ICPC Templates

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1 基础算法

1.1 快速排序

1.1.1 模板

```
#include <iostream>
 1
   #include <algorithm>
   using namespace std;
   #define N 100010
 4
 5
   int num[N];
   int n;
 6
 7
    int quick_sort(int num[], int 1, int r){
 8
        if(1 >= r) return 0;
        int mid = num[l+r >> 1], i = l-1, j = r + 1;
 9
10
        while(i < j){</pre>
11
           while(num[++i] < mid);</pre>
12
           while(num[--j] > mid);
13
           if(i < j) swap(num[i],num[j]);</pre>
14
       }
15
        quick_sort(num, 1, j);
        quick_sort(num, j + 1, r);
16
17
18
    int main(){
19
        scanf("%d", &n);
20
        for(int i =0 ;i < n ; i++) scanf("%d", &num[i]);</pre>
21
        quick_sort(num, 0, n-1);
        for(int i = 0; i < n; i++) printf("%d ", num[i]);</pre>
22
23
        return 0;
24
```

1.1.2 第 k 个数

```
1 #include <iostream>
   using namespace std;
 3
   #define N 100010
4
   int num[N];
 5
 6
   int n,k;
7
    int quick_sort(int num[], int 1, int r,int k){
 9
        if(1>=r) return 0;
       int x = num[l + r>>1], i = l-1, j= r+1;
10
11
       while(i < j){</pre>
12
           while(num[++i] < x);
13
           while(num[--j] > x);
14
           if(i < j) swap(num[i],num[j]);</pre>
15
16
17
       int sk = j - l + 1;
18
       if(k <= sk) quick_sort(num, 1, j, k);</pre>
19
       else quick_sort(num, j+1, r, k - sk);
20 }
```

```
21
22  int main(){
23    scanf("%d %d", &n, &k);
24    for(int i = 0; i < n;i++) scanf("%d", &num[i]);
25    quick_sort(num, 0, n-1, k);
26    printf("%d", num[k-1]);
27    return 0;
28 }</pre>
```

1.2 归并排序

1.2.1 模板

```
1 #include<iostream>
    using namespace std;
   #define N 100010
 5
   int q[N];
 6
   int n;
 7
 8
 9
    int merge_sort(int q[], int 1, int r){
10
        if(1>=r) return 0;
11
       int mid = 1+r >>1;
12
       merge_sort(q,1,mid);
13
        merge_sort(q,mid+1,r);
14
        int i=1,j=mid+1,k=0,tmp[r-l+1];
15
        while(i<=mid&&j<=r){</pre>
16
           if(q[i]<=q[j]) tmp[k++] = q[i++];</pre>
17
           else tmp[k++] = q[j++];
18
        }
19
        while(i<=mid) tmp[k++] = q[i++];</pre>
20
        while(j \le r) tmp[k++] = q[j++];
21
        for(int i=0,j=1;j<=r;i++,j++) q[j] = tmp[i];</pre>
22
23
24
   int main(){
25
       scanf("%d", &n);
26
        for(int i = 0 ;i < n;i++) scanf("%d", &q[i]);</pre>
27
        merge_sort(q,0,n-1);
        for(int i = 0;i <n ;i++) printf("%d ", q[i]);</pre>
28
29
        return 0;
   }
30
```

1.2.2 逆序对数量

```
#include<iostream>
#include<algorithm>
using namespace std;

#define N 100010
```

第7页

```
7
   int q[N];
    int n;
 9
   long long res=0;
10
11
   int merge_sort(int q[], int 1, int r){
        if(1>=r) return 0;
12
13
        int mid = 1 + r >> 1;
14
       merge_sort(q,1,mid);
15
        merge_sort(q,mid+1,r);
16
        int i=1,j=mid+1,k=0,tmp[r-l+1];
17
        while(i<=mid&&j<=r){</pre>
           if(q[i] <= q[j]) tmp[k++] = q[i++];</pre>
18
19
           else {
20
               tmp[k++] = q[j++];
21
               res += (mid - i + 1);
22
           }
23
24
        while(i<=mid) tmp[k++] = q[i++];</pre>
25
        while(j<=r) tmp[k++] = q[j++];</pre>
26
        for(int i=0,j=1;j<=r;i++,j++) q[j] = tmp[i];</pre>
27
28
   int main(){
29
30
        scanf("%d", &n);
        for(int i= 0; i< n ; i++) scanf("%d", &q[i]);</pre>
31
32
       merge_sort(q, 0, n-1);
33
        printf("%ld",res);
34
        return 0;
35
   }
```

1.3 二分

1.3.1 数的范围

```
1 #include <iostream>
 2 #include <algorithm>
    using namespace std;
 4
    #define N 100010
 5
 6
 7
    int n,q,k;
8
   int a[N];
10
    int lower_find(int a[], int 1, int r, int k){
       while(1 < r){
11
           int mid = 1 + r >> 1;
12
13
           if(k <= a[mid]) r = mid;</pre>
           else 1 = mid+1;
14
15
       return a[1] == k ? 1 : -1;
16
17
   }
18
```

```
19
    int upper_find(int a[], int 1, int r, int k){
20
       while(1 < r){
           int mid = l+r+1 >> 1;
21
22
           if(k >= a[mid]) l = mid;
23
           else r = mid -1;
       }
24
25
       return a[1] == k ? 1 : -1;
26
   }
27
28
    int main(){
29
       scanf("%d %d",&n, &q);
        for(int i= 0 ;i< n;i++) scanf("%d", &a[i]);</pre>
30
31
       for(int i =0 ;i < q;i++) {</pre>
           scanf("%d", &k);
32
33
           int res = lower_find(a, 0, n-1, k);
34
           if(res == -1) printf("%d %d", -1, -1);
35
           else{
              printf("%d ", res);
36
37
              res = upper_find(a, 0, n-1, k);
38
              printf("%d", res);
           }
39
40
           if(i != q - 1)
              printf("\n");
41
42
       }
43
   }
```

1.3.2 数的三次方根

```
#include <iostream>
    using namespace std;
 3
    # define N 100010
 5
    double n;
 6
7
    int main(){
 8
       scanf("%lf", &n);
       double 1 = -10000, r = 10000, x=0;
 9
       while(r - 1 >= 10e-8){
10
11
           x = (1 + r) / 2;
12
           if(x * x * x >= n) r = x;
13
           else 1 = x;
14
       printf("%lf", x);
15
16
    }
```

1.4 高精度运算

1.4.1 加法

```
#include <iostream>
#include <algorithm>
#include <string>
```

```
#include <vector>
    using namespace std;
 7
    vector<int> add(vector<int> &A, vector<int> &B){
 8
       vector<int> C;
 9
       int t = 0;
10
       for(int i=0; i < A.size() || i < B.size(); i++){</pre>
11
           if(i < A.size()) t += A[i];</pre>
           if(i < B.size()) t += B[i];</pre>
12
13
           C.push_back(t % 10);
14
           t /= 10;
       }
15
       if(t) C.push_back(1);
16
17
       return C;
18
19
20
   int main(){
21
       vector<int> A,B;
22
       string a,b;
23
       cin >> a >> b;
24
        for(int i= a.size() - 1; i>=0; i--) A.push_back(a[i] - '0');
25
       for(int i= b.size() -1; i>=0;i--) B.push_back(b[i] - '0');
26
       auto C = add(A,B);
27
       for(int i = C.size() -1; i>=0;i--) printf("%d", C[i]);
28
       return 0;
29
   }
```

1.4.2 减法

```
1 #include<iostream>
   #include<string>
   #include<algorithm>
 4
    #include<vector>
    using namespace std;
 6
7
   // A >= B
9
    bool cmp(vector<int> &A, vector<int> &B){
       if(A.size() > B.size()) return true;
10
       else if(A.size() < B.size()) return false;</pre>
11
12
       else{
13
           for(int i = A.size() - 1; i >= 0; i--)
              if(A[i] != B[i]) return A[i] > B[i];
14
15
           return true;
       }
16
17
    }
18
    vector<int> sub(vector<int> &A, vector<int> &B){
19
20
       vector<int> C;
21
       int t = 0;
       for(int i = 0; i < A.size(); i++){</pre>
22
           t = A[i] -t; // 减掉进位
23
```

```
if(i < B.size()) t -= B[i];</pre>
24
25
           C.push_back((t+10)\%10);
           if(t<0) t=1;
26
27
           else t=0;
28
       }
29
       while(C.size() > 1 && C.back() == 0) C.pop_back();
30
       return C;
31
32
33
    int main(){
34
       string a, b;
35
       vector<int> A, B;
36
       cin >> a >> b;
37
       for(int i = a.size() - 1;i>=0; i--) A.push_back(a[i] -'0');
38
        for(int i= b.size() -1; i>=0;i--) B.push_back(b[i] - '0');
39
       vector<int> C;
40
       if(cmp(A,B)){
41
           C = sub(A,B);
42
       }
43
       else{
           printf("%c", '-');
44
45
           C = sub(B,A);
46
       }
47
       for(int i = C.size() -1; i>=0 ; i--) printf("%d", C[i]);
48
       return 0;
49
    }
```

1.4.3 乘法

```
1
   #include<iostream>
   #include<vector>
 3
    #include<algorithm>
 4
    #include<string>
 5
    using namespace std;
 6
7
 8
    vector<int> mul(vector<int> &A, int b){
 9
       vector<int> C;
10
       int t = 0;
11
       for(int i = 0; i < A.size() || t; i++){</pre>
           if(i < A.size()) t += A[i] * b;</pre>
12
13
           C.push_back(t % 10);
14
           t /= 10;
15
       while(C.size() > 1 && C.back() == 0) C.pop_back();
16
17
       return C;
18
    }
19
20
21
    int main(){
22
       string a;
23
       int b;
```

```
24     vector<int> A,B;
25     cin>>a>>b;
26     for(int i = a.size() - 1 ;i >= 0 ; i --) A.push_back(a[i] - '0');
27     // for(int i = b.size() - 1 ;i >= 0 ; i ++) B.push_back(b[i] - '0');
28     auto C = mul(A,b);
29     for(int i = C.size() - 1; i>=0; i--) printf("%d" ,C[i]);
30 }
```

1.4.4 除法

```
#include <iostream>
1
 2 #include <vector>
 3
    #include <string>
   #include <algorithm>
   using namespace std;
 6
 7
    vector<int> div(vector<int> &A, int b, int &r){
8
 9
       vector<int> C;
10
       int t = 0;
       for(int i = A.size() -1; i >=0;i --){
11
           r = r * 10 + A[i];
12
13
           C.push_back(r / b);
           r %= b;
14
15
       }
16
       reverse(C.begin(),C.end());
17
       while(C.size() > 1 && C.back() == 0) C.pop_back();
       return C;
18
19
20
21
22
   int main(){
23
       string a;
       int b, r=0;
24
25
       vector<int> A;
26
       cin >> a >> b;
27
       for(int i = a.size() - 1; i>=0; i--) A.push_back(a[i] - '0');
28
       auto C = div(A,b,r);
29
       for(int i =C.size() -1; i>=0; i--) printf("%d", C[i]);
30
       printf("\n%d", r);
31
       return 0;
32
    }
```

1.5 前缀与差分

1.5.1 前缀和

```
#include<iostream>
using namespace std;
#define N 100010
int a[N];
int S[N];
```

```
int n,m,l,r;
 6
 7
    int main(){
        scanf("%d %d",&n,&m);
 8
 9
        for(int i = 1; i<=n; i++) scanf("%d", &a[i]);</pre>
10
        for(int i =1 ;i<=n;i++) S[i] = S[i-1] + a[i];</pre>
11
        while(m--){
12
           scanf("%d %d", &1, &r);
13
           printf("%d\n" ,S[r] - S[1-1]);
14
        }
15
```

1.5.2 子矩阵的和

```
#include<iostream>
 2
    using namespace std;
 3
   #define N 1010
 4
 5
   int a[N][N];
 6
 7
   int S[N][N];
 9
    int n,m,q,x1,x2,y1,y2;
10
11
    int main(){
12
        scanf("%d %d %d",&n,&m, &q);
13
        for(int i = 1; i<=n;i++)</pre>
14
           for(int j=1; j<=m; j++)</pre>
15
              scanf("%d", &a[i][j]);
16
17
        for(int i = 1;i<=n;i++)</pre>
18
           for(int j = 1; j<=m;j++)</pre>
19
               S[i][j] = S[i-1][j] + S[i][j-1] - S[i-1][j-1] + a[i][j];
20
21
        while(q--){
22
23
           scanf("%d %d %d %d", &x1,&y1, &x2, &y2);
24
           printf("%d\n", S[x2][y2] - S[x1-1][y2] - S[x2][y1-1] + S[x1-1][y1-1]);
25
26
        return 0;
27
    }
```

