

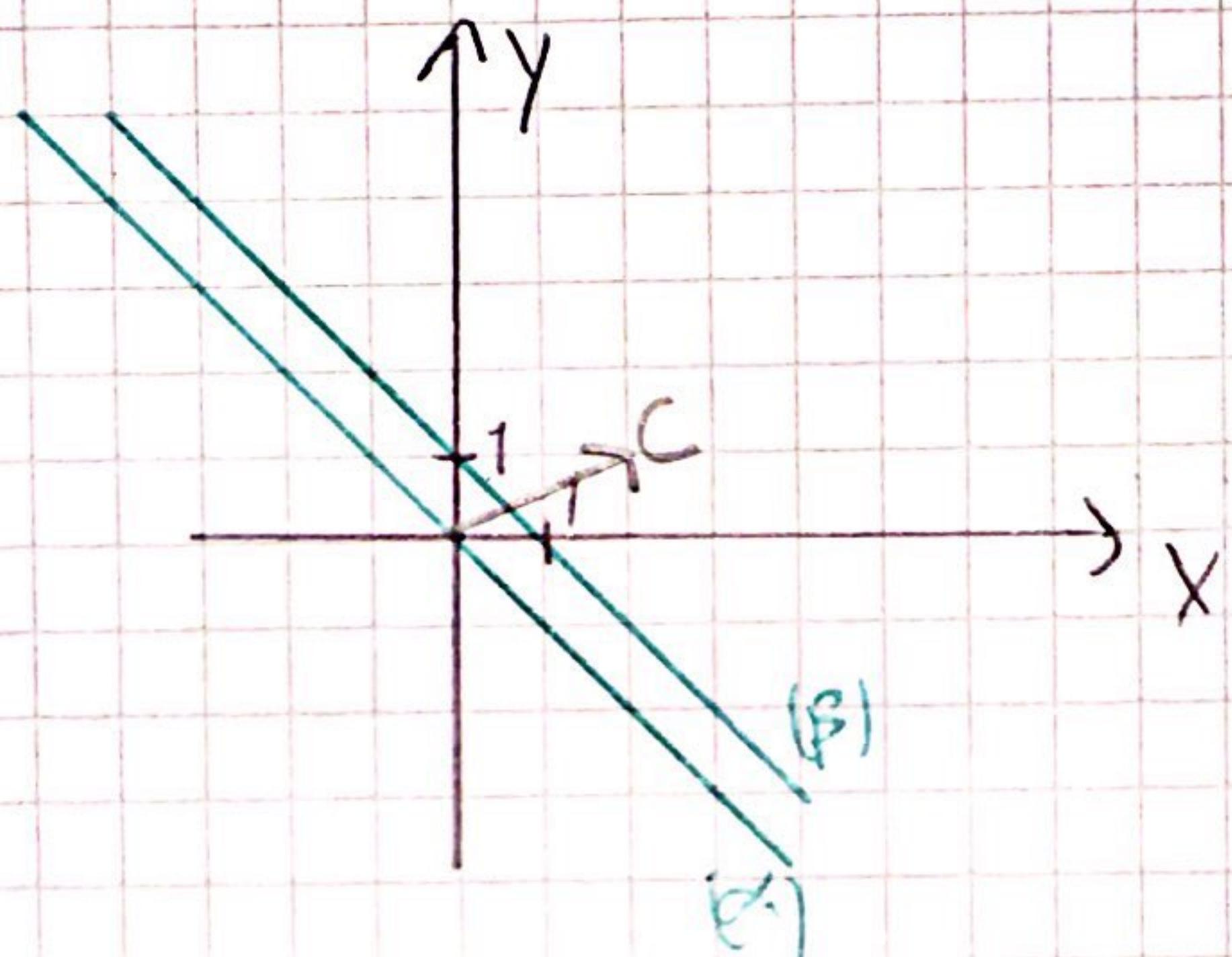
PRACTICO 7

1

a) $\nabla f(x, y) = (2, 1) = C$

(A) $f(x, y) = 0 = 2x + y$

(B) $f(x, y) = 1 = 2x + y$

