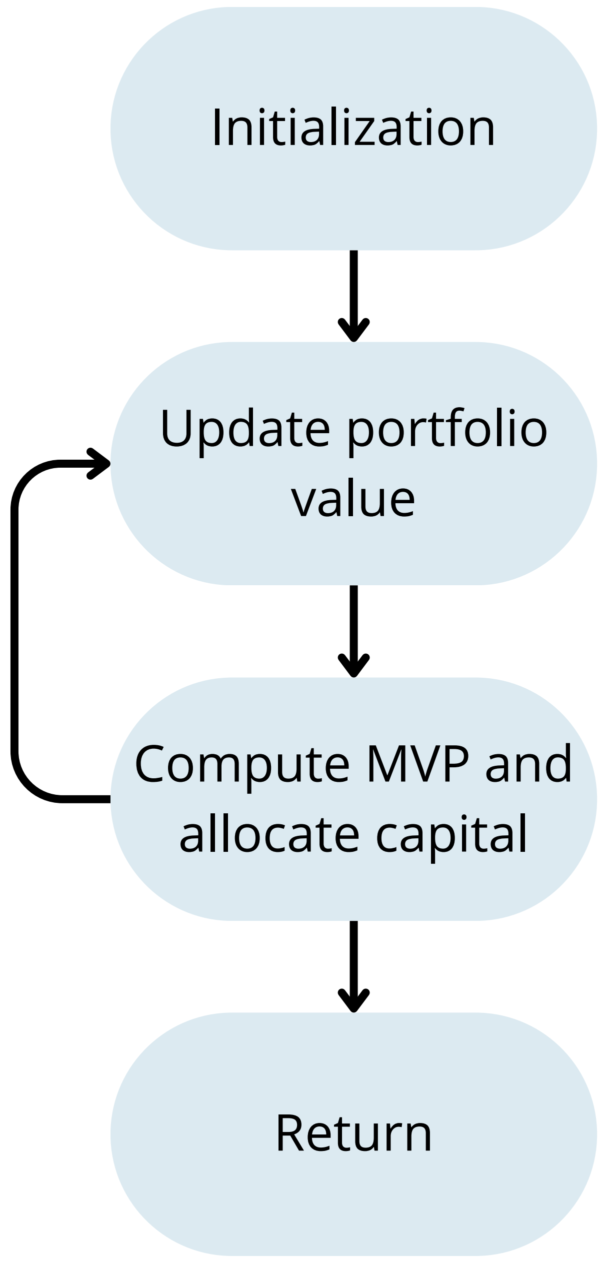
Markowitz Portfolio Construction

Construct the Minimum Variance Portfolio

The portfolio was constructed day by day, each time we:

