



**国家超级计算广州中心**  
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

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# XCPC-Template

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# Contents

<b>0</b>	<b>Preface</b>	<b>5</b>	<b>4</b>	<b>Basic Math</b>	<b>17</b>
0.1	Template	5	4.1	Prime Numbers	17
0.2	Operator Precedence	5	4.1.1	Judging Prime Numbers	17
0.3	Time Complexity	5	4.1.2	Prime Factorization	17
0.4	If <code>&lt;bits/stdc++.h&gt;</code> Failed	6	4.1.3	Euler's Sieve	17
<b>1</b>	<b>Basic Algorithm</b>	<b>7</b>	4.2	Divisor	17
1.1	Quick Sort	7	4.2.1	Find All Divisors	17
1.2	Binary Search	7	4.2.2	The Number of Divisors	17
1.3	Ternary Search	7	4.2.3	The Sum of Divisors	17
1.4	High Precision	7	4.2.4	Euclidean Algorithm	18
1.4.1	High Precision Add	7	4.3	Euler Function	18
1.4.2	High Precision Subsection	8	4.3.1	Simple Method	18
1.4.3	High Precision Multiply	8	4.3.2	Euler's Sieve Method	18
1.4.4	High Precision Divide	8	4.4	Exponentiating by Squaring	18
1.5	Prefix Sum & Difference Array	8	4.5	Extended Euclidean Algorithm	18
1.5.1	1D Prefix Sum	8	4.6	Chinese Remainder Theorem	18
1.5.2	2D Prefix Sum	9	4.7	Gauss-Jordan Elimination	19
1.5.3	1D Difference Array	9	4.7.1	Linear Equation Group	19
1.5.4	2D Difference Array	9	4.7.2	XOR Linear Equation Group	19
<b>2</b>	<b>Basic Data Structures</b>	<b>10</b>	4.8	Combinatorial Counting	19
2.1	Linked List	10	4.8.1	Recurrence Relation	19
2.1.1	Singly Linked List	10	4.8.2	Preprocessing & Inverse Element	19
2.1.2	Bidirectional Linked List	10	4.8.3	Lucas Theorem	20
2.2	Stack & Queue	10	4.8.4	Factorization Method	20
2.2.1	Monotonic Stack	10	4.8.5	Catalan Number	20
2.2.2	Monotonic Queue	10	4.9	Inclusion-Exclusion Principle	20
2.3	KMP	10	4.10	Game Theory	21
2.4	Trie	10	4.10.1	NIM Game	21
2.5	Disjoint-Set	11	<b>5</b>	<b>Basic DP</b>	<b>22</b>
2.6	Hash	11	5.1	Knapsack Problem	22
2.6.1	Simple Hash	11	5.1.1	01 Knapsack	22
2.6.2	String Hash	11	5.1.2	Complete Knapsack	22
2.7	STL	11	5.1.3	Mutiple Knapsack	22
<b>3</b>	<b>Search &amp; Graph Theory</b>	<b>13</b>	5.1.4	Grouped Knapsack	22
3.1	Representation of Tree & Graph	13	5.2	Linear DP	22
3.1.1	Adjacency Matrix	13	5.2.1	LIS	22
3.1.2	Adjacency List	13	5.2.2	LCS	23
3.2	DFS & BFS	13	5.3	Interval DP	23
3.2.1	DFS	13	5.4	Counting DP	23
3.2.2	BFS	13	5.5	Digit DP	23
3.3	Topological Sort	13	5.6	State Compression DP	24
3.4	Shortest Path	13	5.7	Tree DP	24
3.4.1	Dijkstra	13	5.8	Memoized Search	25
3.4.2	Bellman-Ford	13	<b>6</b>	<b>Advanced Basic</b>	<b>27</b>
3.4.3	SPFA	14	6.1	Slow Multiplication	27
3.4.4	Detecting Negative Circle in SPFA	14	6.2	Sum of Geometric Series	27
3.4.5	Floyd	14	6.3	Sort	27
3.5	Minimum Spanning Tree	14	6.3.1	Card Balancing Problem	27
3.5.1	Prim	14	6.3.2	2D Card Balancing Problem	27
3.5.2	Kruskal	15	6.3.3	Dual Heaps	27
3.6	Bipartite Graph	15	6.4	RMQ	28
3.6.1	Coloring Method	15			
3.6.2	Hungarian Algorithm	16			

<b>7</b>	<b>Advanced Data Structures</b>	<b>29</b>	<b>11</b>	<b>Advanced DP</b>	<b>47</b>
7.1	Binary Indexed Tree . . . . .	29	11.1	Advanced Linear DP . . . . .	47
7.2	Segment Tree . . . . .	29	11.1.1	Two-pass grid collection problem	47
7.2.1	Maintain the Maximum . . . . .	29	11.2	Advanced LIS . . . . .	47
7.2.2	Maintain the Maximum Subarray Sum . . . . .	29	11.2.1	Longest Bitonic Subsequence . .	47
7.2.3	Maintain the GCD . . . . .	30	11.2.2	MSIS . . . . .	47
7.2.4	Optimize Range Updates . . . . .	30	11.2.3	LCIS . . . . .	47
7.3	Persistent Data Structure . . . . .	31	11.3	Knapsack Problem . . . . .	47
7.3.1	Persistent Trie . . . . .	31	11.3.1	How To Initialize . . . . .	47
7.3.2	Persistent Segment Tree . . . . .	31	11.3.2	Multiple Knapsack Problem . . .	48
7.4	Treap . . . . .	32	11.3.3	Two-Dimensional Cost Knapsack Problem . . . . .	48
7.5	AC Automaton . . . . .	33	11.3.4	Finding the Actual Solution Set .	48
<b>8</b>	<b>Advanced Search</b>	<b>34</b>	11.3.5	Maximum Linearly Independent Subset . . . . .	49
8.1	Flood-Fill . . . . .	34	11.3.6	Mixed Knapsack Problem . . . .	49
8.2	Multi-source BFS . . . . .	34	11.3.7	Dependent Knapsack Problem .	49
8.3	BFS with Deque . . . . .	34	11.3.8	Number of Solutions . . . . .	49
8.4	Bidirectional BFS . . . . .	35	11.4	FSM . . . . .	50
8.5	A* . . . . .	35	11.4.1	Common FSM . . . . .	50
8.6	DFS Connectivity Model . . . . .	35	11.4.2	Linear DP + KMP . . . . .	50
8.7	IDDFS . . . . .	35	11.4.3	Linear DP + AC Automaton . .	50
8.8	Bidirectional DFS . . . . .	36	11.5	Digit DP . . . . .	51
8.9	IDA* . . . . .	36	11.6	Queue Optimization for DP . . . .	51
<b>9</b>	<b>Advanced Graph Theory</b>	<b>37</b>			
9.1	Detecting Negative Cycles . . . . .	37			
9.2	SPFA-SLF . . . . .	37			
9.3	SPFA-Stack . . . . .	37			
9.4	SPFA & MIN & MAX . . . . .	37			
9.5	Second Shortest Path . . . . .	38			
9.6	Second Minimum Spanning Tree . . . .	38			
9.6.1	brute-force . . . . .	38			
9.6.2	LCA . . . . .	39			
9.7	Difference Constraints . . . . .	40			
9.7.1	Maximum-Shortest Path . . . . .	40			
9.7.2	Minimum-Longest Path . . . . .	41			
9.8	LCA . . . . .	41			
9.9	SCC . . . . .	41			
9.10	DCC . . . . .	42			
9.10.1	e-DCC . . . . .	42			
9.10.2	v-DCC . . . . .	42			
9.11	Bipartite Graph . . . . .	42			
9.11.1	maximum matching . . . . .	42			
9.11.2	minimum vertex cover . . . . .	43			
9.11.3	maximum independent set . . . . .	43			
9.11.4	minimum path cover . . . . .	43			
9.12	Eulerian Circuit & Eulerian Path . . . .	44			
9.12.1	Eulerian Circuit . . . . .	44			
9.12.2	Eulerian Path . . . . .	44			
<b>10</b>	<b>Advanced Math</b>	<b>46</b>			
10.1	Euler's Totient Function . . . . .	46			
10.1.1	GCD . . . . .	46			
10.2	Matrix Multiplication . . . . .	46			