1.5.3 差分

```
1 #include <iostream>
2 using namespace std;
3
4 #define N 100010
5
6 int a[N], b[N];
int n,m,l,r,c;
8
```

```
9
    int insert(int b[],int 1, int r , int c){
10
        b[1] += c;
        b[r+1] -= c;
11
12
    }
13
14
   int main(){
15
        scanf("%d %d", &n, &m);
16
        for(int i=1; i<=n;i++) scanf("%d", &a[i]);</pre>
        for(int i=1; i<=n;i++) insert(b,i,i,a[i]);</pre>
17
        while(m--){
18
19
           scanf("%d %d %d",&l, &r, &c);
20
            insert(b,1,r,c);
        }
21
        for(int i=1;i<=n;i++)</pre>
22
23
           b[i] += b[i-1];
24
        for(int i=1;i<=n;i++) printf("%d ", b[i]);</pre>
25
   }
```

1.5.4 差分矩阵

```
#include <iostream>
    using namespace std;
 3
   #define N 1010
 5
 6
   int a[N][N], b[N][N];
8
    int n,m,q,x1,x2,y1,y2,c;
9
10
    int insert(int x1, int x2, int y1 ,int y2, int c){
11
       b[x1][y1] += c;
       b[x1][y2+1] -= c;
12
13
       b[x2+1][y1] -=c;
14
       b[x2+1][y2+1] +=c;
       return 0;
15
16
   }
17
18
    int main(){
       scanf("%d %d %d",&n,&m,&q);
19
       for(int i = 1;i <=n ;i++)</pre>
20
21
           for(int j = 1; j<=m; j++){</pre>
22
               scanf("%d", &a[i][j]);
23
               insert(i,i,j,j,a[i][j]);
24
25
       while(q--){
26
           scanf("%d %d %d %d %d", &x1,&y1,&x2,&y2,&c);
27
           insert(x1,x2,y1,y2,c);
28
       }
29
30
       for(int i = 1; i<=n ;i++)</pre>
31
           for(int j=1; j<=m; j++)</pre>
              b[i][j] = b[i-1][j] + b[i][j-1] - b[i-1][j-1] + b[i][j];
32
```

1.6 双指针算法

1.6.1 最长连续不重复子序列

```
#include<iostream>
 1
    #include<algorithm>
    using namespace std;
 5
 6
   #define N 100010
 7
 8
    int n,a[N];
 9
    int cnt[N];
10
    int main(){
11
12
       scanf("%d", &n);
13
        int res = 0;
        for(int i=0; i<n;i++) scanf("%d", &a[i]);</pre>
14
15
       for(int i = 0, j=i ; i < n; i++){</pre>
16
17
           ++cnt[a[i]];
           while(cnt[a[i]] > 1) --cnt[a[j++]];
18
19
           res = max(res, i-j + 1);
20
       }
21
22
       printf("%d", res);
23
24
        return 0;
25
```

1.6.2 数组元素的目标和

```
#include<iostream>
#include<algorithm>
using namespace std;

#define N 100010

int n,m,x;
int a[N], b[N];

int main(){
    scanf("%d %d %d", &n, &m, &x);
```

```
12
        for(int i = 0 ;i < n; i++) scanf("%d", &a[i]);</pre>
13
        for(int i = 0 ;i < m; i++) scanf("%d", &b[i]);</pre>
        int i = n-1, j = 0;
14
        while(i \ge 0 \mid | j < m){
15
16
            if(i>=0 && a[i] + b[j] > x) i--;
            else if(j < m && a[i] +b[j] < x) j++;</pre>
17
18
            else if(i \ge 0 && j \le m && a[i] + b[j] == x){
19
               printf("%d %d",i ,j);
                break;
20
21
            };
22
        }
23
        return 0;
24
```

1.6.3 判断子序列

```
1
   #include <iostream>
 2 #include <algorithm>
 3
    #include <string>
   #include <map>
 5
   using namespace std;
 6
    #define N 100010
 7
   int a[N],b[N];
 8
 9
    int n,m;
10
    int main(){
        scanf("%d %d", &n, &m);
11
12
        for(int i = 0 ;i < n ;i ++) scanf("%d", &a[i]);</pre>
13
       for(int i = 0 ;i < m ;i ++) scanf("%d", &b[i]);</pre>
        int i = 0, j = 0;
14
15
       for(; j < m; j++){</pre>
16
           if(i < n && a[i] == b[j]) i++;</pre>
17
       if(i == n) printf("Yes");
18
19
       else printf("No");
20
21
        return 0;
   1
22
```

1.7 位运算

1.7.1 二进制中 1 的个数

```
#include <iostream>
using namespace std;

#define N 100010

int lowbit(int x){
   return x & -x;
}
```

```
int n;
10
11
    int main(){
12
13
        scanf("%d", &n);
14
        int x;
        for(int i = 0 ; i < n; i++ ){</pre>
15
           scanf("%d", &x);
16
17
           int cnt = 0;
           while(x){
18
19
               x = x - lowbit(x);
20
               cnt++;
           }
21
           printf("%d ",cnt);
22
23
        }
24
        return 0;
25
    }
```

1.8 离散化

1.8.1 区间和

假定有一个无限长的数轴,数轴上每个坐标上的数都是 0。现在,我们首先进行 n 次操作,每次操作将某一位置 x 上的数加 c。接下来,进行 m 次询问,每个询问包含两个整数 l 和 r,你需要求出在区间 [l,r] 之间的所有数的和。

```
#include<iostream>
 2
   #include<algorithm>
   #include<vector>
 4
    using namespace std;
 5
 6
   #define N 300000 + 10
 7
    typedef pair<int,int> PII;
    vector<PII> opers,query;
    vector<int> alls;
9
10
    int a[N];
11
12
    int n, m, x,c, l,r;
13
14
    int binary_search(int k){
       int 1 = 0 , r = alls.size() - 1;
15
16
       while(1 < r){
           int mid = 1 + r >> 1;
17
18
           if(k <= alls[mid]) r = mid;</pre>
19
           else 1 = mid + 1;
20
21
       return alls[1] == k ? 1+1 : -1;
22
   }
23
24
   int main(){
25
       scanf("%d %d", &n, &m);
       for(int i = 0 ;i < n; i++){</pre>
26
           scanf("%d %d", &x, &c);
27
```

第 17 页

```
28
           opers.push_back({x,c});
29
           alls.push_back(x);
       }
30
31
32
       for(int i= 0; i < m; i++){</pre>
           scanf("%d %d", &l, &r);
33
34
           query.push_back({1,r});
35
           alls.push_back(1);
36
           alls.push_back(r);
37
38
       sort(alls.begin(),alls.end());
39
       alls.erase(unique(alls.begin(),alls.end()),alls.end());
40
41
       for(auto p : opers){
42
           int index = binary_search(p.first);
43
           a[index] += p.second;
44
45
46
       for(int i = 1; i <= alls.size(); i++) a[i] = a[i-1] + a[i];</pre>
47
48
       for(auto p : query){
49
           int 1 = binary_search(p.first);
50
           int r = binary_search(p.second);
51
           printf("%d\n", a[r] - a[l-1]);
52
       }
53
54
    }
```

1.9 区间合并

1.9.1 区间合并

```
#include <iostream>
    #include <vector>
 3
    #include <algorithm>
   using namespace std;
 4
 6
   typedef pair<int,int> PII;
 7
 8
    int n;
 9
    int 1, r;
    vector<PII> segs;
10
11
    int merge(vector<PII> &segs){
12
13
       sort(segs.begin(),segs.end());
       int st = -2e9, ed = -2e9;
14
15
       vector<PII> res;
       for(auto seg : segs){
16
           if(ed < seg.first){</pre>
17
18
              if(st!=-2e9) res.push_back({st,ed});
19
              st = seg.first;
20
              ed = seg.second;
```

```
21
           }
22
           else{
23
              ed = max(ed, seg.second);
           }
24
25
       }
26
       if(st != -2e9) res.push_back({st,ed});
27
       return res.size();
28
29
30
    int main(){
31
       scanf("%d",&n);
       while(n--){
32
           scanf("%d %d", &1,&r);
33
34
           segs.push_back({1,r});
35
36
       int res = merge(segs);
37
       printf("%d", res);
38
       return 0;
39
    }
```

2 数据结构

2.1 单链表

```
1
    #include <iostream>
 3
    #include <string>
   using namespace std;
 5
    const int N = 100010;
 6
    string c;
 7
    int head,idx,el[N],ne[N],k,x,m;
8
    void init(){
9
       head = -1;
10
       idx = 0;
11
    void add_head(int x){
12
13
       el[idx] = x, ne[idx] = head, head = idx++;
14
15
    void add(int k, int x){
16
       el[idx] = x; ne[idx] = ne[k], ne[k] = idx++;
17
    void del_head(){
18
19
       head = ne[head];
20
21
    void del(int k){
       ne[k] = ne[ne[k]];
22
23
24
    int main(){
25
       cin >> m;
26
       init();
       while(m--){
27
28
           cin >> c;
```

```
29
           if(c == "H"){
30
              cin >> x;
              add_head(x);
31
           }
32
           if(c == "D"){
33
34
              cin >> k;
35
              if(!k) del_head();
36
              else del(k-1);
           }
37
           if(c =="I"){
38
39
              cin >> k >> x;
40
               add(k-1,x);
           }
41
42
43
       for(int i = head; i!=-1;i=ne[i]){
          cout << el[i] << ' ';
44
45
46
       return 0;
47
    }
```

2.2 双链表

```
1
   #include <iostream>
    #include <string>
 3
    using namespace std;
4
 5
    const int N = 100010;
 6
7
    string oper;
8
    int m,k,x,idx,l[N],r[N],el[N];
    void init(){
10
11
       // 0是左节点, 1是右节点;
       r[0] = 1, 1[1]=0;
12
13
       idx = 2;
14
    //在第k个数的右边添加新节点
15
    void add(int k, int x){
16
17
       el[idx] = x, l[idx] = k, r[idx] = r[k], l[r[k]] = idx, r[k] = idx ++;
18
    //删除第k个节点
19
    void del(int k){
20
21
       r[l[k]] = r[k];
22
       1[r[k]] = 1[k];
23
   }
24
    int main(){
25
       cin >> m;
       init();
26
27
       while(m--){
28
          cin >> oper;
          if(oper == "L"){
29
             cin >> x;
30
```

```
31
              add(0,x);
           }
32
33
           if(oper == "R"){
34
              cin >> x;
35
              add(1[1],x);
           }
36
37
           if(oper == "D"){
38
              cin >> k;
39
              del(k+1);
40
           }
           if(oper == "IL"){
41
42
              cin >> k >> x;
43
              add(l[k+1], x);
44
           if(oper == "IR"){
45
46
              cin >> k >> x;
47
              add(k+1, x);
           }
48
49
       }
50
       for(int i = r[0]; i!=1; i = r[i]){
51
           cout << el[i] << ' ';
52
       }
53
       return 0;
    }
54
```

2.3 模拟栈

```
1 | #include <iostream>
    #include <string>
2
   using namespace std;
 4
5
    const int N = 100010;
6
7
    int m;
8
    int s[N], t=0, x;
9
    string oper;
10
    void init(){
11
12
       t = 0;
13
    }
14
15
    void push(int x){
16
       s[++t] = x;
17
   }
18
19
    void pop(){
20
       --t;
21
   }
22
23
    string empty(){
24
       return t == 0 ? "YES" : "NO";
25 }
```

```
26
27
    int query(){
28
        return s[t];
29
    }
30
31
32
    int main(){
33
       cin >> m;
34
        init();
        while(m--){
35
36
           cin >> oper;
           if(oper == "push"){
37
38
               cin >> x;
39
               push(x);
           }
40
           if(oper == "pop"){
41
42
               pop();
           }
43
           if(oper == "empty"){
44
45
               cout << empty() << endl;</pre>
           }
46
47
           if(oper == "query"){
48
               cout << query() << endl;</pre>
49
50
       }
51
    }
```

2.4 模拟队列

```
#include<iostream>
 2
    #include<string>
 3
    #include<algorithm>
4
   using namespace std;
    const int N = 100010;
 6
7
    int q[N], h,t,m,x;
8
    string o;
9
10
    void init(){
11
       h = 0, t = -1;
12
   }
13
14
    void push(int x){
15
       q[++t] = x;
16
   }
17
    void pop(){
18
19
       ++h;
20
    }
21
22
   int query(){
23
       return q[h];
```

```
24
   }
25
26
    string empty(){
27
        return h > t ? "YES" : "NO";
28
29
30
    int main(){
31
       cin >> m;
32
        init();
        while(m--){
33
34
           cin >> o;
           if(o == "push"){
35
36
               cin >> x;
37
               push(x);
           }
38
           if(o == "pop"){
39
40
               pop();
           }
41
42
           if(o == "empty"){
43
               cout << empty() <<endl;</pre>
           }
44
45
           if(o == "query"){
46
               cout << query() << endl;</pre>
47
48
       }
49
    }
```

