PRACTICO 6

1) a)

$$\begin{cases} x+2y+3z=10 & (\alpha) \\ 4y+5z=13 & (\beta) \\ 6z=6 & (\gamma) \end{cases}$$

Por (γ): $6z=6 \Rightarrow \boxed{z=1}$

Luego en (β):

$$4y+5=13$$

$$4y=8 \Rightarrow \boxed{y=2}$$

Luego en (α):

$$x+4+3=10$$

$$\Rightarrow \boxed{x=3}$$

b)

$$\begin{cases} \boxed{x=2} & (\alpha) \\ 2y=8 & (\beta) \\ -x+5y-3z=0 & (\gamma) \end{cases}$$

En $(\beta) \Rightarrow \boxed{y=4}$

Luego en (γ):

$$-2+20-3z=0 \Rightarrow \boxed{z=6}$$

c)

$$\begin{cases} 2x - y + z + 3w = 5 & (\alpha) \\ 4y + 5z + 10v + 2w = 21 & (\beta) \\ -5z + 11w = 6 & (\gamma) \\ 2v - 7w = -5 & (\delta) \\ 40w = 40 & (\varepsilon) \end{cases}$$

Por (ε) $\boxed{w=1}$

Luego en (δ)

$$2v - 7 = -5$$

$$2v = 2 \Rightarrow \boxed{v=1}$$

En (γ):

$$-5z + 11 = 6$$

$$-5z = -5 \Rightarrow \boxed{z=1}$$

En (β):

$$4y + 5 + 10 + 2 = 21$$

$$4y = 4 \Rightarrow \boxed{y=1}$$

En (α)

$$2x - 1 + 1 + 3 = 5$$

$$2x = 2 \Rightarrow \boxed{x=1}$$

(2)

$$\begin{cases} 5A+4B+3C=53 \\ 2A+7B+4C=46 \\ 8A+13B+5C=99 \end{cases}$$

$$\left(\begin{array}{ccc|c} 5 & 4 & 3 & 53 \\ 2 & 7 & 4 & 46 \\ 8 & 13 & 5 & 99 \end{array}\right) \xrightarrow[F_1 \cdot 2]{F_3-4F_2} \left(\begin{array}{ccc|c} 10 & 8 & 6 & 106 \\ 2 & 7 & 4 & 46 \\ 0 & -15 & -11 & -85 \end{array}\right) \xrightarrow{F_1-5F_2}$$

$$\left(\begin{array}{ccc|c} 0 & -27 & -14 & -124 \\ 2 & 7 & 4 & 46 \\ 0 & -15 & -11 & -85 \end{array}\right) \xrightarrow{F_1 \leftrightarrow F_2} \left(\begin{array}{ccc|c} 2 & 7 & 4 & 46 \\ 0 & -27 & -14 & -124 \\ 0 & -15 & -11 & -85 \end{array}\right) \xrightarrow{F_3-\frac{15}{27}F_2}$$

$$\left(\begin{array}{ccc|c} 2 & 7 & 4 & 46 \\ 0 & -27 & -14 & -124 \\ 0 & 0 & -11+\frac{14\cdot 15}{27} & -85+\frac{124\cdot 15}{27} \end{array}\right)$$

$$\begin{cases} (\alpha) & 2A+7B+4C=46 \\ (\beta) & -27B-14C=-124 \\ (\gamma) & \left(-11+\frac{14\cdot 15}{27}\right)C=-85+\frac{124\cdot 15}{27} \end{cases}$$

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Por (γ):

$$C\left(\frac{-11.27+14.15}{27}\right)=\frac{-85.27+124.15}{27}$$

$$C(-11.27+14.15)=-85.27+124.15$$

$$C=\frac{-85.27+124.15}{-11.27+14.15}$$

$$\boxed{C=5}$$

Por (β):

$$-27B-70=-124$$

$$-27B=-54 \Longrightarrow \boxed{B=2}$$

Por (α)

$$2A+14+20=46$$

$$2A+34=46$$

$$2A=12 \Longrightarrow \boxed{A=6}$$

4 a)