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# Part I: Basic Template

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## 0 ★ Preface

### 0.1 Template

```
1 #define itn int
2 #define nit int
3 #define nti int
4 #define tin int
5 #define tni int
6 #define retrun return
7 #define reutrn return
8 #define rutren return
9 #define fastin \
10 ios_base::sync_with_stdio(0); \
11 cin.tie(0), cout.tie(0);
12 #include <bits/stdc++.h>
13 using namespace std;
14 typedef long long LL;
15 typedef long double LD;
16 typedef pair<int, int> PII;
17 typedef pair<long long, long long> PLL;
18 typedef pair<double, double> PDD;
19 typedef vector<int> VI;
20 #ifndef ONLINE_JUDGE
21 #define dbg(args...) \
22 do \
23 { \
24     cout << "\033[32;1m" << #args << " -> \
25 "; \
26     err(args); \
27 } while (0)
28 #define dbg(...)
29 #endif
30 void err()
31 { cout << "\033[39;0m" << endl; }
32 template <template <typename...> class T,
33         typename t, typename... Args>
34 void err(T<t> a, Args... args)
35 {
36     for (auto x : a) cout << x << ' ';
37     err(args...);
38 }
39 template <typename T, typename... Args>
40 void err(T a, Args... args)
41 { cout << a << ' '; err(args...); }
42 const int INF = 0x3f3f3f3f;
43 const int mod = 1e9 + 7;
44 const double eps = 1e-6;
45 int main()
46 {
47     #ifndef ONLINE_JUDGE
48         freopen("test.in", "r", stdin);
49         freopen("test.out", "w", stdout);
50     #endif
51     fastin;
52     return 0;
53 }
```

### 0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

### 0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about  $10^7 \sim 10^8$  operations.
- Below is a guide on how to choose algorithms based on different input size ranges:
  1.  $n \leq 30 \rightarrow$  Exponential complexity: DFS with pruning, State Compression DP
  2.  $n \leq 100 \rightarrow O(n^3)$ : Floyd, DP, Gaussian Elimination
  3.  $n \leq 1000 \rightarrow O(n^2), O(n^2 \log n)$ : DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
  4.  $n \leq 10000 \rightarrow O(n^3)$ : Block Linked List, Mo's Algorithm
  5.  $n \leq 100000 \rightarrow O(n \log n)$ : sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
  6.  $n \leq 1000000 \rightarrow O(n)$ , or small-constant  $O(n \log n)$ : Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
  7.  $n \leq 10000000 \rightarrow O(n)$ : Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
  8.  $n \leq 10^9 \rightarrow O(\sqrt{n})$ : Primality Testing
  9.  $n \leq 10^{18} \rightarrow O(\log n)$ : GCD, Fast Exponentiation, Digit DP
  10.  $n \leq 10^{1000} \rightarrow O((\log n)^2)$ : Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
  11.  $n \leq 10^{100000} \rightarrow O(\log k \cdot \log \log k)$ , where  $k$  is the number of digits: Big Integer Add/Subtract, FFT/NTT

## 0.4 If <bits/stdc++.h> Failed

Replace it with:

```
1  #include <algorithm>
2  #include <bitset>
3  #include <complex>
4  #include <deque>
5  #include <exception>
6  #include <fstream>
7  #include <functional>
8  #include <iomanip>
9  #include <ios>
10 #include <iosfwd>
11 #include <iostream>
12 #include <istream>
13 #include <iterator>
14 #include <limits>
15 #include <list>
16 #include <locale>
17 #include <map>
18 #include <memory>
19 #include <numeric>
20 #include <ostream>
21 #include <queue>
22 #include <set>
23 #include <sstream>
24 #include <stack>
25 #include <stdexcept>
26 #include <streambuf>
27 #include <string>
28 #include <typeinfo>
29 #include <utility>
30 #include <valarray>
31 #include <vector>
32 #include <unordered_map>
33 #include <unordered_set>
```

# 1 ★ Basic Algorithm

## 1.1 Quick Sort

Sort the given array from index 1 to n.

```
1 void quick_sort(int l, int r)
2 {
3     if (l >= r) return;
4     int x = a[(l + r) >> 1], i = l - 1, j = r
        + 1;
5     while (i < j)
6     {
7         do i++; while (a[i] < x);
8         do j--; while (a[j] > x);
9         if (i < j) swap(a[i], a[j]);
10    }
11    quick_sort(l, j);
12    quick_sort(j + 1, r);
13    return;
14 }
```

## 1.2 Binary Search

```
1 // 区间 [l, r] 被划分成 [l, mid] 和 [mid + 1,
    r] 时使用
2 // 大于等于区间的最小值, check 应为 target <=
    a[mid]
3 int bsearch_1(int l, int r)
4 {
5     while (l < r)
6     {
7         int mid = l + r >> 1;
8         if (check(mid)) r = mid;
9         else l = mid + 1;
10    }
11    return l;
12 }
13 // 区间 [l, r] 被划分成 [l, mid - 1] 和 [mid,
    r] 时使用
14 // 小于等于区间的最大值, check 应为 target >=
    a[mid]
15 int bsearch_2(int l, int r)
16 {
17     while (l < r)
18     {
19         // 为什么要 l + r + 1: 因为 l 的更新条
            件是 mid 本身
20         // 当 r == l + 1 时 mid 向下取整必定取
            l, 有可能在满足 check(mid) 时导致无限循环
21         int mid = l + r + 1 >> 1;
22         if (check(mid)) l = mid;
23         else r = mid - 1;
24    }
25    return l;
26 }
27 // 浮点数二分
28 double bsearch_3(double l, double r)
29 {
30     // eps 表示精度, 取决于题目对精度的要求
31     const double eps = 1e-6;
32     while (r - l > eps)
```

```
33 {
34     double mid = (l + r) / 2;
35     if (check(mid)) r = mid;
36     else l = mid;
37 }
38 return l;
39 }
```

## 1.3 Ternary Search

```
1 // 整数三分
2 void tsearch_1(int l, int r)
3 {
4     while (l < r)
5     {
6         int lmid = l + (r - l) / 3, rmid = r -
            (r - l) / 3;
7         lans = cal(lmid), rans = cal(rmid);
8         if (lans <= rans) r = rmid - 1;
9         else l = lmid + 1;
10        if (lans <= rans) l = lmid + 1;
11        else r = rmid - 1;
12    }
13    // 求凹函数的极小值
14    cout << min(lans, rans) << endl;
15    // 求凸函数的极大值
16    cout << max(lans, rans) << endl;
17 }
18 // 浮点数三分
19 void tsearch_2(int l, int r)
20 {
21     const double eps = 1e-6;
22     while (r - l < eps)
23     {
24         double lmid = l + (r - l) / 3;
25         double rmid = r - (r - l) / 3;
26         lans = cal(lmid), rans = cal(rmid);
27         // 求凹函数的极小值
28         if (lans <= rans) r = rmid;
29         else l = lmid;
30         // 求凸函数的极大值
31         if (lans <= rans) l = lmid;
32         else r = rmid;
33    }
34 }
```

