



**国家超级计算广州中心**  
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

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# XCPC-Template

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# Part I: Basic Template

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## 0 ★ Preface

### 0.1 Template

```
1 #define itn int
2 #define nit int
3 #define nti int
4 #define tin int
5 #define tni int
6 #define retrun return
7 #define reutrn return
8 #define rutren return
9 #define fastin \
10     ios_base::sync_with_stdio(0); \
11     cin.tie(0), cout.tie(0);
12 #include <bits/stdc++.h>
13 using namespace std;
14 typedef long long LL;
15 typedef long double LD;
16 typedef pair<int, int> PII;
17 typedef pair<long long, long long> PLL;
18 typedef pair<double, double> PDD;
19 typedef vector<int> VI;
20 #ifndef ONLINE_JUDGE
21 #define dbg(args...) \
22     do \
23     { \
24         cout << "\033[32;1m" << #args << " -> \
25         "; \
26         err(args); \
27     } while (0)
28 #define dbg(...)
29 #endif
30 void err()
31 { cout << "\033[39;0m" << endl; }
32 template <template <typename...> class T,
33         typename t, typename... Args>
34 void err(T<t> a, Args... args)
35 {
36     for (auto x : a) cout << x << ' ';
37     err(args...);
38 }
39 template <typename T, typename... Args>
40 void err(T a, Args... args)
41 { cout << a << ' '; err(args...); }
42 const int INF = 0x3f3f3f3f;
43 const int mod = 1e9 + 7;
44 const double eps = 1e-6;
45 int main()
46 {
47     #ifndef ONLINE_JUDGE
48         freopen("test.in", "r", stdin);
49         freopen("test.out", "w", stdout);
50     #endif
51     fastin;
52     return 0;
53 }
```

### 0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

### 0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about  $10^7 \sim 10^8$  operations.
- Below is a guide on how to choose algorithms based on different input size ranges:
  1.  $n \leq 30 \rightarrow$  Exponential complexity: DFS with pruning, State Compression DP
  2.  $n \leq 100 \rightarrow O(n^3)$ : Floyd, DP, Gaussian Elimination
  3.  $n \leq 1000 \rightarrow O(n^2), O(n^2 \log n)$ : DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
  4.  $n \leq 10000 \rightarrow O(n^{\frac{3}{2}})$ : Block Linked List, Mo's Algorithm
  5.  $n \leq 100000 \rightarrow O(n \log n)$ : sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
  6.  $n \leq 1000000 \rightarrow O(n)$ , or small-constant  $O(n \log n)$ : Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
  7.  $n \leq 10000000 \rightarrow O(n)$ : Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
  8.  $n \leq 10^9 \rightarrow O(\sqrt{n})$ : Primality Testing
  9.  $n \leq 10^{18} \rightarrow O(\log n)$ : GCD, Fast Exponentiation, Digit DP
  10.  $n \leq 10^{1000} \rightarrow O((\log n)^2)$ : Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
  11.  $n \leq 10^{100000} \rightarrow O(\log k \cdot \log \log k)$ , where  $k$  is the number of digits: Big Integer Add/Subtract, FFT/NTT

## 0.4 If <bits/stdc++.h> Failed

Replace it with:

```
1 #include <algorithm>
2 #include <bitset>
3 #include <complex>
4 #include <deque>
5 #include <exception>
6 #include <fstream>
7 #include <functional>
8 #include <iomanip>
9 #include <ios>
10 #include <iosfwd>
11 #include <iostream>
12 #include <istream>
13 #include <iterator>
14 #include <limits>
15 #include <list>
16 #include <locale>
17 #include <map>
18 #include <memory>
19 #include <numeric>
20 #include <ostream>
21 #include <queue>
22 #include <set>
23 #include <sstream>
24 #include <stack>
25 #include <stdexcept>
26 #include <streambuf>
27 #include <string>
28 #include <typeinfo>
29 #include <utility>
30 #include <valarray>
31 #include <vector>
```

# 1 ★ Basic Algorithm

## 1.1 Quick Sort

Sort the given array from index 1 to n.

```
1 void quick_sort(int l, int r)
2 {
3     if (l >= r) return;
4     int x = a[(l + r) >> 1], i = l - 1, j = r
5     + 1;
6     while (i < j)
7     {
8         do i++; while (a[i] < x);
9         do j--; while (a[j] > x);
10        if (i < j) swap(a[i], a[j]);
11    }
12    quick_sort(l, j);
13    quick_sort(j + 1, r);
14    return;
15 }
```

## 1.2 Binary Search

```
1 // 区间 [l, r] 被划分成 [l, mid] 和 [mid + 1,
2 // r] 时使用
3 // 大于等于区间的最小值, check 应为 target <= a
4 // [mid]
5 int bsearch_1(int l, int r)
6 {
7     while (l < r)
8     {
9         int mid = l + r >> 1;
10        if (check(mid)) r = mid;
11        else l = mid + 1;
12    }
13    return l;
14 }
15 // 区间 [l, r] 被划分成 [l, mid - 1] 和 [mid,
16 // r] 时使用
17 // 小于等于区间的最大值, check 应为 target >= a
18 // [mid]
19 int bsearch_2(int l, int r)
20 {
21     while (l < r)
22     {
23         // 为什么要 l + r + 1: 因为 l 的更新条
24         // 件是 mid 本身
25         // 当 r == l + 1 时 mid 向下取整必定取
26         // l, 有可能在满足 check(mid) 时导致无限循环
27         int mid = l + r + 1 >> 1;
28         if (check(mid)) l = mid;
29         else r = mid - 1;
30    }
31    return l;
32 }
33 // 浮点数二分
34 double bsearch_3(double l, double r)
35 {
36     // eps 表示精度, 取决于题目对精度的要求
37     const double eps = 1e-6;
38     while (r - l > eps)
```

```
33 {
34     double mid = (l + r) / 2;
35     if (check(mid)) r = mid;
36     else l = mid;
37 }
38 return l;
39 }
```

## 1.3 High Precision

### 1.3.1 High Precision Add

```
1 string s1, s2;
2 vector<int> a, b, c;
3 void add(vector<int> &a, vector<int> &b)
4 {
5     if (a.size() < b.size())
6     { add(b, a); return; }
7     int t = 0;
8     for (int i = 0; i < a.size(); i++)
9     {
10        t += a[i];
11        if (i < b.size()) t += b[i];
12        c.push_back(t % 10);
13        t /= 10;
14    }
15    while (t)
16        c.push_back(t % 10), t /= 10;
17 }
18 int main()
19 {
20     cin >> s1 >> s2;
21     for (int i = s1.size() - 1; i >= 0; i--)
22         a.push_back(s1[i] - '0');
23     for (int i = s2.size() - 1; i >= 0; i--)
24         b.push_back(s2[i] - '0');
25     add(a, b);
26     for (int i = c.size() - 1; i >= 0; i--)
27         cout << c[i];
28     return 0;
29 }
```

### 1.3.2 High Precision Subsection

```
1 vector<int> a, b, c;
2 string s1, s2;
3 void sub(vector<int> &a, vector<int> &b)
4 {
5     int t = 0;
6     for (int i = 0; i < a.size(); i++)
7     {
8         t = a[i] - t;
9         if (i < b.size()) t -= b[i];
10        c.push_back((t + 10) % 10);
11        if (t < 0) t = 1;
12        else t = 0;
13    }
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
```

```

18 {
19     cin >> s1 >> s2;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     for (int i = s2.size() - 1; i >= 0; i--)
23         b.push_back(s2[i] - '0');
24     if (s1.size() < s2.size())
25         cout << '-', sub(b, a);
26     else if (s1.size() == s2.size() && s1 < s2)
27         cout << '-', sub(b, a);
28     else sub(a, b);
29     for (int i = c.size() - 1; i >= 0; i--)
30         cout << c[i];
31     return 0;
32 }

```

```

16 }
17 int main()
18 {
19     cin >> s1 >> b;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     divide(a, b, r);
23     for (int i = c.size() - 1; i >= 0; i--)
24         cout << c[i];
25     cout << '\n' << r;
26     return 0;
27 }

```

## 1.4 Prefix Sum & Difference Array

### 1.3.3 High Precision Multiply

```

1 string s1, s2;
2 vector<int> a, c;
3 int b;
4 void mul(vector<int> &a, int b)
5 {
6     for (int i = 0, t = 0; i < a.size() || t; i++)
7     {
8         if (i < a.size()) t += a[i] * b;
9         c.push_back(t % 10);
10        t /= 10;
11    }
12    while (c.size() > 1 && c.back() == 0)
13        c.pop_back();
14 }
15 int main()
16 {
17     cin >> s1 >> b;
18     for (int i = s1.size() - 1; i >= 0; i--)
19         a.push_back(s1[i] - '0');
20     mul(a, b);
21     for (int i = c.size() - 1; i >= 0; i--)
22         cout << c[i];
23     return 0;
24 }

```

### 1.3.4 High Precision Divide

```

1 string s1, s2;
2 vector<int> a, c;
3 int b, r;
4 void divide(vector<int> &a, int b, int &r)
5 {
6     r = 0;
7     for (int i = a.size() - 1; i >= 0; i--)
8     {
9         r = r * 10 + a[i];
10        c.push_back(r / b);
11        r %= b;
12    }
13    reverse(c.begin(), c.end());
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();