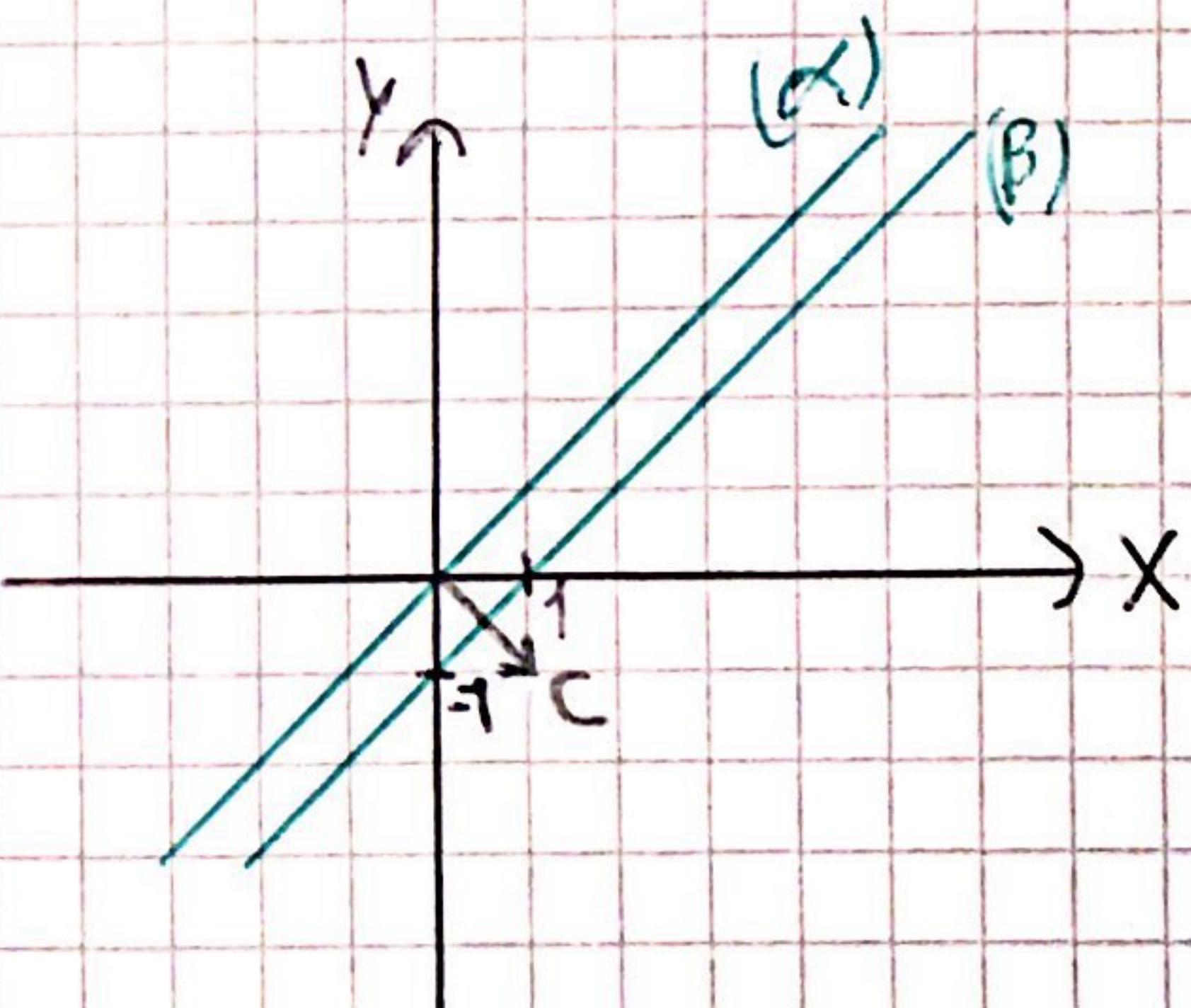


b)

$\nabla g(x, y) = (1, -1)$

(A) $g(x, y) = 0 = x - y$

(B) $g(x, y) = 1 = x - y$

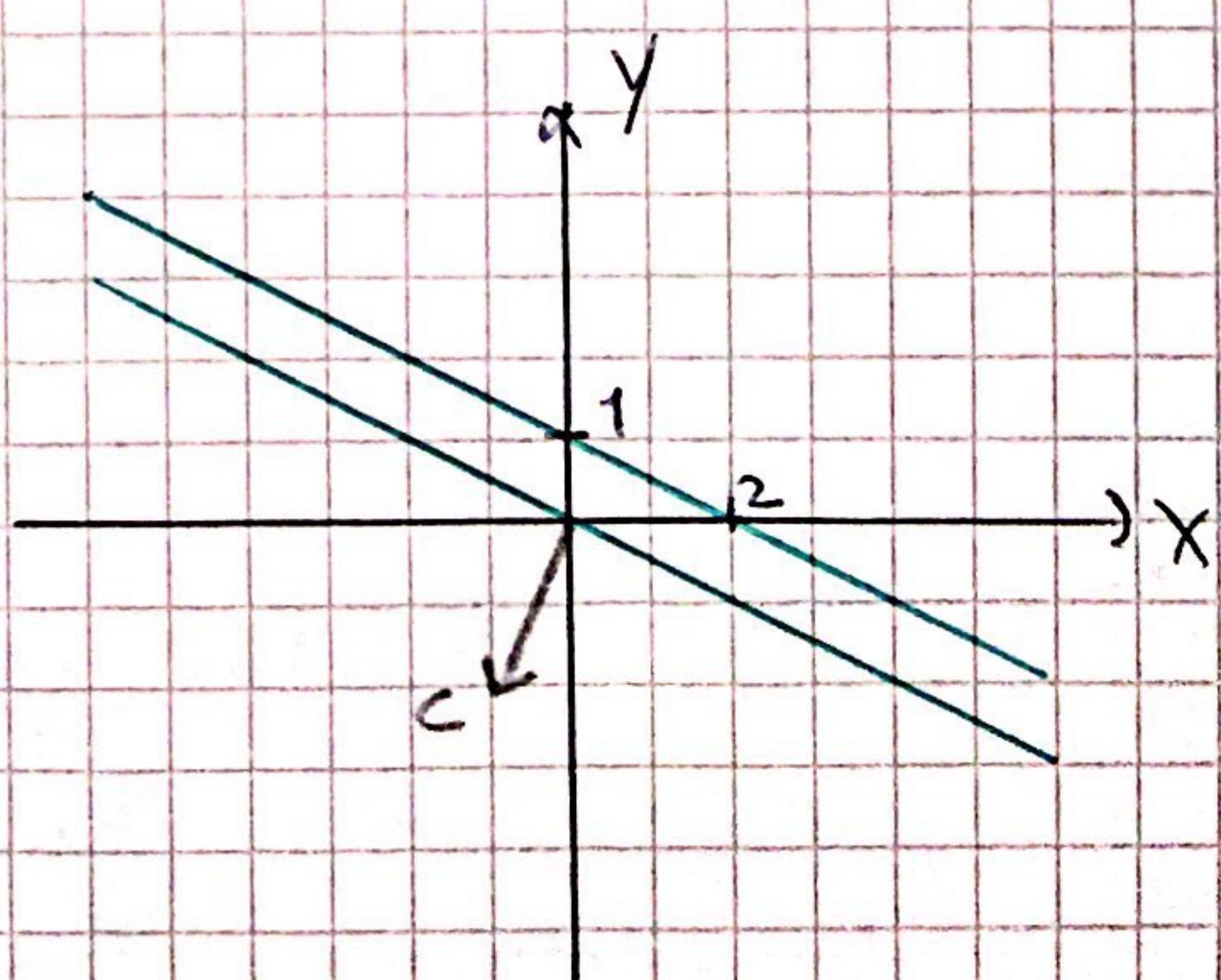


c)