2.5 单调栈

```
#include <iostream>
    using namespace std;
 3
4
    const int N = 100010;
 5
6
    int s[N], a[N], t=0,n;
7
8
9
    int main(){
10
       scanf("%d", &n);
       for(int i = 0 ;i < n; i++) scanf("%d", &a[i]);</pre>
11
       for(int i = 0;i < n; i++){</pre>
12
13
           while(t && s[t] >= a[i]) t--;
           if(t){
14
15
               printf("%d ",s[t]);
16
17
           else{
18
              printf("%d ",-1);
19
           s[++t] = a[i];
20
21
       }
22
       return 0;
    }
23
```

2.6 单调队列

```
#include <iostream>
    using namespace std;
 2
 3
 4
   const int N = 1000010;
 5
    int n,k,q[N],a[N];
   int h=0,t=-1;
 7
 8
    int main(){
10
       scanf("%d %d",&n,&k);
       for(int i = 0; i < n;i++) scanf("%d", &a[i]);</pre>
11
12
       for(int i = 0; i < n;i++) {</pre>
13
14
          while(h<=t && q[h] < i - k + 1) h++; // 去除不在滑动窗口内的下标;
15
          while(h<=t && a[q[t]]>= a[i]) t--; // 单调队列中所有元素小于当前元素;
          q[++t] = i; // 队列储存的是下标;
16
          if(i+1>=k){ // 滑动窗口未完全进入不输出;
17
18
              printf("%d ", a[q[h]]);
          }
19
20
       }
       puts("");
21
22
       h=0, t=-1;
       for(int i = 0; i < n;i++) {</pre>
23
24
          while(h \le t \& q[h] < i - k + 1) h + +;
25
          while(h<=t && a[q[t]]<= a[i]) t--;</pre>
26
          q[++t] = i;
27
          if(i+1>=k){
              printf("%d ", a[q[h]]);
28
29
          }
30
       }
31
32
       return 0;
33
   }
```

2.7 kmp

```
#include <iostream>
    using namespace std;
 3
   const int N = 1000010;
4
    int ne[N],n,m;
 6
7
    char p[N],s[N];
8
    int main(){
9
10
       scanf("%d %s %d %s",&n, p+1, &m, s+1);
11
        for(int i = 2 ,j = 0; i <=n;i++){</pre>
12
           while(j && p[i] != p[j+1]) j = ne[j];
13
           if(p[i] == p[j+1]) j++;
           ne[i] = j;
14
```

```
15
        }
16
        for(int i = 1, j= 0; i<=m;i++){</pre>
           while(j && s[i] != p[j+1]) j=ne[j];
17
18
           if(s[i] == p[j+1]) j++;
19
           if(j == n){
20
               j = ne[j];
21
               printf("%d ", i-n);
22
           }
23
        }
24
```

2.8 Trie 树

```
#include<iostream>
    using namespace std;
 3
 4
    const int N = 2e4 + 10;
 5
 6
    char oper[2],str[N];
 7
    int cnt[N],n, s[N][26],idx=0;
 8
    void insert(char * str){
 9
10
       int p = 0;
11
        for(int i = 0 ; str[i]; i++){
           int c = str[i] - 'a';
12
13
           if(!s[p][c]) s[p][c] = ++idx;
14
           p = s[p][c];
15
       }
16
       ++cnt[p];
17
    }
18
19
    int query(char * str){
20
       int p = 0;
        for(int i = 0; str[i]; i++){
21
           int c = str[i] - 'a';
22
23
           if(!s[p][c]) return 0;
24
           p = s[p][c];
25
26
       return cnt[p];
27
    }
28
    int main(){
29
30
       scanf("%d", &n);
31
       while(n--){
           scanf("%s %s",oper,str);
32
33
           // cout << oper <<endl << str <<endl;
           if(oper[0] == 'I'){
34
               // printf("%d\n",insert())
35
36
               insert(str);
           }
37
           if(oper[0] == 'Q'){
38
              printf("%d\n", query(str));
39
```

2.9 最大异或树

```
#include <iostream>
 2
    using namespace std;
    const int N = 1e5 * 31 + 10;
 4
5
   int n, son[N][2],idx=0;
 6
8
    void insert(int x){
9
       int p = 0;
       for(int i = 31; i>=0;i--){ // 贪心, 从整数最高位开始构造Tried树;
10
11
           int b = x >> i & 1;
12
           if(!son[p][b]) son[p][b] = ++idx;
13
           p = son[p][b];
14
       }
    }
15
16
    int query(int x){
17
18
       int p = 0, res=0;
19
       for(int i = 31;i >=0; i--){
20
           int b = x >> i & 1;
21
           if(!son[p][!b]){
22
              res = 2 * res;
23
              p = son[p][b];
           }
24
25
           else{
26
              res = 2 * res + 1;
27
              p = son[p][!b];
28
           }
29
       }
30
       return res;
31
    }
32
33
    int main(){
34
35
       scanf("%d", &n);
       int res = 0,x=0;
36
       for(int i = 0; i < n;i ++){</pre>
37
38
           scanf("%d", &x);
39
           insert(x);
40
           res = max(res, query(x));
41
       printf("%d", res);
42
43
44
       return 0;
    }
45
```

2.10 并查集

2.10.1 合并集合

```
#include <iostream>
    using namespace std;
    const int N = 1e5 * 31 + 10;
 4
5
 6
    int n, son[N][2],idx=0;
 7
 8
    void insert(int x){
       int p = 0;
 9
10
       for(int i = 31; i>=0;i--){ // 贪心, 从整数最高位开始构造Tried树;
           int b = x >> i & 1;
11
12
           if(!son[p][b]) son[p][b] = ++idx;
13
           p = son[p][b];
14
       }
15
16
17
    int query(int x){
       int p = 0, res=0;
18
19
       for(int i = 31;i >=0; i--){
20
           int b = x >> i & 1;
21
           if(!son[p][!b]){
22
              res = 2 * res;
23
              p = son[p][b];
           }
24
25
           else{
              res = 2 * res + 1;
26
27
              p = son[p][!b];
28
           }
       }
29
30
       return res;
31
    }
32
33
34
    int main(){
35
       scanf("%d", &n);
       int res = 0,x=0;
36
37
       for(int i = 0; i < n;i ++){</pre>
38
           scanf("%d", &x);
39
           insert(x);
40
           res = max(res, query(x));
41
42
       printf("%d", res);
43
44
       return 0;
45
```

2.10.2 连通块中点的数量

```
1 #include <iostream>
```

```
using namespace std;
2
 3
    const int N = 100010;
 4
 5
    char oper[3];
 6
   int p[N], s[N], n, m , x, y;
8
    int find(int x){
 9
       if(p[x] != x) p[x] = find(p[x]);
10
       return p[x];
11
    }
12
13
    int main(){
14
15
       scanf("%d %d", &n, &m);
16
        for(int i = 1; i <=n; i++) p[i] = i,s[i] = 1;</pre>
       while(m--){
17
           scanf("%s",oper);
18
           if(oper[0] == 'C'){
19
              scanf("%d%d",&x, &y);
20
21
               int px = find(x), py = find(y);
22
              if(px != py){
23
                  s[py] += s[px];
24
                  p[px] = py;
              }
25
           }
26
27
           if(oper[0] == 'Q'){
28
               if(oper[1] == '1'){
29
                  scanf("%d%d",&x, &y);
30
                  if(find(x) == find(y)) printf("Yes\n");
31
                  else printf("No\n");
              }
32
              if(oper[1] == '2'){
33
34
                   scanf("%d",&x);
35
                   printf("%d\n", s[find(x)]);
              }
36
37
           }
38
       }
39
    }
```

2.10.3 食物链

```
#include <iostream>
1
   using namespace std;
 2
 4
    const int N = 50010;
 5
 6
 7
    int n,k,d,x,y,res=0,dist[N],p[N];
 8
 9
    int find(int x){
10
       if(p[x] != x){
11
           int t = find(p[x]);
```

```
12
           dist[x] += dist[p[x]];
13
           p[x] = t;
14
       }
15
       return p[x];
16
    }
17
18
    int main(){
19
       scanf("%d%d",&n,&k);
       for(int i = 1;i<=n;i++) p[i] = i, dist[i] = 0;</pre>
20
21
       while(k--){
22
           scanf("%d%d%d",&d,&x,&y);
           if(x > n \mid \mid y > n) res++;
23
           else{
24
25
               if(d==1){
26
                  int px = find(x), py = find(y);
27
                  if(px == py && (dist[x] - dist[y]) % 3) res++;
28
                  else if(px!=py){
29
                      p[px] = py;
30
                      dist[px] = dist[y] - dist[x];
31
                  }
               }
32
               else{
33
34
                  int px = find(x), py =find(y);
35
                  if(px == py && (dist[x] - dist[y] - 1) %3) res++;
                  else if(px!=py){
36
37
                      p[px] = py;
38
                      dist[px] = dist[y] + 1 -dist[x];
39
                  }
40
              }
41
           }
42
       printf("%d",res);
43
44
```

2.11 堆

2.11.1 堆排序

```
#include <iostream>
 1
    #include <algorithm>
 3
    using namespace std;
 4
 5
    const int N = 100010;
 6
 7
    int n,m,h[N];
 8
 9
10
    void down(int u){
11
12
        int t = u;
13
        if(2*u<=n&&h[2*u] < h[t]) t = 2*u;</pre>
14
        if(2*u+1<=n&&h[2*u+1] < h[t]) t = 2*u+1;</pre>
```

```
15
        if(u!=t){
           swap(h[u],h[t]);
16
17
           down(t);
        }
18
19
    }
20
21
22
    int main(){
23
        scanf("%d%d",&n,&m);
        for(int i = 1;i<=n;i++) scanf("%d",&h[i]);</pre>
24
25
        for(int i = n/2;i;i--){
26
           down(i);
27
       }
28
        while(m--){
           printf("%d ", h[1]);
29
30
           swap(h[1],h[n]);
31
           n--;
32
           down(1);
       }
33
34
        return 0;
35
```

2.11.2 模拟堆

```
1 | #include <iostream>
   #include <string.h>
 3
    using namespace std;
 4
    const int N = 100010;
 5
 6
    char o[3];
 7
    // string o;
 8
    int idx=0,n,sz=0,x,k,h[N],hp[N],ph[N];
 9
10
    void heap_swap(int u, int d){
11
12
        \verb|swap(ph[hp[u]],ph[hp[d]]);|\\
13
        swap(h[u],h[d]);
        swap(hp[u],hp[d]);
14
15
    }
16
    int down(int u){
17
18
        int t = u;
19
        if(2*u<=sz && h[2*u] < h[t]) t = 2*u;</pre>
        if(2*u+1<=sz &&h[2*u+1] < h[t]) t = 2*u+1;</pre>
20
        if(u!=t){
21
22
           heap_swap(u,t);
23
           down(t);
24
        }
25
    }
26
27
28 | int up(int u){
```

```
29
       while (u/2 \&\& h[u] < h[u/2]){
30
          heap_swap(u/2,u);
          u >>=1;
31
32
       }
33
    }
34
35
36
    int main(){
       scanf("%d", &n);
37
38
       while(n--){
39
          scanf("%s", o);
           if(!strcmp(o,"I")){ // 插入一个数
40
              scanf("%d", &x);
41
42
              h[++sz] = x;
              hp[sz] = ++idx;
43
44
              ph[idx] = sz;
45
              up(sz);
          }
46
          if(!strcmp(o,"PM")){ // 输出当前集合中的最小值
47
48
              printf("%d\n", h[1]);
          }
49
50
          if(!strcmp(o,"DM")){ // 删除当前集合的最小值
51
              heap_swap(1,sz--);
52
              down(1);
          }
53
54
          if(!strcmp(o,"D")){ // 删除第k个插入的数
55
              scanf("%d", &k);
56
              int u = ph[k];
57
              heap_swap(u, sz--);
58
              up(u);
              down(u);
59
          }
60
61
          if(!strcmp(o,"C")){ // 修第k个插入的数
62
              scanf("%d%d",&k, &x);
              h[ph[k]] = x;
63
64
              up(ph[k]);
65
              down(ph[k]);
66
          }
67
       }
68
```

2.12 哈希表

2.12.1 模拟散列表

```
#include<iostream>
#include <string.h>
using namespace std;

const int N = 100010;
char oper[2];
int idx=0,n,x, el[N],ne[N],h[N], mod =100003;
```

```
8
9
    void insert(int x){
       int t = (x % mod + mod) % mod;
10
11
       el[idx] = x, ne[idx] = h[t], h[t] = idx++;
12
13
    bool query(int x){
14
       int t = (x \% mod + mod) \% mod;
15
       for(int i = h[t]; i!= -1;i = ne[i]){
           if(el[i] == x) return true;
16
17
18
       return false;
    }
19
20
    int main(){
21
       scanf("%d", &n);
22
23
       memset(h,-1,sizeof(h));
24
       while(n--){
           scanf("%s%d",oper,&x);
25
           if(oper[0] == 'I'){
26
27
              insert(x);
28
           }
           if(oper[0] == 'Q'){
29
30
              bool is_exisit = query(x);
              if(is_exisit)
31
                  printf("%s\n","Yes");
32
33
              else
34
                  printf("%s\n","No");
35
           }
36
       }
```