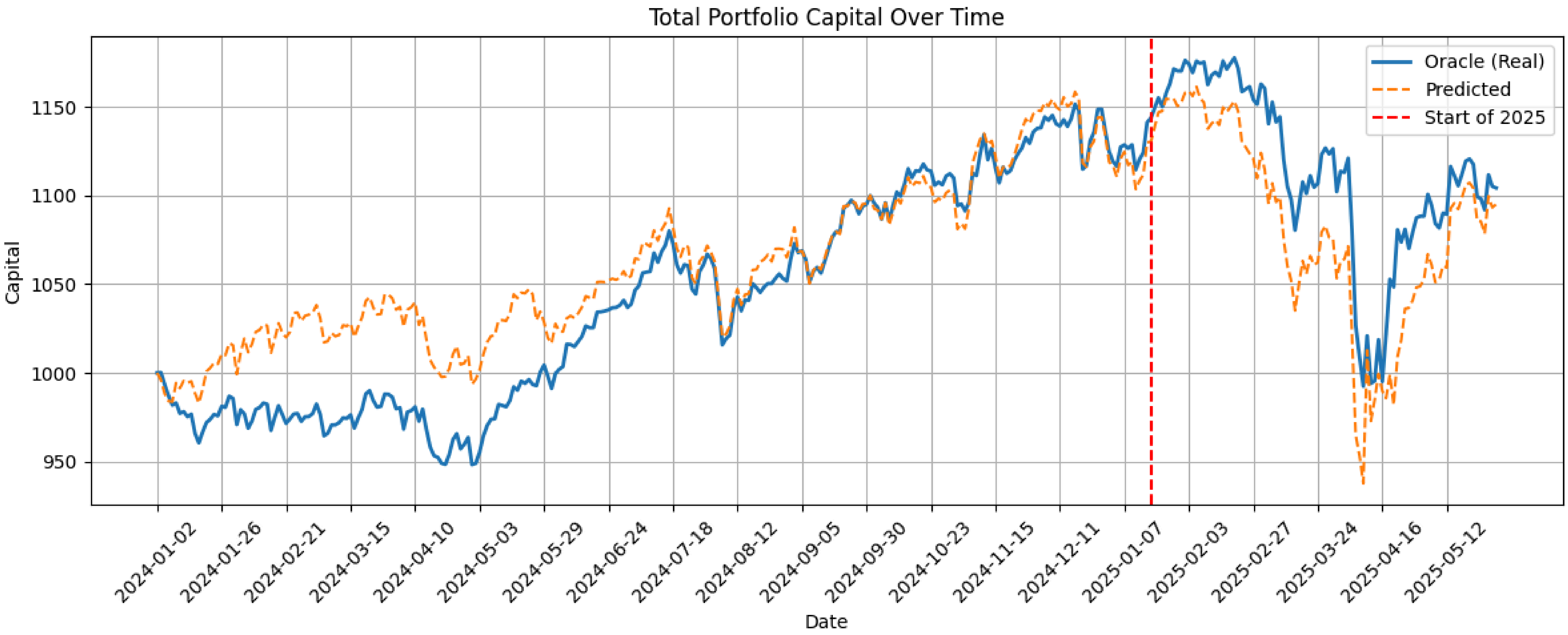
- update the value of the portfolio,
- predict variances and covariance,
- create the daily mvp by implementing the following formula