```

### 1.4.1 1D Prefix Sum

```

1 S[i] = a[1] + a[2] + ... a[i]
2 a[1] + ... + a[r] = S[r] - S[1 - 1]

```

### 1.4.2 2D Prefix Sum

```

1 // S[i, j] = i 行 j 列左上部分所有元素和为:
2 s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1] +
  a[i][j]
3 // 以 (x1, y1) 为左上角, (x2, y2) 为右下角的子
  矩阵的和为:
4 S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1] + S[
  x1 - 1][y1 - 1]

```

### 1.4.3 1D Difference Array

```

1 const int N = 100010;
2 int n, m;
3 int a[N], b[N];
4 void insert(int l, int r, int c)
5 { b[l] += c; b[r + 1] -= c; }
6 int main()
7 {
8     cin >> n >> m;
9     for (int i = 1; i <= n; i++)
10        cin >> a[i];
11     for (int i = 1; i <= n; i++)
12        insert(i, i, a[i]);
13     while (m--)
14     {
15         int l, r, c;
16         cin >> l >> r >> c;
17         insert(l, r, c);
18     }
19     for (int i = 1; i <= n; i++)
20         b[i] += b[i - 1];
21     cout << b[i] << ' ';
22     return 0;
23 }

```



### 1.4.4 2D Difference Array

```
1  const int N = 1010;
2  int n, m, q, a[N][N], b[N][N];
3  void insert(int x1, int y1, int x2, int y2,
4             int c)
5  {
6      b[x1][y1] += c;
7      b[x2 + 1][y2 + 1] += c;
8      b[x1][y2 + 1] -= c;
9      b[x2 + 1][y1] -= c;
10 }
11 int main()
12 {
13     cin >> n >> m >> q;
14     for (int i = 1; i <= n; i++)
15         for (int j = 1; j <= m; j++)
16             cin >> a[i][j];
17     for (int i = 1; i <= n; i++)
18         for (int j = 1; j <= m; j++)
19             insert(i, j, i, j, a[i][j]);
20     while (q--)
21     {
22         int x1, x2, y1, y2, c;
23         cin >> x1 >> y1 >> x2 >> y2 >> c;
24         insert(x1, y1, x2, y2, c);
25     }
26     // 其他过程略
```

## 2 ★ Basic Data Structures

### 2.1 Linked List

#### 2.1.1 Singly Linked List

```
1 const int N = 100010;
2 int n, h[N], e[N], ne[N], idx = 1;
3 void init() { ne[0] = -1; }
4 void insert(int k, int x) // 第 k 个节点后插入
5 { e[idx] = x, ne[idx] = ne[k], ne[k] = idx++; }
6 void del(int k) // 第 k 个节点后删除
7 { ne[k] = ne[ne[k]]; }
```

#### 2.1.2 Bidirectional Linked List

```
1 const int N = 100010;
2 int n, r[N], l[N], e[N], idx = 2;
3 void init() { r[0] = 1; l[1] = 0; }
4 void insert(int k, int x) // 第 k 个节点后插入
5 {
6     e[idx] = x;
7     r[idx] = r[k];
8     l[idx] = k;
9     l[r[k]] = idx;
10    r[k] = idx++;
11 }
12 void remove(int k) // 删除 k 本身
13 { r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

### 2.2 Stack & Queue

#### 2.2.1 Monotonic Stack

```
1 // 常见模型：找出每个数左边离它最近的比它大/小的数
2 int tt = 0;
3 for (int i = 1; i <= n; i++)
4 {
5     while (tt && check(stk[tt], i)) tt--;
6     stk[++tt] = i;
7 }
```

#### 2.2.2 Monotonic Queue

```
1 // 常见模型：找出滑动窗口中的最大值/最小值
2 int hh = 0, tt = -1;
3 for (int i = 0; i < n; i++)
4 {
5     while (hh <= tt && check_out(q[hh]))
6         hh++; // 判断队头是否滑出窗口
7     while (hh <= tt && check(q[tt], i))
8         tt--;
9     q[++tt] = i;
```

```
10 }
```

### 2.3 KMP

```
1 const int N = 100010, M = 1000010;
2 int n, m;
3 char p[N], s[M];
4 void getNext(int ne[])
5 {
6     for (int i = 2, j = 0; i <= n; i++)
7     {
8         while (j && p[j + 1] != p[i])
9             j = ne[j];
10        if (p[j + 1] == p[i]) j++;
11        ne[i] = j;
12    }
13 }
14 int KMP()
15 {
16     int *ne = new int[n + 1];
17     getNext(ne);
18     for (int i = 1, j = 0; i <= m; i++)
19     {
20         while (j && p[j + 1] != s[i])
21             j = ne[j];
22         if (p[j + 1] == s[i]) j++;
23         if (j == n) cout << i - n << ' ';
24     }
25     return -1;
26 }
```

### 2.4 Trie

```
1 const int N = 100010;
2 int trie[N][26], cnt[N], idx = 0;
3 void insert(string &str) // 插入到 Trie 数组
4 {
5     int p = 0;
6     for (auto c : str)
7     {
8         int u = c - 'a';
9         if (!trie[p][u])
10             trie[p][u] = ++idx;
11         p = trie[p][u];
12     }
13     cnt[p]++;
14 }
15 int query(string &str) // 查询字符串出现的次数
16 {
17     int p = 0;
18     for (auto c : str)
19     {
20         int u = c - 'a';
21         if (!trie[p][u]) return 0;
22         p = trie[p][u];
23     }
24     return cnt[p];
25 }
```

## 2.5 Disjoint-Set

```
1  const int N = 100010;
2  int n, m, p[N], Size[N], D[N];
3  void init()
4  {
5      for (int i = 1; i <= n; i++)
6          p[i] = i, Size[i] = 1, D[i] = 0;
7  }
8  int find(int x)
9  {
10     if (p[x] != x)
11     {
12         int u = find(p[x]);
13         D[x] += D[p[x]]; // 视具体情况计算
14         p[x] = u;
15     }
16     return p[x];
17 }
18 void merge(int a, int b, int distance)
19 {
20     int x = find(a), y = find(b);
21     if (x != y)
22     {
23         p[x] = y;
24         D[x] = distance; // 视具体情况计算
25         Size[y] += Size[x];
26     }
27 }
```

## 2.6 Hash

### 2.6.1 Simple Hash

```
1  // (1) 拉链法
2  int h[N], e[N], ne[N], idx;
3  void insert(int x)
4  {
5      int k = (x % N + N) % N;
6      e[idx] = x, ne[idx] = h[k], h[k] = idx++;
7  }
8  bool find(int x)
9  {
10     for (int i = h[(x % N + N) % N]; i != -1; i = ne[i])
11         if (e[i] == x) return true;
12     return false;
13 }
14 // (2) 开放寻址法
15 int find(int x)
16 {
17     int t = (x % N + N) % N;
18     while (h[t] != null && h[t] != x)
19         { t++; if (t == N) t = 0; }
20     return t;
21 }
```

### 2.6.2 String Hash

```
1  typedef unsigned long long ULL;
2  ULL h[N], p[N];
3  void init()
4  {
5      p[0] = 1;
6      for (int i = 1; i <= n; i++) { h[i] = h[i - 1] * P + str[i]; p[i] = p[i - 1] * P; }
7  }
8  ULL get(int l, int r) { return h[r] - h[l - 1] * p[r - l + 1]; }
```

## 2.7 STL

```
1  // vector
2  size()      返回元素个数
3  empty()     返回是否为空
4  clear()     清空
5  front()/back()
6  push_back()/pop_back()
7  begin()/end()
8  []
9  支持比较运算, 按字典序
10 // pair<int, int>
11 first       第一个元素
12 second      第二个元素
13 支持比较运算, 以first为第一关键字, 以second为第二关键字 (字典序)
14 // string
15 size()/length() 返回字符串长度
16 empty()
17 clear()
18 substr(起始下标, (子串长度)) 返回子串
19 c_str() 返回字符串所在字符数组的起始地址
20 // queue
21 size()
22 empty()
23 push()      向队尾插入一个元素
24 front()     返回队头元素
25 back()      返回队尾元素
26 pop()       弹出队头元素
27 // priority_queue
28 size()
29 empty()
30 push()      插入一个元素
31 top()       返回堆顶元素
32 pop()       弹出堆顶元素
33 定义成小根堆的方式: priority_queue<int, vector<int>, greater<int>> q;
34 // stack
35 size()
36 empty()
37 push()      向栈顶插入一个元素
38 top()       返回栈顶元素
39 pop()       弹出栈顶元素
40 // deque
41 size()
42 empty()
43 clear()
44 front()/back()
45 push_back()/pop_back()
46 push_front()/pop_front()
47 begin()/end()
```

