A Rapid Review of the Quantum Inspired and Quantum Algorithms we have today

Quantum Computing is an entirely new model of computation that harnesses the principles of quantum states such as superposition, interference and entanglement to decidedly complicated calculations. The earliest quantum computer was a quantum mechanical model of a Turing Machine proposed by physicist Paul Benioff. A breakthrough in the field occurred when Peter Shor developed a quantum algorithm for factoring integers with the potential to decrypt RSA-encrypted communications. The factorization problem and its feasibility to be solved in polynomial time by a classical computer is one of the biggest unsolved problems in computer science today, and as such would suggest that quantum computers despite their severely hindered state today, can simulate things classical computers simply cannot do. Even if a full blown quantum computer is not realizable, there are several techniques from quantum algorithms we can study and use them to improve our classical algorithms of today, to provide more efficient algorithms and a new purview of computation as we know it.

ACM Reference Format:

. 2022. A Rapid Review of the Quantum Inspired and Quantum Algorithms we have today. 1, 1 (March 2022), 5 pages. https://doi.org/10.1145/nnnnnnn.nnnnnnn

1 PROCEDURE

I looked at quantum analogues for some of the classical algorithms at the forefront of complexity theory and machine learning. While classical algorithms are simply not efficient enough for certain kinds of problems, quantum algorithms (save a few) are not feasible with the technology we possess today. Quantum inspired algorithms are the middle ground that is at the cutting edge today, and can help provide valuable insight to further advance the field of quantum computing

2 THE FACTORIZED FORM OF UNITARY COUPLED CLUSTER THEORY

In quantum chemistry, the minimum eigenvalue of the Hermitian Matrix characterizing that molecule is the ground state energy of that particular system. While the quantum phase estimation algorithm is an appropriate technique that can be used to find said minimum eigenvalue, its implementation on useful problems requires circuit depths exceeding the limits of hardware available in the N.I.S.Q (Noisy Intermediate scale Quantum) era. The variational quantum solver algorithm is one such solution to this problem using quantum circuits of much shallower depth. The factorized form of unitary coupled cluster theory (UCC) is a promising wave-function ansatz for the VQE algorithm. For weakly correlated molecules, the factorized form of the UCC provides similar accuracy to conventional coupled cluster theory (CC) and for strongly correlated molecules, where CC often breaks down, UCC comes out on top by a large margin.

[1]

Author's address:

Manuscript submitted to ACM

3 SEPARABLE NON-NEGATIVE MATRIX FACTORIZATION

Non-negative Matrix Factorization (NMF) is a problem that asks for the decomposition of a non-negative matrix into the product of two smaller-sized non-negative matrices, which has been shown to be a problem for which no efficient algorithms exist as of today. In the classical model of computation, we introduce the Separability Assumption. that assumes all data points exist in a conical hull. Doing so makes NMF a tractable problem and has potential in text analysis and image processing, however it is still widely impractical for large datasets. Based on recent development in dequantization techniques, the authors propose a new classical algorithm for the separable NMF problem. Their algorithm runs in polynomial time in the rank and logarithmic time in the size of the matrix. Consequently doing so achieves an exponential speedup in the low-rank setting.

[2]

4 QUANTUM-INSPIRED ALGORITHMS FROM RANDOMIZED NUMERICAL LINEAR ALGEBRA

The authors of this paper create classical (non-quantum) dynamic data structures supporting queries for recommender systems and a method of least-squares regression that is comparable to their quantum analogues. More significantly, they achieve these improvements by arguing that the previous quantum-inspired algorithms for these problems are doing leverage score sampling or ridge-leverage score sampling. With this recognition, they use the body of work in numerical linear algebra to obtain algorithms for these problems that are simpler or faster (or both) than existing approaches.

[3]

5 QUANTUM ALGORITHMS FOR SOLVING LINEAR EQUATIONS

Solving linear systems of equations is a common problem that arises both on its own and as a subroutine in more complex problems: given a matrix A and a vector $\vec{b}$, find a vector $\vec{x}$ such that $A\vec{x} = \vec{b}$. Instead of trying to compute an approximation of the expected value of some operator associated with $\vec{x}$ itself. Here, the authors exhibit a quantum algorithm for this task that runs in $\text{poly}(\log N, \kappa)$ time, an exponential improvement over the best classical algorithm.

[10]

6 RECOMMENDATION SYSTEMS

The authors come up with a classical algorithm to Kerenidis and Prakash's quantum recommendation system, believed to be one of the strongest candidates for provably exponential speedups in quantum machine learning. Their result is that an mn matrix in a data structure supporting certain ℓ^2-norm sampling operations, outputs a similar sample from a rank-k approximation of that matrix in time $O(\text{poly}(k)\log(mn))$, which is only polynomially slower than the quantum algorithm which does not in fact demonstrate an exponential speedup over the classical analogue.