## 1.4 High Precision

### 1.4.1 High Precision Add

```
1 string s1, s2;
2 vector<int> a, b, c;
3 void add(vector<int> &a, vector<int> &b)
4 {
5     if (a.size() < b.size())
6     { add(b, a); return; }
7     int t = 0;
8     for (int i = 0; i < a.size(); i++)
9     {
10        t += a[i];
```

```

11     if (i < b.size()) t += b[i];
12     c.push_back(t % 10);
13     t /= 10;
14 }
15 while (t)
16     c.push_back(t % 10), t /= 10;
17 }
18 int main()
19 {
20     cin >> s1 >> s2;
21     for (int i = s1.size() - 1; i >= 0; i--)
22         a.push_back(s1[i] - '0');
23     for (int i = s2.size() - 1; i >= 0; i--)
24         b.push_back(s2[i] - '0');
25     add(a, b);
26     for (int i = c.size() - 1; i >= 0; i--)
27         cout << c[i];
28     return 0;
29 }

```

```

5 {
6     for (int i = 0, t = 0; i < a.size() || t; i++)
7     {
8         if (i < a.size()) t += a[i] * b;
9         c.push_back(t % 10);
10        t /= 10;
11    }
12    while (c.size() > 1 && c.back() == 0)
13        c.pop_back();
14 }
15 int main()
16 {
17     cin >> s1 >> b;
18     for (int i = s1.size() - 1; i >= 0; i--)
19         a.push_back(s1[i] - '0');
20     mul(a, b);
21     for (int i = c.size() - 1; i >= 0; i--)
22         cout << c[i];
23     return 0;
24 }

```

## 1.4.2 High Precision Subsection

```

1 vector<int> a, b, c;
2 string s1, s2;
3 void sub(vector<int> &a, vector<int> &b)
4 {
5     int t = 0;
6     for (int i = 0; i < a.size(); i++)
7     {
8         t = a[i] - t;
9         if (i < b.size()) t -= b[i];
10        c.push_back((t + 10) % 10);
11        if (t < 0) t = 1;
12        else t = 0;
13    }
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> s2;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     for (int i = s2.size() - 1; i >= 0; i--)
23         b.push_back(s2[i] - '0');
24     if (s1.size() < s2.size())
25         cout << '-', sub(b, a);
26     else if (s1.size() == s2.size() && s1 < s2)
27         cout << '-', sub(b, a);
28     else sub(a, b);
29     for (int i = c.size() - 1; i >= 0; i--)
30         cout << c[i];
31     return 0;
32 }

```

## 1.4.4 High Precision Divide

```

1 string s1, s2;
2 vector<int> a, c;
3 int b, r;
4 void divide(vector<int> &a, int b, int &r)
5 {
6     r = 0;
7     for (int i = a.size() - 1; i >= 0; i--)
8     {
9         r = r * 10 + a[i];
10        c.push_back(r / b);
11        r %= b;
12    }
13    reverse(c.begin(), c.end());
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> b;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     divide(a, b, r);
23     for (int i = c.size() - 1; i >= 0; i--)
24         cout << c[i];
25     cout << '\n' << r;
26     return 0;
27 }

```

## 1.4.3 High Precision Multiply

```

1 string s1, s2;
2 vector<int> a, c;
3 int b;
4 void mul(vector<int> &a, int b)

```

## 1.5 Prefix Sum & Difference Array

### 1.5.1 1D Prefix Sum

```

1 S[i] = a[1] + a[2] + ... a[i]
2 a[l] + ... + a[r] = S[r] - S[l - 1]

```



### 1.5.2 2D Prefix Sum

```
1 // S[i, j] = i 行 j 列左上部分所有元素和为:
2 s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1]
  + a[i][j]
3 // 以 (x1, y1) 为左上角, (x2, y2) 为右下角的
  子矩阵的和为:
4 S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1] +
  S[x1 - 1][y1 - 1]
```

```
23         insert(x1, y1, x2, y2, c);
24     }
25     // 其他过程略
26 }
```

### 1.5.3 1D Difference Array

```
1 const int N = 100010;
2 int n, m;
3 int a[N], b[N];
4 void insert(int l, int r, int c)
5 { b[l] += c; b[r + 1] -= c; }
6 int main()
7 {
8     cin >> n >> m;
9     for (int i = 1; i <= n; i++)
10         cin >> a[i];
11     for (int i = 1; i <= n; i++)
12         insert(i, i, a[i]);
13     while (m--)
14     {
15         int l, r, c;
16         cin >> l >> r >> c;
17         insert(l, r, c);
18     }
19     for (int i = 1; i <= n; i++)
20         b[i] += b[i - 1],
21         cout << b[i] << ' ';
22     return 0;
23 }
```

### 1.5.4 2D Difference Array

```
1 const int N = 1010;
2 int n, m, q, a[N][N], b[N][N];
3 void insert(int x1, int y1, int x2, int y2,
4             int c)
5 {
6     b[x1][y1] += c;
7     b[x2 + 1][y2 + 1] += c;
8     b[x1][y2 + 1] -= c;
9     b[x2 + 1][y1] -= c;
10 }
11 int main()
12 {
13     cin >> n >> m >> q;
14     for (int i = 1; i <= n; i++)
15         for (int j = 1; j <= m; j++)
16             cin >> a[i][j];
17     for (int i = 1; i <= n; i++)
18         for (int j = 1; j <= m; j++)
19             insert(i, j, i, j, a[i][j]);
20     while (q--)
21     {
22         int x1, x2, y1, y2, c;
23         cin >> x1 >> y1 >> x2 >> y2 >> c;
```

## 2 ★ Basic Data Structures

### 2.1 Linked List

#### 2.1.1 Singly Linked List

```
1  const int N = 100010;
2  int n, h[N], e[N], ne[N], idx = 1;
3  void init() { ne[0] = -1; }
4  void insert(int k, int x) // 第 k 个节点后
    插入
5  { e[idx] = x, ne[idx] = ne[k], ne[k] = idx
    ++; }
6  void del(int k) // 第 k 个节点后删除
7  { ne[k] = ne[ne[k]]; }
```

#### 2.1.2 Bidirectional Linked List

```
1  const int N = 100010;
2  int n, r[N], l[N], e[N], idx = 2;
3  void init() { r[0] = 1; l[1] = 0; }
4  void insert(int k, int x) // 第 k 个节点后插
    入
5  {
6      e[idx] = x;
7      r[idx] = r[k];
8      l[idx] = k;
9      l[r[k]] = idx;
10     r[k] = idx++;
11 }
12 void remove(int k) // 删除 k 本身
13 { r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

### 2.2 Stack & Queue

#### 2.2.1 Monotonic Stack

```
1  // 常见模型：找出每个数左边离它最近的比它大/小
    的数
2  int tt = 0;
3  for (int i = 1; i <= n; i++)
4  {
5      while (tt && check(stk[tt], i)) tt--;
6      stk[++tt] = i;
7  }
```

#### 2.2.2 Monotonic Queue

```
1  // 常见模型：找出滑动窗口中的最大值/最小值
2  int hh = 0, tt = -1;
3  for (int i = 0; i < n; i++)
4  {
5      while (hh <= tt && check_out(q[hh]))
6          hh++; // 判断队头是否滑出窗口
7      while (hh <= tt && check(q[tt], i))
8          tt--;
```