$\nabla h(x, y) = (-1, -2)$

(A) $h(x, y) = 0 = -x - 2y$

(B) $h(x, y) = 1 = -x - 2y$



2

a)

funciónmaximizar \rightarrow minimizarfunción de costo: $f(x) = -3x_1 - 5x_2 + 4x_3$ Variable(s)

$$\bar{x}_1 = x_1 - 1 \geq 0$$

$$\bar{x}_2 = 7 - x_2 \geq 0$$

$$\text{Lugar: } z = -3x_1 - 5x_2 + 4x_3 = -3(\bar{x}_1 + 1) - 5(7 - \bar{x}_2) + 4x_3 = -3\bar{x}_1 + 5\bar{x}_2 + 4x_3$$

$$\left\{ \begin{array}{l} 7(\bar{x}_1 + 1) - 2(7 - \bar{x}_2) - 3x_3 \geq 4 \\ -2(\bar{x}_1 + 1) + 4(7 - \bar{x}_2) + 8x_3 = -3 \end{array} \right.$$

$$\left\{ \begin{array}{l} 5(\bar{x}_1 + 1) - 3(7 - \bar{x}_2) - 2x_3 \leq 9 \\ \bar{x}_1, \bar{x}_2, x_3 \geq 0 \end{array} \right.$$

Restricciones

$$\text{I) } 7\bar{x}_1 + 2\bar{x}_2 - 3x_3 + 7 - 14 \geq 4$$

$$7\bar{x}_1 + 2\bar{x}_2 - 3x_3 \geq 11$$

$$7\bar{x}_1 + 2\bar{x}_2 - 3x_3 - s_1 = 11 \quad \text{con } s_1 \geq 0$$

$$\text{II) } 5\bar{x}_1 + 3\bar{x}_2 - 2x_3 + 5 - 21 \leq 9$$

$$5\bar{x}_1 + 3\bar{x}_2 - 2x_3 \leq 25$$

$$5\bar{x}_1 + 3\bar{x}_2 - 2x_3 + s_2 = 25 \quad \text{con } s_2 \geq 0$$

Finalmente:

$$\min: \bar{z} = -3\bar{x}_1 + 5\bar{x}_2 + 4\bar{x}_3 - 38$$

$$\text{s. a.: } 7\bar{x}_1 + 2\bar{x}_2 - 3\bar{x}_3 - s_1 = 11$$

$$-2\bar{x}_1 - 4\bar{x}_2 + 8\bar{x}_3 = -29$$

$$5\bar{x}_1 + 3\bar{x}_2 - 2\bar{x}_3 + s_2 = 25$$

$$\bar{x}_1, \bar{x}_2, \bar{x}_3, s_1, s_2 \geq 0$$

b)

Variables

$$\bar{x}_1 = x_1 - 2 \geq 0$$

$$x_3 = \bar{x}_3 - \hat{x}_3 \text{ con } \bar{x}_3, \hat{x}_3 \geq 0$$

Luego:

$$\bar{z} = x_1 - 5x_2 - 7x_3 = (\bar{x}_1 + 2) - 5x_2 - 7(\bar{x}_3 - \hat{x}_3) = \bar{x}_1 - 5x_2 - 7\bar{x}_3 + 7\hat{x}_3 + 2$$

$$\begin{cases} 5(\bar{x}_1 + 2) - 2x_2 + 6(\bar{x}_3 - \hat{x}_3) \geq 5 \\ 2(\bar{x}_1 + 2) + 4x_2 - 9(\bar{x}_3 - \hat{x}_3) = 3 \\ 7(\bar{x}_1 + 2) + 3x_2 + 5(\bar{x}_3 - \hat{x}_3) \leq 9 \\ \bar{x}_1, \bar{x}_3, \hat{x}_3, x_2 \geq 0 \end{cases}$$

Restricciones

$$I) 5\bar{x}_1 - 2x_2 + 6\bar{x}_3 - 6\hat{x}_3 + 10 \geq 5$$

$$5\bar{x}_1 - 2x_2 + 6\bar{x}_3 - 6\hat{x}_3 \geq -5$$

$$5\bar{x}_1 - 2x_2 + 6\bar{x}_3 - 6\hat{x}_3 - s_1 = -5 \quad \text{con } s_1 \geq 0$$

II)

$$7\bar{x}_1 + 3\bar{x}_2 + 5\bar{x}_3 - 5\hat{x}_3 + 14 \leq 9$$

$$7\bar{x}_1 + 3\bar{x}_2 + 5\bar{x}_3 - 5\hat{x}_3 \leq -5$$

$$7\bar{x}_1 + 3\bar{x}_2 + 5\bar{x}_3 - 5\hat{x}_3 + S_2 = -5 \quad \text{con } S_2 \geq 0$$