$$\left(\begin{array}{ccc|c} 2 & -2 & 1 & -1 \\ 1 & 1 & 3 & 6 \\ 0 & 4 & 1 & 9 \end{array}\right) \xrightarrow{2F_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & -1 \\ 2 & 2 & 6 & 12 \\ 0 & 4 & 1 & 9 \end{array}\right) \xrightarrow{F_2-F_1}$$

$$\left(\begin{array}{ccc|c} 2 & -2 & 1 & -1 \\ 0 & 4 & 5 & 13 \\ 0 & 4 & 1 & 9 \end{array}\right) \xrightarrow{F_3-F_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 1 \\ 0 & 4 & 5 & 13 \\ 0 & 0 & -4 & -4 \end{array}\right)$$

$$\begin{array}{l} (\alpha) \\ (\beta) \\ (\gamma) \end{array} \begin{cases} x-2y+z=-1 \\ 4y+5z=13 \\ -4z=-4 \end{cases}$$

Por (γ):

$$-4z=-4 \Rightarrow \boxed{z=1}$$

En (β):

$$4y+5=13$$
$$4y=8 \Rightarrow \boxed{y=2}$$

En (α):

$$x-4+1=-1 \Rightarrow \boxed{x=2}$$

b)

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 2 & -2 & 1 \\ 1 & 1 & 3 \\ 0 & 4 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \qquad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}$$

$$U_{1j} = a_{1j} \Longrightarrow \begin{cases} U_{11} = a_{11} = 2 \\ U_{12} = a_{12} = -2 \\ U_{13} = a_{13} = 1 \end{cases}$$

$$l_{i1} = \frac{a_{i1}}{U_{11}} \Longrightarrow \begin{cases} l_{11} = \frac{a_{11}}{U_{11}} = 1 \\ l_{21} = \frac{a_{21}}{U_{11}} = \frac{1}{2} \\ l_{31} = \frac{a_{31}}{U_{11}} = 0 \end{cases}$$

Luego:

$$(\alpha): U_{kj} = a_{kj} - \sum_{m=1}^{k-1} l_{km} U_{mj} \qquad (\beta): l_{ik} = \frac{1}{U_{kk}}\left(a_{ik} - \sum_{m=1}^{k-1} l_{im} U_{mk}\right)$$

para $j = k, \dots, n$ para $i = k+1, \dots, n$

<u>$k=2$</u>

usando (α):

$$U_{2j} = a_{2j} - \sum_{m=1}^{1} l_{2m} U_{mj} = a_{2j} - l_{21} \cdot U_{1j} \quad , \ j \geq 2$$

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$j=2 \Rightarrow U_{22}=a_{22}-l_{21}U_{12}=1-\left(\frac{1}{2}\cdot(-2)\right)=1+1=2$

$j=3 \Rightarrow U_{23}=a_{23}-l_{21}U_{11}=3-\left(\frac{1}{2}\cdot 1\right)=3-\frac{1}{2}=\frac{5}{2}$

Usando (β):

$$l_{i2}=\frac{1}{U_{22}}\left(a_{i2}-\sum_{m=1}^{1} l_{im}U_{m2}\right)=\frac{1}{U_{22}}\left(a_{i2}-\ i1U_{i2}\right) \qquad i\geq 3$$

$$i=3 \Rightarrow l_{32}=\frac{1}{U_{22}}(a_{32}-l_{31}U_{32})=\frac{1}{2}(4-0)=2$$

$\underline{K=3}$

usando (α):

$$U_{3j}=a_{3j}-\sum_{m=1}^{2} l_{3m}U_{mj}=a_{3j}-(l_{31}U_{1j}+l_{32}U_{2j}) \quad , \quad j\geq 3$$

$$j=3 \Rightarrow U_{33}=a_{33}-[l_{31}U_{13}+l_{32}U_{23}]=1-\left(0+2\cdot\frac{5}{2}\right)=-4$$