```

48 []
49 // set, map, multiset, multimap: 基于平衡二叉树
    (红黑树) 动态维护有序序列
50 size()
51 empty()
52 clear()
53 begin()/end()
54 ++, -- 返回前驱和后继, 时间复杂度  $O(\log n)$ 
55 // set/multiset
56     insert() 插入一个数
57     find()    查找一个数
58     count()   返回某一个数的个数
59     erase()
60         (1) 输入是一个数x, 删除所有x,  $O(k + \log n)$ 
61         (2) 输入一个迭代器, 删除这个迭代器
62     lower_bound()/upper_bound()
63         lower_bound(x) 返回大于等于x的最小的数的
        迭代器
64         upper_bound(x) 返回大于x的最小的数的迭
        代器
65 // map/multimap
66     insert() 插入的数是一个pair
67     erase()   输入的参数是pair或者迭代器
68     find()
69     []        注意multimap不支持此操作。 时间复
        杂度是  $O(\log n)$ 
70     lower_bound()/upper_bound()
71 // unordered_set, unordered_map,
    unordered_multiset, unordered_multimap
72 增删改查的时间复杂度是  $O(1)$ 
73 不支持 lower_bound()/upper_bound(), 迭代器的++
    , --
74 // bitset
75 bitset<10000> s;
76 ~, &, |, ^
77 >>, <<
78 ==, !=
79 []
80 count()    返回有多少个1
81 any()      判断是否至少有一个1
82 none()     判断是否全为0
83 set()      把所有位置成1
84 set(k, v)  将第k位变成v
85 reset()    把所有位变成0
86 flip()     等价于~
87 flip(k)    把第k位取反

```

## 3 ★ Search & Graph Theory

### 3.1 Representation of Tree & Graph

#### 3.1.1 Adjacency Matrix

```
1 // g[a][b] = a->b
```

#### 3.1.2 Adjacency List

```
1 int h[N], e[N], ne[N], idx;
2 void init() { memset(h, -1, sizeof h); }
3 void add(int a, int b) { e[idx] = b, ne[idx] = h[a], h[a] = idx++; }
```

## 3.2 DFS & BFS

#### 3.2.1 DFS

```
1 int dfs(int u)
2 {
3     st[u] = true; // 表示点 u 已经被遍历过
4     for (int i = h[u]; i != -1; i = ne[i])
5         { int j = e[i]; if (!st[j]) dfs(j); }
6 }
```

#### 3.2.2 BFS

```
1 queue<int> q;
2 st[1] = true; q.push(1);
3 while (q.size())
4 {
5     int t = q.front(); q.pop();
6     for (int i = h[t]; i != -1; i = ne[i])
7         if (!st[e[i]]) { st[e[i]] = true; q.push(e[i]); }
8 }
```

## 3.3 Topological Sort

```
1 const int N = 100010;
2 int e[2 * N], ne[2 * N], h[N], d[N], idx;
3 int n, m, q[N];
4 void init() { memset(h, -1, sizeof h); }
5 void add(int a, int b) { e[idx] = b, ne[idx] = h[a], h[a] = idx++; d[b]++; }
6 bool topSort()
7 {
8     int hh = 0, tt = -1;
9     for (int i = 1; i <= n; i++)
10         if (!d[i]) q[++tt] = i;
11     while (hh <= tt)
```

```
12         for (int i = h[q[hh++]]; ~i; i = ne[i])
13             if (--d[e[i]] == 0) q[++tt] = e[i];
14     return tt == n - 1;
15 }
```

## 3.4 Shortest Path

#### 3.4.1 Dijkstra

```
1 const int N = 1010;
2 int n, dist[N];
3 int h[N], w[N], e[N], ne[N], idx;
4 bool st[N];
5 void add(int a, int b, int c) { e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++; }
6 int dijkstra() // 需要初始化 dist 与 h
7 {
8     dist[1] = 0;
9     priority_queue<PII, vector<PII>, greater<PII>> heap;
10    heap.push({0, 1});
11    while (heap.size())
12    {
13        auto t = heap.top();
14        heap.pop();
15        int ver = t.second, distance = t.first;
16        if (st[ver]) continue;
17        st[ver] = true;
18        for (int i = h[ver]; i != -1; i = ne[i])
19            if (dist[e[i]] > distance + w[i])
20            {
21                dist[e[i]] = distance + w[i];
22                heap.push({dist[e[i]], e[i]});
23            }
24    }
25    if (dist[n] == 0x3f3f3f3f) return -1;
26    return dist[n];
27 }
```

#### 3.4.2 Bellman-Ford

```
1 const int N = 100010;
2 int n, m, dist[N], backup[N];
3 struct Edge
4 {
5     int a, b, w;
6 } edges[N];
7 int bellman_ford()
8 {
9     memset(dist, 0x3f, sizeof dist);
10    dist[1] = 0;
11    for (int i = 0; i < n; i++)
12    {
13        memcpy(backup, dist, sizeof dist);
14        for (int j = 0; j < m; j++)
15        {
```

```

16         int a = edges[j].a, b = edges[j].b
17         , w = edges[j].w;
18         dist[b] = min(dist[b], backup[a] +
19             w);
20     }
21     if (dist[n] > 0x3f3f3f3f / 2) return -1;
22     return dist[n];
23 }

```

### 3.4.3 SPFA

```

1  const int N = 100010;
2  int n, m, dist[N];
3  int e[2 * N], ne[2 * N], w[2 * N], h[N], idx;
4  bool vis[N];
5  void spfa()    // 需要初始化 dist 与 h
6  {
7      queue<int> q;
8      q.push(1); vis[1] = true;
9      while (q.size())
10     {
11         int t = q.front();
12         q.pop();
13         vis[t] = false;
14         for (int i = h[t]; ~i; i = ne[i])
15             if (dist[e[i]] > dist[t] + w[i])
16             {
17                 dist[e[i]] = dist[t] + w[i];
18                 if (!vis[e[i]]) vis[e[i]] =
19                     true, q.push(j);
20             }
21     }
22     dist[n] > INF / 2 ? cout << "impossible" :
23     cout << dist[n];
24 }

```

### 3.4.4 Detecting Negative Circle SPFA

```

1  void spfa()    // 只需要初始化 h
2  {
3      queue<int> q;
4      // 基于虚拟原点假设, 所有点放入队列
5      for (int i = 1; i <= n; i++) q.push(i), st
6      [i] = true;
7      while (q.size())
8      {
9          int t = q.front();
10         q.pop();
11         vis[t] = false;
12         for (int i = h[t]; ~i; i = ne[i])
13             if (dist[e[i]] > dist[t] + w[i])
14             {
15                 dist[e[i]] = dist[t] + w[i];
16                 // 新增
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n) return true
19                 if (!st[j]) q.push(j), st[j] =
20                 true;
21             }
22     }
23 }

```

```

20     }
21     return false;
22 }

```

### 3.4.5 Floyd

```

1  const int N = 210;
2  int g[N][N], n, m, k;
3  int main()
4  {
5      cin >> n >> m >> k;
6      memset(g, 0x3f, sizeof g);
7      for (int i = 1; i <= n; i++) g[i][i] = 0;
8      while (m--)
9      {
10         int a, b, c;
11         cin >> a >> b >> c;
12         g[a][b] = min(g[a][b], c);
13     }
14     for (int k = 1; k <= n; k++)
15         for (int i = 1; i <= n; i++)
16             for (int j = 1; j <= n; j++)
17                 g[i][j] = min(g[i][k] + g[k][j]
18                     , g[i][j]);
19     // 后续代码略
20     return 0;
21 }

```

## 3.5 Minimum Spanning Tree

### 3.5.1 Prim

```

1  const int N = 510;
2  int n, m, g[N][N], dist[N];
3  bool vis[N];
4  void prim()
5  {
6      int res = 0;
7      for (int i = 0; i < n; i++)
8      {
9          int t = -1;
10         for (int j = 1; j <= n; j++)
11             if (!vis[j] && (t == -1 || dist[j]
12                 < dist[t])) t = j;
13         if (i && dist[t] == INF) { res = INF;
14             break; }
15         if (i) res += dist[t];
16         vis[t] = true;
17         for (int j = 1; j <= n; j++) dist[j] =
18             min(dist[j], g[t][j]);
19     }
20     res == INF ? cout << "impossible" : cout
21     << res;
22 }
23 int main()
24 {
25     memset(g, 0x3f, sizeof g);
26     memset(dist, 0x3f, sizeof dist);
27     cin >> n >> m;
28     while (m--)
29     {

```

```

26     int a, b, c;
27     cin >> a >> b >> c;
28     g[a][b] = min(g[a][b], c);
29     g[b][a] = min(g[b][a], c);
30 }
31 prim();
32 return 0;
33 }

```

### 3.5.2 Kruskal

```

1  const int N = 100010;
2  int n, m;
3  int p[N];
4  struct Edge
5  {
6      int a, b, w;
7      bool operator<(const Edge &e) const {
8          return w < e.w; };
9  } edge[2 * N];
10 void init() { for (int i = 1; i <= n; i++) p[i] = i; }
11 int find(int x)
12 {
13     if (x != p[x]) p[x] = find(p[x]);
14     return p[x];
15 }
16 void merge(int x, int y) { p[find(x)] = find(y); }
17 void kruskal()
18 {
19     int res = 0, cnt = 0;
20     for (int i = 1; i <= m; i++)
21         if (find(edge[i].a) != find(edge[i].b))
22         {
23             merge(edge[i].a, edge[i].b);
24             res += edge[i].w;
25             cnt++;
26         }
27     if (cnt < n - 1) res = INF;
28     res == INF ? cout << "impossible" : cout << res;
29 }
30 int main()
31 {
32     init();
33     cin >> n >> m;
34     for (int i = 1; i <= m; i++) cin >> edge[i].a >> edge[i].b >> edge[i].w;
35     sort(edge + 1, edge + m + 1);
36     kruskal();
37     return 0;
38 }

```

```

2  int n, m;
3  int e[M], ne[M], h[N], color[N], idx;
4  bool dfs(int u, int c)
5  {
6      color[u] = c;
7      for (int i = h[u]; ~i; i = ne[i])
8          if (color[e[i]] == -1)
9              {
10                 if (!dfs(e[i], !c)) return false;
11             }
12         else if (color[e[i]] == c) return false;
13     return true;
14 }
15 bool check()
16 {
17     for (int i = 1; i <= n; i++)
18         if (color[i] == -1)
19             if (!dfs(i, 0)) return false;
20     return true;
21 }
22 int main()
23 {
24     // 注意另外初始化 h 与 color
25     cin >> n >> m;
26     while (m--)
27     {
28         int a, b;
29         cin >> a >> b;
30         add(a, b), add(b, a);
31     }
32     // 其余过程略
33 }

```

## 3.6 Bipartite Graph

### 3.6.1 Coloring Method

To check if a given graph is bipartite.