Finalmente:

$$\min: \hat{z} = \bar{x}_1 - 5\bar{x}_2 - 7\bar{x}_3 + 7\hat{x}_3 + 2$$

$$\text{S.O.: } 5\bar{x}_1 - 2\bar{x}_2 + 6\bar{x}_3 - 6\hat{x}_3 - S_1 = -5,$$

$$3\bar{x}_1 + 4\bar{x}_2 - 9\bar{x}_3 - 9\hat{x}_3 = -3$$

$$7\bar{x}_1 + 3\bar{x}_2 + 5\bar{x}_3 - 5\hat{x}_3 + S_2 = -5$$

$$\bar{x}_1, \bar{x}_2, \bar{x}_3, \hat{x}_3, S_1, S_2 \geq 0$$

3

a) $C = \nabla f = (3, 1)$

$$\textcircled{1} \quad x - y \leq 1$$

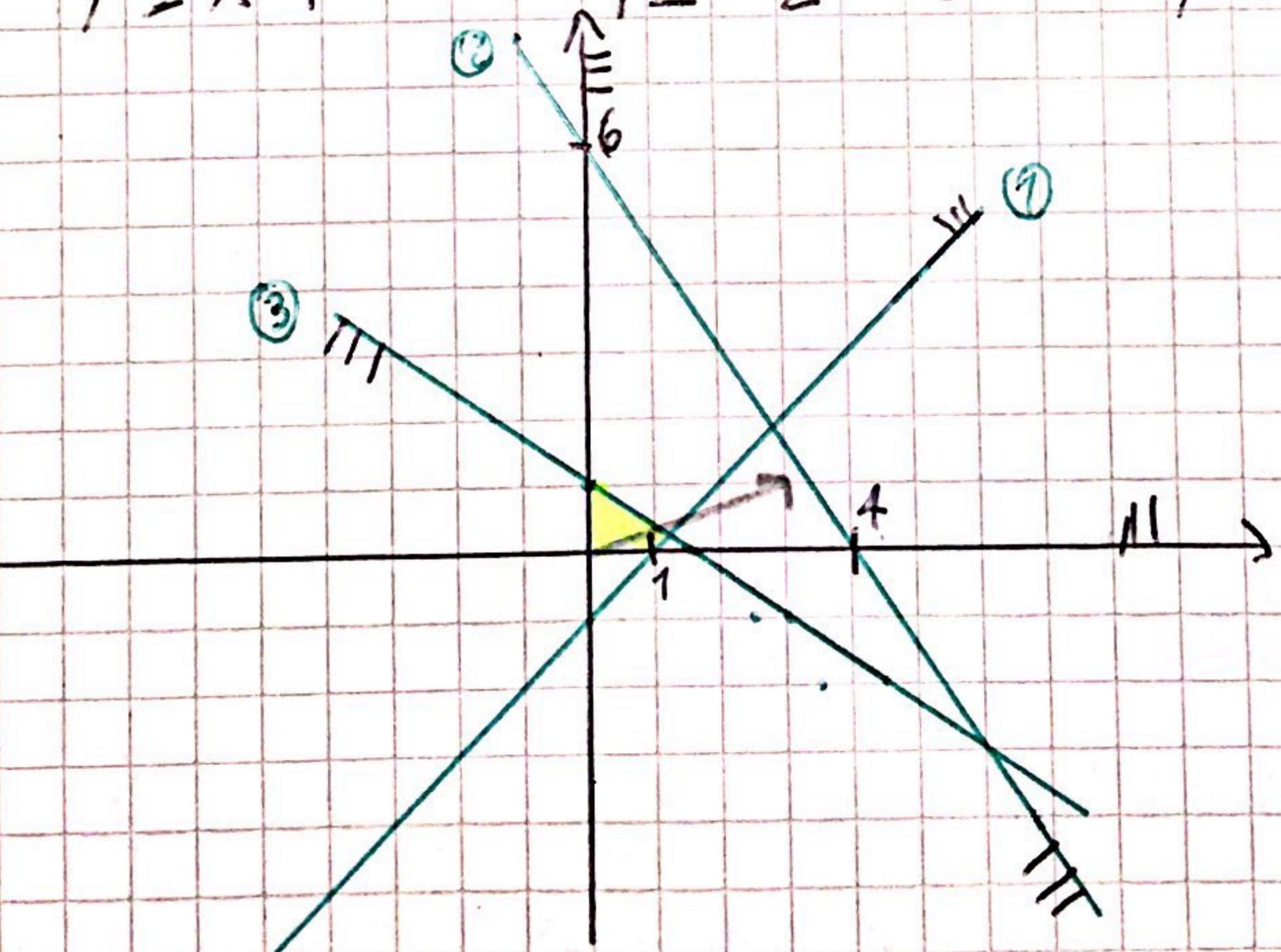
$$y \geq x - 1$$

$$\textcircled{2} \quad 3x + 2y \leq 12$$

$$y \leq -\frac{3}{2}x + 6$$

$$\textcircled{3} \quad 2x + 3y \leq 3$$

$$y \leq -\frac{2}{3}x + 1$$

minimo: $(0,0)$

b)