Luego:

$$L=\begin{pmatrix}1 & 0 & 0\\ \frac{1}{2} & 1 & 0\\ 0 & 2 & 1\end{pmatrix}, \quad U=\begin{pmatrix}2 & -2 & 1\\ 0 & 2 & \frac{5}{2}\\ 0 & 0 & -4\end{pmatrix}$$

$$Ax=b \Leftrightarrow L.Ux=b \Leftrightarrow \begin{cases} Ly=b\\ Ux=y\end{cases}$$

$$\begin{cases} y_1=-1\\ \frac{y_1}{2}+y_2=6 \Rightarrow y_2-\frac{1}{2}=6 \Rightarrow y=\frac{13}{2}\\ 2y_2+y_3=9 \Rightarrow 13+y_3=9 \Rightarrow y_2=-4\end{cases}$$

$$y = \begin{pmatrix} -1 \\ \frac{13}{2} \\ -4 \end{pmatrix}$$

Luego:

$$\begin{cases} 2x_1 - 2x_2 + x_3 = -1 \\ 2x_2 + \frac{5}{2}x_3 = \frac{13}{2} \\ -4x_3 = -4 \implies \boxed{x_3 = 1} \end{cases}$$

$2x_2 = \frac{13}{2} - \frac{5}{2} \implies \boxed{x_2 = 2}$

$2x_1 - 4 + 1 = -1 \implies \boxed{x_1 = 1}$

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a) Verdadero:

$$\det(A) = \det(LU) = \det(L) \cdot \det(U) = 1 \cdots 1 \det(U) = \det(U)$$

b) Es de notar que no se puede demostrar por teorema.

Sabemos que $U_{1j} = a_{1j} \implies U_{11} = 0$

Además $U_{11} l_{i1} = a_{i1}$ para $i=2$: $0 \cdot l_{21} = 1$, absurdo.

$\therefore$ No tiene descomposición LU.

c)

$$\begin{pmatrix} 0 & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \cdot \begin{pmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{pmatrix} = A$$

$$\begin{array}{cc|cc} & & U_{11} & U_{12} \\ & & 0 & U_{22} \\ \hline 1 & 0 & U_{11} & U_{12} \\ l_{21} & 1 & l_{21}U_{11} & l_{21}U_{12}+U_{22} \end{array}$$

$$\Rightarrow \begin{cases} U_{11} = 0 \\ U_{12} = a \\ l_{21}U_{11} = 0 \quad \rightsquigarrow \text{supongamos } l_{21} = 1,\ U_{11} = 0 \\ l_{21}U_{12} + U_{22} = 6 \end{cases}$$

$\hookrightarrow \; l_{21}U_{12} + U_{22} = 6 \Rightarrow U_{12} + U_{22} = 6$

$U_{22} = 6 - U_{12} \Rightarrow U_{22} = (6-a)$

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \rightsquigarrow \text{triangular inferior}$$

$$U = \begin{pmatrix} 0 & a \\ 0 & 6-a \end{pmatrix} \rightsquigarrow \text{Triangular superior}$$

$\therefore$ Falso, si tiene

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$\underline{r \neq 0}$

Sea $A' = A(1{:}2, 1{:}2) = \begin{pmatrix} 1 & 1 \\ 0 & r \end{pmatrix}$, $A'' = A(1{:}1, 1{:}1) = (1)$

$\det(A') = 1 \cdot r \neq 0 \Rightarrow$ es no singular (a)

$\det(A'') = 1 \neq 0 \Rightarrow$ es no singular (β)

Por (α) y (β) se cumple el teorema $\Rightarrow$ existen únicas matrices LU.

$\underline{\Gamma = 0}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{pmatrix} = L \cdot U$$

$$\begin{array}{ccc|ccc}
 & & & 1 & 1 & 1 \\
 & & & & U_{22} & U_{23} \\
 & L \cdot U & & 0 & 0 & U_{33} \\
\hline
1 & 0 & 0 & 1 & 1 & 1 \\
l_{21} & 1 & 0 & l_{21} & l_{21}+U_{22} & l_{21}+U_{23} \\
l_{31} & l_{32} & 1 & l_{31} & l_{31}+l_{32}U_{22} & l_{31}+l_{32}U_{23}+U_{33}
\end{array}$$