```

1  const int N = 100010, M = 200010;

```

### 3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```
1  const int N = 510, M = 100010;
2  int n1, n2, m;
3  int e[M], ne[M], h[N], match[N], idx;
4  bool vis[N];
5  bool find(int x)
6  {
7      for (int i = h[x]; ~i; i = ne[i])
8          if (!vis[e[i]])
9              {
10                 vis[e[i]] = true;
11                 if (match[e[i]] == 0 || find(match[e[i]])
12                     ))
13                     {
14                         match[e[i]] = x;
15                         return true;
16                     }
17             }
18     return false;
19 }
20 int main()
21 {
22     // 注意初始化 h
23     cin >> n1 >> n2 >> m;
24     while (m--)
25     {
26         int a, b;
27         cin >> a >> b;
28         add(a, b);
29     }
30     int res = 0;
31     for (int i = 1; i <= n1; i++)
32     {
33         memset(vis, false, sizeof vis);
34         if (find(i)) res++;
35     }
36     cout << res;
37     return 0;
38 }
```



## 4 ★ Basic Math

### 4.1 Prime Numbers

#### 4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0) return false;
6     return true;
7 }
```

#### 4.1.2 Prime Factorization

```
1 void divide(int x)
2 {
3     for (int i = 2; i <= x / i; i++)
4         if (x % i == 0)
5             { // 此条件成立时 i 一定是质数
6                 int s = 0;
7                 while (x % i == 0) x /= i, s++;
8                 cout << i << ' ' << s << '\n';
9             }
10    if (x > 1) cout << x << ' ' << 1 << '\n'
11 }
```

#### 4.1.3 Euler's Sieve

```
1 int primes[N], cnt;
2 bool st[N];
3 void get_primes(int n)
4 {
5     for (int i = 2; i <= n; i++)
6     {
7         if (!st[i]) primes[cnt++] = i;
8         for (int j = 0; primes[j] <= n / i; j++)
9             {
10                st[primes[j] * i] = true;
11                if (i % primes[j] == 0) break;
12            }
13     }
14 }
```

## 4.2 Divisor

### 4.2.1 Find All Divisors

```
1 vector<int> get_divisors(int x)
2 {
3     vector<int> res;
4     for (int i = 1; i <= x / i; i++)
5         if (x % i == 0)
```

```
6             {
7                 res.push_back(i);
8                 if (i != x / i) res.push_back(x /
9             )
10    sort(res.begin(), res.end());
11    return res;
12 }
```

### 4.2.2 The Number of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 int main()
4 {
5     cin >> n;
6     unordered_map<int, int> h;
7     while (n--)
8     {
9         int x;
10        cin >> x;
11        for (int i = 2; i <= x / i; i++)
12            while (x % i == 0) { h[i]++; x = x
13                / i; }
14        if (x > 1) h[x]++;
15    }
16    long long res = 1;
17    for (auto iter = h.begin(); iter != h.end()
18        ); iter++)
19        res = res * (iter->second + 1) % mod;
20    cout << res;
21    return 0;
22 }
```

### 4.2.3 The Sum of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 long long getSum(int x, int c)
4 {
5     long long s = 1;
6     while(c--) s = (s * x + 1) % mod;
7     return s;
8 }
9 int main()
10 {
11     cin >> n;
12     unordered_map<int, int> h;
13     while (n--)
14     {
15         int x;
16         cin >> x;
17         for (int i = 2; i <= x / i; i++)
18             while (x % i == 0) { h[i]++; x = x
19                 / i; }
20         if (x > 1) h[x]++;
21     }
22     long long res = 1;
23     for (auto iter = h.begin(); iter != h.end()
24         ); iter++)
25         res = res * getSum(iter->first, iter->
26             second) % mod;
```

```

24     cout << res;
25     return 0;
26 }

```

## 4.2.4 Euclidean Algorithm

```

1 int gcd(int a, int b)
2 { return a % b == 0 ? b : gcd(b, a % b); }

```

## 4.3 Euler Function

### 4.3.1 Simple Method

```

1 int phi(int x)
2 {
3     int res = x;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0)
6             {
7                 res = res / i * (i - 1);
8                 while (x % i == 0) x /= i;
9             }
10    if (x > 1) res = res / x * (x - 1);
11    return res;
12 }

```

### 4.3.2 Euler's Sieve Method

```

1 const int N = 1000010;
2 int n, primes[N], phi[N], cnt;
3 bool st[N];
4 void getEuler()
5 {
6     phi[1] = 1;
7     for (int i = 2; i <= n; i++)
8     {
9         if (!st[i])
10            {
11                primes[cnt++] = i;
12                // i 是质数，它只会被本身整除，所以
                // 直接赋值 i - 1
13                phi[i] = i - 1;
14            }
15            for (int j = 0; primes[j] <= n / i; j++)
16            {
17                st[i * primes[j]] = true;
18                if (i % primes[j] == 0)
19                {
20                    // 如果 i % primes[j] == 0 成
                    // 立表示 primes[j] 是 i 的最小质因子
                    // 也是 primes[j] * i 的最小质
                    // 因子
21                    // 1 - 1 / primes[j] 这一项在
                    // phi[i] 中计算过了，只需将基数 N 修正为
                    // primes[j] 倍
22                    phi[i * primes[j]] = phi[i] *
                    // primes[j];
23                    break;
24                }

```

```

25     }
26     // 否则，primes[j] 不是 i 的质因
    // 子，只是 primes[j] * i 的最小质因子
27     // 不仅需要将基数 N 修正为 primes[j]
    // 倍
28     // 还需要补上 1 - 1 / primes[j] 的
    // 分子项，因此最终结果为 phi[i] * (primes[j]
    // - 1)
29     phi[primes[j] * i] = phi[i] * (
    // primes[j] - 1);
30 }
31 }
32 }

```

## 4.4 Exponentiating by Squaring

```

1 LL qmi(int m, int k, int p)
2 {
3     LL res = 1 % p, t = m;
4     while (k)
5     {
6         if (k & 1) res = res * t % p;
7         t = t * t % p;
8         k >>= 1;
9     }
10    return res;
11 }

```

## 4.5 Extended Euclidean Algorithm

```

1 int exgcd(int a, int b, int &x, int &y)
2 {
3     if (!b)
4     {
5         x = 1;
6         y = 0;
7         return a;
8     }
9     int d = exgcd(b, a % b, y, x);
10    y -= (a / b) * x;
11    return d;
12 }

```

## 4.6 Chinese Remainder Theorem

```

1 LL exgcd(LL a, LL b, LL &x, LL &y)
2 {
3     if (!b) { x = 1, y = 0; return a; }
4     LL d = exgcd(b, a % b, y, x);
5     y -= a / b * x;
6     return d;
7 }
8 int main()
9 {

```

```

10 int n;
11 cin >> n;
12 LL x = 0, m1, a1;
13 cin >> m1 >> a1;
14 for (int i = 0; i < n - 1; i++)
15 {
16     LL m2, a2;
17     cin >> m2 >> a2;
18     LL k1, k2;
19     LL d = exgcd(m1, m2, k1, k2);
20     if ((a2 - a1) % d) { x = -1; break; }
21     k1 *= (a2 - a1) / d;
22     k1 = (k1 % (m2 / d) + m2 / d) % (m2 /
23 d);
24     x = k1 * m1 + a1;
25     LL m = abs(m1 / d * m2);
26     a1 = k1 * m1 + a1;
27     m1 = m;
28 }
29 if (x != -1)
30     x = (a1 % m1 + m1) % m1;
31 cout << x << '\n';
32 return 0;
33 }

```

## 4.7 Gauss-Jordan Elimination

### 4.7.1 Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++) // 找
            绝对值最大的行
8             if (fabs(a[i][c]) > fabs(a[t][c]))
9                 t = i;
10        if (fabs(a[t][c]) < eps) // 此
            时没必要对该列该行处理
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[t][i], a[r][i]); // 将
            绝对值最大的行换到最顶端
14        for (int i = n; i >= c; i--)
15            a[r][i] /= a[r][c]; // 将
            当前行的首位变成1
16        for (int i = r + 1; i < n; i++) // 用
            当前行将下面所有的列消成0
17            if (fabs(a[i][c]) > eps)
18                for (int j = n; j >= c; j--)
19                    a[i][j] -= a[r][j] * a[i][c];
20        r++;
21    }
22    if (r < n)
23    {
24        for (int i = r; i < n; i++)
25            if (fabs(a[i][n]) > eps)
26                return 2; // 无解
27        return 1; // 有无穷多组解
28    }

