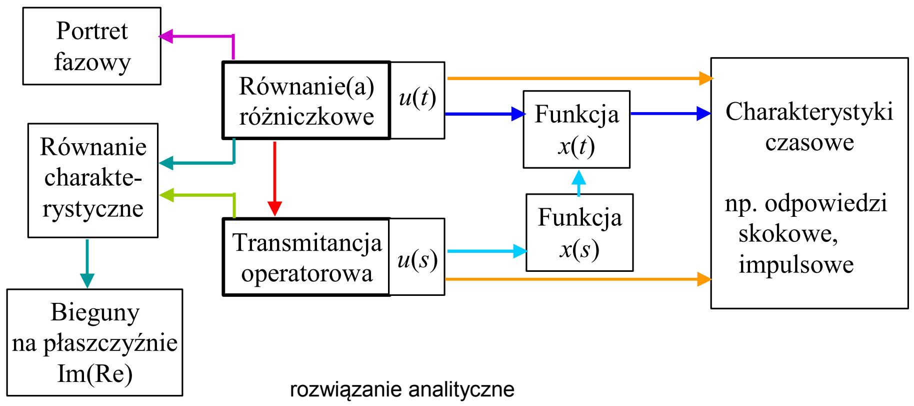
Opis własności dynamicznych (1)



rozwiązanie analityczne
charakterystyka czasowa
położenie biegunów
portret fazowy
przekształcenie Laplace'a,
  położenie biegunów
  transformata rozwiązania, przekształcenie odwrotne
rozwiązanie symulacyjne
charakterystyki częstotliwościowe

Układ równań stanu / transmitancja – własności obiektu

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 \\ b_1 & -b_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$s^2 + (a_1 + b_2)s + a_1 b_2 - b_1 a_2 = 0$$

$$s\mathbf{x}(s) = \mathbf{A}\mathbf{x}(s) + \mathbf{B}\mathbf{u}(s)$$

$$\mathbf{x}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{u}(s)$$

$$\mathbf{x}(s) = \mathbf{G}(s)\mathbf{u}(s)$$

$$\begin{cases} (s + a_1)x_1(s) = a_2 x_2(s) + u_1(s) \\ (s + b_2)x_2(s) = b_1 x_1(s) + u_2(s) \end{cases}$$

$$\begin{cases} M_1(s)x_1(s) = a_2 x_2(s) + u_1(s) \\ M_2(s)x_2(s) = b_1 x_1(s) + u_2(s) \end{cases}$$

$$x_1(s) = \frac{M_2(s)}{M(s)} u_1(s) + \frac{a_2}{M(s)} u_2(s)$$

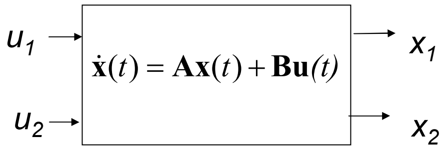
$$x_2(s) = \frac{b_1}{M(s)} u_1(s) + \frac{M_1(s)}{M(s)} u_2(s)$$

$$s^2 + (a_1 + b_2)s + a_1 b_2 - b_1 a_2 = 0$$

$$M(s) = M_1(s) M_2(s) - b_1 a_2$$

Układ równań stanu / transmitancja – „struktura” obiektu

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-a_1 & a_2\\ b_1 & -b_2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$$

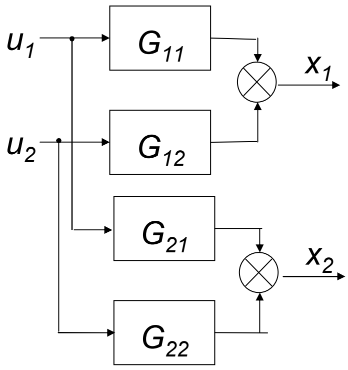


$$X_1(s)=\frac{s+b_2}{M(s)}U_1(s)+\frac{a_2}{M(s)}U_2(s)$$

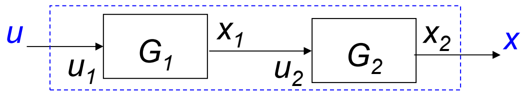
$$X_2(s)=\frac{b_1}{M(s)}U_1(s)+\frac{s+a_1}{M(s)}U_2(s)$$

$$M(s)=s^2+(a_1+b_2)s+a_1b_2-b_1a_2$$

$$X_1(s)=G_{11}(s)U_1(s)+G_{12}(s)U_2(s)$$
$$X_2(s)=G_{21}(s)U_1(s)+G_{22}(s)U_2(s)$$