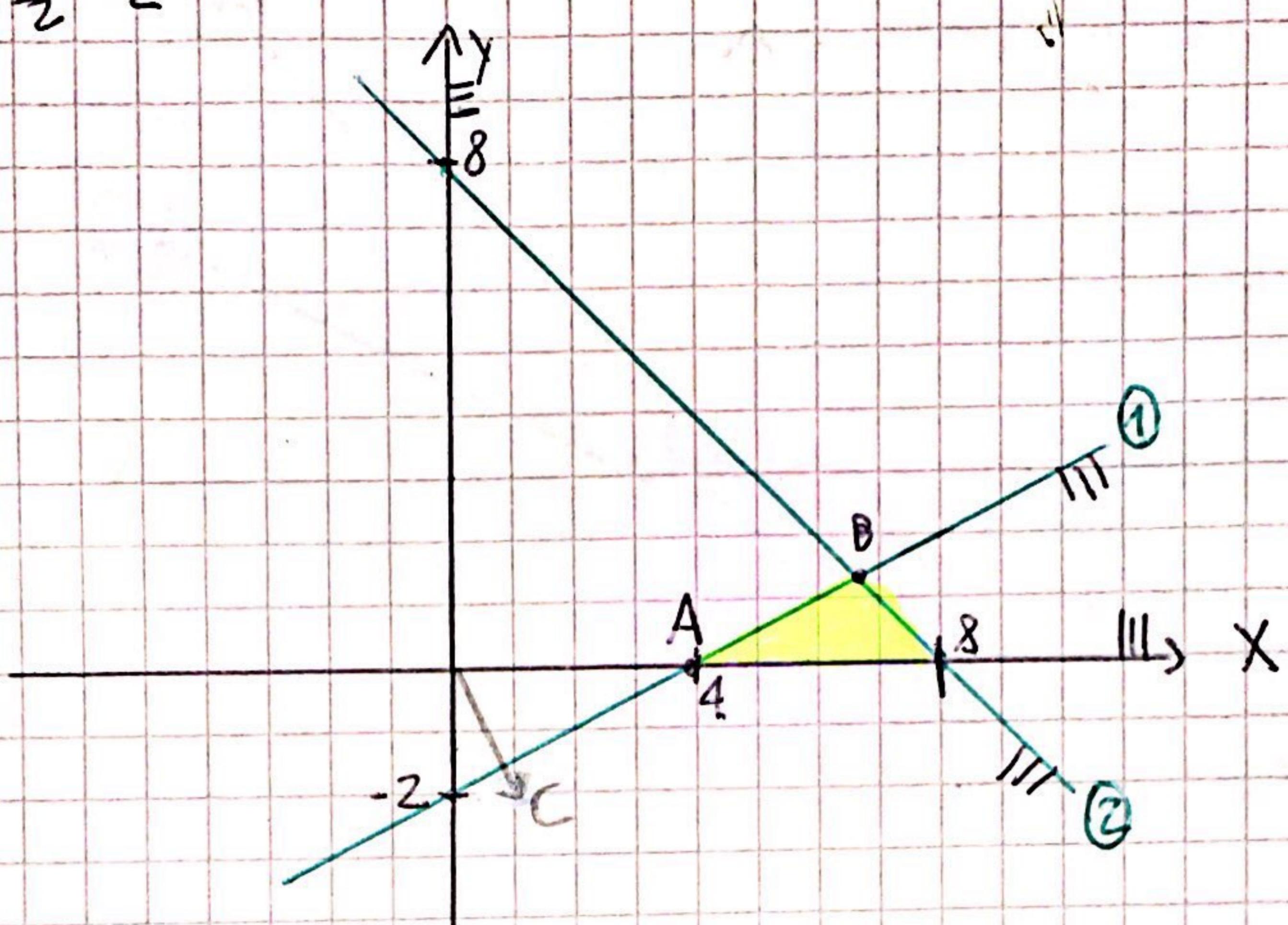
$$C = \nabla f = (1, -2)$$

$$\textcircled{1} \quad x - 2y \geq 4$$

$$y \leq \frac{x}{2} - 2$$

$$\textcircled{2} \quad x + y \leq 8$$

$$y \leq -x + 8$$



Dado que $C \perp \textcircled{1}$:

minimo: segmento \overline{AB}

c)