2.12.2 字符串哈希

```
#include<iostream>
    using namespace std;
 3
    const int N = 100010;
 5
    typedef unsigned long long ULL;
 6
    ULL p[N],h[N];
 7
    int n,m,l1,r1,l2,r2,P=1331;
8
    char str[N];
9
    ULL get(int 1,int r){
10
11
        return h[r] - h[l-1] * p[r-l+1];
12
   }
13
    int main(){
       scanf("%d%d",&n,&m);
15
       scanf("%s",str+1);
16
17
       p[0] = 1;
       for(int i=1;i <=n;i++){</pre>
18
19
           p[i] = p[i-1] * P;
```

```
20     h[i] = h[i-1] * P + str[i];
21     }
22     while(m--){
23         scanf("%d%d%d%d",&l1,&r1,&l2,&r2);
24         if(get(l1,r1) == get(l2,r2)) puts("Yes");
25         else puts("No");
26     }
27 }
```

3 搜索与图论

3.1 DFS

3.1.1 排列数字

```
1 #include<iostream>
 2
    using namespace std;
   const int N = 8;
 4
 5
 6
   int n,p[N],st[N];
 7
 8
    void dfs(int u){
 9
       if(u == n){
           for(int i = 0; i < n; i++) printf("%d ",p[i]);</pre>
10
           puts("");
11
12
       }
13
14
       for(int i = 1; i<=n ; i++){</pre>
15
           if(!st[i]){
16
               st[i] = true;
17
               p[u] = i;
18
               dfs(u+1);
19
               st[i] = false;
           }
20
       }
21
22
23
24
   int main(){
25
       scanf("%d",&n);
26
        dfs(0);
27
    }
```

3.1.2 n 皇后问题

```
#include <iostream>
using namespace std;

const int N = 20;

char q[N][N];
```

```
7
    bool dg[N],udg[N],col[N];
8
    int n;
 9
    void dfs(int u ){
10
11
       if(u == n){
           for(int i = 0 ; i < n; i++) puts(q[i]);</pre>
12
13
           puts("");
14
       }
15
       for(int i = 0 ; i < n; i ++){</pre>
16
17
           if(!col[i] && !dg[u+i] && !udg[n-u+i]){
18
               q[u][i] = 'Q';
               col[i] = dg[u+i] = udg[n-u+i] = true; // 用截距表示对角线与反对角线
19
20
               dfs(u+1);
21
               col[i] = dg[u+i] = udg[n-u+i] = false;
22
               q[u][i] = '.';
23
           }
       }
24
25
   }
26
27
    int main(){
       scanf("%d",&n);
28
29
       for(int i = 0 ;i<n;i++)</pre>
30
           for(int j = 0; j < n; j++)</pre>
31
               q[i][j] = '.';
32
       dfs(0);
33
    }
```

3.2 BFS

3.2.1 走迷宫

```
#include <iostream>
    #include <string.h>
 3
    using namespace std;
 4
 5
    typedef pair<int,int> PII;
    const int N = 110;
 6
 7
 8
    int n,m,h=0,t=-1,d[N][N],g[N][N],dx[4]=\{-1,0,1,0\},dy[4]=\{0,1,0,-1\};
 9
    PII q[N*N];
10
    int bfs(PII u){
11
       memset(d,-1,sizeof(d));
12
       d[u.first][u.second] = 0;
13
       q[++t] = u;
14
15
       while(h<=t){</pre>
           auto e = q[h++];
16
           for(int i = 0 ;i < 4;i++){</pre>
17
18
               int x = e.first + dx[i], y = e.second + dy[i];
19
               if(x \le n \&\& x \ge 1 \&\& y \le m \&\& y \ge 1 \&\& d[x][y] == -1 \&\& !g[x][y]){
20
                   d[x][y] = d[e.first][e.second] + 1;
```

```
21
                   q[++t] = \{x,y\};
               }
22
23
            }
24
25
       return d[n][m];
26
27
28
29
    int main(){
30
        scanf("%d%d",&n,&m);
31
        for(int i = 1 ;i <= n ; i++)</pre>
            for(int j = 1; j<= m; j++)</pre>
32
               scanf("%d",&g[i][j]);
33
34
35
        printf("%d",bfs({1,1}));
36
    | }
```

3.2.2 八重码

```
#include <iostream>
    #include <queue>
 3
    #include <string>
   #include <unordered_map>
 5
    using namespace std;
 7
    int dx[4] = \{-1,0,1,0\}, dy[4] = \{0,1,0,-1\};
 8
    queue<string> q;
    string u;
10
    unordered_map<string, int> d;
11
12
13
    int bfs(string u){
14
       q.push(u);
15
       d[u] = 0;
       string end = "12345678x";
16
17
       while(!q.empty()){
18
           auto t = q.front();
19
           q.pop();
           if(t == end) return d[t];
20
21
           int k = t.find('x');
           int a = k / 3, b = k % 3;
22
23
           int dist = d[t];
24
           for(int i = 0 ; i < 4;i++){</pre>
25
               int x = a + dx[i], y = b + dy[i];
26
               if(x)=0 \&\& x < 3 \&\& y>=0 \&\& y < 3){
27
                  swap(t[x * 3 + y], t[k]);
28
                  if(!d.count(t)){
                      d[t] = dist + 1;
29
30
                      q.push(t);
31
32
                  swap(t[x * 3 + y], t[k]);
33
              }
```

```
34
           }
35
36
        }
37
        return -1;
38
39
40
41
    int main(){
42
        char c;
43
        for(int i = 0 ; i < 9;i++){</pre>
44
           scanf("%c ",&c);
45
           u+=c;
        }
46
47
        // cout << u << endl;
48
        printf("%d",bfs(u));
49
    }
```

3.3 树与图的 DFS

3.3.1 树的重心

```
#include <iostream>
 2
   #include <string.h>
   using namespace std;
 4
5
   const int N = 100010, M = 2 * N; // 无向图边数比有向图边数多两倍;
 6
 7
   int h[N], el[M], ne[M], idx = 0, n,x,y;
 8
   int ans = N;
9
   bool st[N];
10
11
   void insert(int x ,int y){
12
13
       el[idx] = y, ne[idx] = h[x], h[x] = idx ++; //头插法
14
15
   // 计算以u为根节点的树的节点数(包括u自身)
16
   int dfs(int u){
17
       st[u] = true;
18
19
       int sum = 1, res = 0; // 当前树的节点数,连通子图的节点数最大值;
20
       for(int i = h[u]; i != -1; i = ne[i]){
          if(!st[el[i]]){
21
22
             int s = dfs(el[i]);
23
             sum += s;
24
             res = max(res, s);
25
          }
26
      }
27
28
      res = max(res, n - sum); // 比较剩余父连通图的节点数
29
       ans = min(ans, res);
30
       return sum;
31 }
```

```
32
33
34
    int main(){
35
       scanf("%d", &n);
36
       memset(h,-1,sizeof(h));
       for(int i = 0; i < n-1; i++){// n-1条边, 题意
37
38
          scanf("%d%d",&x,&y);
39
          insert(x,y);
           insert(y,x);// 无向图插两条边
40
41
       }
42
       dfs(1);
       printf("%d", ans);
43
44
```

3.4 树与图的 BFS

3.4.1 图中点的层次

```
#include <iostream>
 1
 2
    #include <string.h>
   #include <queue>
 4
   using namespace std;
 5
    const int N = 100010, M = 100010;
   int n,m,h[N],el[N], ne[M], idx=0, x, y ,d[N];
 7
    queue<int> q;
8
9
    void insert(int x, int y){
10
       el[idx] = y, ne[idx] = h[x], h[x] = idx ++;
11
12
   }
13
    int bfs(int u){
14
15
       q.push(u);
       memset(d, -1, sizeof(d));
16
       d[u] = 0;
17
18
       while(!q.empty()){
19
           int t = q.front();
20
           q.pop();
21
           for(int i = h[t]; i!=-1; i = ne[i]){
22
              if(d[el[i]] == -1){
23
                  d[el[i]] = d[t] + 1;
24
                  q.push(el[i]);
25
              }
           }
26
27
       }
       return d[n];
28
29
    }
30
31
    int main(){
       scanf("%d%d",&n,&m);
32
33
       memset(h, -1,sizeof(h));
34
       for(int i = 0 ; i < m ; i++){</pre>
```

3.5 拓扑排序

3.5.1 有向图的拓扑排序

```
#include <iostream>
    #include <string.h>
   using namespace std;
 3
 4
    const int N = 100010, M = 100010;
 6
    int idx=0,el[M],ne[M],h[N],x,y,n,m,degree[N];
 7
    int q[M], hh = 0, tt = -1;
 8
 9
    void insert(int x, int y){
10
        el[idx] = y, ne[idx] = h[x], h[x] = idx ++;
11
    }
12
13
    int topsort(){
14
        for(int i = 1 ; i <= n ; i++){</pre>
15
           if(!degree[i]) q[++tt] = i;
16
       }
17
        while(hh<=tt){</pre>
           int t = q[hh++];
18
19
           for(int i = h[t]; i != -1; i = ne[i]){
20
               int j = el[i];
21
               --degree[j];
22
               if(!degree[j]){
23
                  q[++tt] = j;
               }
24
25
           }
26
       }
27
        return tt == n-1;
28
    }
29
30
    int main(){
31
        memset(h,-1,sizeof(h));
        memset(degree,0,sizeof(degree));
32
33
        scanf("%d%d",&n,&m);
        for(int i = 0; i < m; i++){</pre>
34
35
           scanf("%d%d",&x,&y);
           insert(x,y);
36
37
           ++degree[y];
        }
38
39
        if(topsort()){
           for(int i = 0; i<=tt;i++){</pre>
40
               printf("%d ", q[i]);
41
           }
42
```

```
43 | }
44 | else{
45 | printf("%d",-1);
46 | }
47 |}
```

3.6 Dijkstra

3.6.1 求最短路 1

```
#include<iostream>
    #include<string.h>
   using namespace std;
 3
   const int N = 510, M = 100010;
 6
   int d[N][N], dist[M], n, m, x, y, z;
 7
    bool st[N];
 8
9
10
    int dijkstra(int u){
11
       memset(dist, 0x3f, sizeof dist);
12
       dist[u] =0;
13
       for(int i = 0; i<n;i++){</pre>
14
           int t = -1;
15
           for(int j = 1; j <= n; j++){</pre>
16
              if(!st[j] && ((t == -1) || dist[t] > dist[j]))
17
                  t = j;
           }
18
19
20
           st[t] =true;
21
22
           for(int j=1;j<=n;j++){</pre>
23
              dist[j] = min(dist[j],dist[t] + d[t][j]);
           }
24
25
       if(dist[n] == 0x3f3f3f3f) return -1;
26
27
       else return dist[n];
28
   }
29
30
    int main(){
31
       memset(d, 0x3f, sizeof d);
       scanf("%d%d",&n,&m);
32
33
       for(int i = 0 ; i < m ;i ++){</pre>
           scanf("%d%d%d",&x,&y,&z);
34
35
           d[x][y] = min(d[x][y],z);
36
37
       int res = dijkstra(1);
       printf("%d",res);
38
39
       return 0;
40
```

3.6.2 求最短路 2

```
#include <iostream>
    #include <vector>
    #include <queue>
   #include <string.h>
 5
    using namespace std;
 7
   const int N = 150010,M = 150010;
 8
    typedef pair<int,int> PII;
10
   int dist[N],el[M],ne[M],w[M],h[N],n,m,x,y,z,idx=0;
    bool st[N];
11
   priority_queue<PII,vector<PII>,greater<PII>> q;
12
13
    void add(int x, int y, int z){
14
15
       el[idx] = y, w[idx] = z, ne[idx] = h[x], h[x] = idx++;
16
   }
17
18
   int dijkstra(int u){
19
       memset(dist, 0x3f, sizeof dist);
       dist[u] = 0;
20
21
       q.push({dist[u], u});
22
       while(q.size()){
23
           auto t = q.top();
24
           q.pop();
25
           int ver = t.second, d = t.first;
           if(st[ver]) continue;
26
27
           st[ver] = true;
28
           for(int i = h[ver]; i != -1; i = ne[i]){
29
              int y = el[i];
30
              if(dist[y] > d + w[i]){
31
                  dist[y] = d + w[i];
32
                  q.push({dist[y],y});
              }
33
34
           }
35
       }
       if(dist[n] == 0x3f3f3f3f) return -1;
36
37
       return dist[n];
38
39
   }
40
41
   int main(){
       scanf("%d%d",&n,&m);
42
43
       memset(h, -1, sizeof h);
44
       for(int i = 0; i < m;i++){</pre>
           scanf("%d%d%d",&x,&y,&z);
45
46
           add(x,y,z);
47
       printf("%d",dijkstra(1));
48
49
```