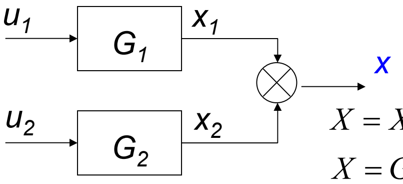


Transmitancja - połączenia elementarne

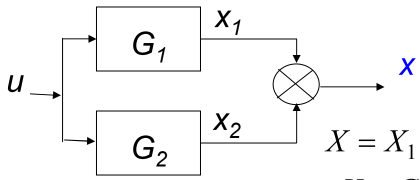


$$X=G_1G_2U$$

$$X_1=G_1U_1 \qquad X_2=G_2U_2=G_2X_1$$
$$X_2=G_1G_2U_1$$



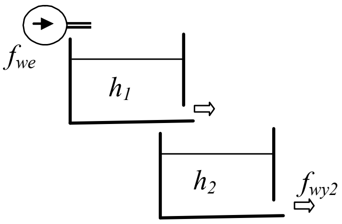
$$X=X_1+X_2$$
$$X=G_1U_1+G_2U_2$$

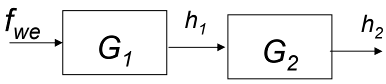


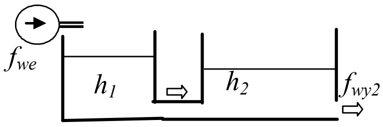
$$X=(G_1+G_2)U$$

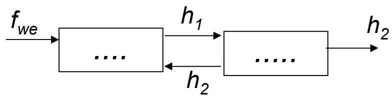
$$X=X_1+X_2$$
$$X=G_1U+G_2U$$

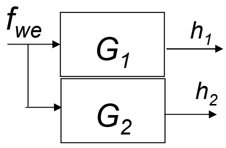
Kaskady niewspółdziałające i współdziałające











$$h_1(s) = \frac{\cdots}{M_1(s)} f_{we}(s)$$

$$h_2(s) = \frac{\cdots}{M_1(s)M_2(s)} f_{we}(s)$$

$$h_1(s) = \frac{\cdots}{M(s)} f_{we}(s)$$

$$h_2(s) = \frac{\cdots}{M(s)} f_{we}(s)$$

$$M(s) = M_1(s)M_2(s) + k$$

Człon inercyjny

$$G(s) = \frac{k}{Ts+1} \quad ,T > 0$$

$$a_1\dot{x}(t) + a_0 x(t) = b_0 u(t)$$

$$G(s) = \frac{b_0}{a_1 s + a_0} = \frac{b_0}{a_0\left(\frac{a_1}{a_0}s+1\right)} \quad ,k = \frac{b_0}{a_0} \quad ,T = \frac{a_1}{a_0}$$

r.s.: $a_0 x = b_0 u \rightarrow x = b_0 u / a_0$ $\qquad \lim_{s\to 0} s\frac{k}{Ts+1}\frac{u_k}{s} = ku_k$ $\qquad , u(t) = u_k$

r.ch.: $a_1 s + a_0 = 0 \rightarrow s_1 = \frac{-a_0}{a_1}$ $\qquad Ts + 1 = 0 \rightarrow s_1 = \frac{-1}{T}$ $\qquad s_1 < 0 \rightarrow T > 0$

Rozwiązanie ogólne dla u_k: $\boxed{x(t) = Ae^{(-1/T)t} + ku_k}$

a) $x_s(t) = Ae^{s_1 t} = Ae^{(-1/T)t}$

b) $x_w(t) = b_0 u_k / a_0 = ku_k$

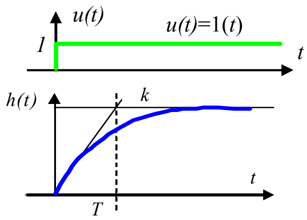
Odpowiedź skokowa:

c) $u_k = 1$

$$\boxed{\begin{aligned} x(t) &= -ke^{(-1/T)t} + k \\ x(t) &= k\left(1 - e^{(-1/T)t}\right) \end{aligned}}$$

d) $u(0)=0$, $x(0)=0$

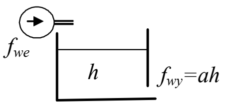
$0 = Ae^{-1/T\cdot 0} + k \rightarrow A = -k$