```

```

29     for (int i = n - 1; i >= 0; i--)
30         for (int j = i + 1; j < n; j++)
31             a[i][n] -= a[i][j] * a[j][n];
32     return 0; // 有解
33 }

```

### 4.7.2 XOR Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++)
8             if (a[i][c])
9                 t = i;
10        if (!a[t][c])
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[r][i], a[t][i]);
14        for (int i = r + 1; i < n; i++)
15            if (a[i][c])
16                for (int j = n; j >= c; j--)
17                    a[i][j] ^= a[r][j];
18        r++;
19    }
20    if (r < n)
21    {
22        for (int i = r; i < n; i++)
23            if (a[i][n])
24                return 2;
25        return 1;
26    }
27    for (int i = n - 1; i >= 0; i--)
28        for (int j = i + 1; j < n; j++)
29            a[i][n] ^= a[i][j] * a[j][n];
30    return 0;
31 }

```

## 4.8 Combinatorial Counting

### 4.8.1 Recurrence Relation

```

1 void init()
2 {
3     for (int i = 0; i < N; i++)
4         for (int j = 0; j <= i; j++)
5             if (!j) c[i][j] = 1;
6             else c[i][j] = (c[i - 1][j] + c[i
7 - 1][j - 1]) % mod;
8 }

```

### 4.8.2 Preprocessing & Inverse Element

```

1 const int N = 100010, mod = 1e9 + 7;
2 int n, fact[N], infact[N];
3 int qmi(int a, int b, int p)
4 {

```

```

5   int res = 1;
6   while (b)
7   {
8       if (b & 1)
9           res = (LL)res * a % p;
10      a = (LL)a * a % p;
11      b >>= 1;
12  }
13  return res;
14 }
15 int main()
16 {
17     fact[0] = infact[0] = 1;
18     for (int i = 1; i < N; i++)
19     {
20         fact[i] = (LL)fact[i - 1] * i % mod;
21         infact[i] = (LL)infact[i - 1] * qmi(i,
22             mod - 2, mod) % mod;
23     }
24     // 此后 C(a, b) = (LL)fact[a] * infact[b]
25     // mod * infact[a - b] % mod
26 }

```

### 4.8.3 Lucas Theorem

```

1  int qmi(int a, int k, int p)
2  {
3      int res = 1 % p;
4      while (k)
5      {
6          if (k & 1)
7              res = (LL)res * a % p;
8          a = (LL)a * a % p;
9          k >>= 1;
10     }
11     return res;
12 }
13 int C(int a, int b, int p)
14 {
15     if (a < b) return 0;
16     LL x = 1, y = 1;
17     // x = a * (a - 1) * (a - 2) * ... * (a -
18     // b + 1) = a! / (a - b)! (mod p)
19     // y = 1 * 2 * ... * b = b! (mod p)
20     for (int i = a, j = 1; j <= b; i--, j++)
21     { x = (LL)x * i % p; y = (LL)y * j % p; }
22     return x * (LL)qmi(y, p - 2, p) % p;
23 }
24 int lucas(LL a, LL b, int p)
25 {
26     if (a < p && b < p)
27         return C(a, b, p);
28     return (LL)C(a % p, b % p, p) * lucas(a /
29         p, b / p, p) % p;
30 }

```

### 4.8.4 Factorization Method

```

1  const int N = 5010;
2  int n, primes[N], sum[N], cnt;
3  bool st[N];
4  void getPrimes(int n) { // 略 }

```

```

5  // 求 n! 中 p 的幂次
6  int get(int n, int p)
7  {
8      int res = 0;
9      while (n) { res += n / p; n /= p; }
10     return res;
11 }
12 void mul(vector<int> &a, int b) { // 高精度
13     // 乘, 略 }
14 int main()
15 {
16     int a, b;
17     cin >> a >> b;
18     getPrimes(a);
19     for (int i = 0; i < cnt; i++)
20     {
21         int p = primes[i];
22         sum[i] = get(a, p) - get(b, p) - get(a
23             - b, p);
24     }
25     vector<int> res;
26     res.push_back(1);
27     for (int i = 0; i < cnt; i++)
28     {
29         for (int j = 0; j < sum[i]; j++)
30             mul(res, primes[i]);
31     }
32     for (int i = res.size() - 1; i >= 0; i--)
33         cout << res[i];
34 }

```

### 4.8.5 Catalan Number

```

1  const int N = 100010, mod = 1e9 + 7;
2  int qmi(int a, int k, int p) { // 略 }
3  int main()
4  {
5      int n;
6      cin >> n;
7      int a = n * 2, b = n, res = 1;
8      for (int i = a; i > a - b; i--)
9          res = (LL)res * i % mod;
10     for (int i = 1; i <= b; i++)
11         res = (LL)res * qmi(i, mod - 2, mod) %
12         mod;
13     res = (LL)res * qmi(n + 1, mod - 2, mod) %
14     mod;
15 }

```

## 4.9 Inclusion-Exclusion Principle

```

1  const int N = 20;
2  int n, m, res = 0, p[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < m; i++)
7          cin >> p[i];
8      // 使用二进制数字表示数字选取情况
9      for (int i = 1; i < 1 << m; i++)
10     {

```

```

11     int t = 1, cnt = 0;
12     // 遍历每个被选取的质数
13     for (int j = 0; j < m; j++)
14         if (i >> j & 1)
15             {
16                 cnt++;
17                 // 一个质数能被选取的条件应该是
其累乘积不超过目标数字
18                 if ((LL)t * p[j] > n)
19                     { t = -1; break; }
20                 t *= p[j];
21             }
22     if (t != -1)
23         // 容斥原理公式中奇数个并集系数为 1
, 反之则为 -1
24         if (cnt % 2) res += n / t;
25         else res -= n / t;
26     }
27     cout << res;
28 }

```

## 4.10 Game Theory

### 4.10.1 NIM Game

```

1  const int N = 110, M = 100010;
2  int k, n, s[N], f[M];
3  int sg(int x)
4  {
5      if (f[x] != -1) return f[x];
6      // 到达节点得 SG 函数集合
7      unordered_set<int> S;
8      // 能取走石子就说明能到达, 并且递归向下求解
9      for (int i = 0; i < k; i++)
10         {
11             int sum = s[i];
12             if (x >= sum) S.insert(sg(x - sum));
13         }
14      // SG 从小到达遍历并返回, 找到最小的、不包含
在 SG 函数集合中的自然数
15      for (int i = 0;; i++)
16         if (!S.count(i))
17             return f[x] = i;
18  }
19
20 int main()
21 {
22     cin >> k;
23     for (int i = 0; i < k; i++) cin >> s[i];
24     cin >> n;
25     memset(f, -1, sizeof f);
26     int res = 0;
27     // 每一堆石子都是一个入度为 0 的起始点
28     for (int i = 0; i < n; i++)
29     {
30         int x;
31         cin >> x;
32         res ^= sg(x);
33     }
34     res ? cout << "Yes" : cout << "No";
35     return 0;
36 }

```

## 5 ★ Basic DP

### 5.1 Knapsack Problem

#### 5.1.1 01 Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = m; j >= v[i]; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.2 Complete Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = v[i]; j <= m; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.3 Mutiple Knapsack

```
1  const int N = 25000;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      int cnt = 0;
7      for (int i = 1; i <= n; i++)
8      {
9          int a, b, s;
10         cin >> a >> b >> s;
11         int k = 1;
12         while (k <= s)
13         {
14             cnt++;
15             v[cnt] = a * k, w[cnt] = b * k;
16             s -= k, k *= 2;
17         }
18         if (s > 0)
19         {
20             cnt++;
21             v[cnt] = a * s, w[cnt] = b * s;
```

```
22     }
23 }
24 n = cnt;
25 for (int i = 1; i <= n; i++)
26     for (int j = m; j >= v[i]; j--)
27         f[j] = max(f[j], f[j - v[i]] + w[i]);
28 cout << f[m];
29 }
```

#### 5.1.4 Grouped Knapsack

```
1  const int N = 120;
2  int n, m, s[N], v[N][N], w[N][N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> s[i];
9          for (int j = 1; j <= s[i]; j++)
10             cin >> v[i][j] >> w[i][j];
11     }
12     for (int i = 1; i <= n; i++)
13         for (int j = m; j >= 0; j--)
14             for (int k = 1; k <= s[i]; k++)
15                 if (v[i][k] <= j)
16                     f[j] = max(f[j], f[j - v[i][k]] + w[i][k]);
17     cout << f[m];
18 }
```

## 5.2 Linear DP

### 5.2.1 LIS

Here is an  $O(n^2)$  solution:

```
1  const int N = 1010;
2  int n, a[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[j] < a[i])
13                 f[i] = max(f[i], f[j] + 1);
14     }
15     int res = 0;
16     for (int i = 1; i <= n; i++)
17         res = max(res, f[i]);
18     cout << res;
19 }
```

Another is an  $O(n \log n)$  solution:

```
1  const int N = 100010;
2  int n, a[N], q[N];
```

```

3 int main()
4 {
5     cin >> n;
6     for (int i = 1; i <= n; i++) cin >> a[i];
7     int len = 0;
8     q[len] = -INF;
9     for (int i = 1; i <= n; i++)
10    {
11        int l = 0, r = len;
12        while (l < r)
13        {
14            int mid = l + r + 1 >> 1;
15            if (q[mid] < a[i]) l = mid;
16            else r = mid - 1;
17        }
18        len = max(r + 1, len);
19        q[r + 1] = a[i];
20    }
21    cout << len;
22 }

```

## 5.2.2 LCS

```

1 const int N = 1010;
2 int n, m, f[N][N];
3 char a[N], b[N];
4 int main()
5 {
6     cin >> n >> m >> (a + 1) >> (b + 1);
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= m; j++)
9         {
10            f[i][j] = max(f[i - 1][j], f[i][j - 1]);
11            if (a[i] == b[j])
12                f[i][j] = max(f[i][j], f[i - 1][j - 1] + 1);
13        }
14    cout << f[n][m];
15 }

```

## 5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```

1 const int N = 310;
2 int n, s[N], f[N][N];
3 int main()
4 {
5     cin >> n;
6     for (int i = 1; i <= n; i++)
7         cin >> s[i], s[i] += s[i - 1];
8     for (int len = 2; len <= n; len++)
9         for (int i = 1; i + len - 1 <= n; i++)
10        {
11            int l = i, r = i + len - 1;
12            f[l][r] = INF;
13            for (int k = l; k < r; k++)
14                f[l][r] = min(f[l][r], f[l][k]
15                    + f[k + 1][r] + s[r] - s[l - 1]);
16        }
17 }

```

```

16     cout << f[1][n];
17 }

```

## 5.4 Counting DP

```

1 const int N = 1010, M = 1e9 + 7;
2 int n, f[N][N];
3 int main()
4 {
5     cin >> n;
6     f[0][0] = 1;
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= i; j++)
9             f[i][j] = (f[i - 1][j - 1] + f[i - 1][j]) % M;
10    int ans = 0;
11    for (int i = 1; i <= n; i++)
12        ans = (ans + f[n][i]) % M;
13    cout << ans;
14 }

```

## 5.5 Digit DP

```

1 // 求数 n 的位数
2 int get(int n)
3 {
4     int res = 0;
5     while (n) n /= 10, res++;
6     return res;
7 }
8 int count(int n, int i)
9 {
10    int res = 0, dgt = get(n);
11    for (int j = 1; j <= dgt; j++)
12    {
13        // p 为当前遍历位次(第 j 位)的数大小
14        // <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
15        // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
16        int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
17        // 求要选的数在 i 的左边的数小于 1 的情况:
18        // 1)、当 i 不为 0 时 xxx : 0...0 ~ 1 - 1, 即 1 * (右边的数的位数) == 1 * p 种选法
19        // 2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0...1 ~ 1 - 1, 即 (1 - 1) * (右边的数的位数) == (1 - 1) * p 种选法
20        if (i) res += 1 * p;
21        else res += (1 - 1) * p;
22        // 求要选的数在 i 的左边的数等于 1 的情况: (即视频中的 xxx == 1 时)
23        // 1)、i > dj 时 0 种选法
24        // 2)、i == dj 时 yyy : 0...0 ~ r 即 r + 1 种选法
25        // 3)、i < dj 时 yyy : 0...0 ~ 9...9 即 10^(右边的数的位数) == p 种选法
26        if (i == dj) res += r + 1;
27    }
28 }

```

```

26     if (i < dj) res += p;
27 }
28 return res;
29 }
30 int main()
31 {
32     int a, b;
33     while (cin >> a >> b, a)
34     {
35         if (a > b) swap(a, b);
36         for (int i = 0; i <= 9; ++i)
37             cout << count(b, i) - count(a - 1,
38 i) << ' ';
39         // 利用前缀和思想: [1, r] 的和 = s[r] -
40 s[1 - 1]
41         cout << '\n';
42     }
43 }
44 }

```

```

37     for (int k = 0; k < 1 << n; k
38 ++))
39         // 满足两个条件: 两列的摆放
40         // 互不冲突; 两列摆放状态的结合状态是一个可取的
41         // 状态则累加情况数
42         if (!(j & k) && st[j | k])
43             f[i][j] += f[i - 1][k]
44 ];
45 // 输出摆放好第 m 列且第 (m + 1) 列没有
46 // 任何方格的状态数
47 cout << f[m][0] << '\n';
48 }
49 }

```

## 5.7 Tree DP

## 5.6 State Compression DP

```

1  const int N = 12, M = 1 << 12;
2  int n, m;
3  LL f[N][M];
4  bool st[M];
5  int main()
6  {
7      while (cin >> n >> m, n || m)
8      {
9          memset(f, 0, sizeof f);
10         for (int i = 0; i < 1 << n; i++)
11         {
12             st[i] = true;
13             // 统计连续 0 的个数, 若连续 0 为奇
14             // 数个就不能正好放得下竖放的方格
15             int cnt = 0;
16             for (int j = 0; j < n && st[i]; j
17 ++))
18                 if (i >> j & 1)
19                 {
20                     // 当前格子被使用
21                     // 如果连续 0 的数量为奇数
22                     // 个, 当前格子被使用的后果就是导致格子重合, 所以
23                     // 不可取
24                     if (cnt & 1)
25                         st[i] = false;
26                     // 刷新状态
27                     cnt = 0;
28                 }
29                 else cnt++;
30             // 最后再判断一次, 防止漏判
31             if (cnt & 1)
32                 st[i] = false;
33         }
34         // 没有摆放任何棋子的状态默认只有 1 种
35         // 取法
36         f[0][0] = 1;
37         // 遍历每一列
38         for (int i = 1; i <= m; i++)
39         {
40             // 遍历当前列的每一种用二进制数字表
41             // 示的摆放状态: 1 指横向摆放, 0 指空位
42             for (int j = 0; j < 1 << n; j++)
43                 // 遍历上一列的每一种用二进制数

```

```

1  // Don't use I/O functions from stdio.h!!!
2  #define itn int
3  #define nit int
4  #define nti int
5  #define tin int
6  #define tni int
7  #define retrun return
8  #define reutrn return
9  #define rutren return
10 #define INF 0x3f3f3f3f
11 #include <bits/stdc++.h>
12 using namespace std;
13 typedef pair<int, int> PII;
14 typedef long long LL;
15
16 const int N = 6010;
17
18 int n;
19 int e[N], ne[N], happy[N], h[N], idx;
20 int f[N][2];
21 bool has_father[N];
22 void add(int a, int b)
23 { e[idx] = b, ne[idx] = h[a], h[a] = idx++; }
24 void dfs(int u)
25 {
26     f[u][1] = happy[u];
27     for (int i = h[u]; ~i; i = ne[i])
28     {
29         dfs(e[i]);
30         f[u][0] += max(f[e[i]][0], f[e[i]][1])
31 ;
32         f[u][1] += f[e[i]][0];
33     }
34 }
35 int main()
36 {
37     memset(h, -1, sizeof h);
38     cin >> n;
39     for (int i = 1; i <= n; i++) cin >> happy[
40 i];
41     for (int i = 0; i < n - 1; i++)
42     {
43         int a, b;
44         cin >> a >> b;
45         has_father[a] = true;
46         add(b, a);

```



```

45     }
46     int root = 1;
47     while (has_father[root]) root++;
48     dfs(root);
49     cout << max(f[root][0], f[root][1]);
50 }

```

## 5.8 Memoized Search

```

1  const int N = 310;
2  int n, m,
3  h[N][N], f[N][N],
4  dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1, 0};
5  int dp(int x, int y)
6  {
7      int &v = f[x][y];
8      if (v != -1) return v;
9      v = 1;
10     for (int i = 0; i < 4; i++)

```

```

11     {
12         int a = x + dx[i], b = y + dy[i];
13         if (a >= 1 && a <= n && b >= 1 && b <=
14             m && h[a][b] < h[x][y])
15             v = max(v, dp(a, b) + 1);
16     }
17     return v;
18 }
19 int main()
20 {
21     cin >> n >> m;
22     for (int i = 1; i <= n; i++)
23         for (int j = 1; j <= m; j++)
24             cin >> h[i][j];
25     memset(f, -1, sizeof f);
26     int res = 0;
27     for (int i = 1; i <= n; i++)
28         for (int j = 1; j <= m; j++)
29             res = max(res, dp(i, j));
30     cout << res;

```



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## Part II: Advanced Template

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## 6 ★ Advanced Basic

### 6.1 Slow Multiplication

```
1 LL mul(LL a, LL b, LL p)
2 {
3     LL ans = 0;
4     while (b)
5     {
6         if (b & 1) ans = (ans + a) % p;
7         a = a * 2 % p; b >>= 1;
8     }
9     return ans;
10 }
```