$$\nabla f = (1, 2)$$

$$\textcircled{1} \quad 2x + y \geq 12$$

$$y \geq -2x + 12$$

$$\textcircled{2} \quad x + y \geq 5$$

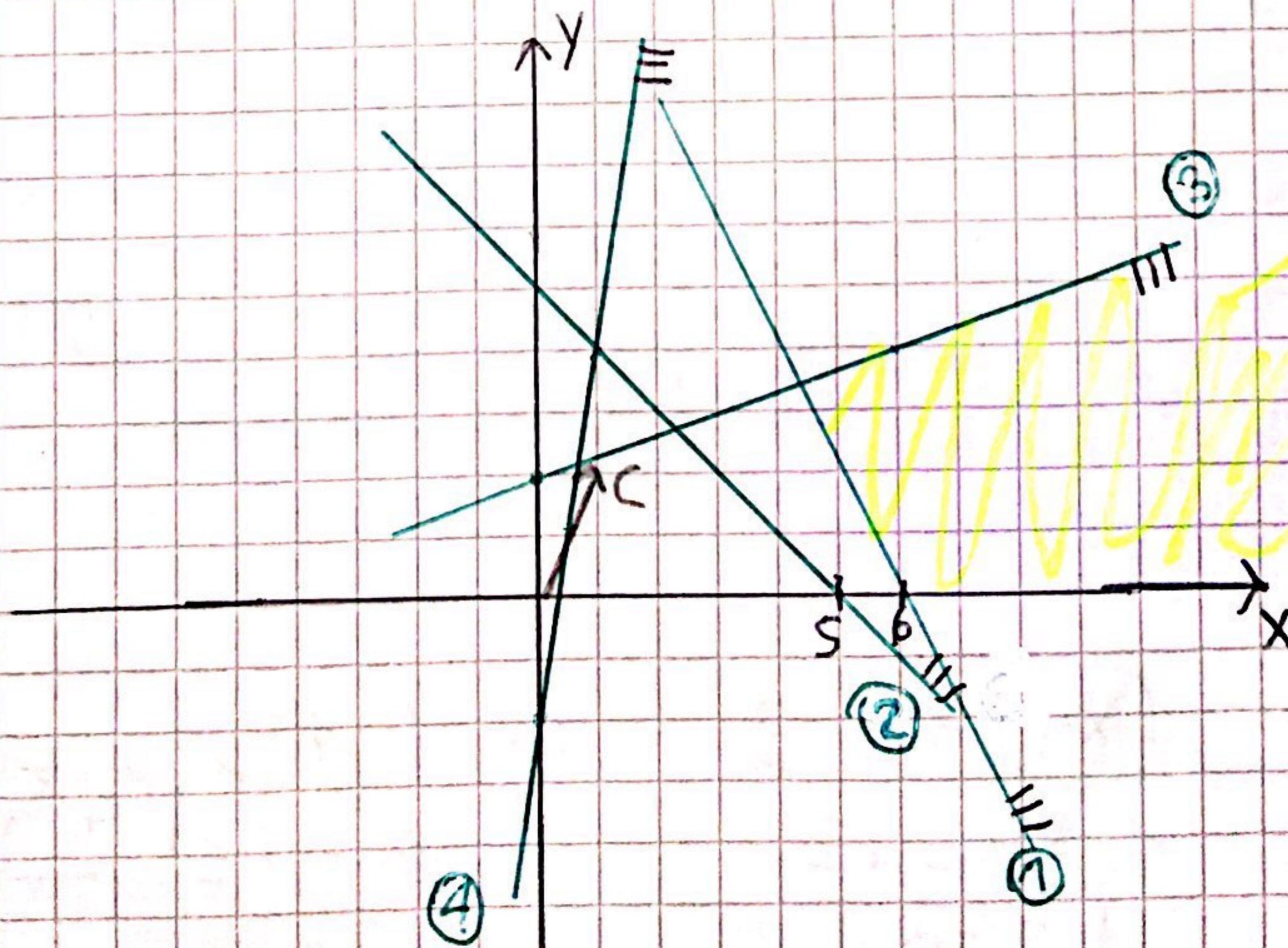
$$y \geq -x + 5$$

$$\textcircled{3} \quad -x + 3y \leq 3$$

$$y \leq \frac{x}{3} + 1$$

$$\textcircled{4} \quad 6x - y \geq 2$$

$$y \leq 6x - 2$$



\therefore No tiene max

d)

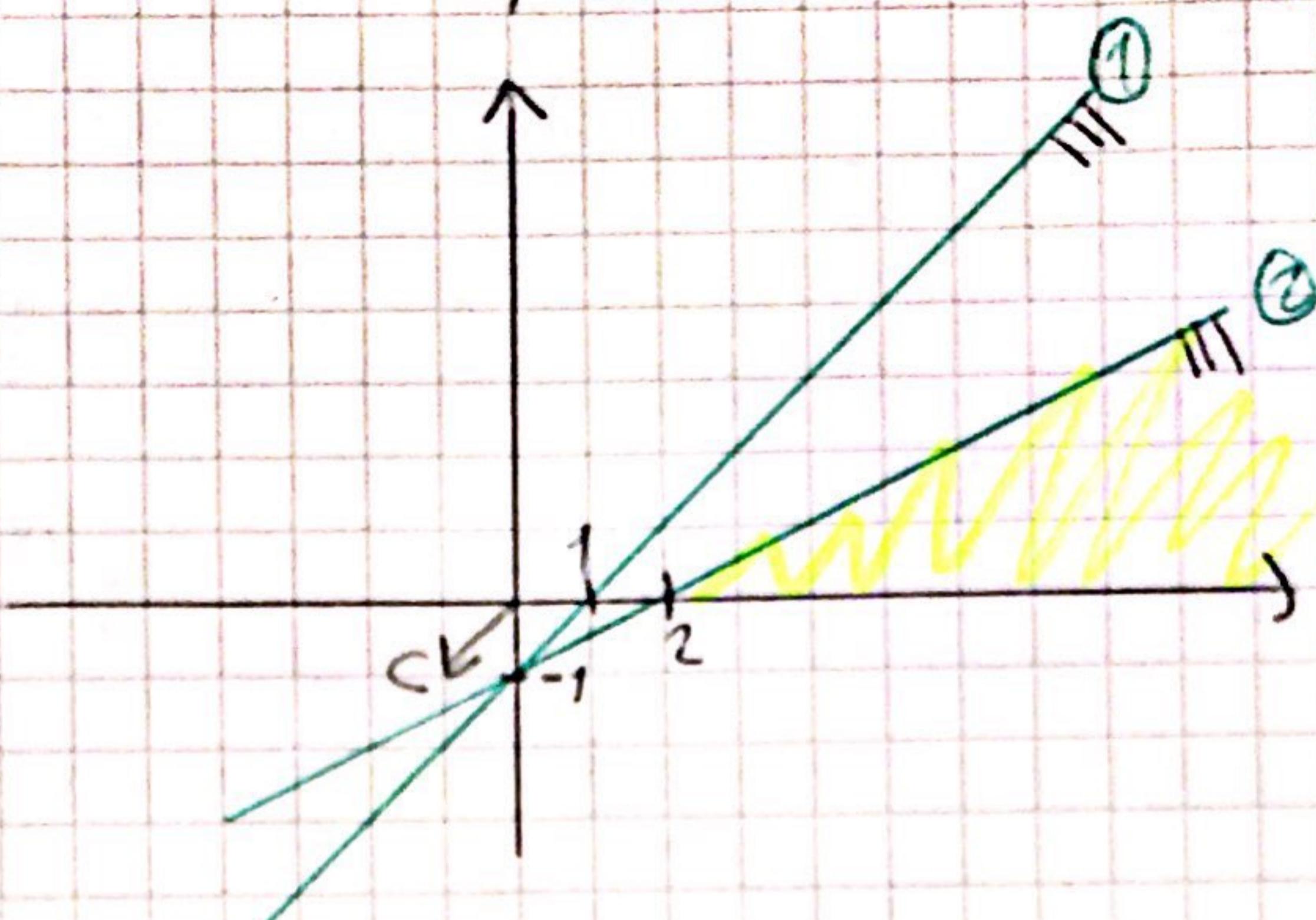
$$\nabla f = (-1, -1)$$

$$① x - y \geq 1$$

$$y \leq x - 1$$

$$② x - 2y \geq 2$$

$$y \leq \frac{x}{2} - 1$$



∴ No tiene mínimo

4)

$$\underline{A} \quad \begin{cases} x + y + z = 3 \\ y - z = 2 \\ x - 2y = 1 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & 2 \\ 1 & -2 & 0 & 1 \end{array} \right) \xrightarrow{F_3 - F_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & 0 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & 0 & 1 \end{array} \right) \xrightarrow{F_1 - F_2} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & -3 & 0 & 1 \end{array} \right) \xrightarrow{F_3 + 3F_2} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -4 & 4 \end{array} \right) \xrightarrow{-\frac{F_3}{4}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{F_1 - 2F_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array} \right) \quad P_1: (3, 1, -1)$$