3.7 Bellman-Ford

3.7.1 有边数限制的最短路

```
#include<iostream>
 1
 2
    #include<string.h>
 3
    using namespace std;
 4
 5
   const int N = 510, M = 10010;
 6
    int n,m,k,dist[N],backup[N];
 7
8
    struct Edge{
9
       int x,y,w;
10
    }edges[M];
11
    int bellman_ford(int u ){
12
13
       memset(dist,0x3f,Sizeof dist);
14
       dist[u] = 0;
15
       for(int i = 0 ; i < k; i++){</pre>
16
           memcpy(backup,dist,sizeof backup);
17
           for(int j = 0; j < m; j++){
               int x = edges[j].x,y = edges[j].y, w= edges[j].w;
18
19
              dist[y] = min(dist[y], backup[x] + w);
20
           }
21
       }
       if(dist[n] > 0x3f3f3f3f / 2) return -1;
22
23
        return dist[n];
24
   }
25
26
    int main(){
27
       scanf("%d%d%d",&n,&m,&k);
       for(int i = 0 ; i < m ; i++){</pre>
28
           scanf("%d%d%d", &edges[i].x, &edges[i].y, &edges[i].w);
29
30
       }
31
       int res = bellman_ford(1);
       if(res == -1) printf("impossible");
32
33
       else printf("%d", res);
34
       return 0;
35
```

3.8 SPFA

3.8.1 SPFA 求最短路

```
#include<iostream>
#include<string.h>
using namespace std;

const int N = 100010, M = 100010;
int idx=0,el[M], w[M], ne[M], h[N], dist[N], n, m, x, y, z;
int hh=0,tt=-1,q[N];
bool st[N];
```

```
void add(int x, int y, int z){
11
       el[idx] = y, w[idx] = z, ne[idx] = h[x], h[x] = idx++;
   }
12
13
14
   int spfa(int u){
       memset(dist, 0x3f3f3f , sizeof dist);
15
       dist[u] = 0;
16
17
       q[++tt] =u;
       while(hh <= tt){</pre>
18
19
           int t = q[hh++];
20
           st[t] = false;
21
           for(int i = h[t]; i!= -1; i = ne[i]){
22
              int j = el[i];
               if(dist[j] > dist[t] + w[i]){
23
24
                  dist[j] = dist[t] + w[i];
25
                  if(!st[j]){
26
                      q[++tt] = j;
27
                      st[j] = true;
28
                  }
29
              }
           }
30
31
       if(dist[n] == 0x3f3f3f3f) return -1;
32
33
       return dist[n];
34
   | }
35
36
    int main(){
37
       memset(h, -1, sizeof h);
       scanf("%d%d",&n,&m);
38
39
       for(int i = 0 ; i < m ;i++){</pre>
           scanf("%d%d%d",&x,&y,&z);
40
41
           add(x,y,z);
42
       }
43
       int res = spfa(1);
       if(res == -1 ) puts("impossible");
44
45
       else printf("%d",res);
46
```

3.8.2 SPFA 判断负环

```
1 #include <iostream>
   #include<string.h>
   #include <queue>
   using namespace std;
 5
 6
   const int N = 100010, M = 100010;
 7
8
    int n,m,x,y,z,idx=0, h[N],el[M],ne[M],w[M],dist[N],cnt[N];
9
    bool st[N];
10
    queue<int> q;
11
12 | void add(int x, int y, int z){
```

```
el[idx] = y, w[idx] = z, ne[idx] = h[x], h[x] = idx ++;
13
14
    }
15
16
    bool spfa(){
17
       for(int i = 1; i<=n; i++){</pre>
18
           q.push(i);
19
           st[i] = true;
20
21
       while(q.size()){
22
           int t = q.front();
23
           q.pop();
           st[t] = false;
24
25
           for(int i = h[t];i!=-1;i=ne[i]){
               int j = el[i];
26
27
               if(dist[j] > dist[t] + w[i]){
28
                  dist[j] = dist[t] + w[i];
29
                  cnt[j] = cnt[t] + 1;
30
                  if(cnt[j] >=n) return true;
31
                  if(!st[j]){}
32
                      q.push(j);
33
                      st[j] = true;
                  }
34
35
              }
           }
36
       }
37
38
       return false;
39
40
41
    int main(){
42
       memset(h, -1, sizeof h);
       scanf("%d%d",&n,&m);
43
44
       for(int i = 0 ; i < m; i++){</pre>
45
           scanf("%d%d%d",&x,&y,&z);
46
           add(x,y,z);
47
       }
48
       if(spfa()) puts("Yes");
49
       else puts("No");
50
    }
```

3.9 Floyd

3.9.1 Floyd 求最短路

```
#include <iostream>
#include <string.h>
using namespace std;

const int N = 210, M = 20010, INF = 0x3f3f3f3f3f;
int n, m, k, d[N][N], x, y, z;

void floyd(){
```

```
10
        for(int k = 1;k <= n; k++)</pre>
11
           for(int i = 1; i <= n; i++)</pre>
               for(int j = 1; j <= n; j++)</pre>
12
13
                   d[i][j] = min(d[i][j],d[i][k] + d[k][j]);
14
15
16
17
    int main(){
        scanf("%d%d%d",&n,&m,&k);
18
19
20
        for(int i = 1; i<=n ;i ++)</pre>
21
           for(int j = 1; j <= n; j++){</pre>
               if(i == j) d[i][j] = 0;
22
               else d[i][j] = INF;
23
           }
24
25
        for(int i = 0; i < m ;i++){</pre>
26
           scanf("%d%d%d",&x, &y, &z);
27
28
           d[x][y] = min(d[x][y], z);
        }
29
30
31
        floyd();
32
33
        while(k -- ){
           scanf("%d%d",&x, &y);
34
35
           if(d[x][y] > INF / 2) puts("impossible");
36
           else printf("%d\n", d[x][y]);
37
        }
        return 0;
38
39
40
```

3.10 最小生成树

3.10.1 Kruskal

```
1 #include<iostream>
   #include<algorithm>
   using namespace std;
 4
    const int N = 1e5 + 10, M = 2e5 + 10;
 5
    int n,m,x,y,z, p[N];
 6
7
    struct Edge{
8
       int x,y,z;
 9
       bool operator < (const Edge &e) const{</pre>
10
           return z < e.z;</pre>
11
       }
   }edges[M];
12
13
   int find(int x){
14
15
       if(p[x] != x) p[x] = find(p[x]);
16
       return p[x];
```

```
17
   }
18
19
    int main(){
        scanf("%d%d",&n,&m);
20
21
        for(int i = 0 ;i < m; i++){</pre>
           scanf("%d%d%d", &x, &y, &z);
22
23
           edges[i] = \{x,y,z\};
24
25
26
        for(int i = 1; i <= N; i++) p[i] =i;</pre>
27
28
        sort(edges , edges + m);
29
30
        int cnt = 0, res = 0;
31
32
        for(int i = 0; i<m; i++){</pre>
33
           int x = edges[i].x, y = edges[i].y, z = edges[i].z;
34
35
           int px = find(x), py = find(y);
36
37
           if(px != py){
38
               p[px] = py;
39
               cnt ++;
40
               res += z;
           }
41
42
        }
43
44
        if(cnt != n-1) puts("impossible");
45
        else printf("%d",res);
46
```

3.10.2 Prim

```
#include<iostream>
   #include<string.h>
 3
   using namespace std;
 5
   const int N = 520, M = 2e105 + 10, INF = 0x3f3f3f3f;
    int d[N][N], dist[N], n, m, x, y, z;
 6
7
   bool st[N];
8
9
    int prime(){
       memset(dist, 0x3f, sizeof dist);
10
11
       int res = 0;
12
       for(int i = 0 ; i<n ;i++){</pre>
13
           int t = -1;
14
           for(int j = 1; j<=n ; j++){</pre>
              if(!st[j] && ((t == -1 ) || dist[t] > dist[j])){
15
16
                  t = j;
17
18
           }
19
           st[t] = true;
```

```
20
           if(i && dist[t] == INF) return -1;
21
           if(i) res+= dist[t];
22
23
           for(int j = 1; j<=n; j++){</pre>
24
               dist[j] = min(dist[j],d[t][j]);
25
           }
26
       }
27
       return res;
28
    }
29
30
   int main(){
       scanf("%d%d",&n,&m);
31
       memset(d, 0x3f, sizeof d);
32
33
       for(int i = 0 ; i < m ;i ++){</pre>
           scanf("%d%d%d", &x, &y, &z);
34
35
           d[x][y] = d[y][x] = min(d[x][y],z);
       }
36
37
       int res = prime();
38
       if(res == -1) puts("impossible");
       else printf("%d", res);
39
40
```

3.11 二分图

3.11.1 染色法判定二分图

```
#include<iostream>
    #include<string.h>
 3
    using namespace std;
   const int N = 1e5 + 10, M = 2e5 + 10;
 5
 6
7
    int idx=0,el[M],ne[M],h[N],color[N],n,m,x,y;
 8
9
    void add(int x, int y){
       el[idx] = y, ne[idx] = h[x], h[x] = idx++;
10
11
    }
12
13
14
    bool dfs(int u, int c){
15
       color[u] = c;
       for(int i = h[u]; i != -1; i = ne[i]){
16
17
           int j = el[i];
18
           if(!color[j]){
19
              if(!dfs(j, 3-c)) return false;
20
21
           else if(color[j] == c) return false;
       }
22
23
       return true;
24
   }
25
26 | int main(){
```

```
27
       memset(h, -1, sizeof h);
       scanf("%d%d",&n,&m);
28
29
       while(m --){
           scanf("%d%d",&x,&y);
30
31
           add(x,y),add(y,x);
32
       }
33
34
       bool flag = true;
35
       for(int i = 1 ; i <=n ; i++){</pre>
36
           if(!color[i]){
37
               if(!dfs(i, 1)){
                  flag = false;
38
                  break;
39
40
               }
           }
41
42
       }
43
       if(flag) puts("Yes");
44
       else puts("No");
45
    }
```

3.11.2 匈牙利算法求二分最大匹配

```
1 #include<iostream>
 2
    #include<string.h>
   using namespace std;
 5
    const int N = 510, M = 1e5 + 10;
 6
   int n1,n2,m,x,y,match[N],el[M], ne[M], h[N], idx=0;
 7
    bool st[N];
 8
    void add(int x, int y){
 9
       el[idx] = y, ne[idx] = h[x], h[x] = idx ++;
10
    }
11
12
    bool find(int u){
13
14
       for(int i = h[u];i!=-1;i=ne[i]){
15
           int j = el[i];
           if(!st[j]){
16
17
              st[j] = true;
              if(match[j] == 0 || find(match[j])){
18
19
                  match[j] = u;
20
                  return true;
              }
21
           }
22
23
24
       return false;
25
    }
26
27
   int main(){
28
       memset(h, -1, sizeof h);
29
       scanf("%d%d%d",&n1, &n2, &m);
       for(int i = 0 ; i < m; i++){</pre>
30
```

```
31
           scanf("%d%d",&x,&y);
32
           add(x,y);
33
       }
34
35
       int res = 0;
36
37
       for(int i = 1 ; i<=n1; i++){</pre>
38
           memset(st, false, sizeof st);
           if(find(i)){
39
40
               res ++;
41
           }
        }
42
43
        printf("%d\n",res);
44
        return 0;
45
46
    }
```

4 数论

4.1 质数

4.1.1 试除法判定质数

```
#include<iostream>
    using namespace std;
 3
    const int N = 110;
 4
   int m,x;
 5
    bool st[N];
 6
7
    bool is_prime(int x){
 8
       if(x < 2) return false;</pre>
       for(int i = 2; i <= x/i ; i++){</pre>
9
           if(x % i == 0) return false;
10
11
       }
       return true;
12
   }
13
14
15
    int main(){
       scanf("%d", &m);
16
17
       while(m -- ){
           scanf("%d", &x);
18
           if(is_prime(x)) puts("Yes");
19
20
           else puts("No");
21
       }
    }
22
```

4.1.2 分解质因数

```
#include<iostream>
using namespace std;
const int N = 110;
```

```
int n, x;
 4
 5
 6
 7
    void get_prime(int x){
 8
        for(int i = 2; i <= x/i ; i++){</pre>
 9
            if(x \% i == 0){
10
               int s = 0;
11
               while(x \% i == 0){
                   x /= i;
12
13
                   s++;
14
               }
               cout << i << ' ' << s <<endl;
15
           }
16
17
        if(x > 1) cout << x <<' ' << 1 << endl;</pre>
18
        cout << endl;</pre>
19
20
   }
21
22
   int main(){
23
        scanf("%d",&n);
24
        while(n --){
25
            scanf("%d", &x);
26
           get_prime(x);
27
28
        return 0;
29
    }
```