Człon inercyjny - przykłady

$$G(s)=\frac{k}{Ts+1}\quad ,T>0$$

Obiekty
z samowyrównywaniem



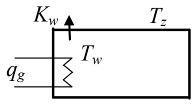
$$A\dot{h}(t)=f_{we}(t)-ah(t)$$

$$h(s)=\frac{1}{As+a}f_{we}(s)$$

$$k=\frac{1}{a}$$

$$T=\frac{A}{a}>0$$

Gdy $A\uparrow$, to …



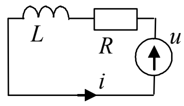
$$C_V\dot{T}_w(t)=q_g(t)-K_{cw}\left(T_w(t)-T_z(t)\right)$$

$$T_w(s)=\frac{1}{C_Vs+K_{cw}}q_g(s)+\frac{K_{cw}}{C_Vs+K_{cw}}T_z(s)$$

$$k=\frac{1}{K_{cw}}\qquad k=1$$

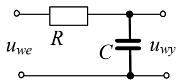
$$T=\frac{C_v}{K_{cw}}>0$$

Gdy $K_{cw}\uparrow$, to …
Gdy $V\uparrow$, to …



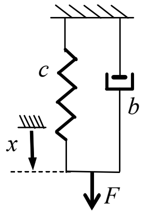
$$sLi(s)+Ri(s)=u(s)$$

$$i(s)=\frac{1}{sL+R}u(s)$$



$$u_{wy}(s)=\frac{1}{sC}\frac{u_{we}(s)}{R+1/(sC)}$$

$$u_{wy}(s)=\left(\frac{1}{sCR+1}\right)u_{we}(s)$$



$$b\dot{x}(t)+cx(t)=F(t)$$

$$x(s)=\frac{1}{sb+c}F(s)$$

Człon całkujący

$$G(s)=\frac{1}{T_is}=k_i\frac{1}{s}$$

$$a_1\dot{x}(t)=b_0u(t)$$

$$G(s)=\frac{b_0}{a_1s}\quad ,T_i=\frac{a_1}{b_0}$$

r.s.: $\quad \lim_{s\to 0} s\frac{1}{T_is}\frac{u_k}{s}=\infty \quad , u(t)=u_k$

$\lim_{s\to 0} s\frac{1}{T_is}1=\frac{1}{T_i} \quad , u(t)=\delta(t)$

r.ch.: $a_1s=0 \ \rightarrow \ s=0$

Rozwiązanie ogólne dla u_k: $\boxed{x(t)=k_iu_kt+x_0}$

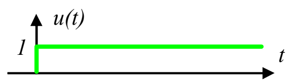
a) $x(t)=k_i\int u_k\,dt+x_0$

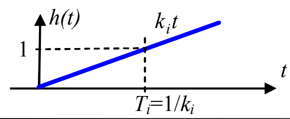
Odpowiedź skokowa: $\boxed{x(t)=k_it}$

b) u_k =1

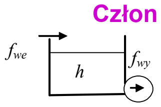
c) $u(0)$=0, $x(0)$=0

$0=k_it+x_k$



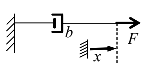


Człon całkujący - przykłady



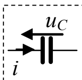
$$A\dot{h}(t)=f_{we}(t)-f_{wy}(t)$$

$$h_1(s)=\frac{1}{As}f_{we}(s)-\frac{1}{As}f_{wy}(s)$$



$$b\dot{x}(t)=F(t)$$

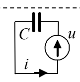
$$x(s)=\frac{1}{sb}F(s)$$



$$u_C(t)=\frac{1}{C}\int i(t)dt$$

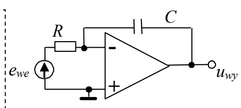
$$u_C(s)=\frac{1}{sC}i(s)$$

$$\left(u_C(t)=\frac{1}{C}q(t)\right)$$



$$\frac{1}{sC}i(s)=u(s)$$

$$i(s)=sCu(s)$$



$$u_{wy}=\frac{-1}{sRC}e_{we}$$

Człon oscylacyjny

$$a_2\ddot{x}(t)+a_1\dot{x}(t)+a_0x(t)=b_0u(t)$$

$$G(s)=\frac{k_1}{s^2+2\,\xi\,\omega_n s+\omega_n^2}\quad,\omega_n>0$$

$$G(s)=\frac{k}{T_n^2 s^2+2\,\xi\,T_n\,s+1}\quad,T_n>0$$

$$G(s)=\frac{b_0}{a_2s^2+a_1s+a_0}=\frac{b_0}{a_2\left(s^2+\frac{a_1}{a_2}s+\frac{a_0}{a_2}\right)}=\frac{b_0}{a_0\left(\frac{a_2}{a_0}s^2+\frac{a_1}{a_0}s+1\right)}$$