B)

$$\begin{cases} x + y + z = 3 \\ y - z = 2 \\ x = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_1 - F_3} \left(\begin{array}{ccc|c} 0 & 0 & 2 & 3 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_1 - F_2} \left(\begin{array}{ccc|c} 0 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{2F_2} \left(\begin{array}{ccc|c} 0 & 0 & 2 & 1 \\ 0 & 2 & -2 & 4 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_2 + F_1} \left(\begin{array}{ccc|c} 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 5 \\ 1 & 0 & 0 & 0 \end{array} \right) \quad P_2: (0, \frac{5}{2}, \frac{1}{2})$$

$$\text{C} \quad \left\{ \begin{array}{l} y-z=2 \\ x-2z=1 \\ x=0 \end{array} \right. \quad \left(\begin{array}{ccc|c} 0 & 1 & -1 & 2 \\ 1 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_2 - F_1} \left(\begin{array}{ccc|c} 0 & 1 & -1 & 2 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{2F_1} \left(\begin{array}{ccc|c} 0 & 1 & -1 & 2 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 0 & 2 & -2 & 4 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_1 + F_2} \left(\begin{array}{ccc|c} 0 & 0 & -2 & 5 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \quad P_3: (0, -\frac{1}{2}, -\frac{5}{2})$$

$$\text{D} \quad \left\{ \begin{array}{l} x+y+z=3 \\ x-2y=1 \\ x=0 \end{array} \right. \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_1 - F_3} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{2F_1} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 3 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 0 & 2 & 2 & 6 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \xrightarrow{F_1 + F_2} \left(\begin{array}{ccc|c} 0 & 0 & 2 & 7 \\ 0 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{array} \right) \quad P_4: (0, -\frac{1}{2}, \frac{7}{2})$$

Veamos si P_1, P_2, P_3, P_4 cumplen con todas las restricciones.

$$\underline{P_1} \quad x=3 \geq 0 \quad \checkmark$$

$$\underline{P_2} \quad 0 - 2 \cdot \frac{5}{2} = -5 \leq 1 \quad \checkmark$$

P₃

$$-\frac{1}{2} - \frac{5}{2} = -3 \leq 3 \quad \checkmark$$

P₄

$$-\frac{1}{2} - \frac{7}{2} = -4 \leq 3 \quad \checkmark$$

7

a)

Lo llevo a forma est醤dar:

$$2x_1 + x_2 \leq 100 \quad \rightsquigarrow 2x_1 + x_2 + x_3 = 100 \quad \text{con } x_3 \geq 0$$

$$x_1 + x_2 \leq 80 \quad \rightsquigarrow x_1 + x_2 + x_4 = 80 \quad \text{con } x_4 \geq 0$$

$$x_1 \leq 40 \quad \rightsquigarrow x_1 + x_5 = 40 \quad \text{con } x_5 \geq 0$$

Entonces:

$$\begin{cases} 2x_1 + x_2 + x_3 = 100 \\ x_1 + x_2 + x_4 = 80 \\ x_1 + x_5 = 40 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

Luego:

$$A = \begin{pmatrix} 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} \in \mathbb{R}^{3 \times 5}$$

$$b = \begin{pmatrix} 100 \\ 80 \\ 40 \end{pmatrix}$$

$$\text{haz: } \binom{5}{3} = \frac{5!}{3!2!} = 10 \text{ casos}$$

Caso 1

$$B = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad X_B = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \Rightarrow X_B = B^{-1} \cdot b = (40, 40, -20)$$

Luego:

$$N = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Por lo tanto la solución asociada a esta base:

$$(x_1, x_2, x_3, x_4, x_5) = (40, 40, -20, 0, 0)$$

la cual es infactible pues $x_3 < 0$

Caso 2

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Luego:

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1} \cdot b = (40, 20, 20)$$

Solución asociada a esta base: $(x_1, x_2, x_3, x_4, x_5) = (40, 20, 0, 20, 0)$

Cumple con las restricciones \therefore es factible

y corresponde al vértice $(40, 20)$

Caso 3

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad X_B = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Luego:

$$N = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1} \cdot b = (20, 60, 20)$$

Solución asociada a esta base: $(x_1, x_2, x_3, x_4, x_5) = (20, 60, 0, 0, 20)$

Cumple con las restricciones \leadsto es factible

y corresponde al vértice $(20, 60)$

Caso 4

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$N = \begin{pmatrix} 2 & 0 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_1 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

B no es invertible $\Rightarrow B \cdot X_B = b$ no corresponde a una sol. factible

Caso 5

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_2 \\ x_3 \\ x_5 \end{pmatrix}$$

$$N = \begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_1 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1}b = (80, 20, 40)$$

La sol. básica asociada a B: $(x_1, x_2, x_3, x_4, x_5) = (0, 80, 20, 0, 40)$

no cumple con las restricciones no es factible

y corresponde al vértice (0, 80)

Caso 6

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix}$$

Luego:

$$N = \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1}b = (100, -20, 40)$$

Sol. básica asociada a B: $(0, 100, 0, -20, 40)$

No es factible pues $x_4 < 0$

Caso 7

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_1 \\ x_3 \\ x_4 \end{pmatrix}$$

$$N = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_2 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1}b = (40, 20, 40)$$

Solución básica asociada a B: $(x_1, x_2, x_3, x_4, x_5) = (40, 0, 20, 40, 0)$

→ cumple con las restricciones → es factible

y está asociada al vértice (40, 0)

Caso 8

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad X_B = \begin{pmatrix} x_1 \\ x_3 \\ x_5 \end{pmatrix}$$

$$N = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} x_2 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1}b = (80, -60, -40)$$