$$w^* = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho}$$



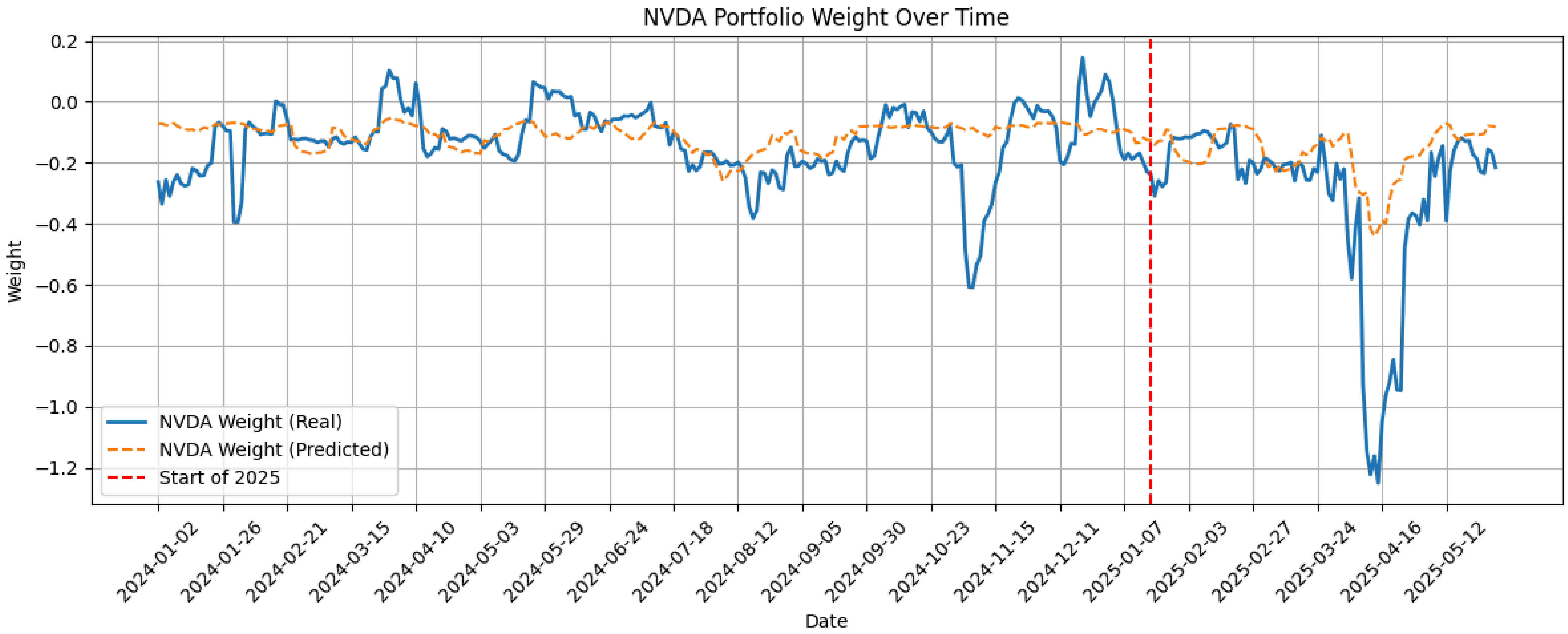
Markowitz Portfolio Construction

Capital of the portfolio over time



Markowitz Portfolio Construction

Weight of NVDA in the portfolio over time



Stress Testing Framework

Value at Risk - Expected Shortfall

These are the formulas implemented to calculate Value at Risk and Expected Shortfall (α=95%) considering the returns:

$$VaR_\alpha(X) = \mu + (\sigma * \Phi^{-1}(1-\alpha))$$

$$ES_\alpha(X) = \mathbb{E}[X | X \leq VaR_\alpha(X)]$$

Stress Testing Framework

Standard Case

To obtain the variance of our portoflio we calculate

$$V_t = w_t^T \Sigma_t w_t$$

Stress Testing Framework

Dynamic Scenario Generation

The models we used for simulating the portfolio returns are:

- ARMA - Mean Estimator
    - Assuming zero mean return it is possible to exploit:

$$\mathbb{E}[Y_t^2] = \mathbb{E}[(Y_t - 0)^2] = \mathbb{E}[(Y_t - \mathbb{E}[Y_t])^2] = \mathrm{Var}(Y_t) = \sigma_t^2$$

- GARCH - Mean and Volatility Estimator

The best parameters for both models are found by checking the minimum BIC.