4.1.3 筛质数

```
1
   #include<iostream>
   using namespace std;
4
    const int N = 1e6 + 10;
   int primes[N];
   bool st[N];
 6
7
    int cnt;
8
9
    void get_primes(int n){
       for(int i = 2; i <= n ; i++){</pre>
10
11
           if(!st[i]) primes[cnt++] = i;
           for(int j = 0 ; primes[j] <= n/i ; j++){</pre>
12
13
               st[primes[j] * i] = true;
14
               if(i % primes[j] == 0) break;
           }
15
16
       }
17
    }
18
19
    int main(){
20
       int n;
21
       scanf("%d", &n);
22
       get_primes(n);
       printf("%d\n",cnt);
23
```

24 }

4.2 约数

4.2.1 试除法求约数

```
#include<bits/stdc++.h>
 2
   using namespace std;
 3
 4
    vector<int> get_divisor(int x) {
 5
        vector<int> res;
        for (int i = 1; i <= x / i; ++ i )</pre>
 6
 7
           if (x \% i == 0) {
 8
               res.push_back(i);
 9
               if (i != x / i) res.push_back(x / i);
10
           }
        sort(res.begin(), res.end());
11
12
        return res;
13
14
    int main() {
15
16
        int n;
17
        cin >> n;
18
19
        while (n -- ) {
20
           int x;
21
           cin >> x;
22
           auto res = get_divisor(x);
23
           for (auto x : res) cout << x << ' ';</pre>
24
25
           cout << endl;</pre>
26
27
        return 0;
28
    }
```

4.2.2 约数个数

```
1 #include < iostream >
 2
   #include < unordered_map >
   using namespace std;
 3
    const int MOD = 1e9+7;
 4
 5
    typedef long long LL;
    unordered_map<int,int> primes;
 6
    int n = 0;
 7
8
9
    int main(){
10
       scanf("%d",&n);
       LL res = 1;
11
12
       while(n--){
13
           int x;
           scanf("%d",&x);
14
15
           for(int i = 2 ; i <= x / i ; i++){</pre>
```

```
16
               while(x \% i == 0){
17
                 x /= i;
18
                 primes[i]++;
19
20
           }
21
           if( x > 1) primes[x] ++;
22
23
        for(auto p : primes) res = res * (p.second + 1) % MOD;
        printf("%lld\n", res);
24
        return 0;
25
26
27
```

4.2.3 约数之和

```
#include <iostream>
 2
   #include <unordered_map>
    using namespace std;
 4
5
   const int MOD = 1e9 + 7;
 6
    typedef long long LL;
 7
    int n;
8
    unordered_map<int, int> primes;
    int main(){
10
11
       scanf("%d",&n);
       while(n--){
12
13
           int x;
           scanf("%d", &x);
14
15
           for(int i = 2 ; i <= x/i ; i++){</pre>
16
              while(x \% i == 0){
                  x /= i ;
17
                  primes[i] ++;
18
19
              }
           }
20
21
           if(x > 1) primes[x] ++;
22
       }
23
       LL res = 1;
       for(auto x : primes){
24
25
           int p = x.first, m = x.second;
26
27
           while(m --) t = (t * p + 1) % MOD;
28
           res = res * t % MOD;
29
       printf("%lld", res);
30
31
       return 0;
32
```

4.2.4 最大公约数

```
1 | #include<iostream>
```

```
2
    #include<algorithm>
    using namespace std;
 5
    int gcd(int a, int b){
 6
       if(a < b) swap(a,b);
 7
       return b ? gcd(b, a%b) : a;
 8
    }
 9
10
    int n;
11
12
    int main(){
       scanf("%d",&n);
13
       while(n--){
14
15
           int a, b;
           scanf("%d%d",&a, &b);
16
17
           printf("%d\n", gcd(a,b));
18
       }
19
       return 0;
20
   }
```

4.3 欧拉函数

4.3.1 欧拉函数

```
1 #include<iostream>
   using namespace std;
    const int N = 1e4 + 10;
 3
 4
   int primes[N], n, phi[N];
 5
    int main(){
 6
 7
       scanf("%d", &n);
 8
       while(n -- ){
           int x;
 9
           scanf("%d", &x);
10
11
           int res = x;
12
           for(int i = 2 ; i <= x / i ; i++){</pre>
13
             if(x \% i == 0){
14
                 res = res / i * (i-1); // 防止溢出;
                 while(x \% i == 0) x /= i;
15
             }
16
17
18
           if( x > 1) res = res / x * (x-1);
19
           printf("%d\n", res);
20
       }
21
```

4.3.2 筛法求欧拉函数

```
#include<iostream>
using namespace std;

const int N = 1e6 + 10;
```

```
5
    typedef long long LL;
    int phi[N], primes[N],cnt;
 8
    bool st[N];
 9
    LL get_eulers(int n){
10
11
       phi[1] = 1;
12
        for(int i = 2; i <= n ; i++){</pre>
13
           if(!st[i]){
14
               primes[cnt++] = i;
15
               phi[i] = i -1;
           }
16
           for(int j = 0 ; primes[j] <= n / i ; j ++){</pre>
17
               int t = primes[j] * i;
18
               st[t] = true;
19
20
               if(i % primes[j] == 0){
21
                  phi[t] = phi[i] * primes[j];
22
23
               }
               phi[t] = phi[i] * (primes[j] - 1);
24
           }
25
26
       }
27
       LL res = 0;
28
        for(int i = 1; i<=n ;i++){</pre>
29
           res += phi[i];
30
       }
31
        return res;
32
   | }
33
34
    int main(){
        int n;
35
        scanf("%d", &n);
36
37
        printf("%lld\n",get_eulers(n));
38
```

4.4 快速幂

4.4.1 快速幂

```
#include<iostream>
 2
    using namespace std;
 3
4
    int n, a, b ,p;
 5
   typedef long long LL;
 7
    LL qmi(int a,int b, int p){
8
       LL res = 1;
9
       while(b){
10
           if(b & 1){
11
              res = (LL)res * a % p;
12
           }
13
           a = (LL) a * a % p;
```

```
14
           b >>= 1;
15
16
       return res;
17
    }
18
   int main(){
19
20
       scanf("%d", &n);
21
       while(n--){
           scanf("%d%d%d", &a, &b, &p);
22
23
           printf("%lld\n", qmi(a,b,p));
24
       }
    }
25
```

4.4.2 快速幂求逆元

```
#include<iostream>
 2
    using namespace std;
 3
4
    typedef long long LL;
 5
 6
    LL pmi(int a, int b, int p){
 7
       LL res = 1;
 8
       while(b){
 9
           if(b&1){
10
              res = (LL)res * a % p;
11
12
           a = (LL) a * a % p;
13
           b >>=1;
14
15
       return res;
16
    }
17
18
    int main(){
19
       int n ;
20
       scanf("%d",&n);
       while(n -- ){
21
22
           int a, p;
23
           scanf("%d%d",&a,&p);
24
           LL res = pmi(a, p -2, p);
25
           if(a % p) printf("%lld\n", res);
           else puts("impossible");
26
       }
27
28
29
    }
```

4.5 扩展欧几里得算法

4.5.1 扩展欧几里得算法

```
#include<iostream>
using namespace std;
```

```
4
   /*
5
   (a,b) = (b, a\%b) ==>
   ax + by = d
 6
    (a - a//b * b)x + by = d
 7
    ax + b(y - a //b *x) = d
    y' = y - a//b *x
9
10
    */
11
    int exgcd(int a, int b, int &x, int &y){
       if(!b){
12
13
           x = 1, y = 0;
14
           return a;
       }
15
16
       int d = exgcd(b,a%b,y,x);
17
       y -= (a/b)*x;
       return d;
18
19
    }
20
   int main(){
21
22
       int n;
       scanf("%d",&n);
23
       while(n--){
24
25
           int a, b, x ,y;
           scanf("%d%d",&a,&b);
26
27
           exgcd(a,b,x,y);
28
           printf("%d %d\n",x,y);
29
       }
30
       return 0;
31
   |}
```

4.5.2 线性同余方程

```
/*
1
 2
           a * x === b % m
 3
        ==> ax = my + b
        ==> ax - my = b
 4
 5
        ==> ax + my'= b
 6
           gcd(a, m) | b 则有解
 7
         x = x0 * b / d % m 相当于倍增
8
    */
9
   #include<bits/stdc++.h>
   using namespace std;
10
11
   using LL = long long;
12
13
    int exgcd(int a, int b, int & x, int & y) {
14
15
       if (!b) {
16
          x = 1, y = 0;
17
          return a;
18
19
       int d = exgcd(b, a % b, y, x);
20
       y -= a / b * x;
21
       return d;
```

```
22
   }
23
24
    int main() {
25
        int n;
26
        cin >> n;
        while (n -- ) {
27
28
           int a, b, m;
29
           cin >> a >> b >> m;
30
           int x, y;
           int d = exgcd(a, m, x, y);
31
32
           if (b % d) cout << "impossible" << endl;</pre>
           else cout << (LL)b / d * x % m << endl;</pre>
33
34
       }
35
    }
```

4.6 中国剩余定理

4.6.1 表达整数的奇怪方式

```
#include<iostream>
    using namespace std;
4
    typedef long long LL;
 5
 6
   LL exgcd(LL a, LL b, LL &x, LL &y){
 7
       if(!b){
 8
           x = 1, y = 0;
 9
           return a;
10
       }
11
       LL d = exgcd(b, a\%b, y, x);
12
       y = (a/b) * x;
13
       return d;
   }
14
15
16
    int main(){
17
       int n;
18
       LL a1,m1;
19
       scanf("%d",&n);
       scanf("%lld%lld", &a1, &m1);
20
21
       bool has_ans = true;
22
       for(int i = 0 ; i < n - 1 ;i++){</pre>
23
           LL a2,m2,k1,k2;
24
           scanf("%lld%lld", &a2, &m2);
25
           LL d = exgcd(a1,a2,k1,k2);
           if((m2-m1) % d){
26
27
              has_ans = false;
28
              break;
29
           }
30
           int t = a2 / d;
           k1 *= (m2-m1) / d;
31
32
           k1 = (k1 \% t + t) \% t;
33
```

```
34     m1 = a1 * k1 + m1;
35     a1 = abs(a1 / d * a2);
36     }
37     if(has_ans) printf("%lld\n", (m1 % a1 + a1) % a1);
38     else printf("%d\n",-1);
39     return 0;
40 }
```

4.7 高斯消元

4.7.1 高斯消元解线性方程组

4.7.2 高斯消元解异或线性方程组

```
#include<iostream>
 1
 2
   using namespace std;
 3
    const int N = 110;
 4
 5
   int n,a[N][N];
 6
 7
 8
    int solve(){
 9
        int r, c;
10
        for(r=0,c=0; c < n; c ++){
           int t = r;
11
12
           for(int i = r ; i < n ; i++)</pre>
13
               if(a[i][c]){
14
                   t = i;
15
                   break;
16
               }
17
18
           if(!a[t][c]) continue;
19
20
           for(int i = c; i < n + 1; i++) swap(a[t][i], a[r][i]);</pre>
21
22
           for(int i = r + 1; i < n; i++)</pre>
23
               if(a[i][c]){
24
                   for(int j = n ; j >= c; j --)
25
                      a[i][j] ^= a[r][j];
26
27
           r ++;
28
        if(r < n){
29
30
           for(int i = r ; i < n; i++)</pre>
31
               if(a[i][n]) return 2;
32
           return 1;
33
34
        for(int i = n -1 ; i >=0 ; i--)
35
           for(int j = i + 1; j < n ; j++)</pre>
36
               a[i][n] ^= a[i][j] & a[j][n];
37
        return 0;
```

```
38
   }
39
40
    int main(){
41
        cin >> n;
42
        for(int i = 0 ; i < n ; i++)</pre>
           for(int j = 0; j < n + 1; j++)
43
44
               cin >> a[i][j];
45
46
        int t = solve();
47
        if(t ==0){
48
           for(int i = 0; i < n; i ++)</pre>
               cout << a[i][n] << endl;</pre>
49
50
       }
51
        else if(t == 1) puts("Multiple sets of solutions");
        else puts("No solution");
52
53
        return 0;
54
   }
```

4.8 求组合数

4.8.1 上三角法

```
1
    #include<iostream>
    using namespace std;
 3
   const int N = 2010, mod = 1e9 + 7;
 4
 5
   int C[N][N], n;
 6
 7
    void solve(){
 8
        for(int a = 0 ; a < N; a ++)</pre>
           for(int b = 0 ; b <= a ; b++){</pre>
 9
               if(!b) C[a][b] = 1;
10
               else{
11
                   C[a][b] = (C[a-1][b] + C[a-1][b-1]) \% mod;
12
13
               }
14
           }
15
    }
16
17
    int main(){
18
       cin >> n;
19
        int a, b;
       solve();
20
21
        while(n--){
           cin >> a >> b;
22
23
           cout << C[a][b] <<endl;</pre>
24
25
        return 0;
26
```