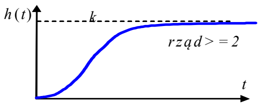
Jeśli $a_0/a_2>0$, to: $\omega_n^2=\frac{a_0}{a_2}$, $2\xi\omega_n=\frac{a_1}{a_2}$, $k_1=\frac{b_0}{a_2}$ $\qquad T^2=\frac{a_2}{a_0}$, $2\xi T=\frac{a_1}{a_0}$, $k=\frac{b_0}{a_0}$

r.s.: $a_0x=b_0u$ $\qquad \lim_{s\to0} s\frac{k_1}{s^2+2\xi\omega_n s+\omega_n^2}\frac{u_0}{s}=\frac{k_1u_0}{\omega_n^2}$, $u(t)=u_0$

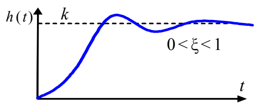
r.ch.: $a_2s^2+a_1s+a_0=0\rightarrow s_1,s_2$ $\qquad s^2+2\xi\omega_n s+\omega_n^2=0$ $\qquad T^2s^2+2\xi Ts+1=0$

$\mathrm{Im}(s_i)=0,\ \mathrm{Re}(s_i)<0$

$$\frac{k}{(T_1s+1)(T_2s+1)}\longleftarrow G(s)=\frac{b_0}{a_2(s-s_1)(s-s_2)}\longrightarrow\frac{k_1}{s^2+2\,\xi\,\omega\,s+\omega^2}$$



Człon inercyjny
II rzędu



Człon oscylacyjny znormalizowany

$$\frac{\omega_n^2}{s^2+2\,\xi\,\omega_n s+\omega_n^2}\quad,\omega_n>0 \qquad \frac{1}{T_n^2s^2+2\,\xi\,T_n\,s+1}\quad,T_n>0$$

$$\frac{\sigma^2+\omega_r^2}{s^2+2\,\sigma s+\sigma^2+\omega_r^2}$$

$$s_1=-\xi\omega_n+\omega_n\sqrt{\xi^2-1}$$
$$s_2=-\xi\omega_n-\omega_n\sqrt{\xi^2-1}$$

Gdy $\xi^2-1<0$

$$s_1=\alpha\ +j\omega_r$$
$$s_2=\alpha\ -j\omega_r$$

gdzie:

$$\alpha=-\xi\omega_n$$
$$\omega_r=\omega_n\sqrt{1-\xi^2}$$

$$s_1=-\sigma\ +j\omega_r$$
$$s_2=-\sigma\ -j\omega_r$$

gdzie:

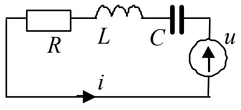
$$\sigma=\xi\omega_n$$
$$\omega_r=\omega_n\sqrt{1-\xi^2}$$

$$\frac{\sigma^2+\omega_r^2}{s^2+2\,\sigma s+\sigma^2+\omega_r^2}=\frac{(\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}{s^2+2\,\xi\omega_n s+(\xi\omega_n)^2+\left(\omega_n\sqrt{1-\xi^2}\right)^2}=\frac{\omega_n^2}{s^2+2\,\xi\omega_n s+\omega_n^2}$$