Under strong input assumptions, the classical recommendation system resulting from this new classical algorithm produces recommendations exponentially faster than previous classical systems, which run in time linear in m and n. The main insight here is the use of simple routines to manipulate ℓ^2 norm sampling distributions, which play the role of quantum superpositions in the classical setting.

[13]

7 QUANTUM-INSPIRED SUBLINEAR CLASSICAL ALGORITHMS FOR SOLVING LOW-RANK LINEAR SYSTEMS

The authors present classical sublinear-time algorithms for solving low-rank linear systems of equations. The algorithms proposed are inspired by the HHL quantum algorithm [10] for solving linear systems and the recent breakthrough by Tang [13] of dequantizing the quantum algorithm for recommendation systems.

8 QUANTUM INSPIRED ALGORITHM FOR SUPPORT VECTOR MACHINES

Support Vector Machines are powerful and flexible supervised learning models that analyzes data for both the purposes of classification and regression. Their algorithm complexity scales polynomially with the dimension of data space and the number of data points. As with other algorithms, a quantum SVM algorithm was proposed to solve the problem of large-scale data sets. The quantum algorithm is claimed to acheive an exponential speedup for Least Squares Support Vector Machines (LS-SVM). Drawing on techniques from the quantum SVM algorithm, the authors present a quantum-inspired classical algorithm for an LS-SVM. They propose indirect sampling, an improved fast sampling technique, for sampling the kernel matrix and classifying.

[5]

9 QUANTUM INSPIRED ALGORITHMS FOR MULTIVARIATE ANALYSIS

The main focus of this paper is developing quantum inspired algorithms for multivariate analysis. The authors study the encoding of differentiable multivariate functions distributions in quantum registers. They present eight quantum-inspired numerical analysis algorithms, including Fourier sampling, interpolation, differentiation and integration of partial derivative equations. These algorithms combine classical ideas: finite-differences, spectral methods—with the efficient encoding of quantum registers, and well known algorithms, such as the Quantum Fourier Transform. When these heuristic methods work, they provide an exponential speed-up over other classical algorithms, such as Monte Carlo integration, finite-difference and fast Fourier transforms (FFT).

10 QUANTUM INSPIRED LOW RANK STOCHASTIC REGRESSION

The authors construct a classical analogue of the quantum matrix inversion algorithm [10] for low-rank matrices. They implement the pseudoinverse of a low-rank matrix and sample from the solution to the problem $Ax = b$ using fast sampling techniques. The pseudo-inverse is implemented by finding an approximate singular value decomposition of A via subsampling, then inverting the singular values. Their result suggests that more low-rank quantum algorithms can be effectively "dequantised" into classical length-square sampling algorithms.

[7]

11 AN IMPROVED QUANTUM INSPIRED ALGORITHM FOR LINEAR REGRESSION

We give a classical algorithm for linear regression analogous to the quantum matrix inversion algorithm [10] for low-rank matrices when the input matrix is stored in a data structure applicable for QRAM-based state preparation. This results from this paper could lead to feasible implementations of classical regression in quantum-inspired settings, for comparison against future quantum computers.

12 A QUANTUM INSPIRED EVOLUTIONARY ALGORITHM

This paper proposes a novel evolutionary algorithm inspired by quantum computing, called a quantum-inspired evolutionary algorithm (QEA). Like other evolutionary algorithms, QEA is also characterized by the representation of the individual, evaluation function, and population dynamics. A Q-gate is introduced as a variation operator to drive the individuals toward better solutions. To demonstrate its effectiveness and applicability, experiments were carried out on the knapsack problem, a well-known combinatorial optimization problem. The results show that QEA performs well, even with a small population, without premature convergence (for which several mitigation strategies have been adopted), compared to the conventional genetic algorithm.

13 QUANTUM INSPIRED ALGORITHM FOR THE SINGULAR VALUE TRANSFORMATION

Pursuing the line of research associated with [13] and the quantum algorithm for recommendation systems by Kerendish and Prakash, the authors of this paper develop quantum-inspired algorithms for a large class of matrix transformations that are defined via the singular value decomposition of the matrix. In particular, we obtain classical algorithms with complexity polynomially related (in most parameters) to the complexity of the best quantum algorithms for singular value transformation so far.

[11]

14 A QUANTUM INSPIRED GENETIC ALGORITHM FOR THE TRAVELLING SALESMAN PROBLEM

The TSP is one of the most known combinatorial optimisation problems. It is about finding the shortest Hamiltonian cycle relating N cities, where a Hamiltonian cycle is simply a Hamiltonian path which is a cycle and a Hamiltonian path is a regular path(directed or undirected) that visits each vertex in a graph exactly once. The paper involves techniques that extend the standard genetic algorithms by combining them with the concepts of quantum bits, the superposition of states and interference. The obtained results from the application of the proposed algorithm on some instances of the TSP are significantly better than those provided by standard genetic algorithms.