Stress Testing Framework

ARMA

$$Y_t = \mu + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0, \sigma^2)$$

The best parameters are p=3 and q=2

But high **heteroskedasticity**: **3.55**

Stress Testing Framework

GARCH

$$Y_t = \mu + \sum_{i=1}^{r} \phi_i Y_{t-i} + \epsilon_t$$

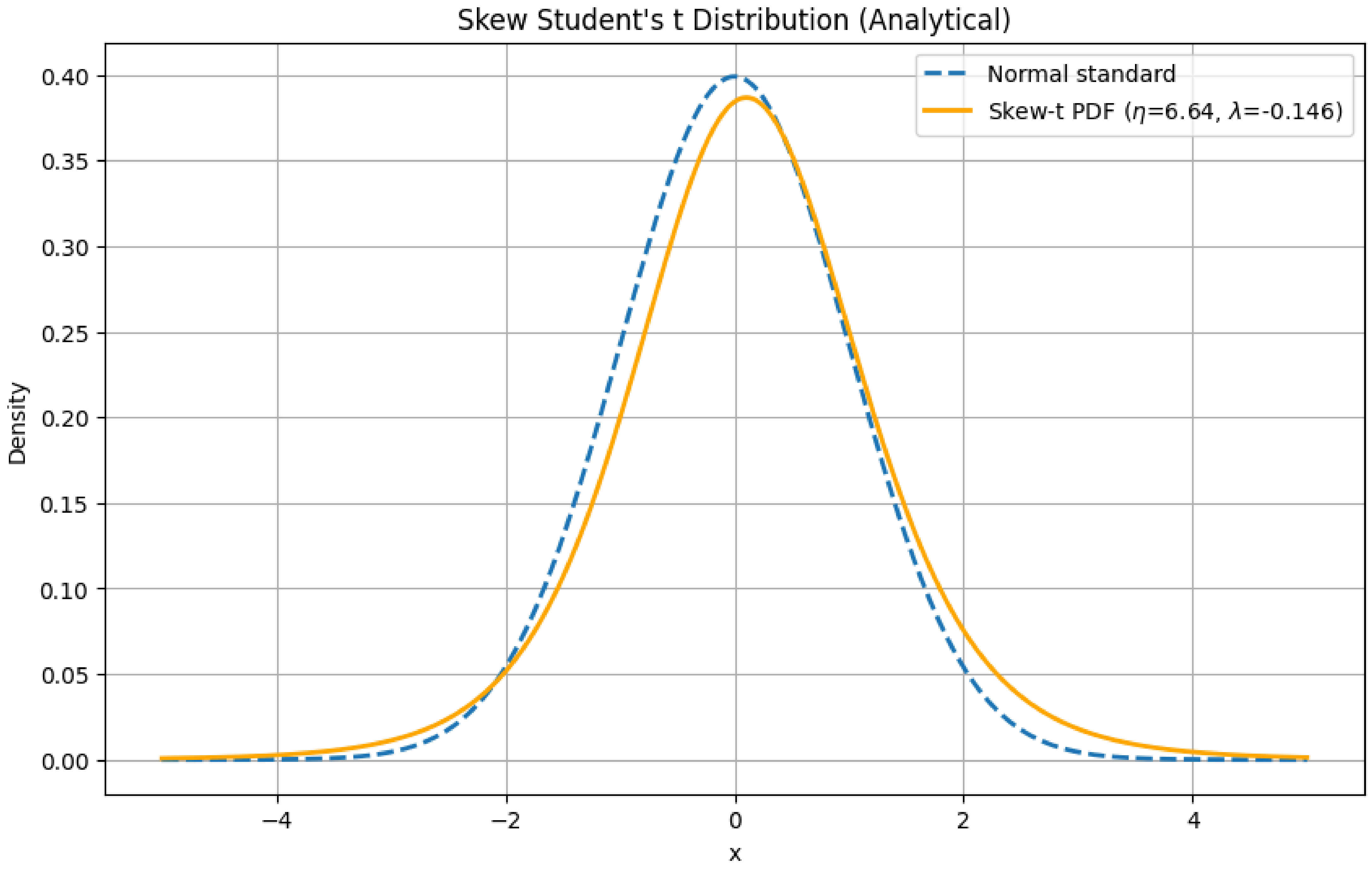
$$\epsilon_t = \sigma_t^2 z \quad z \sim Dist$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

By fixing an AR(r) with r=3 as mean estimator, the best parameters are p=1, q=1 and Dist=SkewT