4.8.2 小费马定理与逆元法

```
#include<iostream>
    #include<algorithm>
 2
    #include<string.h>
   using namespace std;
 5
 6
   typedef long long LL;
7
8
    const int N = 1e5 + 10, mod = 1e9 + 7;
    LL qmi(LL a, LL k, LL p){
10
       LL res =1;
11
12
       while(k){
           if(k & 1) res = (LL) res * a % p;
13
           a = (LL) a * a % p;
14
15
           k >>= 1;
16
17
       return res;
18
   }
19
20
   int main(){
21
       int n;
22
       cin >> n;
23
       LL fact[N], infact[N];
       fact[0] = infact[0] = 1;
24
25
       for(int i =1 ; i < N ;i ++){</pre>
           fact[i] = fact[i-1] * i % mod;
26
27
           infact[i] = infact[i-1] * qmi(i, mod - 2, mod) % mod;
28
29
       LL a, b;
       while(n--){
30
31
          cin >> a >> b;
32
          cout << fact[a] * infact[b] % mod * infact[a-b] % mod << endl;</pre>
       }
33
34
       return 0;
35
```

4.8.3 卢卡斯定理

```
#include<iostream>
   using namespace std;
2
4
   typedef long long LL;
 5
   LL qmi(LL a ,LL k, LL p){
 6
 7
       LL res = 1;
 8
       while(k){
 9
          if(k & 1) res = (LL) res * a % p;
10
          a = (LL) a * a % p;
11
          k >>= 1;
13
       return res;
14 }
```

```
15
16
   LL C(LL a, LL b, LL p){
17
       LL res = 1;
18
       for(int i = 1, j = a ; i <= b; i++, j--){</pre>
19
          res = (LL)res * j % p;
20
          res = (LL) res * qmi(i, p-2, p) % p;
21
       }
22
       return res;
23
   }
24
25
   LL lucica(LL a, LL b, LL p){
26
      if(a 
27
      return C(a % p, b % p, p) * lucica(a / p, b / p, p) % p;
   }
28
29
30
   int main(){
       int n;
31
32
       LL a,b,p;
       scanf("%d", &n);
33
34
       while(n--){
          scanf("%lld%lld", &a,&b,&p);
35
          printf("%lld\n", lucica(a,b,p));
36
37
       }
38
   }
```

4.8.4 高精度

```
#include <iostream>
   #include<vector>
 3
   using namespace std;
 5
    const int N = 5010;
 6
    typedef long long LL;
 7
    int primes[N], sum[N], cnt=0;
8
9
    bool st[N];
10
    int get(int n, int p){
11
       int res = 0;
12
13
       while(n){
14
           res += n / p;
15
           n \neq p;
       }
16
17
       return res;
18
19
20
    void get_primes(int n){
       for(int i = 2 ; i <= n; i++){</pre>
21
22
           if(!st[i]) primes[cnt++] = i;
23
           for(int j = 0 ; primes[j] <= n / i ; j ++){</pre>
24
               st[primes[j] * i] = true;
              if(i % primes[j] == 0) break;
25
```

```
26
           }
        }
27
28
    }
29
30
    vector<int> mul(vector<int> &a , int b){
31
        vector<int> c ;
32
        int t = 0;
33
        for(int i = 0; i < a.size(); i++){</pre>
           t += a[i] * b;
34
35
           c.push_back(t%10);
36
           t /= 10;
       }
37
38
       while(t) {
39
           c.push_back(t % 10);
40
           t /= 10;
41
       }
42
        return c;
    }
43
44
45
    int main(){
46
        int a, b;
47
        scanf("%d%d",&a,&b);
48
        get_primes(a);
49
        for(int i = 0 ; i < cnt; i++){</pre>
50
           int p = primes[i];
51
           sum[i] = get(a,p) - get(a-b,p) - get(b,p);
52
53
       vector<int> res;
54
       res.push_back(1);
55
        for(int i = 0 ; i < cnt; i++)</pre>
56
           for(int j = 0; j < sum[i]; j ++){</pre>
57
               int p = primes[i];
58
               res = mul(res, p);
           }
59
60
61
        for(int i = res.size() -1; i>=0 ; i--)
62
           printf("%d",res[i]);
63
        return 0;
64
```

4.8.5 卡特兰数

```
#include <iostream>
#include <algorithm>
using namespace std;

typedef long long LL;
const LL mod = 1e9 + 7;

typedef long long LL;

typedef long long LL;
```

```
11
12
    LL qmi(LL a, LL k, LL p){
       LL res = 1;
13
14
       while(k){
15
           if(k & 1) res = (LL) res * a % p;
16
           a = (LL) a * a % p;
17
           k >>= 1;
18
       }
19
       return res;
20
    }
21
    int main(){
22
23
       int n;
       scanf("%d", &n);
24
25
       int a = 2 *n, b = n;
       LL res =1;
26
       for(int i = a; i > a- b ; i--) res = (LL) res * i % mod;
27
       for(int i = 1; i <= b ; i++) res = (LL) res * qmi(i, mod - 2, mod) % mod;</pre>
28
       res = (LL) res * qmi(n+1, mod - 2, mod) % mod;
29
       printf("%lld\n",res);
30
       scanf("%d", &n);
31
32
33
   }
```

4.9 容斥原理

4.9.1 能被整除的数

```
#include<iostream>
   using namespace std;
 2
 3
    const int N = 20;
   typedef long long LL;
 5
    int p[N];
 6
 7
    int main(){
 8
       int n, m;
        scanf("%d%d",&n,&m);
 9
        int res = 0;
10
        for(int i = 0; i < m; i++) scanf("%d", &p[i]);</pre>
11
12
        for(int i = 1 ; i < 1 << m; i++){</pre>
13
           int t = 1, cnt = 0;
           for(int j = 0 ; j < m; j++){</pre>
14
15
               if(i >> j & 1){
                   if((LL)t * p[j] > n){
16
                      t = -1;
17
18
                      break;
19
                  }
20
                   cnt++;
                   t *= p[j];
21
               }
22
23
24
           if(t != -1){
```

4.10 博弈论

4.10.1 Nim 游戏

```
#include<iostream>
 2
    using namespace std;
 3
 4
5
    int main(){
 6
       int n;
 7
       scanf("%d",&n);
 8
       int x;
       scanf("%d", &x);
 9
       for(int i = 0; i < n -1; i++){</pre>
10
11
           int y;
12
           scanf("%d",&y);
13
           x ^=y;
14
       }
15
       if(x) puts("Yes");
       else puts("No");
16
17
       return 0;
18
   }
```

4.10.2 台阶-Nim 游戏

```
#include<iostream>
    using namespace std;
 2
 3
 4
 5
   int main(){
 6
       int n;
 7
        scanf("%d",&n);
 8
        int res = 0;
 9
        for(int i = 1; i <= n ; i ++){</pre>
10
           int x;
           scanf("%d",&x);
11
12
           if(i%2) res ^= x;
13
14
       if(res) puts("Yes");
15
       else puts("No");
        return 0;
16
17
```

4.10.3 集合-Nim 游戏

```
#include<iostream>
    #include<algorithm>
 2
    #include<unordered_set>
   #include<string.h>
 5
   using namespace std;
 8
    const int N = 110, M = 10010;
    int s[N], f[M], n, k;
10
11
12
    int sg(int x){
        if(f[x] != -1) return f[x];
13
14
15
       unordered_set<int> S;
       for(int i = 0; i < k; i++){</pre>
16
           if(x >= s[i]) S.insert(sg(x - s[i]));
17
18
19
       for(int i = 0; i < n; i++){</pre>
           if(!S.count(i)) {
20
21
               return f[x] = i;
22
23
       }
24
   }
25
   int main(){
26
27
       memset(f, -1, sizeof f);
28
       scanf("%d", &k);
29
       for(int i = 0; i < k; i ++) scanf("%d", &s[i]);</pre>
30
       int res = 0;
31
       scanf("%d", &n);
32
       for(int i = 0 ; i < n; i ++) {</pre>
          int x;
33
34
          scanf("%d", &x);
35
          res = sg(x);
       }
36
37
       if(res) puts("Yes");
38
       else puts("No");
       return 0;
39
40
```

4.10.4 拆分-Nim 游戏

```
#include<iostream>
#include<unordered_set>
#include<algorithm>
#include<string.h>
using namespace std;
const int N = 110;
int s[N],f[N], n;
```

```
int sg(int x){
 8
 9
        if(f[x]!=-1) return f[x];
10
11
       unordered_set<int> S;
12
       for(int i = 0 ; i < x; i++)</pre>
13
14
           for(int j = 0 ; j <= i; j++)</pre>
15
               S.insert(sg(i) ^ sg(j));
16
17
        for(int i = 0; ; i++)
18
           if(!S.count(i)) return f[x] = i;
19
    }
20
   int main(){
21
        scanf("%d", &n);
22
23
       int res = 0;
       memset(f, -1, sizeof f);
24
        for(int i = 0 ; i < n; i++){</pre>
25
26
           int x;
           scanf("%d", &x);
27
28
           res = sg(x);
29
       }
        if(res) puts("Yes");
30
31
        else puts("No");
32
    }
```

5 动态规划

5.1 背包问题

5.1.1 01 背包问题

```
#include<iostream>
 1
 2
    using namespace std;
 3
   const int N = 1010, M = 1010;
 4
 5
    int n,m,f[M], v[N], w[N];
 6
 7
    int main(){
 8
       cin >> n >> m;
 9
        for(int i = 1; i <=n; i++){</pre>
           cin >> v[i] >> w[i];
10
11
12
       for(int i = 1; i <=n ;i++)</pre>
13
14
           for(int j = m; j >= v[i]; j--)
15
               f[j] = max(f[j], f[j - v[i]] + w[i]);
        cout << f[m] << endl;</pre>
16
17
        return 0;
18
    }
```

5.1.2 完全背包问题

```
#include<iostream>
    using namespace std;
 2
 3
 4
 5
    const int N = 1010, M = 1010;
   // int n,m, f[N][M], v[N], w[N];
   int n,m, f[M], v[N], w[N];
 8
 9
    int main(){
10
        cin >> n >> m;
        for(int i = 1 ; i <= n ; i++) cin >> v[i] >> w[i];
11
12
        for(int i = 1; i<=n ; i++)</pre>
13
           for(int j = v[i]; j <=m; j++)</pre>
14
15
               f[j] = max(f[j], f[j - v[i]] + w[i]);
16
       // for(int i = 1; i <= n; i++)
17
       // for(int j = 0; j <= m; j++)
18
19
        // for(int k = 0; k * v[i] <= j; k++){
20
       // f[i][j] = \max(f[i][j], f[i-1][j - k*v[i]] + k * w[i]);
21
       // }
22
23
       cout << f[m] <<endl;</pre>
24
   |}
```

5.1.3 多重背包问题

```
1 #include<iostream>
   using namespace std;
   const int N = 110, M = 110;
 4
    int n, m , f[N][M], v[N], w[N], s[N];
 6
   int main(){
 7
       cin >> n >> m;
 8
        for(int i = 1; i <=n; i++) cin >> v[i] >> w[i] >> s[i];
 9
10
        for(int i = 1; i<=n; i++)</pre>
11
           for(int j = 0; j <=m ; j++)</pre>
               for(int k = 0; k <=s[i] && k * v[i] <= j; k++)</pre>
12
13
                   f[i][j] = max(f[i][j], f[i-1][j - k * v[i]] + k * w[i]);
14
        cout << f[n][m] << endl;</pre>
15
    }
```

5.1.4 多重背包问题 2

```
#include<iostream>
using namespace std;

const int N = 10010, M = 2010;
```

```
5
    int n,m,cnt=0,f[M], v[N], w[N];
 8
    int main(){
 9
        cin >> n >> m;
        for(int i = 1 ; i <=n; i++){</pre>
10
11
           int a,b,s;
12
           cin >> a >> b >> s;
           int k = 1;
13
14
           while(k <= s){</pre>
15
               v[++cnt] = k * a;
               w[cnt] = k * b;
16
17
               s -= k;
18
               k *=2;
           }
19
20
           if(s > 0){
21
               v[++cnt] = s * a;
22
               w[cnt] = s * b;
           }
23
        }
24
25
        for(int i = 1; i <= cnt; i ++)</pre>
26
           for(int j = m; j >= v[i]; j--)
27
28
               f[j] = max(f[j], f[j - v[i]] + w[i]);
29
30
        cout << f[m] <<endl;</pre>
31
        return 0;
32
```

5.1.5 分组背包问题

```
#include<iostream>
 2
    using namespace std;
    const int N = 110, M = 110;
   int n,m,f[M],w[N],v[N],s[N];
 5
    int main(){
 6
 7
        cin >> n >> m;
        for(int i = 1; i<=n; i++){</pre>
 8
 9
           cin >> s[i];
10
           for(int j = 1; j <=s[i]; j++) cin >> v[j] >> w[j];
           for(int j = m; j >=0; j--)
11
               for(int k = 1; k <= s[i]; k++)</pre>
12
13
                   if(j \ge v[k])
                      f[j] = max(f[j], f[j - v[k]] + w[k]);
14
15
       }
16
17
        cout << f[m] << endl;</pre>
18
        return 0;
19
```