Luego:

$$\begin{cases}
\boxed{l_{21} = 0} & (\alpha) \\
l_{21}+U_{22} = 0 & (\beta) \\
l_{21}+U_{23} = 2 & (\gamma) \\
l_{31} = 0 & (\delta) \\
l_{31}+l_{32}U_{22} = 0 & (\varepsilon) \\
l_{31}+l_{32}U_{23}+U_{33} = 0 & (\zeta)
\end{cases}$$

En (β):

$$l_{21}+U_{22}=0 \underset{\text{Por }(\alpha)}{\Longrightarrow} \boxed{U_{22}=0}$$

En (γ)

$$l_{21}+U_{23}=2 \underset{\text{Por }\alpha}{\Longrightarrow} \boxed{U_{23}=2}$$

En (ε):

$$l_{31}+l_{32}U_{22} \underset{\text{Por }(\delta)}{=} l_{32}U_{22}=0 \Longrightarrow \boxed{l_{32}=0 \vee U_{22}=0} \quad \underline{A}$$

En (ζ):

$$l_{31}+l_{32}U_{23}+U_{33}=3$$

$$\boxed{2l_{32}+U_{33}=3} \quad \underline{B}$$

Por $\underline{A}$ y $\underline{B}$ existen infinitas soluciones al sistema, por lo tanto existen infinitas descomposiciones LU.

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$$A=\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix} \quad M=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} \quad N=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$$

Busco M^{-1}:

$$\left(\begin{array}{cc|cc}1 & 0 & 1 & 0\\ 1 & 1 & 0 & 1\end{array}\right) \xrightarrow{F_2-F_1} \left(\begin{array}{cc|cc}1 & 0 & 1 & 0\\ 0 & 1 & -1 & 1\end{array}\right) \Longrightarrow M^{-1}=\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$$

Busco $M^{-1}.N$:

$$\begin{array}{cc|cc} & & 0 & 1 \\ & & 0 & 0 \\ \hline 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & -1 \end{array} \Longrightarrow M^{-1}N = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}$$

Busco autovectores:

$$\det(\lambda . Id - M^{-1}N) = \det\begin{pmatrix} \lambda & 1 \\ 0 & \lambda+1 \end{pmatrix} = \lambda(\lambda+1) = 0 \Longrightarrow \lambda = 0 \vee \lambda = -1$$

$\rho(M^{-1}N) = 1 \Longrightarrow$ El método no es independiente del X^0 inicial.

Busco Autoespacios:

$$(\lambda Id - M^{-1}N) \cdot \begin{pmatrix} X \\ y \end{pmatrix} = (0, 0)$$

$\underline{\lambda = 0}$

$$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ y \end{pmatrix} = (0, 0) \Longrightarrow y = 0$$

Es decir que los autovectores asociados al autovector 0 son de la forma:

$$(x, y) = (x, 0) = x(1, 0)$$

$$V_0 = \{t(1,0) / t \in \mathbb{R}\}$$

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$\underline{\lambda=1}$

$$\begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (0,0) \Rightarrow x+y=0 \Rightarrow x=-y$$

Luego:

$$(x,y) = (x,-x) = x(1,-1)$$

$$V_1 = \{t(1,-1) / t \in \mathbb{R}\}$$

El metodo va a converger dado un punto inicial X^0, si este es combinación lineal de los generadores de los autoespacios asociados a los autovalores λ tales que $|\lambda|<1$.
En este caso si son combinación lineal de $(1,0)$.

a) $X^0=(2,0)=2(1,0) \Rightarrow$ Si va a converger

b) $X^0=(-0{,}03, 0{,}03) \neq k(1,0)\ \forall k\in\mathbb{R} \Rightarrow$ No converge

c) $X^0=(0,1) \neq k(1,0)\ \forall k\in\mathbb{R} \Rightarrow$ No converge

9 a) $$X^{(k+1)} = (M^{-1}N)\,x^{(k)} + M^{-1}b$$

Jacobi

$$M=\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \qquad N=\begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$$

$$M^{-1}=\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$M^{-1}\cdot N$:

$$\begin{array}{cc|cc} & & 0 & -2 \\ & & -1 & 0 \\ \hline \frac{1}{3} & 0 & 0 & -\frac{2}{3} \\ 0 & \frac{1}{3} & -\frac{1}{2} & 0 \end{array}$$