Stress Testing Framework

SkewT



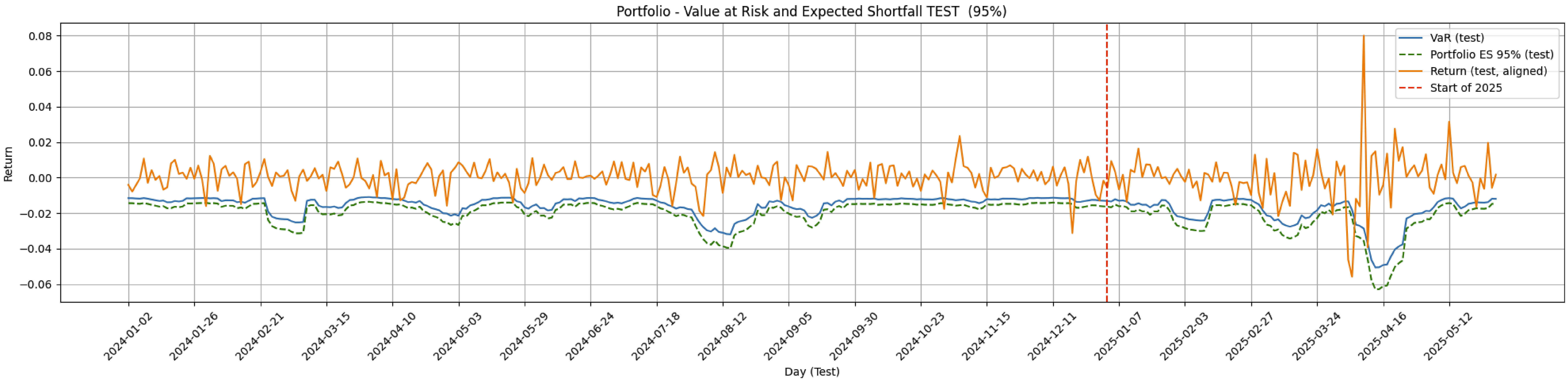
Model Evaluation and Backtesting

Backtesting

	Stress Case VaR Breach Rate	Stress Case - ARMA VaR Breach Rate	Stress Case - GARCH VaR Breach Rate
Train	5.1837%	6.4084%	5.8958%
Test	3.1073%	6.4972%	5.3672%
2024	2.3810%	5.5556%	4.7619%
2025	4.9020%	8.8235%	6.8627%