5.2 线性 DP

5.2.1 数字三角形

```
#include<iostream>
 2
    using namespace std;
 4
    const int N = 510;
 5
 6
    int n,a[N][N], f[N][N], INF = 1e9;
 7
 8
    int main(){
 9
10
        cin >> n;
        for(int i = 1 ; i <=n; i++)</pre>
11
12
            for(int j = 1; j <=i; j++)</pre>
13
               cin >> a[i][j];
        for(int i = 1; i <=n; i++)</pre>
14
15
            for(int j = 0; j <= i+1; j++)</pre>
16
               f[i][j] = -INF;
        f[1][1] = a[1][1];
17
        for(int i = 1; i <=n ; i++)</pre>
18
19
            for(int j = 1; j <= i; j++)</pre>
20
                f[i][j] = max(f[i-1][j-1] + a[i][j], f[i-1][j] + a[i][j]);
21
22
        int res = -INF;
        for(int i = 1; i<=n; i++)</pre>
23
24
            res = max(res, f[n][i]);
25
26
        cout << res <<endl;</pre>
27
        return 0;
28
```

5.2.2 最长上升子序列

```
#include<iostream>
    using namespace std;
 2
    const int N = 1010;
 4
   int a[N],f[N];
 5
   int main(){
 6
 7
        int n;
 8
        cin >> n;
 9
        int res = 1;
10
        for(int i = 1 ; i<=n; i++) cin >> a[i];
        for(int i = 1; i <=n ; i++){</pre>
11
12
           f[i] =1;
13
           for(int j = 1; j < i; j ++){</pre>
               if(a[i] > a[j]) f[i] = max(f[i], f[j] + 1);
14
15
           }
16
           res = max(res, f[i]);
17
        }
18
        cout << res << endl;</pre>
```

5.2.3 最长上升子序列 2

```
#include<iostream>
 2
    using namespace std;
    const int N = 100010;
 4
    int a[N], q[N], n;
 5
 6
 7
    int main(){
 8
        scanf("%d", &n);
 9
        for(int i = 0; i < n; i++) scanf("%d", &a[i]);</pre>
10
        q[0] = -1e9;
11
        int len = 0;
        for(int i = 0; i < n; i++){</pre>
12
13
           int 1 = 0, r = len;
           while(1 < r){
14
15
               int mid = 1 + r + 1 >>1;
16
               if(q[mid] < a[i]) 1 = mid;</pre>
17
               else r = mid -1;
           }
18
19
           len = max(len, r+1);
20
           q[r+1] = a[i];
21
22
        cout << len << endl;</pre>
23
    }
```

5.2.4 最长公共子序列

```
#include<iostream>
    using namespace std;
 2
 3
 5
    const int N = 1010, M = 1010;
 6
    char a[N], b[M];
 7
 8
    int n,m,f[N][M];
 9
10
    int main(){
       scanf("%d%d", &n, &m);
11
        scanf("%s%s", a + 1, b + 1);
12
13
14
        for(int i = 1; i <= n ;i ++)</pre>
15
           for(int j =1 ; j <=m ; j ++){</pre>
               f[i][j] = max(f[i-1][j], f[i][j-1]);
16
17
               if(a[i] == b[j])
18
                   f[i][j] = max(f[i][j], f[i-1][j-1] + 1);
19
           }
20
        cout << f[n][m] <<endl;</pre>
```

Template By Aether

21 }

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5.2.5 最短编辑距离

```
#include<iostream>
   using namespace std;
 2
   const int N = 1010, M = 1010;
 5
   int n, m, f[N][M];
    char a[N], b[M];
 6
 7
 8
    int main(){
 9
        scanf("%d%s", &n, a + 1);
        scanf("%d%s", &m ,b + 1);
10
11
12
        for(int i =1 ; i<=n; i++) f[i][0] = i;</pre>
13
        for(int i = 1; i <=m; i++) f[0][i] = i;</pre>
14
15
        for(int i =1 ; i <=n ; i++)</pre>
16
           for(int j = 1 ; j <=m ; j++){</pre>
               f[i][j] = min(f[i-1][j] + 1, f[i][j-1] + 1);
17
               if(a[i] == b[j])
18
19
                  f[i][j] = min(f[i][j], f[i-1][j-1]);
20
               else
21
                  f[i][j] = min(f[i][j], f[i-1][j-1] + 1);
22
           }
23
        printf("%d\n", f[n][m]);
24
        return 0;
25
```

5.3 区间 DP

5.3.1 石子合并

```
#include<iostream>
    using namespace std;
 2
 3
   const int N = 310;
 5
   int n, m, f[N][N], s[N];
 7
    int main(){
 8
       cin >> n;
        for(int i = 1 ; i<=n; i++) cin >> s[i];
 9
10
        for(int i = 1; i <=n ; i++) s[i] += s[i-1];</pre>
        for(int len = 2; len <= n; len ++){</pre>
11
          for(int 1 = 1; 1 + len - 1 <= n; 1++){</pre>
12
13
              int r = 1 + len -1;
              f[1][r] = 1e9;
14
15
              for(int k = 1; k < r; k++)</pre>
                   f[1][r] = min(f[1][r], f[1][k] + f[k+1][r] + s[r] - s[1-1]);
17
          }
18
        }
```

5.4 计数类 DP

5.4.1 整数划分

```
/*
 1
 2 | f[i][j] = f[i-1][j] + f[i-1][j-i] + f[i-1][j-2*i] + ... f[i-1][j-s * i];
   f[i][j-i] = f[i-1][j-i] + f[i-1][j-2*i] + ... f[i-1][j - s * i];
 4 | f[i][j] = f[i-1][j] + f[i][j-i];
 5 | */
 6
   #include<iostream>
7
   using namespace std;
   const int N = 1010, mod = 1e9 + 7;
 8
9
    int n, f[N];
10
   int main(){
       scanf("%d", &n);
11
12
       f[0] = 1;
13
       for(int i = 1; i <=n ; i++)</pre>
           for(int j = i; j <=n; j++)</pre>
14
15
              f[j] = (f[j] + f[j - i]) \% mod;
       printf("%d\n", f[n]);
17
       return 0;
18
```

5.5 数位统计 DP

5.5.1 计数问题

```
1 #include<iostream>
   #include<cstring>
   using namespace std;
   const int N = 100;
 5
   int dp[N][N][2][2], digit[N];
 6
 7
 8
       x: 统计的数字
9
       1: 第几位
       lead: 前导0
10
       sum: 当前整数包含x的个数
11
       limit: 是否达到上限
12
13
   int f(int x, int 1, int sum, bool lead, bool limit){
14
15
      // 当前数字统计完了
16
       if(!1) return sum;
17
       // 记忆化搜索
18
       if(dp[l][sum][lead][limit] != -1) return dp[l][sum][lead][limit];
19
       int res =0;
       int k = limit ? digit[1] : 9;
20
21
       for(int i = 0; i <= k; i ++){</pre>
```

```
22
          bool ne_limit = limit && (i == k);
23
          if(lead &&!i) // 含前导0, 跳过继续搜
              res += f(x, 1-1, sum, true, ne_limit);
24
25
          else // 不含前导零,判断当前位是否为x
26
              res += f(x, 1-1, sum + (i == x), false, ne_limit);
27
       }
28
       // 返回第1~1位中的统计结果
29
       return dp[l][sum][lead][limit] = res;
30
   | }
31
32
   int solve(int n, int x){
33
       int len = 0;
       memset(dp, -1 ,sizeof dp);
34
35
       while(n){
36
          digit[++len] = n % 10;
37
          n /= 10;
38
39
       return f(x, len, 0, true, true);
40
   }
41
42
    int main(){
43
       int a, b;
       while(scanf("%d%d",&a, &b) && a && b){
44
45
          if(a > b) swap(a, b);
          for(int i = 0 ; i <= 9; i++)</pre>
46
              printf("%d ", solve(b,i) - solve(a-1,i));
47
48
          puts("");
49
       }
50
       return 0;
51
```

5.6 状态压缩 DP

5.6.1 最短 Hamilton 路径

```
#include<iostream>
 1
   #include<cstring>
    using namespace std;
    const int N = 21, M = 1 << N;</pre>
 5
    int dp[M][N], a[N][N], n;
 6
 7
    int main(){
 8
        scanf("%d", &n);
 9
        for(int i = 0; i < n; i++)</pre>
           for(int j = 0; j < n; j++)</pre>
10
               scanf("%d", &a[i][j]);
11
12
        memset(dp, 0x3f, sizeof dp);
        dp[1][0] = 0;
13
        for(int i= 0; i < 1 << n; i++)</pre>
14
15
           for(int j = 0; j < n; j++)
               if(i >> j & 1){
16
17
                   for(int k = 0; k < n; k ++)
```

5.6.2 蒙德里安的梦想

```
#include<iostream>
 1
 2 #include<cstring>
 3
    using namespace std;
   const int N = 12, M = 1 << N;</pre>
   int n,m;
 5
 6
    long long f[N][M];
 7
    bool st[M];
 8
 9
    int main(){
        while(scanf("%d%d", &n, &m) && (n || m)){
10
           memset(f, 0, sizeof f);
11
12
           memset(st, 0, sizeof st);
13
           for(int i = 0; i < 1 << n; i++){</pre>
               int cnt = 0;
14
15
               st[i] = true;
               for(int j = 0; j < n; j ++){
16
                   if(i >> j & 1){
17
                      if(cnt & 1) st[i] = false;
18
19
                      cnt = 0;
                   }
20
21
                   else cnt ++;
22
               if(cnt & 1) st[i] = false;
23
24
           }
           f[0][0] = 1;
25
26
           for(int i =1; i<=m; i++)</pre>
               for(int j = 0; j < 1 << n; j++)</pre>
27
                   for(int k = 0; k < 1 << n; k++)</pre>
28
29
                      if((j \& k) == 0 \&\& st[j | k])
30
                          f[i][j] += f[i-1][k];
31
           printf("%lld\n", f[m][0]);
        }
32
33
        return 0;
34
    }
```

5.7 树形 DP

5.7.1 没有上司的舞会

```
#include<iostream>
#include<cstring>
using namespace std;
```

```
const int N = 6010;
    int dp[N][2], happy[N], el[N], ne[N], h[N], idx = 0, n;
    bool has_father[N];
 6
 7
 8
    void insert(int x, int y){
       el[idx] = y, ne[idx] = h[x], h[x] = idx ++;
9
10
   }
11
    int dfs(int u){
12
13
       dp[u][1] = happy[u]; // 初始化, 只选u的快乐值
14
       for(int i = h[u]; i!=-1; i = ne[i]){
15
           int j = el[i];
16
           dfs(j);
17
           dp[u][0] += max(dp[j][0], dp[j][1]);
18
           dp[u][1] += dp[j][0];
19
       }
20
       return max(dp[u][0], dp[u][1]);
21
   }
22
23
   int main(){
24
       memset(h, -1, sizeof h);
25
       scanf("%d", &n);
26
       for(int i = 1; i <=n ;i ++) scanf("%d", &happy[i]);</pre>
27
       for(int i = 0; i < n-1; i++){</pre>
           int 1,k;
28
29
           scanf("%d%d", &1, &k);
30
           insert(k, 1); // k是1的祖先, 符合头插法
31
           has_father[1] = true;
32
       }
       int root =1 ;
33
34
       while(has_father[root]) root++;
       printf("%d\n", dfs(root));
35
36
       return 0;
37
```

5.8 记忆化搜索

5.8.1 滑雪

```
#include<iostream>
 2
   #include<cstring>
 3 using namespace std;
    const int N = 310, M=310;
 5
   int h[N][M], f[N][M], n, m;
   int dx[4] = \{-1,0,1,0\}, dy[4] = \{0,1,0,-1\};
 6
8
    int dfs(int x, int y){
9
       int &v = f[x][y];
10
       if(v != -1) return v;
11
       v = 1;
12
        for(int i=0; i < 4; i++){</pre>
13
           int a = x + dx[i], b = y + dy[i];
```

```
14
           if(a<=n && a >= 1 && b <=m && b >=1 && h[x][y] > h[a][b])
15
              v = max(v, dfs(a,b) + 1);
16
       }
17
       return v;
18
   }
19
20
   int main(){
       memset(f, -1, sizeof f);
21
       scanf("%d%d", &n, &m);
22
23
       for(int i = 1; i <= n; i++)</pre>
24
           for(int j = 1; j <=m; j++)</pre>
               scanf("%d", &h[i][j]);
25
26
27
       int res = 0;
28
       for(int i = 1; i <=n; i++)</pre>
29
           for(int j = 1; j<=m; j++)</pre>
30
              res = max(res, dfs(i,j));
       printf("%d\n", res);
31
32 }
```