Człon oscylacyjny - przykłady

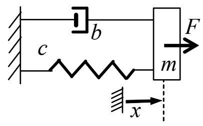
$$G(s)=\frac{k_1}{s^2+2\,\xi\,\omega_n s+\omega_n^2}\quad,\omega_n>0$$

$$G(s)=\frac{k}{T_n^2 s^2+2\,\xi\,T_n\,s+1}\quad,T_n>0$$



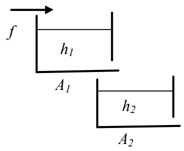
$$sLi(s)+Ri(s)+\frac{1}{sC}i(s)=u(s)$$

$$\frac{i(s)}{u(s)}=\frac{sC}{s^2LC+sRC+1}$$



$$m\ddot{x}(t)+b\dot{x}(t)+cx(t)=F(t)$$

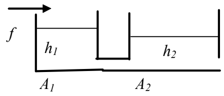
$$\frac{x(s)}{F(s)}=\frac{1}{ms^2+bs+c}$$



$$\begin{cases}A_1\dot{h}_1(t)=f(t)-a_1h_1(t)\\ A_2\dot{h}_2(t)=a_1h_1(t)-a_2h_2(t)\end{cases}$$

$$\frac{h_2(s)}{f(s)}=\frac{1}{(A_1s+a_1)(A_2s+a_2)}$$

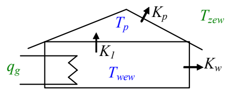
$\xi>1$



$$\begin{cases}A_1\dot{h}_1(t)=f(t)-a_1\big(h_1(t)-h_2(t)\big)\\ A_2\dot{h}_2(t)=a_1\big(h_1(t)-h_2(t)\big)-a_2h_2(t)\end{cases}$$

$$\frac{h_2(s)}{f(s)}=\frac{a_1}{(A_1s+a_1)(A_2s+a_1+a_2)-a_1^2}$$

$\xi=?$



$\xi=?$

Człon proporcjonalny

$$a_0x(t)=b_0u(t)$$

$$G(s)=k\qquad,k=\frac{b_0}{a_0}$$

$$G(s)=k$$

r.s.: $a_0x=b_0u$

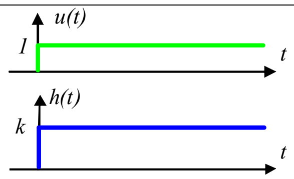
r.ch.:

Odpowiedź skokowa:

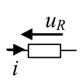
a) $u_k=1$

b) $u(0)=0$, $x(0)=0$

$$\boxed{x(t)=k1(t)}$$

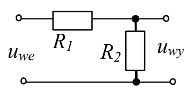


Człon proporcjonalny - przykłady



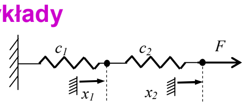
$$u_R(t)=Ri(t)$$

$$\frac{u_R(s)}{i(s)}=R$$



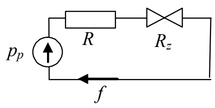
$$u_{wy}(t)=\left(\frac{R_2}{R_1+R_2}\right)u_{we}(t)$$

$$\frac{u_{wy}(s)}{u_{we}(s)}=\frac{R_2}{R_1+R_2}$$



$$\begin{cases}0=c_1x_1(t)+c_2\big(x_1(t)-x_2(t)\big)\\ F(t)=c_2\big(x_2(t)-x_1(t)\big)\end{cases}$$

$$x_1=\frac{F}{c_1}\qquad x_2=\frac{c_1+c_2}{c_1c_2}F$$



$$p_p(t)=(R+R_z)f(t)$$

$$f=\frac{1}{R+R_z}p_p$$

Człon różniczkujący

$a_0 x(t) = b_1 \dot{u}(t)$

$G(s) = T_d s \qquad , T_d = \dfrac{b_1}{a_0}$

$$\boxed{G(s) = T_d s}$$

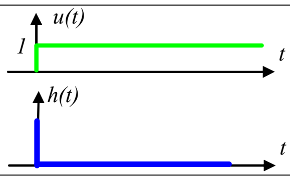
r.s.: $\quad a_0 x = 0$

r.ch.:

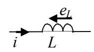
Odpowiedź skokowa:
a) $u_k = 1$

b) $u(0)=0$, $x(0)=0$

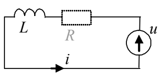
$\boxed{x(t) = T_d \delta(t)}$



Człon różniczkujący - przykłady



$e_L(t) = -L\dfrac{di(t)}{dt}$



$L\dfrac{di(t)}{dt} = u(t)$

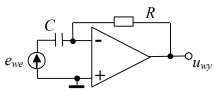
$sLi(s) = u(s)$

$i(s) = \dfrac{1}{sL} u(s)$

$L\dfrac{di(t)}{dt} + Ri(t) = u(t)$

$sLi(s) + Ri(s) = u(s)$

$i(s) = \dfrac{1}{sL+R} u(s)$



$u_{wy} = -sRC\, e_{we}$

Podstawowe obiekty (człony) dynamiki

$a_0 x(t) = b_0 u(t)$

$$G(s) = k$$

$a_1 \dot{x}(t) + a_0 x(t) = b_0 u(t)$

$$G(s) = \frac{k}{Ts+1} \quad , T > 0$$

$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = b_0 u(t)$

$$G(s) = \frac{k1}{s^2 + 2\,\xi\,\omega_n\, s + \omega_n^2}, \omega_n > 0$$

$$G(s) = \frac{k}{T_n^2 s^2 + 2\,\xi\,T_n\, s + 1}, T_n > 0$$

$a_1 \dot{x}(t) = b_0 u(t)$

$$G(s) = \frac{1}{T_i s}$$

$a_0 x(t) = b_1 \dot{u}(t)$

$$G(s) = T_d s$$

k, k_1 – współczynniki wzmocnienia członu
T – stała czasowa
T_o – opóźnienie
T_i – czas całkowania
T_d – czas różniczkowania
ω_n – pulsacja drgań własnych nietłumionych
T_n – okres drgań własnych nietłumionych
(współczynnik okresu drgań własnych)

$\omega_n = \dfrac{1}{T_n} \qquad \omega_n = 2\pi f = \dfrac{2\pi}{T}$

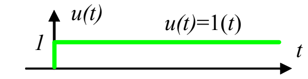
ξ – współczynnik tłumienia względnego

$$\boxed{G(s) = \frac{L(s)}{M(s)}}$$

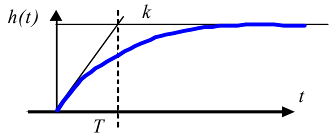
$x(t) = u(t - T_0)$

$$G(s) = e^{-T_0 s}$$

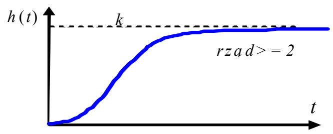
Odpowiedzi skokowe członów podstawowych



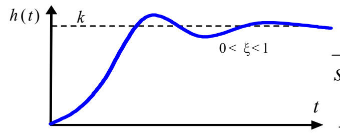
k



$$\frac{k}{Ts+1}$$

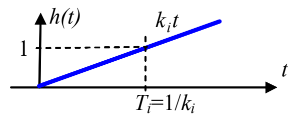


$$\frac{k}{(T_1 s+1)(T_2 s+1)}$$



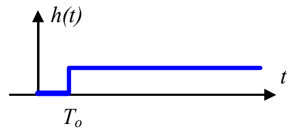
$$\frac{k}{s^2+2\,\xi\,\omega_n\,s+\omega_n^2}$$

$$\frac{K}{T_n^2 s^2+2\,\xi T_n\,s+1}$$



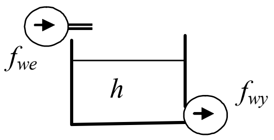
$$\frac{1}{T_i s}=\frac{k_i}{s}$$

$T_d s$



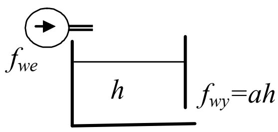
$e^{-T_o s}$

Przykłady obiektów



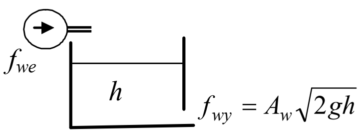
$$A\dot{h}(t)=f_{we}(t)-f_{wy}(t)$$

$$h_1(s)=\frac{1}{As}f_{we}(s)-\frac{1}{As}f_{wy}(s)$$

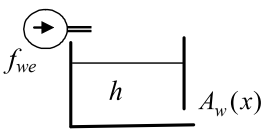


$$A\dot{h}(t)=f_{we}(t)-ah(t)$$

$$h(s)=\frac{1}{As+a}f_{we}(s)$$



$$A\dot{h}(t)=f_{we}(t)-A_w\sqrt{2gh(t)}$$



$$A\dot{h}(t)=f_{we}(t)-A_w(x(t))\sqrt{2gh(t)}$$

Człony o zadanych parametrach

1) Cz.inercyjny z biegunem s_1:

$$G(s)=\frac{a}{s-s_1}=\frac{a}{-s_1\left(\frac{1}{-s_1}s+1\right)}=\frac{k}{Ts+1}$$

a) wzmocnienie członu inercyjnego = 1

$$k=1 \quad \rightarrow \frac{a}{-s_1}=1 \ \rightarrow a=-s_1$$

b) wzmocnienie układu K_0:

$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad \frac{a}{-s_1}=K_0 \rightarrow a=\ldots$$

2) Cz.oscylacyjny o tłumieniu* ½ i pulsacji** 2:

$$G(s)=\frac{a}{s^2+2\xi\omega s+\omega^2}=\frac{a}{s^2+2s+4}$$

$$=\frac{a}{(s-s_1)(s-s_2)}=\frac{b}{(T_1s+1)(T_2s+1)}$$

a) wzmocnienie członu oscylacyjnego = 1

$$a=1$$

b) wzmocnienie układu K_0:

$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad \frac{a}{\omega^2}=K_0 \rightarrow a=\ldots$$

3) Cz.oscylacyjny o tłumieniu ½ i okresie** 2:

$$G(s)=\frac{a}{T^2s^2+2\xi Ts+1}=\frac{a}{4s^2+2s+1}$$

a) wzmocnienie członu oscylacyjnego = 1

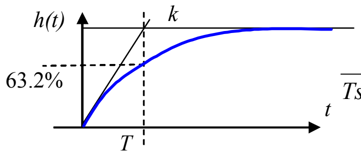
$$a=1$$

b) wzmocnienie układu K_0:

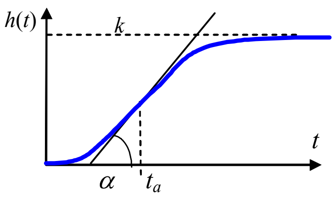
$$\lim_{s\to 0} sG(s)\frac{1}{s}=K_0 \quad \rightarrow \quad a=K_0$$

* tłumienie - współczynnik tłumienia względnego
** pulsacja/okres – pulsacja/okres drgań własnych nietłumionych

Identyfikacja modelu na podstawie odpowiedzi na wymuszenie skokowe

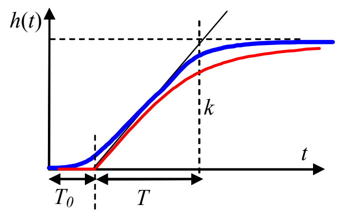


$$\frac{k}{Ts+1}\quad , \ k=\frac{\Delta x}{\Delta u}$$

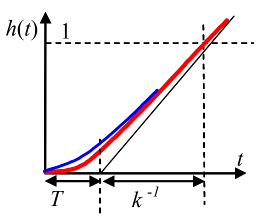


$$\frac{k}{(1+sT)^n}$$

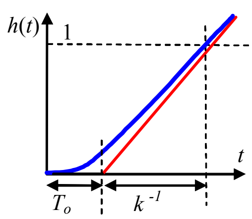
$$\frac{k}{(1+sT)^n}e^{-sT_o}$$



$$\frac{k}{Ts+1}e^{-sT_o}$$

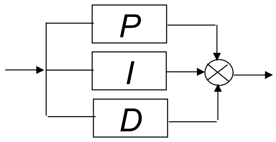


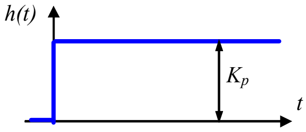
$$\frac{k}{s(Ts+1)}$$



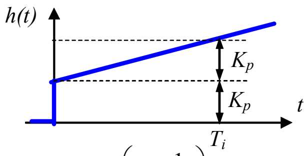
$$\frac{k}{s}e^{-sT_o}$$

Regulator PID – odpowiedzi skokowe

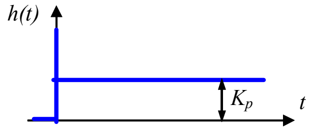




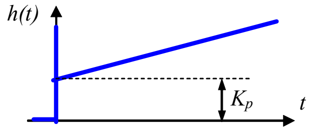
$$\mathrm{P}: K_p$$



$$\mathrm{PI}: K_p\left(1+\frac{1}{T_i s}\right)$$

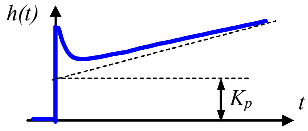


$$\mathrm{PD}: K_p\left(1+T_d s\right)$$



$$\mathrm{PID}: K_p\left(1+\frac{1}{T_i s}+T_d s\right)$$

idealny



$$\mathrm{PID}: K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{Ts+1}\right)$$

rzeczywisty