```
9      q[++tt] = i;
10 }
```

### 2.3 KMP

```
1  const int N = 100010, M = 1000010;
2  int n, m;
3  char p[N], s[M];
4  void getNext(int ne[])
5  {
6      for (int i = 2, j = 0; i <= n; i++)
7      {
8          while (j && p[j + 1] != p[i])
9              j = ne[j];
10         if (p[j + 1] == p[i]) j++;
11         ne[i] = j;
12     }
13 }
14 int KMP()
15 {
16     int *ne = new int[n + 1];
17     getNext(ne);
18     for (int i = 1, j = 0; i <= m; i++)
19     {
20         while (j && p[j + 1] != s[i])
21             j = ne[j];
22         if (p[j + 1] == s[i]) j++;
23         if (j == n) cout << i - n << ' ';
24     }
25     return -1;
26 }
```

### 2.4 Trie

```
1  const int N = 100010;
2  int trie[N][26], cnt[N], idx = 0;
3  void insert(string &str) // 插入到 Trie
    数组
4  {
5      int p = 0;
6      for (auto c : str)
7      {
8          int u = c - 'a';
9          if (!trie[p][u])
10             trie[p][u] = ++idx;
11             p = trie[p][u];
12     }
13     cnt[p]++;
14 }
15 int query(string &str) // 查询字符串出现
    的次数
16 {
17     int p = 0;
18     for (auto c : str)
19     {
20         int u = c - 'a';
21         if (!trie[p][u]) return 0;
22         p = trie[p][u];
23     }
24     return cnt[p];
```

## 2.5 Disjoint-Set

```

1  const int N = 100010;
2  int n, m, p[N], Size[N], D[N];
3  void init()
4  {
5      for (int i = 1; i <= n; i++)
6          p[i] = i, Size[i] = 1, D[i] = 0;
7  }
8  int find(int x)
9  {
10     if (p[x] != x)
11     {
12         int u = find(p[x]);
13         D[x] += D[p[x]]; // 视具体情况计算
14         p[x] = u;
15     }
16     return p[x];
17 }
18 void merge(int a, int b, int distance)
19 {
20     int x = find(a), y = find(b);
21     if (x != y)
22     {
23         p[x] = y;
24         D[x] = distance; // 视具体情况计算
25         Size[y] += Size[x];
26     }
27 }

```

## 2.6 Hash

### 2.6.1 Simple Hash

```

1  // (1) 拉链法
2  int h[N], e[N], ne[N], idx;
3  void insert(int x)
4  {
5      int k = (x % N + N) % N;
6      e[idx] = x, ne[idx] = h[k], h[k] = idx++;
7  }
8  bool find(int x)
9  {
10     for (int i = h[(x % N + N) % N]; i != -1; i = ne[i])
11         if (e[i] == x) return true;
12     return false;
13 }
14 // (2) 开放寻址法
15 int find(int x)
16 {
17     int t = (x % N + N) % N;
18     while (h[t] != null && h[t] != x)
19         t++; if (t == N) t = 0;
20     return t;
21 }

```

### 2.6.2 String Hash

```

1  typedef unsigned long long ULL;
2  ULL h[N], p[N];
3  void init()
4  {
5      p[0] = 1;
6      for (int i = 1; i <= n; i++) { h[i] = h[i - 1] * P + str[i]; p[i] = p[i - 1] * P; }
7  }
8  ULL get(int l, int r) { return h[r] - h[l - 1] * p[r - l + 1]; }

```

## 2.7 STL

```

1  // vector
2  size()      返回元素个数
3  empty()     返回是否为空
4  clear()     清空
5  front()/back()
6  push_back()/pop_back()
7  begin()/end()
8  []
9  支持比较运算, 按字典序
10 // pair<int, int>
11 first       第一个元素
12 second      第二个元素
13 支持比较运算, 以first为第一关键字, 以second为第二关键字 (字典序)
14 // string
15 size()/length() 返回字符串长度
16 empty()
17 clear()
18 substr(起始下标, (子串长度)) 返回子串
19 c_str() 返回字符串所在字符数组的起始地址
20 // queue
21 size()
22 empty()
23 push()      向队尾插入一个元素
24 front()     返回队头元素
25 back()      返回队尾元素
26 pop()       弹出队头元素
27 // priority_queue
28 size()
29 empty()
30 push()      插入一个元素
31 top()       返回堆顶元素
32 pop()       弹出堆顶元素
33 定义成小根堆的方式: priority_queue<int, vector<int>, greater<int>> q;
34 // stack
35 size()
36 empty()
37 push()      向栈顶插入一个元素
38 top()       返回栈顶元素
39 pop()       弹出栈顶元素
40 // deque
41 size()
42 empty()
43 clear()
44 front()/back()
45 push_back()/pop_back()

```

```

46 push_front()/pop_front()
47 begin()/end()
48 []
49 // set, map, multiset, multimap: 基于平衡二叉
    树 (红黑树) 动态维护有序序列
50 size()
51 empty()
52 clear()
53 begin()/end()
54 ++, -- 返回前驱和后继, 时间复杂度  $O(\log n)$ 
55 // set/multiset
56 insert() 插入一个数
57 find() 查找一个数
58 count() 返回某一个数的个数
59 erase()
60 (1) 输入是一个数x, 删除所有x,  $O(k + \log n)$ 
    (2) 输入一个迭代器, 删除这个迭代器
61 lower_bound()/upper_bound()
62 lower_bound(x) 返回大于等于x的最小的数的
    迭代器
63 upper_bound(x) 返回大于x的最小的数的迭
    代器
64 // map/multimap
65 insert() 插入的数是一个pair
66 erase() 输入的参数是pair或者迭代器
67 find()
68 [] 注意multimap不支持此操作。 时间复
    杂度是  $O(\log n)$ 
69 lower_bound()/upper_bound()
70 // unordered_set, unordered_map,
    unordered_multiset, unordered_multimap
71 增删改查的时间复杂度是  $O(1)$ 
72 不支持 lower_bound()/upper_bound(), 迭代器的
    ++, --
73 // bitset
74 bitset<10000> s;
75 ~, &, |, ^
76 >>, <<
77 ==, !=
78 []
79 count() 返回有多少个1
80 any() 判断是否至少有一个1
81 none() 判断是否全为0
82 set() 把所有位置成1
83 set(k, v) 将第k位变成v
84 reset() 把所有位变成0
85 flip() 等价于~
86 flip(k) 把第k位取反

```

## 3 ★ Search & Graph Theory

### 3.1 Representation of Tree & Graph

#### 3.1.1 Adjacency Matrix

```
1 // g[a][b] = a->b
```

#### 3.1.2 Adjacency List

```
1 int h[N], e[N], ne[N], idx;
2 void init() { memset(h, -1, sizeof h); }
3 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++; }
```

## 3.2 DFS & BFS

#### 3.2.1 DFS

```
1 int dfs(int u)
2 {
3     st[u] = true; // 表示点 u 已经被遍历过
4     for (int i = h[u]; i != -1; i = ne[i])
5     { int j = e[i]; if (!st[j]) dfs(j); }
6 }
```

#### 3.2.2 BFS

```
1 queue<int> q;
2 st[1] = true; q.push(1);
3 while (q.size())
4 {
5     int t = q.front(); q.pop();
6     for (int i = h[t]; i != -1; i = ne[i])
7     { if (!st[e[i]]) { st[e[i]] = true; q.
            push(e[i]); }
8 }
```

## 3.3 Topological Sort

```
1 const int N = 100010;
2 int e[2 * N], ne[2 * N], h[N], d[N], idx;
3 int n, m, q[N];
4 void init() { memset(h, -1, sizeof h); }
5 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++, d[b]++; }
6 bool topSort()
7 {
8     int hh = 0, tt = -1;
9     for (int i = 1; i <= n; i++)
10         if (!d[i]) q[++tt] = i;
11     while (hh <= tt)
```