$M^{-1}\cdot b$:

$$\begin{array}{cc|c} & & 5 \\ & & 3 \\ \hline \frac{1}{3} & 0 & \frac{5}{3} \\ 0 & \frac{1}{2} & \frac{3}{2} \end{array}$$

Sea $X^k=(x^k,y^k)$:

$$X^{(k+1)}=\begin{pmatrix} 0 & -\frac{2}{3} \\ -\frac{1}{2} & 0 \end{pmatrix}\cdot\begin{pmatrix} x^{(k)} \\ y^{(k)} \end{pmatrix}+\begin{pmatrix} \frac{5}{3} \\ \frac{3}{2} \end{pmatrix}$$

$$X^{(k+1)}=\begin{pmatrix} -\frac{2y^{(k)}}{3}+\frac{5}{3} \\ -\frac{1}{2}x^{(k)}+\frac{3}{2} \end{pmatrix}$$

Gauss-Seidel

$$M=\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \qquad N=\begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$$

Calculo M^{-1}:

$$\left(\begin{array}{cc|cc} 3 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \xrightarrow{F_1-3F_2} \left(\begin{array}{cc|cc} 0 & -6 & 1 & -3 \\ 1 & 2 & 0 & 1 \end{array}\right) \xrightarrow{3F_2} \left(\begin{array}{cc|cc} 0 & -6 & 1 & -3 \\ 3 & 6 & 0 & 3 \end{array}\right)$$

$$\xrightarrow{F_2+F_1} \left(\begin{array}{cc|cc} 0 & -6 & 1 & -3 \\ 3 & 0 & 1 & 0 \end{array}\right) \xrightarrow[\frac{1}{3}\cdot F_2]{\left(-\frac{1}{6}\right)\cdot F_1} \left(\begin{array}{cc|cc} 0 & 1 & -\frac{1}{6} & \frac{1}{2} \\ 1 & 0 & \frac{1}{3} & 0 \end{array}\right) \xrightarrow{F_1 \leftrightarrow F_2}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} \end{array}\right) \Longrightarrow M^{-1}=\begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{6} & \frac{1}{2} \end{pmatrix}$$

$M^{-1}N$:

$$\begin{array}{cc|cc} & & 0 & -2 \\ & & 0 & 0 \\ \hline \frac{1}{3} & 0 & 0 & -\frac{2}{3} \\ -\frac{1}{6} & \frac{1}{2} & 0 & \frac{1}{3} \end{array}$$

$M^{-1}b$:

$$\begin{array}{cc|c} & & 5 \\ & & 3 \\ \hline \frac{1}{3} & 0 & \frac{5}{3} \\ -\frac{1}{6} & \frac{1}{2} & \frac{2}{3} \end{array}$$

Sea $X^{(k)} = (x^k, y^k)$

$$X^{(k+1)} = \begin{pmatrix} 0 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} x^k \\ y^k \end{pmatrix} + \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \end{pmatrix}$$

$$X^{(k+1)} = \begin{pmatrix} \frac{-2y^{(k)}}{3} + \frac{5}{3} \\ \frac{y^{(k)}}{3} + \frac{2}{3} \end{pmatrix}$$

b) La matriz es diagonalmente dominante $\Longrightarrow$ La sucesión converge

10 a) $X^{(k+1)} = (M^{-1}N)\, X^{(k)} + M^{-1} \cdot b$

$$M = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & -3 & 0 \\ -2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Luego:

$$\begin{array}{ccc|ccc}
 & & & 0 & -3 & 0 \\
 & & & -2 & 0 & 1 \\
 & M^{-1}\cdot N & & 0 & 1 & 0 \\
\hline
\frac{1}{4} & 0 & 0 & 0 & -\frac{3}{4} & 0 \\
0 & \frac{1}{4} & 0 & -\frac{1}{2} & 0 & \frac{1}{4} \\
0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0
\end{array}
\qquad
\begin{array}{ccc|c}
 & & & b_1 \\
 & & & b_2 \\
 & M^{-1}\cdot b & & b_3 \\
\hline
\frac{1}{4} & 0 & 0 & \frac{b_1}{4} \\
0 & \frac{1}{4} & 0 & \frac{b_2}{4} \\
0 & 0 & \frac{1}{4} & \frac{b_3}{4}
\end{array}$$