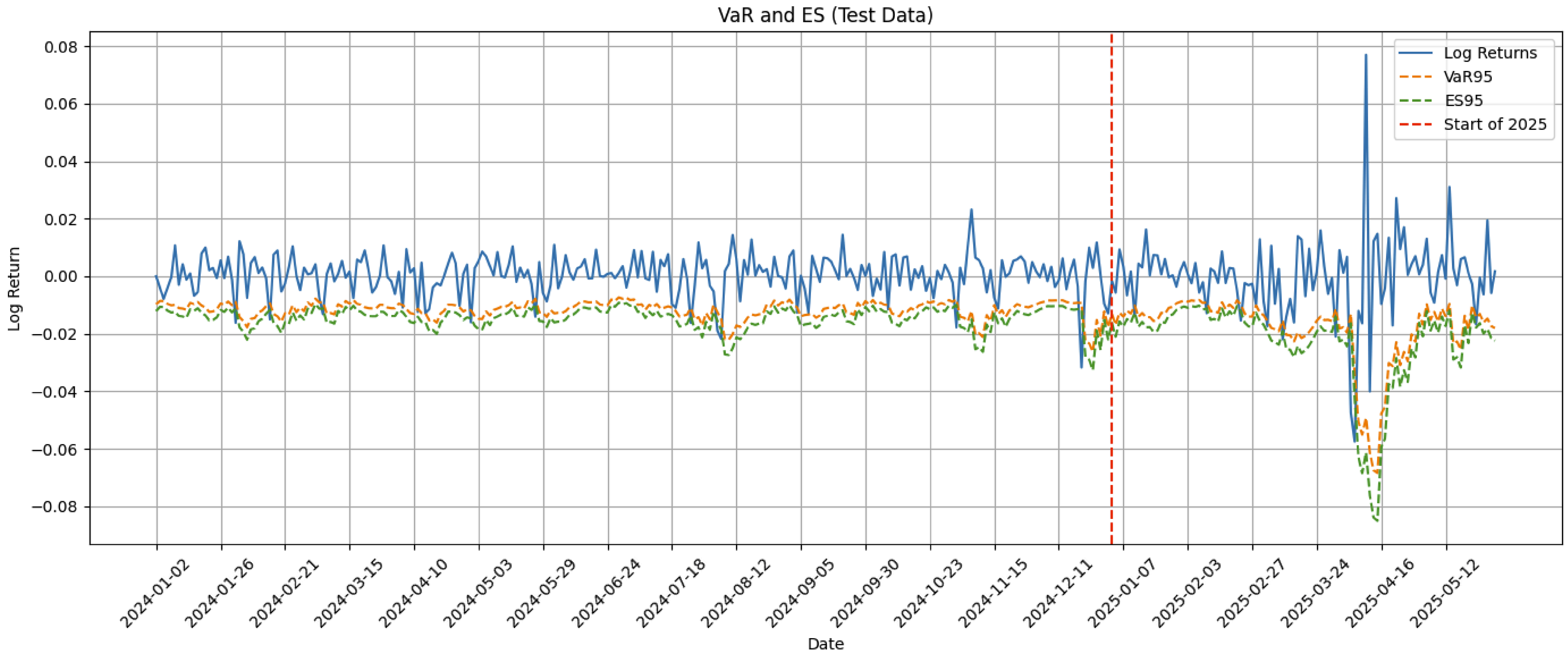
Model Evaluation and Backtesting

Asses Performance - Standard Case



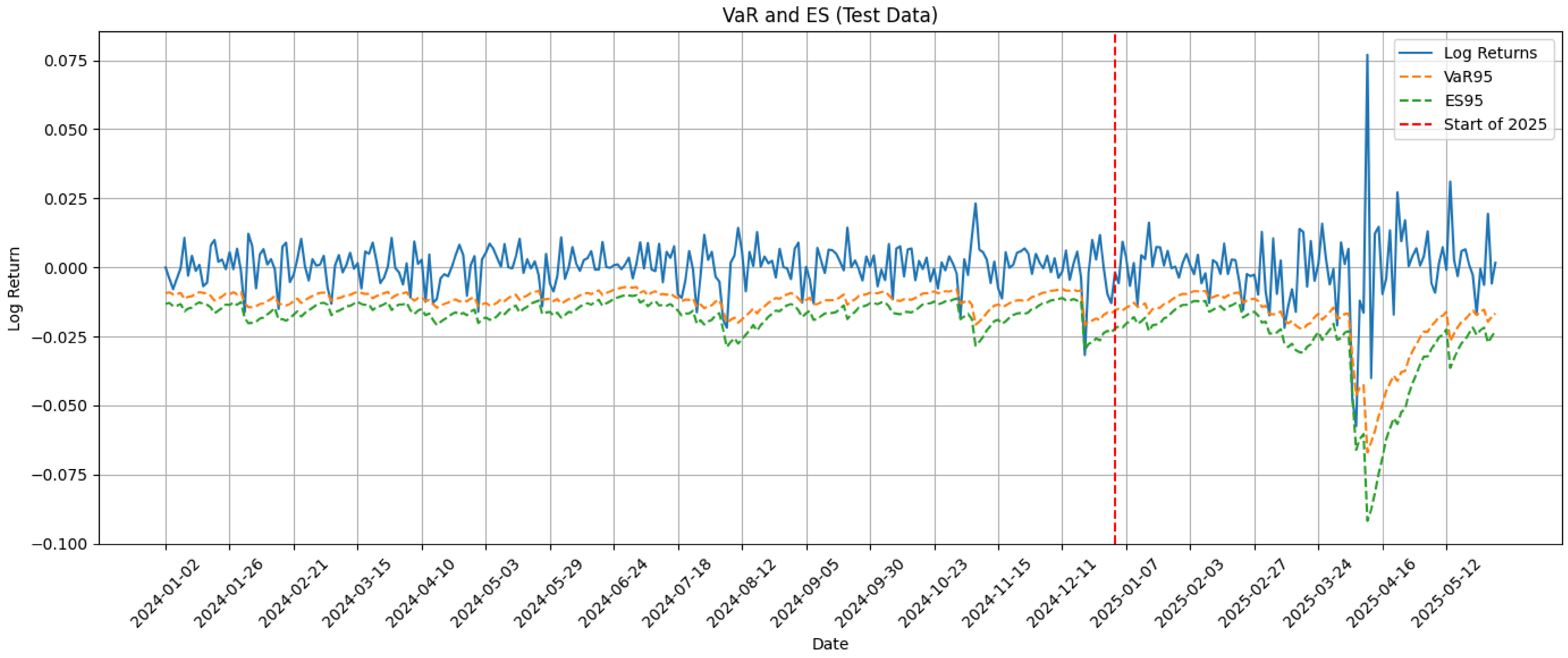
Model Evaluation and Backtesting

Asses Performance - ARMA



Model Evaluation and Backtesting

Asses Performance - GARCH



Thank you

Riccardo Fidanza

Enrico Loda

Luca Panariello