```
12         for (int i = h[q[hh++]]; ~i; i = ne[i]
13             )
14             if (--d[e[i]] == 0) q[++tt] = e[i];
15     return tt == n - 1;
16 }
```

## 3.4 Shortest Path

#### 3.4.1 Dijkstra

```
1 const int N = 1010;
2 int n, dist[N];
3 int h[N], w[N], e[N], ne[N], idx;
4 bool st[N];
5 void add(int a, int b, int c) { e[idx] = b,
    w[idx] = c, ne[idx] = h[a], h[a] = idx
    ++; }
6 int dijkstra() // 需要初始化 dist 与 h
7 {
8     dist[1] = 0;
9     priority_queue<PII, vector<PII>, greater<
        PII>> heap;
10    heap.push({0, 1});
11    while (heap.size())
12    {
13        auto t = heap.top();
14        heap.pop();
15        int ver = t.second, distance = t.first
            ;
16        if (st[ver]) continue;
17        st[ver] = true;
18        for (int i = h[ver]; i != -1; i = ne[i]
19            )
20            if (dist[e[i]] > distance + w[i])
21            {
22                dist[e[i]] = distance + w[i];
23                heap.push({dist[e[i]], e[i]});
24            }
25        if (dist[n] == 0x3f3f3f3f) return -1;
26        return dist[n];
27    }
```

#### 3.4.2 Bellman-Ford

```
1 const int N = 100010;
2 int n, m, dist[N], backup[N];
3 struct Edge
4 {
5     int a, b, w;
6 } edges[N];
7 int bellman_ford()
8 {
9     memset(dist, 0x3f, sizeof dist);
10    dist[1] = 0;
11    for (int i = 0; i < n; i++)
12    {
13        memcpy(backup, dist, sizeof dist);
14        for (int j = 0; j < m; j++)
15        {
```

```

16         int a = edges[j].a, b = edges[j].b
           , w = edges[j].w;
17         dist[b] = min(dist[b], backup[a] +
18             w);
19     }
20     if (dist[n] > 0x3f3f3f3f / 2) return -1;
21     return dist[n];
22 }

```

### 3.4.3 SPFA

```

1  const int N = 100010;
2  int n, m, dist[N];
3  int e[2 * N], ne[2 * N], w[2 * N], h[N], idx
   ;
4  bool vis[N];
5  void spfa()    // 需要初始化 dist 与 h
6  {
7      queue<int> q;
8      q.push(1); vis[1] = true;
9      while (q.size())
10     {
11         int t = q.front();
12         q.pop();
13         vis[t] = false;
14         for (int i = h[t]; ~i; i = ne[i])
15             if (dist[e[i]] > dist[t] + w[i])
16             {
17                 dist[e[i]] = dist[t] + w[i];
18                 if (!vis[e[i]]) vis[e[i]] =
19                     true, q.push(j);
20             }
21     dist[n] > INF / 2 ? cout << "impossible" :
22         cout << dist[n];

```

### 3.4.4 Detecting Negative Circle in SPFA

```

1  void spfa()    // 只需要初始化 h
2  {
3      queue<int> q;
4      // 基于虚拟原点假设, 所有点放入队列
5      for (int i = 1; i <= n; i++) q.push(i), st
6          [i] = true;
7      while (q.size())
8      {
9          int t = q.front();
10         q.pop();
11         vis[t] = false;
12         for (int i = h[t]; ~i; i = ne[i])
13             if (dist[e[i]] > dist[t] + w[i])
14             {
15                 dist[e[i]] = dist[t] + w[i];
16                 // 新增
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n) return true
19                 if (!st[j]) q.push(j), st[j] =
20                     true;

```

```

19     }
20 }
21 return false;
22 }

```

### 3.4.5 Floyd

```

1  const int N = 210;
2  int g[N][N], n, m, k;
3  int main()
4  {
5      cin >> n >> m >> k;
6      memset(g, 0x3f, sizeof g);
7      for (int i = 1; i <= n; i++) g[i][i] = 0;
8      while (m--)
9      {
10         int a, b, c;
11         cin >> a >> b >> c;
12         g[a][b] = min(g[a][b], c);
13     }
14     for (int k = 1; k <= n; k++)
15         for (int i = 1; i <= n; i++)
16             for (int j = 1; j <= n; j++)
17                 g[i][j] = min(g[i][k] + g[k][j]
18                     , g[i][j]);
19     // 后续代码略
20 }

```

## 3.5 Minimum Spanning Tree

### 3.5.1 Prim

```

1  const int N = 510;
2  int n, m, g[N][N], dist[N];
3  bool vis[N];
4  void prim()
5  {
6      int res = 0;
7      for (int i = 0; i < n; i++)
8      {
9          int t = -1;
10         for (int j = 1; j <= n; j++)
11             if (!vis[j] && (t == -1 || dist[j]
12                 < dist[t])) t = j;
13         if (i && dist[t] == INF) { res = INF;
14             break; }
15         if (i) res += dist[t];
16         vis[t] = true;
17         for (int j = 1; j <= n; j++) dist[j] =
18             min(dist[j], g[t][j]);
19     }
20     res == INF ? cout << "impossible" : cout
21         << res;
22 }
23 int main()
24 {
25     memset(g, 0x3f, sizeof g);
26     memset(dist, 0x3f, sizeof dist);
27     cin >> n >> m;
28     while (m--)

```

```

25 {
26     int a, b, c;
27     cin >> a >> b >> c;
28     g[a][b] = min(g[a][b], c);
29     g[b][a] = min(g[b][a], c);
30 }
31 prim();
32 return 0;
33 }

```

### 3.5.2 Kruskal

```

1  const int N = 100010;
2  int n, m;
3  int p[N];
4  struct Edge
5  {
6      int a, b, w;
7      bool operator<(const Edge &e) const {
8          return w < e.w; };
9  } edge[2 * N];
10 void init() { for (int i = 1; i <= n; i++) p[i] = i; }
11 int find(int x)
12 {
13     if (x != p[x]) p[x] = find(p[x]);
14     return p[x];
15 }
16 void merge(int x, int y) { p[find(x)] = find(y); }
17 void kruskal()
18 {
19     int res = 0, cnt = 0;
20     for (int i = 1; i <= m; i++)
21         if (find(edge[i].a) != find(edge[i].b))
22         {
23             merge(edge[i].a, edge[i].b);
24             res += edge[i].w;
25             cnt++;
26         }
27     if (cnt < n - 1) res = INF;
28     res == INF ? cout << "impossible" : cout << res;
29 }
30 int main()
31 {
32     init();
33     cin >> n >> m;
34     for (int i = 1; i <= m; i++) cin >> edge[i].a >> edge[i].b >> edge[i].w;
35     sort(edge + 1, edge + m + 1);
36     kruskal();
37     return 0;
38 }

```