Solución básica asociada a B: $(x_1, x_2, x_3, x_4, x_5) = (80, 0, -60, 0, -40)$

No es factible pues $x_3, x_5 < 0$

Caso 9

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad X_B = \begin{pmatrix} X_1 \\ X_4 \\ X_5 \end{pmatrix}$$

$$N = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$X_B = B^{-1}b = (50, 30, -10)$$

Solución básica asociada a B: $(X_1, X_2, X_3, X_4, X_5) = (50, 0, 0, 30, -10)$

No es factible pues $X_5 < 0$

Caso 10

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_B = \begin{pmatrix} X_3 \\ X_4 \\ X_5 \end{pmatrix}$$

$$N = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad X_N = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

$$X_B = B^{-1}b = (10, 80, 40)$$

Solución básica asociada a B: $(X_1, X_2, X_3, X_4, X_5) = (0, 0, 100, 80, 40)$

→ Cumple las restricciones → es factible.

Y este es asociado al $(0, 0)$.

b) Tales sol. bas. factibles vienen asociadas a un vértice, estos son:

$$(40,20), (20,60), (0,80), (40,0), (0,0)$$

3)

2)

Lo llevo a la forma estandar:

$$x_1 + x_2 \leq 12 \rightsquigarrow x_1 + x_2 + x_3 = 12 \quad \text{con } x_3 \geq 0$$

$$2x_1 + x_2 \leq 12 \rightsquigarrow 2x_1 + x_2 + x_4 = 12 \quad \text{con } x_4 \geq 0$$

Entonces:

$$\min z = 40x_1 + 30x_2$$

$$\text{s.a. } x_1 + x_2 + x_3 = 12$$

$$2x_1 + x_2 + x_4 = 12$$

$$x_i \geq 0 \quad \forall i$$

$$A \in \mathbb{R}^{2 \times 4} \Rightarrow B \in \mathbb{R}^{2 \times 2} \Rightarrow x_3 \in \mathbb{R}^2$$

luego:

	x_1	x_2	x_3	x_4	$L1$
$-z$	40	30	0	0	0
x_3	1	1	1	0	12
x_4	2	1	0	1	12

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Test de optimidad: no hay ningún $Z < 0$ y por lo tanto esta es la sol. básica factible óptima.

$$\approx (0,0,12,12)$$

minimizador: (0,0)

Valor óptimo: $f(0,0) = 0$

9

frágil: $5T_F + 3T_N \approx \frac{250}{T}$

Poco frágil: $3T_F + 5T_N \approx \frac{300}{T}$

Resistente: $5T_F + 5T_N \approx \frac{400}{T}$

entonces:

$$f(x) = x_1 \cdot 250 + x_2 \cdot 300 + x_3 \cdot 400$$

$$\text{Restricciones: } 5x_1 + 3x_2 + 5x_3 \leq 100 \text{ ~total de rojo}$$

$$3x_1 + 5x_2 + 5x_3 \leq 80 \text{ ~total de negro}$$

En forma estándar:

$$\min: Z = -250x_1 - 300x_2 - 400x_3$$

$$\text{s.a: } 5x_1 + 3x_2 + 5x_3 + x_4 = 100$$

$$3x_1 + 5x_2 + 5x_3 + x_5 = 80$$

$$x_i \geq 0$$

$-z$	x_1	x_2	x_3	x_4	x_5	LD
-250	-300	-400	0	0	0	0
x_4	5	3	5	1	0	100
x_5	3	5	5	0	1	80

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Test de optimidad: existe $\hat{c}_j < 0$. Elijo x_3 para entrar a la base, pues $\hat{c}_3 < 0$ y ademá es el menor de todos.

Test del cociente mínimo:

$$\min \left\{ \frac{100}{5}, \frac{80}{5} \right\} = \min \{ 20, 16 \} = 16$$

$\downarrow \quad \downarrow$

$x_4 \quad x_5$

$\Rightarrow x_5$ sale de la base.

$F_2 - F_3$	x_1	x_2	x_3	x_4	x_5	LD
$F_1 + 80F_3$	-7	-10	100	0	0	6400
\rightarrow	x_4	2	-2	0	1	-1
	x_5	3	5	5	0	1

$\frac{F_3}{5}$	x_1	x_2	x_3	x_4	x_5	LD
	-7	-10	100	0	0	6400
	x_4	2	-2	0	1	-1
	x_3	$\frac{3}{5}$	1	1	0	$\frac{1}{5}$

Test de optimidad: existe $c_j < 0$, x_1 entra a la base.

Test del cociente mínimo:

$$\min \left\{ \frac{20}{2}, \frac{16}{\frac{3}{5}} \right\} = \min \left\{ 10, 26.66 \right\} = 10$$

$\Rightarrow x_4$ sale de la base.

$$F_1 + SF_2$$

$$\underbrace{F_3 - \frac{3}{10}F_2}_{\rightarrow}$$

	x_1	x_2	x_3	x_4	x_5	LD
$-Z$	0	90	0	5	75	6500
x_4	2	-2	0	1	-1	10
x_3	0	$\frac{8}{5}$	1	$-\frac{3}{10}$	$\frac{1}{2}$	10

$$\frac{F_2}{2} \rightarrow$$

	x_1	x_2	x_3	x_4	x_5	LD
$-Z$	0	90	0	5	75	6500
x_1	1	-1	0	$\frac{1}{2}$	$-\frac{1}{2}$	10
x_3	0	$\frac{8}{5}$	1	$-\frac{3}{10}$	$\frac{1}{2}$	10

Test de optimilidad: no hay ningún $G \leq 0$

\therefore Terminó el algoritmo

Solución básica: $(x_1, x_2, x_3, x_4, x_5) = (10, 0, 10, 0, 0)$

\downarrow
Solución óptima

\therefore Deberían hacer 10 toneladas de frágil, 10 toneladas de resistentes y ninguna de poco frágil.