### 6.2 Sum of Geometric Series

```
1 const int mod = 9901;
2 int a, b;
3 int qmi(int a, int k)
4 {
5     int res = 1;
6     a %= mod;
7     while (k)
8     {
9         if (k & 1)
10             res = res * a % mod;
11         a = a * a % mod;
12         k >>= 1;
13     }
14     return res;
15 }
16 int sum(int p, int k)
17 {
18     if (k == 1) return 1;
19     if (k % 2 == 0)
20         return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
21     return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
22 }
23 int main()
24 {
25     // 以  $a^b$  约数之和为例求等比数列和
26     cin >> a >> b;
27     int res = 1;
28     for (int i = 2; i <= a / i; i++)
29         if (a % i == 0)
30         {
31             int s = 0;
32             while (a % i == 0) a /= i, s++;
33             res = res * sum(i, b * s + 1) % mod;
34         }
35     if (a > 1) res = res * sum(a, b + 1) % mod;
36 }
```

## 6.3 Sort

### 6.3.1 Card Balancing Problem

```
1 cin >> n;
2 for (int i = 1; i <= n; i++)
3     cin >> a[i], avg += a[i];
4 avg /= n;
5 for (int i = 1; i <= n; i++)
6     if (a[i] != avg)
7         a[i + 1] += a[i] - avg, ans++;
8 cout << ans;
```

### 6.3.2 2D Card Balancing Problem

```
1 const int N = 100010;
2 int row[N], col[N], c[N], s[N];
3 LL work(int n, int a[])
4 {
5     for (int i = 1; i <= n; i++)
6         s[i] = s[i - 1] + a[i];
7     if (s[n] % n) return -1;
8     int avg = s[n] / n;
9     c[1] = 0;
10    for (int i = 2; i <= n; i++)
11        c[i] = s[i - 1] - (i - 1) * avg;
12    sort(c + 1, c + n + 1);
13    LL res = 0;
14    for (int i = 1; i <= n; i++)
15        res += abs(c[i] - c[(n + 1) / 2]);
16    return res;
17 }
18 int main()
19 {
20     int n, m, cnt;
21     cin >> n >> m >> cnt;
22     while (cnt--)
23     {
24         int x, y;
25         cin >> x >> y;
26         row[x]++; col[y]++;
27     }
28     LL r = work(n, row);
29     LL c = work(m, col);
30     if (r != -1 && c != -1)
31         cout << "both " << r + c;
32     else if (r != -1)
33         cout << "row " << r;
34     else if (c != -1)
35         cout << "column " << c;
36     else cout << "impossible";
37 }
```

### 6.3.3 Dual Heaps

```
1 if (down.empty() || x <= down.top())
2     down.push(x);
3 else up.push(x);
4 if (down.size() > up.size() + 1)
5     up.push(down.top()), down.pop();
```

```

6  if (up.size() > down.size())
7      down.push(up.top()), up.pop();
8  if (i % 2)
9  {
10     cout << down.top() << ' ';
11     if (++cnt % 10 == 0) cout << '\n';
12 }

```

## 6.4 RMQ

```

1  const int N = 200010, M = 18;
2  int n, m, w[N], f[N][M];
3  void init()
4  {
5      for (int j = 0; j < M; j++)
6          for (int i = 1; i + (1 << j) - 1 <= n;
7              i++)
8              if (!j) f[i][j] = w[i];
9              else // 也可以是最小值
10                 f[i][j] = max(f[i][j - 1], f[i
11                     + (1 << j - 1)][j - 1]);
12 }
13 int query(int l, int r)
14 {
15     int len = r - l + 1;
16     int k = log(len) / log(2);
17     return max(f[l][k], f[r - (1 << k) + 1][k
18         ]);
19 }

```

## 7 ★ Advanced Data Structures

### 7.1 Binary Indexed Tree

```
1 // 支持区间修改、区间查询
2 // 利用变差分求二阶区间和
3 const int N = 100010;
4 int n, m, a[N];
5 LL tr1[N], tr2[N];
6 int lowbit(int x) { return x & -x; }
7 void add(LL tr[], LL x, LL c)
8 {
9     for (int i = x; i <= n; i += lowbit(i))
10         tr[i] += c;
11 }
12 LL sum(LL tr[], LL x)
13 {
14     LL res = 0;
15     for (int i = x; i; i -= lowbit(i))
16         res += tr[i];
17     return res;
18 }
19 LL prefix_sum(LL x)
20 { return sum(tr1, x) * (x + 1) - sum(tr2, x); }
21 int main()
22 {
23     cin >> n >> m;
24     for (int i = 1; i <= n; i++)
25         cin >> a[i];
26     for (int i = 1; i <= n; i++)
27     {
28         int b = a[i] - a[i - 1];
29         add(tr1, i, b);
30         add(tr2, i, (LL)i * b);
31     }
32     while (m--)
33     {
34         char op[2];
35         int l, r, d;
36         cin >> op >> l >> r;
37         if (*op == 'Q')
38             cout << prefix_sum(r) - prefix_sum
39             (l - 1) << '\n';
40         else
41         {
42             cin >> d;
43             add(tr1, l, d), add(tr2, l, (LL)l
44             * d),
45             add(tr1, r + 1, -d),
46             add(tr2, r + 1, (LL)-(r + 1) * d);
47         }
48     }
49 }
```

## 7.2 Segment Tree

### 7.2.1 Maintain the Maximum

```
1 struct Node
2 { int l, r, v; } tr[N * 4];
```

```
3 void pushup(int u)
4 {
5     tr[u].v = max(tr[u << 1].v, tr[u << 1 |
6     1].v);
7 }
8 void build(int u, int l, int r)
9 {
10     tr[u] = {l, r};
11     if (l == r) return;
12     int mid = l + r >> 1;
13     build(u << 1, l, mid),
14     build(u << 1 | 1, mid + 1, r);
15 }
16 int query(int u, int l, int r)
17 {
18     if (tr[u].l >= l && tr[u].r <= r)
19         return tr[u].v;
20     int mid = tr[u].l + tr[u].r >> 1;
21     int v = 0;
22     if (l <= mid)
23         v = query(u << 1, l, r);
24     if (r > mid)
25         v = max(v, query(u << 1 | 1, l, r));
26     return v;
27 }
28 void modify(int u, int x, int v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31         tr[u].v = v;
32     else
33     {
34         int mid = tr[u].l + tr[u].r >> 1;
35         if (x <= mid)
36             modify(u << 1, x, v);
37         else
38             modify(u << 1 | 1, x, v);
39         pushup(u);
40     }
41 }
```

### 7.2.2 Maintain the Maximum Subarray Sum

```
1 struct Node
2 { int l, r, sum, lmax, rmax, tmax; } tr[N *
3     4];
4 void pushup(Node &u, Node &l, Node &r)
5 {
6     u.sum = l.sum + r.sum;
7     u.lmax = max(l.lmax, l.sum + r.lmax);
8     u.rmax = max(r.rmax, r.sum + l.rmax);
9     u.tmax = max(max(l.tmax, r.tmax), l.rmax +
10     r.lmax);
11 }
12 void pushup(int u)
13 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
14 void build(int u, int l, int r)
15 {
16     if (l == r)
17         tr[u] = {l, r, w[r], w[r], w[r], w[r]};
18     else
19     {
20         tr[u] = {l, r};
21     }
22 }
```

```

19     int mid = l + r >> 1;
20     build(u << 1, l, mid),
21     build(u << 1 | 1, mid + 1, r);
22     pushup(u);
23 }
24 }
25 void modify(int u, int x, int v)
26 {
27     if (tr[u].l == x && tr[u].r == x)
28         tr[u] = {x, x, v, v, v, v};
29     else
30     {
31         int mid = tr[u].l + tr[u].r >> 1;
32         if (x <= mid)
33             modify(u << 1, x, v);
34         else
35             modify(u << 1 | 1, x, v);
36         pushup(u);
37     }
38 }
39 Node query(int u, int l, int r)
40 {
41     if (tr[u].l >= l && tr[u].r <= r)
42         return tr[u];
43     else
44     {
45         int mid = tr[u].l + tr[u].r >> 1;
46         if (r <= mid)
47             return query(u << 1, l, r);
48         else if (l > mid)
49             return query(u << 1 | 1, l, r);
50         else
51         {
52             auto left = query(u << 1, l, r);
53             auto right = query(u << 1 | 1, l,
54                                 r);
55             Node res;
56             pushup(res, left, right);
57             return res;
58         }
59     }

```

### 7.2.3 Maintain the GCD

```

1 struct Node
2 { int l, r; LL sum, d; } tr[N * 4];
3 LL gcd(LL a, LL b)
4 { return b ? gcd(b, a % b) : a; }
5 void pushup(Node &u, Node &l, Node &r)
6 {
7     u.sum = l.sum + r.sum;
8     u.d = gcd(l.d, r.d);
9 }
10 void pushup(int u)
11 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
12 void build(int u, int l, int r)
13 {
14     if (l == r)
15     {
16         LL b = w[r] - w[r - 1];
17         tr[u] = {l, r, b, b};
18     }
19     else