```

1  const int N = 100010, M = 200010;
2  int n, m;
3  int e[M], ne[M], h[N], color[N], idx;
4  bool dfs(int u, int c)
5  {
6      color[u] = c;
7      for (int i = h[u]; ~i; i = ne[i])
8          if (color[e[i]] == -1)
9          {
10             if (!dfs(e[i], !c)) return false;
11         }
12         else if (color[e[i]] == c) return false;
13     return true;
14 }
15 bool check()
16 {
17     for (int i = 1; i <= n; i++)
18         if (color[i] == -1)
19             if (!dfs(i, 0)) return false;
20     return true;
21 }
22 int main()
23 {
24     // 注意另外初始化 h 与 color
25     cin >> n >> m;
26     while (m--)
27     {
28         int a, b;
29         cin >> a >> b;
30         add(a, b), add(b, a);
31     }
32     // 其余过程略
33 }

```

## 3.6 Bipartite Graph

### 3.6.1 Coloring Method

To check if a given graph is bipartite.

### 3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```
1  const int N = 510, M = 100010;
2  int n1, n2, m;
3  int e[M], ne[M], h[N], match[N], idx;
4  bool vis[N];
5  bool find(int x)
6  {
7      for (int i = h[x]; ~i; i = ne[i])
8          if (!vis[e[i]])
9              {
10                 vis[e[i]] = true;
11                 if (match[e[i]] == 0 || find(match[e[i]]))
12                     {
13                         match[e[i]] = x;
14                         return true;
15                     }
16             }
17      return false;
18  }
19  int main()
20  {
21      // 注意初始化 h
22      cin >> n1 >> n2 >> m;
23      while (m--)
24      {
25          int a, b;
26          cin >> a >> b;
27          add(a, b);
28      }
29      int res = 0;
30      for (int i = 1; i <= n1; i++)
31      {
32          memset(vis, false, sizeof vis);
33          if (find(i)) res++;
34      }
35      cout << res;
36      return 0;
37  }
```



## 4 ★ Basic Math

### 4.1 Prime Numbers

#### 4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0) return false;
6     return true;
7 }
```

#### 4.1.2 Prime Factorization

```
1 void divide(int x)
2 {
3     for (int i = 2; i <= x / i; i++)
4         if (x % i == 0)
5             { // 此条件成立时 i 一定是质数
6                 int s = 0;
7                 while (x % i == 0) x /= i, s++;
8                 cout << i << ' ' << s << '\n';
9             }
10    if (x > 1) cout << x << ' ' << 1 << '\n'
11 }
```

#### 4.1.3 Euler's Sieve

```
1 int primes[N], cnt;
2 bool st[N];
3 void get_primes(int n)
4 {
5     for (int i = 2; i <= n; i++)
6     {
7         if (!st[i]) primes[cnt++] = i;
8         for (int j = 0; primes[j] <= n / i; j++)
9             {
10                st[primes[j] * i] = true;
11                if (i % primes[j] == 0) break;
12            }
13    }
14 }
```

## 4.2 Divisor

#### 4.2.1 Find All Divisors

```
1 vector<int> get_divisors(int x)
2 {
3     vector<int> res;
4     for (int i = 1; i <= x / i; i++)
5         if (x % i == 0)
```

```
6         {
7             res.push_back(i);
8             if (i != x / i) res.push_back(x /
9             i);
10        }
11    sort(res.begin(), res.end());
12    return res;
13 }
```

#### 4.2.2 The Number of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 int main()
4 {
5     cin >> n;
6     unordered_map<int, int> h;
7     while (n--)
8     {
9         int x;
10        cin >> x;
11        for (int i = 2; i <= x / i; i++)
12            while (x % i == 0) { h[i]++; x = x
13            / i; }
14        if (x > 1) h[x]++;
15    }
16    long long res = 1;
17    for (auto iter = h.begin(); iter != h.end()
18        ); iter++)
19        res = res * (iter->second + 1) % mod;
20    cout << res;
21    return 0;
22 }
```

#### 4.2.3 The Sum of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 long long getSum(int x, int c)
4 {
5     long long s = 1;
6     while(c--) s = (s * x + 1) % mod;
7     return s;
8 }
9 int main()
10 {
11    cin >> n;
12    unordered_map<int, int> h;
13    while (n--)
14    {
15        int x;
16        cin >> x;
17        for (int i = 2; i <= x / i; i++)
18            while (x % i == 0) { h[i]++; x = x
19            / i; }
20        if (x > 1) h[x]++;
21    }
22    long long res = 1;
23    for (auto iter = h.begin(); iter != h.end()
24        ); iter++)
25        res = res * getSum(iter->first, iter->
26        second) % mod;
```

```

24     cout << res;
25     return 0;
26 }

```

#### 4.2.4 Euclidean Algorithm

```

1  int gcd(int a, int b)
2  { return a % b == 0 ? b : gcd(b, a % b); }

```

### 4.3 Euler Function

#### 4.3.1 Simple Method

```

1  int phi(int x)
2  {
3      int res = x;
4      for (int i = 2; i <= x / i; i++)
5          if (x % i == 0)
6              {
7                  res = res / i * (i - 1);
8                  while (x % i == 0) x /= i;
9              }
10     if (x > 1) res = res / x * (x - 1);
11     return res;
12 }

```

#### 4.3.2 Euler's Sieve Method

```

1  const int N = 1000010;
2  int n, primes[N], phi[N], cnt;
3  bool st[N];
4  void getEuler()
5  {
6      phi[1] = 1;
7      for (int i = 2; i <= n; i++)
8          {
9              if (!st[i])
10                 {
11                     primes[cnt++] = i;
12                     // i 是质数, 它只会被本身整除, 所以
13                     // 直接赋值 i - 1
14                     phi[i] = i - 1;
15                 }
16                 for (int j = 0; primes[j] <= n / i; j++)
17                     {
18                         st[i * primes[j]] = true;
19                         if (i % primes[j] == 0)
20                             {
21                                 // 如果 i % primes[j] == 0 成
22                                 // 立表示 primes[j] 是 i 的最小质因子
23                                 // 也是 primes[j] * i 的最小质
24                                 // 因子
25                                 // 1 - 1 / primes[j] 这一项在
26                                 // phi[i] 中计算过了, 只需将基数 N 修正为
27                                 // primes[j] 倍
28                                 phi[primes[j] * i] = phi[i] *
29                                 primes[j];
30                                 break;
31                             }
32                     }
33             }
34     }
35 }

```

```

25     }
26     // 否则, primes[j] 不是 i 的质因
27     // 子, 只是 primes[j] * i 的最小质因子
28     // 不仅需要将基数 N 修正为 primes[j]
29     // 倍
30     // 还需要补上 1 - 1 / primes[j] 的
31     // 分子项, 因此最终结果为 phi[i] * (primes[j]
32     // - 1)
33     phi[primes[j] * i] = phi[i] * (
34     primes[j] - 1);
35 }
36 }
37 }

```

### 4.4 Exponentiating by Squaring

```

1  LL qmi(int m, int k, int p)
2  {
3      LL res = 1 % p, t = m;
4      while (k)
5          {
6              if (k & 1) res = res * t % p;
7              t = t * t % p;
8              k >>= 1;
9          }
10     return res;
11 }

```

### 4.5 Extended Euclidean Algorithm

```

1  int exgcd(int a, int b, int &x, int &y)
2  {
3      if (!b)
4          {
5              x = 1;
6              y = 0;
7              return a;
8          }
9      int d = exgcd(b, a % b, y, x);
10     y -= (a / b) * x;
11     return d;
12 }

```

### 4.6 Chinese Remainder Theorem

```

1  LL exgcd(LL a, LL b, LL &x, LL &y)
2  {
3      if (!b) { x = 1, y = 0; return a; }
4      LL d = exgcd(b, a % b, y, x);
5      y -= a / b * x;
6      return d;
7  }
8  int main()
9  {

```