Entonces, siedo $X^{k} = (x^{(k)}, y^{(k)}, z^{(k)})$

$$X^{(k+1)} = \begin{pmatrix} 0 & -\frac{3}{4} & 0 \\ -\frac{1}{2} & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 \end{pmatrix} \cdot \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} + \begin{pmatrix} \frac{b_1}{4} \\ \frac{b_2}{4} \\ \frac{b_3}{4} \end{pmatrix}$$

$$X^{(k+1)} = \begin{pmatrix} -\frac{3}{4} y^{(k)} + \frac{b_1}{4} \\ -\frac{1}{2} x^{(k)} + \frac{1}{4} z^{(k)} + \frac{b_2}{4} \\ \frac{1}{4} y^{(k)} + \frac{b_3}{4} \end{pmatrix}$$

b) Sí es convergente, y además es independiente del X^{0}, pues la matriz A es diagonalmente dominante

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$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & & a_{2n} \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{pmatrix}$$

Jacobi

$M_J = D$

$N_J = -(L+ \ ) = -U$, $L=0$

Gauss-Seidel

$M_G = D+L = D$, $L=0$

$N_G = -U$

$$\therefore \ M_J^{-1} \cdot N_J = M_G^{-1} \cdot N_G$$

Sea $M_G^{-1} \cdot N_G = M^{-1}N = T$ ↳ matriz triangular con ceros en la diagonal $\Rightarrow$ Es nilpotente de orden n

$\Rightarrow T^n = 0$

Luego la fórmula de iteración:

$$X^{(n+1)} = TX^{(n)} + h \qquad \text{donde } h = M^{-1}b$$

Luego:

$$X^{(1)} = TX^{(0)} + h$$

$$X^{(2)} = TX^{(1)} + h = T(TX^{(0)} + h) + h = T^2X^{(0)} + h(T+1)$$

$$X^{(3)} = TX^{(2)} + h = T^3X^{(0)} + h(T^2 + T + 1)$$

$$\vdots$$

$$X^{(n)} = T^nX^{(0)} + h(T^{n-1} + T^{n-2} + \cdots + 1) \qquad (\alpha)$$

$$X^{(n+1)} = T^{n+1} \cdot X^{(0)} + h(T^n + T^{n-1} + \cdots + 1) \qquad (\beta)$$

En (α):

$$X^{(n)} = \underbrace{T^n}_{=0} X^{(0)} + (T^{n-1} + T^{n-2} + \cdots + 1) = (T^{n-1} + T^{n-2} + \cdots + 1).h$$

En (β)

$$X^{(n+1)} = \underbrace{T^{n+1}}_{=0} X^{(0)} + (\underbrace{T^n}_{=0} + T^{n-1} + \cdots + 1).h = (T^{n-1} + T^{n-2} + \cdots + 1).h$$

Pues $T^{n+1} = T^n . T = 0.T$

$$\Longrightarrow \boxed{X^{(n+1)} = X^{(n)}}$$

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Usando el teorema de diagonalmente dominante me va a dar valores de "a" en los que el método converge, pero es posible que no sean <u>todos</u> los valores de "a".

Usaremos el teorema del radio espectral.

$$M = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 & a & 1 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & -a & -a^2 \\ 0 & 0 & -a \\ 0 & 0 & 0 \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ 0 & -a & 1 \end{pmatrix}$$

Luego:

$$\begin{array}{ccc|ccc} & & & 0 & -a & -a^2 \\ & & & 0 & 0 & -a \\ & M^{-1}N & & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & -a & -a^2 \\ -a & 1 & 0 & 0 & a^2 & a^3-a \\ 0 & -a & 1 & 0 & 0 & a^2 \end{array}$$

Calculo los autovalores:

$$\det(\lambda Id - M^{-1}N) = \begin{vmatrix} \lambda & -a^2 & -a^2 \\ 0 & \lambda-a^2 & a^3-a \\ 0 & 0 & \lambda-a^2 \end{vmatrix} = \lambda(\lambda-a^2)^2 = 0 \Longrightarrow \lambda = 0 \vee \lambda = a^2$$