```

```

20     {
21         tr[u].l = l, tr[u].r = r;
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
28 void modify(int u, int x, LL v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31     {
32         LL b = tr[u].sum + v;
33         tr[u] = {x, x, b, b};
34     }
35     else
36     {
37         int mid = tr[u].l + tr[u].r >> 1;
38         if (x <= mid)
39             modify(u << 1, x, v);
40         else
41             modify(u << 1 | 1, x, v);
42         pushup(u);
43     }
44 }
45 Node query(int u, int l, int r)
46 {
47     if (tr[u].l >= l && tr[u].r <= r)
48         return tr[u];
49     else
50     {
51         int mid = tr[u].l + tr[u].r >> 1;
52         if (r <= mid)
53             return query(u << 1, l, r);
54         else if (l > mid)
55             return query(u << 1 | 1, l, r);
56         else
57         {
58             auto left = query(u << 1, l, r);
59             auto right = query(u << 1 | 1, l,
60                                 r);
61             Node res;
62             pushup(res, left, right);
63             return res;
64         }
65     }

```

### 7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```

1 struct Node
2 { int l, r; LL sum, add; } tr[N * 4];
3 void pushup(int u)
4 { tr[u].sum = tr[u << 1].sum + tr[u << 1 | 1].
5   sum; }
6 void pushdown(int u)
7 {
8     auto &root = tr[u],
9     &left = tr[u << 1],
10    &right = tr[u << 1 | 1];
11    if (root.add)

```

```

11 {
12     left.add += root.add,
13     left.sum += (LL)(left.r - left.l + 1)
14     * root.add;
15     right.add += root.add,
16     right.sum += (LL)(right.r - right.l +
17     1) * root.add;
18     root.add = 0;
19 }
20 void build(int u, int l, int r)
21 {
22     if (l == r) tr[u] = {l, r, w[r], 0};
23     else
24     {
25         tr[u] = {l, r};
26         int mid = l + r >> 1;
27         build(u << 1, l, mid);
28         build(u << 1 | 1, mid + 1, r);
29         pushup(u);
30     }
31 }
32 void modify(int u, int l, int r, int d)
33 {
34     if (tr[u].l >= l && tr[u].r <= r)
35     {
36         tr[u].sum += (LL)(tr[u].r - tr[u].l +
37         1) * d;
38         tr[u].add += d;
39     }
40     else
41     {
42         pushdown(u);
43         int mid = tr[u].l + tr[u].r >> 1;
44         if (l <= mid)
45             modify(u << 1, l, r, d);
46         if (r > mid)
47             modify(u << 1 | 1, l, r, d);
48         pushup(u);
49     }
50 }
51 LL query(int u, int l, int r)
52 {
53     if (tr[u].l >= l && tr[u].r <= r)
54         return tr[u].sum;
55     pushdown(u);
56     int mid = tr[u].l + tr[u].r >> 1;
57     LL sum = 0;
58     if (l <= mid)
59         sum += query(u << 1, l, r);
60     if (r > mid)
61         sum += query(u << 1 | 1, l, r);
62     return sum;
63 }

```

## 7.3 Persistent Data Structure

### 7.3.1 Persistent Trie

```

1 const int N = 600010, M = N * 25;
2 int n, m, s[N], root[N], idx;
3 int trie[M][2], max_id[M];
4 void insert(int i, int k, int p, int q)

```

```

5 {
6     if (k < 0)
7     {
8         max_id[q] = i;
9         return;
10    }
11    int v = s[i] >> k & 1;
12    if (p)
13        trie[q][v ^ 1] = trie[p][v ^ 1];
14    trie[q][v] = ++idx;
15    insert(i, k - 1, trie[p][v], trie[q][v]);
16    max_id[q] = max(max_id[trie[q][0]], max_id[trie[q][1]]);
17 }
18 int query(int root, int C, int L)
19 {
20     int p = root;
21     for (int i = 23; i >= 0; i--)
22     {
23         int v = C >> i & 1;
24         if (max_id[trie[p][v ^ 1]] >= L)
25             p = trie[p][v ^ 1];
26         else
27             p = trie[p][v];
28     }
29     return C ^ s[max_id[p]];
30 }
31 // insert(i, 23, root[i - 1], root[i]);
32 // query(root[r - 1], l - 1, x ^ s[n]);

```

### 7.3.2 Persistent Segment Tree

```

1 const int N = 100010, M = 10010;
2 int n, m, a[N], root[N], idx;
3 vector<int> nums;
4 struct Node
5 {
6     int l, r;
7     int cnt;
8 } tr[N * 4 + N * 17];
9 int find(int x)
10 {
11     return lower_bound(nums.begin(), nums.end(), x) - nums.begin();
12 }
13 int build(int l, int r)
14 {
15     int p = ++idx;
16     if (l == r)
17         return p;
18     int mid = l + r >> 1;
19     tr[p].l = build(l, mid), tr[p].r = build(mid + 1, r);
20     return p;
21 }
22 int insert(int p, int l, int r, int x)
23 {
24     int q = ++idx;
25     tr[q] = tr[p];
26     if (l == r)
27     {
28         tr[q].cnt++;
29         return q;
30     }

```

```

31     int mid = l + r >> 1;
32     if (x <= mid)
33         tr[q].l = insert(tr[p].l, l, mid, x);
34     else
35         tr[q].r = insert(tr[p].r, mid + 1, r,
36             x);
37     tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].
38         cnt;
39     return q;
40 }
41 int query(int q, int p, int l, int r, int k)
42 {
43     if (l == r)
44         return r;
45     int cnt = tr[tr[q].l].cnt - tr[tr[p].l].
46         cnt;
47     int mid = l + r >> 1;
48     if (k <= cnt)
49         return query(tr[q].l, tr[p].l, l, mid,
50             k);
51     else
52         return query(tr[q].r, tr[p].r, mid +
53             1, r, k - cnt);
54 }

```

## 7.4 Treap

```

1  const int N = 100010, INF = 1e8;
2  int n, root, idx;
3  struct Node
4  {
5      int l, r;
6      int key, val;
7      int cnt, size;
8  } tr[N];
9  void pushup(int p)
10 {
11     tr[p].size = tr[tr[p].l].size + tr[tr[p].r
12         ].size + tr[p].cnt;
13 }
14 int get_node(int key)
15 {
16     tr[++idx].key = key;
17     tr[idx].val = rand();
18     tr[idx].cnt = tr[idx].size = 1;
19     return idx;
20 }
21 void zig(int &p)
22 {
23     int q = tr[p].l;
24     tr[p].l = tr[q].r, tr[q].r = p, p = q;
25     pushup(tr[p].r), pushup(p);
26 }
27 void zag(int &p)
28 {
29     int q = tr[p].r;
30     tr[p].r = tr[q].l, tr[q].l = p, p = q;
31     pushup(tr[p].l), pushup(p);
32 }
33 void build()
34 {
35     get_node(-INF), get_node(INF);
36     root = 1, tr[1].r = 2;

```

```

36     pushup(root);
37     if (tr[1].val < tr[2].val)
38         zag(root);
39 }
40 void insert(int &p, int key)
41 {
42     if (!p) p = get_node(key);
43     else if (tr[p].key == key)
44         tr[p].cnt++;
45     else if (tr[p].key > key)
46     {
47         insert(tr[p].l, key);
48         if (tr[tr[p].l].val > tr[p].val)
49             zig(p);
50     }
51     else
52     {
53         insert(tr[p].r, key);
54         if (tr[tr[p].r].val > tr[p].val)
55             zag(p);
56     }
57     pushup(p);
58 }
59 void remove(int &p, int key)
60 {
61     if (!p) return;
62     if (tr[p].key == key)
63     {
64         if (tr[p].cnt > 1)
65             tr[p].cnt--;
66         else if (tr[p].l || tr[p].r)
67         {
68             if (!tr[p].r || tr[tr[p].l].val >
69                 tr[tr[p].r].val)
70             {
71                 zig(p);
72                 remove(tr[p].r, key);
73             }
74             else
75             {
76                 zag(p);
77                 remove(tr[p].l, key);
78             }
79         }
80         p = 0;
81     }
82     else if (tr[p].key > key)
83         remove(tr[p].l, key);
84     else
85         remove(tr[p].r, key);
86     pushup(p);
87 }
88 int get_rank_by_key(int p, int key)
89 {
90     if (!p) return 0;
91     if (tr[p].key == key)
92         return tr[tr[p].l].size + 1;
93     if (tr[p].key > key)
94         return get_rank_by_key(tr[p].l, key);
95     return tr[tr[p].l].size + tr[p].cnt +
96         get_rank_by_key(tr[p].r, key);
97 }
98 int get_key_by_rank(int p, int rank)
99 {
100     if (!p) return INF;

```



```

100     if (tr[tr[p].l].size >= rank)
101         reutrn get_key_by_rank(tr[p].l, rank);
102     if (tr[tr[p].l].size + tr[p].cnt >= rank)
103         reutrn tr[p].key;
104     return get_key_by_rank(tr[p].r, rank - tr[
tr[p].l].size - tr[p].cnt);
105 }
106 int get_prev(int p, int key)
107 {
108     if (!p) return -INF;
109     if (tr[p].key >= key)
110         reutrn get_prev(tr[p].l, key);
111     return max(tr[p].key, get_prev(tr[p].r,
key));
112 }
113 int get_next(int p, int key)
114 {
115     if (!p) reutrn INF;
116     if (tr[p].key <= key)
117         return get_next(tr[p].r, key);
118     return min(tr[p].key, get_next(tr[p].l,
key));
119 }

```

## 8 ★ Advanced Search

## 9 ★ Advanced Graph Theory



## 11 ★ Advanced DP