```

10 int n;
11 cin >> n;
12 LL x = 0, m1, a1;
13 cin >> m1 >> a1;
14 for (int i = 0; i < n - 1; i++)
15 {
16     LL m2, a2;
17     cin >> m2 >> a2;
18     LL k1, k2;
19     LL d = exgcd(m1, m2, k1, k2);
20     if ((a2 - a1) % d) { x = -1; break; }
21     k1 *= (a2 - a1) / d;
22     k1 = (k1 % (m2 / d) + m2 / d) % (m2 /
23 d);
24     x = k1 * m1 + a1;
25     LL m = abs(m1 / d * m2);
26     a1 = k1 * m1 + a1;
27     m1 = m;
28 }
29 if (x != -1)
30     x = (a1 % m1 + m1) % m1;
31 cout << x << '\n';
32 return 0;
33 }

```

## 4.7 Gauss-Jordan Elimination

### 4.7.1 Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++) // 找
            // 绝对值最大的行
8             if (fabs(a[i][c]) > fabs(a[t][c]))
9                 t = i;
10        if (fabs(a[t][c]) < eps) // 此
            // 时没必要对该列该行处理
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[t][i], a[r][i]); // 将
            // 绝对值最大的行换到最顶端
14        for (int i = n; i >= c; i--)
15            a[r][i] /= a[r][c]; // 将
            // 当前行的首位变成1
16        for (int i = r + 1; i < n; i++) // 用
            // 当前行将下面所有的列消成0
17            if (fabs(a[i][c]) > eps)
18                for (int j = n; j >= c; j--)
19                    a[i][j] -= a[r][j] * a[i][c];
20        r++;
21    }
22    if (r < n)
23    {
24        for (int i = r; i < n; i++)
25            if (fabs(a[i][n]) > eps)
26                return 2; // 无解
27        return 1; // 有无穷多组解
28    }
29 }

```

```

29 for (int i = n - 1; i >= 0; i--)
30     for (int j = i + 1; j < n; j++)
31         a[i][n] -= a[i][j] * a[j][n];
32 return 0; // 有解
33 }

```

### 4.7.2 XOR Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++)
8             if (a[i][c])
9                 t = i;
10        if (!a[t][c])
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[r][i], a[t][i]);
14        for (int i = r + 1; i < n; i++)
15            if (a[i][c])
16                for (int j = n; j >= c; j--)
17                    a[i][j] ^= a[r][j];
18        r++;
19    }
20    if (r < n)
21    {
22        for (int i = r; i < n; i++)
23            if (a[i][n])
24                return 2;
25        return 1;
26    }
27    for (int i = n - 1; i >= 0; i--)
28        for (int j = i + 1; j < n; j++)
29            a[i][n] ^= a[i][j] * a[j][n];
30    return 0;
31 }

```

## 4.8 Combinatorial Counting

### 4.8.1 Recurrence Relation

```

1 void init()
2 {
3     for (int i = 0; i < N; i++)
4         for (int j = 0; j <= i; j++)
5             if (!j) c[i][j] = 1;
6             else c[i][j] = (c[i - 1][j] + c[i - 1][j - 1]) % mod;
7 }

```

### 4.8.2 Preprocessing & Inverse Element

```

1 const int N = 100010, mod = 1e9 + 7;
2 int n, fact[N], infact[N];
3 int qmi(int a, int b, int p)
4 {

```

```

5   int res = 1;
6   while (b)
7   {
8       if (b & 1)
9           res = (LL)res * a % p;
10      a = (LL)a * a % p;
11      b >>= 1;
12  }
13  return res;
14  }
15  int main()
16  {
17      fact[0] = infact[0] = 1;
18      for (int i = 1; i < N; i++)
19      {
20          fact[i] = (LL)fact[i - 1] * i % mod;
21          infact[i] = (LL)infact[i - 1] * qmi(i,
22              mod - 2, mod) % mod;
23      }
24      // 此后  $C(a, b) = (LL)fact[a] * infact[b]$ 
```

### 4.8.3 Lucas Theorem

```

1   int qmi(int a, int k, int p)
2   {
3       int res = 1 % p;
4       while (k)
5       {
6           if (k & 1)
7               res = (LL)res * a % p;
8           a = (LL)a * a % p;
9           k >>= 1;
10      }
11      return res;
12  }
13  int C(int a, int b, int p)
14  {
15      if (a < b) return 0;
16      LL x = 1, y = 1;
17      //  $x = a * (a - 1) * (a - 2) * \dots * (a - b + 1) = a! / (a - b)!$  (mod p)
18      //  $y = 1 * 2 * \dots * b = b!$  (mod p)
19      for (int i = a, j = 1; j <= b; i--, j++)
20          { x = (LL)x * i % p; y = (LL)y * j % p; }
21      return x * (LL)qmi(y, p - 2, p) % p;
22  }
23  int lucas(LL a, LL b, int p)
24  {
25      if (a < p && b < p)
26          return C(a, b, p);
27      return (LL)C(a % p, b % p, p) * lucas(a /
28          p, b / p, p) % p;

```

### 4.8.4 Factorization Method

```

1   const int N = 5010;
2   int n, primes[N], sum[N], cnt;
3   bool st[N];
4   void getPrimes(int n) { // 略 }

```

```

5   // 求  $n!$  中  $p$  的幂次
6   int get(int n, int p)
7   {
8       int res = 0;
9       while (n) { res += n / p; n /= p; }
10      return res;
11  }
12  void mul(vector<int> &a, int b) { // 高精度
13      // 乘, 略 }
14  int main()
15  {
16      int a, b;
17      cin >> a >> b;
18      getPrimes(a);
19      for (int i = 0; i < cnt; i++)
20      {
21          int p = primes[i];
22          sum[i] = get(a, p) - get(b, p) - get(a
23              - b, p);
24      }
25      vector<int> res;
26      res.push_back(1);
27      for (int i = 0; i < cnt; i++)
28          for (int j = 0; j < sum[i]; j++)
29              mul(res, primes[i]);
30      for (int i = res.size() - 1; i >= 0; i--)
31          cout << res[i];

```

### 4.8.5 Catalan Number

```

1   const int N = 100010, mod = 1e9 + 7;
2   int qmi(int a, int k, int p) { // 略 }
3   int main()
4   {
5       int n;
6       cin >> n;
7       int a = n * 2, b = n, res = 1;
8       for (int i = a; i > a - b; i--)
9           res = (LL)res * i % mod;
10      for (int i = 1; i <= b; i++)
11          res = (LL)res * qmi(i, mod - 2, mod) %
12              mod;
13      res = (LL)res * qmi(n + 1, mod - 2, mod) %
14          mod;

```

## 4.9 Inclusion-Exclusion Principle

```

1   const int N = 20;
2   int n, m, res = 0, p[N];
3   int main()
4   {
5       cin >> n >> m;
6       for (int i = 0; i < m; i++)
7           cin >> p[i];
8       // 使用二进制数字表示数字选取情况
9       for (int i = 1; i < 1 << m; i++)
10      {

```

```

11     int t = 1, cnt = 0;
12     // 遍历每个被选取的质数
13     for (int j = 0; j < m; j++)
14         if (i >> j & 1)
15         {
16             cnt++;
17             // 一个质数能被选取的条件应该是
其累乘积不超过目标数字
18             if ((LL)t * p[j] > n)
19                 { t = -1; break; }
20             t *= p[j];
21         }
22     if (t != -1)
23         // 容斥原理公式中奇数个并集系数为 1
, 反之为 -1
24         if (cnt % 2) res += n / t;
25         else res -= n / t;
26     }
27     cout << res;
28 }

```

## 4.10 Game Theory

### 4.10.1 NIM Game