Por teorema del radio espectral:

$$\rho(M^{-1}N) < 1 \Rightarrow a^2 < 1 \Rightarrow \boxed{a \in (-1,1)}$$

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$$A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & -15 \\ 0 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 11 \\ 13 \\ 1 \end{pmatrix}$$

Jacobi

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & -3 & -2 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{pmatrix}$$

Primero veamos que sea independiente del X^0:

$$\det(\lambda I_d - M^{-1}N) = \begin{vmatrix} \lambda & -3 & -2 \\ 0 & \lambda & 15 \\ 0 & 0 & \lambda \end{vmatrix} = \lambda^3 = 0 \Rightarrow \lambda = 0$$

$\Rightarrow \rho(M^{-1}N) < 1 \Rightarrow$ Converge independientemente de X^0.

$$X^{(k+1)} = \begin{pmatrix} 0 & -3 & -2 \\ 0 & 0 & 15 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{pmatrix} + \begin{pmatrix} 11 \\ 13 \\ 1 \end{pmatrix} = \begin{pmatrix} -3y^{(k)} - 3z^{(k)} + 11 \\ 15z^{(k)} + 13 \\ 1 \end{pmatrix}$$

$X^0 = (0,0,0)$, Luego:

$$X^1 = \begin{pmatrix} 11 \\ 13 \\ 1 \end{pmatrix}$$

$$X^2 = \begin{pmatrix} -26-2+11 \\ 15+13 \\ 1 \end{pmatrix} = \begin{pmatrix} 17 \\ 28 \\ 1 \end{pmatrix}$$

$$X^3 = \begin{pmatrix} -64-2+11 \\ 15+13 \\ 1 \end{pmatrix} = \begin{pmatrix} -55 \\ 28 \\ 1 \end{pmatrix}$$

$$X^4 = \begin{pmatrix} -55 \\ 28 \\ 1 \end{pmatrix}$$

Solución: $x = (-55, 28, 1)$

14 a)

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix} \qquad b = \begin{pmatrix} 7 \\ 2 \\ 5 \end{pmatrix}$$

Por Jacobi:

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & -2 & 2 \\ -1 & 0 & -1 \\ -2 & -2 & 0 \end{pmatrix}$$

$$\det(\lambda Id - M^{-1}N) = \begin{vmatrix} \lambda & -2 & 2 \\ -1 & \lambda & -1 \\ -2 & -2 & \lambda \end{vmatrix} = [\lambda^3 + 4 - 4] - [-4\lambda + 2\lambda + 2\lambda]$$

$$= \lambda^3 = 0 \Longrightarrow \lambda = 0$$

$$\begin{matrix} \lambda & -2 & 2 \\ -1 & \lambda & -1 \end{matrix}$$

Por teorema del radio espectral converge, por lo tanto el número de iteraciones es finito.

b)

$$X^{(k+1)} = \begin{pmatrix} -2y+2z+7 \\ -x-z+2 \\ -2x-2y+5 \end{pmatrix}$$

$$X^{0} = \begin{pmatrix} 1 \\ 2,1 \\ -1 \end{pmatrix}$$

$$X^{1} = \begin{pmatrix} -4,2-2+7 \\ -1+1+2 \\ -2-4,2+5 \end{pmatrix} = \begin{pmatrix} 0,8 \\ 2 \\ -1,2 \end{pmatrix}$$

$$X^{2} = \begin{pmatrix} -4-2,4+7 \\ -0,8+1,2+2 \\ -1,64+5 \end{pmatrix} = \begin{pmatrix} 0,6 \\ 2,4 \\ -0,6 \end{pmatrix}$$

$$X^{3} = \begin{pmatrix} -4,8-1,2+7 \\ -0,6+0,6+2 \\ -1,2-4,8+5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$X^{4} = \begin{pmatrix} -4-2+7 \\ -1+0+1+2 \\ -2-4+5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Solución: $X=(1,2,-1)$

Fallo: 3, 5, 11