[12]

15 QUANTUM INSPIRED TRAINING FOR BOLTZMANN MACHINE

Deep Boltzmann machines are models that possess a larger number of hidden layers with directionless connections between their nodes. A DBM learns features hierarchically from raw data and the features extracted in one layer are applied as hidden variables as input to the subsequent layer. They also incorporate a Markov random field for the layer-wise pre-training for unlabeled data and then provides feedback from the upper layer to the backward layers. The training algorithm is then subsequently improved using the method of backpropagation.

The authors of this paper present an efficient classical algorithm for training deep Boltzmann machines that uses rejection sampling in order to estimate the gradients of the training objective function. They obtain rigorous bounds on the errors in the approximate gradients; in turn, we find that choosing the instrumental distribution to minimize the $\alpha = 2$ divergence with the Gibbs state(equivalent probability distribution that stays invariant) minimizes the asymptotic algorithmic complexity. Finally our algorithm can train full Boltzmann machines and scales more favorably (in terms of complexity) with the number of layers in a DBM than greedy contrastive divergence training.

REFERENCES

[1] Jia Chen, Hai-Ping Cheng, and James K. Freericks. 2021. Quantum-Inspired Algorithm for the Factorized Form of Unitary Coupled Cluster Theory. *Journal of Chemical Theory and Computation* 17, 2 (2021), 841–847. https://doi.org/10.1021/acs.jctc.0c01052 PMID: 33503376.

[2] Zhihuai Chen, Yinan Li, Xiaoming Sun, Pei Yuan, and Jialin Zhang. 2019. A Quantum-inspired Classical Algorithm for Separable Non-negative Matrix Factorization. *CoRR* abs/1907.05568 (2019). arXiv:1907.05568 http://arxiv.org/abs/1907.05568

[3] Nadiia Chepurko, Kenneth L. Clarkson, Lior Horesh, and David P. Woodruff. 2020. Quantum-Inspired Algorithms from Randomized Numerical Linear Algebra. *CoRR* abs/2011.04125 (2020). arXiv:2011.04125 https://arxiv.org/abs/2011.04125

[4] Nai-Hui Chia, Han-Hsuan Lin, and Chunhao Wang. 2018. Quantum-inspired sublinear classical algorithms for solving low-rank linear systems. *CoRR* abs/1811.04852 (2018). arXiv:1811.04852 http://arxiv.org/abs/1811.04852

[5] Chen Ding, Tian-Yi Bao, and He-Liang Huang. 2021. Quantum-Inspired Support Vector Machine. *IEEE Transactions on Neural Networks and Learning Systems* (2021), 1–13. https://doi.org/10.1109/TNNLS.2021.3084467

[6] Juan José García-Ripoll. 2021. Quantum-inspired algorithms for multivariate analysis: from interpolation to partial differential equations. *Quantum* 5 (Apr 2021), 431. https://doi.org/10.22331/q-2021-04-15-431

[7] András Gilyén, Seth Lloyd, and Ewin Tang. 2018. Quantum-inspired low-rank stochastic regression with logarithmic dependence on the dimension. *CoRR* abs/1811.04909 (2018). arXiv:1811.04909 http://arxiv.org/abs/1811.04909

[8] András Gilyén, Zhao Song, and Ewin Tang. 2020. An improved quantum-inspired algorithm for linear regression. *CoRR* abs/2009.07268 (2020). arXiv:2009.07268 https://arxiv.org/abs/2009.07268

[9] Kuk-Hyun Han and Jong-Hwan Kim. 2002. Quantum-inspired evolutionary algorithm for a class of combinatorial optimization. *IEEE Transactions on Evolutionary Computation* 6, 6 (2002), 580–593. https://doi.org/10.1109/TEVC.2002.804320

[10] Aram W. Harrow, Avinatan Hassidim, and Seth Lloyd. 2009. Quantum Algorithm for Linear Systems of Equations. *Physical Review Letters* 103, 15 (Oct 2009). https://doi.org/10.1103/physrevlett.103.150502

[11] Dhawal Jethwani, François Le Gall, and Sanjay K. Singh. 2019. Quantum-Inspired Classical Algorithms for Singular Value Transformation. *CoRR* abs/1910.05699 (2019). arXiv:1910.05699 http://arxiv.org/abs/1910.05699

[12] H. Talbi, A. Draa, and M. Batouche. 2004. A new quantum-inspired genetic algorithm for solving the travelling salesman problem. In *2004 IEEE International Conference on Industrial Technology, 2004. IEEE ICIT '04.*, Vol. 3. 1192–1197 Vol. 3. https://doi.org/10.1109/ICIT.2004.1490730

[13] Ewin Tang. 2018. A quantum-inspired classical algorithm for recommendation systems. *CoRR* abs/1807.04271 (2018). arXiv:1807.04271 http://arxiv.org/abs/1807.04271

[14] Nathan Wiebe, Ashish Kapoor, Christopher E. Granade, and Krysta M. Svore. 2015. Quantum Inspired Training for Boltzmann Machines. *CoRR* abs/1507.02642 (2015). arXiv:1507.02642 http://arxiv.org/abs/1507.02642