```

1  const int N = 110, M = 100010;
2  int k, n, s[N], f[M];
3  int sg(int x)
4  {
5      if (f[x] != -1) return f[x];
6      // 到达节点得 SG 函数集合
7      unordered_set<int> S;
8      // 能取走石子就说明能到达, 并且递归向下求解
9      for (int i = 0; i < k; i++)
10     {
11         int sum = s[i];
12         if (x >= sum) S.insert(sg(x - sum));
13     }
14     // SG 从小到大遍历并返回, 找到最小的、不包含
在 SG 函数集合中的自然数
15     for (int i = 0;; i++)
16         if (!S.count(i))
17             return f[x] = i;
18 }
19
20 int main()
21 {
22     cin >> k;
23     for (int i = 0; i < k; i++) cin >> s[i];
24     cin >> n;
25     memset(f, -1, sizeof f);
26     int res = 0;
27     // 每一堆石子都是一个入度为 0 的起始点
28     for (int i = 0; i < n; i++)
29     {
30         int x;
31         cin >> x;
32         res ^= sg(x);
33     }
34     res ? cout << "Yes" : cout << "No";
35     return 0;
36 }

```

## 5 ★ Basic DP

### 5.1 Knapsack Problem

#### 5.1.1 01 Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = m; j >= v[i]; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.2 Complete Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = v[i]; j <= m; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.3 Mutiple Knapsack

```
1  const int N = 25010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         for (int k = 1; k <= s; k *= 2)
11         {
12             for (int j = m; j >= k * v; j--)
13                 f[j] = max(f[j], f[j - k * v] + k * w);
14             s -= k;
15         }
16         if (s)
17             for (int j = m; j >= s * v; j--)
18                 f[j] = max(f[j], f[j - s * v] + s * w);
19     }
```

```
20     cout << f[m];
21 }
```

#### 5.1.4 Grouped Knapsack

```
1  const int N = 120;
2  int n, m, s[N], v[N][N], w[N][N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> s[i];
9          for (int j = 1; j <= s[i]; j++)
10             cin >> v[i][j] >> w[i][j];
11     }
12     for (int i = 1; i <= n; i++)
13         for (int j = m; j >= 0; j--)
14             for (int k = 1; k <= s[i]; k++)
15                 if (v[i][k] <= j)
16                     f[j] = max(f[j], f[j - v[i][k]] + w[i][k]);
17     cout << f[m];
18 }
```

## 5.2 Linear DP

### 5.2.1 LIS

Here is an  $O(n^2)$  solution:

```
1  const int N = 1010;
2  int n, a[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[j] < a[i])
13                 f[i] = max(f[i], f[j] + 1);
14     }
15     int res = 0;
16     for (int i = 1; i <= n; i++)
17         res = max(res, f[i]);
18     cout << res;
19 }
```

Another is an  $O(n \log n)$  solution:

```
1  const int N = 100010;
2  int n, a[N], q[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++) cin >> a[i];
7      int len = 0;
8      q[len] = -INF;
9      for (int i = 1; i <= n; i++)
```

```

10 {
11     int l = 0, r = len;
12     while (l < r)
13     {
14         int mid = l + r + 1 >> 1;
15         if (q[mid] < a[i]) l = mid;
16         else r = mid - 1;
17     }
18     len = max(r + 1, len);
19     q[r + 1] = a[i];
20 }
21 cout << len;
22 }

```

## 5.2.2 LCS

```

1 const int N = 1010;
2 int n, m, f[N][N];
3 char a[N], b[N];
4 int main()
5 {
6     cin >> n >> m >> (a + 1) >> (b + 1);
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= m; j++)
9             {
10                 f[i][j] = max(f[i - 1][j], f[i][j
11                     - 1]);
12                 if (a[i] == b[j])
13                     f[i][j] = max(f[i][j], f[i -
14                         1][j - 1] + 1);
15             }
16     cout << f[n][m];
17 }

```

## 5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```

1 const int N = 310;
2 int n, s[N], f[N][N];
3 int main()
4 {
5     cin >> n;
6     for (int i = 1; i <= n; i++)
7         cin >> s[i], s[i] += s[i - 1];
8     for (int len = 2; len <= n; len++)
9         for (int i = 1; i + len - 1 <= n; i++)
10            {
11                int l = i, r = i + len - 1;
12                f[l][r] = INF;
13                for (int k = l; k < r; k++)
14                    f[l][r] = min(f[l][r], f[l][k]
15                        + f[k + 1][r] + s[r] - s[l - 1]);
16            }
17     cout << f[1][n];
18 }

```

## 5.4 Counting DP

```

1 const int N = 1010, M = 1e9 + 7;
2 int n, f[N][N];
3 int main()
4 {
5     cin >> n;
6     f[0][0] = 1;
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= i; j++)
9             f[i][j] = (f[i - 1][j - 1] + f[i -
10                 j][j]) % M;
11     int ans = 0;
12     for (int i = 1; i <= n; i++)
13         ans = (ans + f[n][i]) % M;
14     cout << ans;
15 }

```

## 5.5 Digit DP

```

1 // 求数 n 的位数
2 int get(int n)
3 {
4     int res = 0;
5     while (n) n /= 10, res++;
6     return res;
7 }
8 int count(int n, int i)
9 {
10     int res = 0, dgt = get(n);
11     for (int j = 1; j <= dgt; j++)
12     {
13         // p 为当前遍历位次(第 j 位)的数大小
14         // <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
15         // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
16         int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
17         // 求要选的数在 i 的左边的数小于 l 的情况:
18         // 1)、当 i 不为 0 时 xxx : 0...0 ~ l - 1, 即 l * (右边的数的位数) == l * p 种选法
19         // 2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0...1 ~ l - 1, 即 (l - 1) * (右边的数的位数) == (l - 1) * p 种选法
20         if (i) res += l * p;
21         else res += (l - 1) * p;
22         // 求要选的数在 i 的左边的数等于 l 的情况: (即视频中的 xxx == l 时)
23         // 1)、i > dj 时 0 种选法
24         // 2)、i == dj 时 yyy : 0...0 ~ r 即 r + 1 种选法
25         // 3)、i < dj 时 yyy : 0...0 ~ 9...9 即 10^(右边的数的位数) == p 种选法
26         if (i == dj) res += r + 1;
27         if (i < dj) res += p;
28     }
29     return res;
30 }
31 int main()
32 {
33     int a, b;

```

```

33 while (cin >> a >> b, a)
34 {
35     if (a > b) swap(a, b);
36     for (int i = 0; i <= 9; ++i)
37         cout << count(b, i) - count(a - 1,
38             i) << ' ';
39     // 利用前缀和思想: [l, r] 的和 = s[r] -
40     // s[l - 1]
41     cout << '\n';
42 }
43 }

```

## 5.6 State Compression DP

```

1  const int N = 12, M = 1 << 12;
2  int n, m;
3  LL f[N][M];
4  bool st[M];
5  int main()
6  {
7      while (cin >> n >> m, n || m)
8      {
9          memset(f, 0, sizeof f);
10         for (int i = 0; i < 1 << n; i++)
11         {
12             st[i] = true;
13             // 统计连续 0 的个数, 若连续 0 为奇
14             // 数个就不能正好放得下竖放的方格
15             int cnt = 0;
16             for (int j = 0; j < n && st[i]; j
17                 ++
18                 {
19                     if (i >> j & 1)
20                     {
21                         // 当前格子被使用
22                         // 如果连续 0 的数量为奇数
23                         // 个, 当前格子被使用的后果就是导致格子重合,
24                         // 所以不可取
25                         if (cnt & 1)
26                             st[i] = false;
27                         // 刷新状态
28                         cnt = 0;
29                     }
30                     else cnt++;
31                     // 最后再判断一次, 防止漏判
32                     if (cnt & 1)
33                         st[i] = false;
34                 }
35             // 没有摆放任何棋子的状态默认只有 1 种取
36             // 法
37             f[0][0] = 1;
38             // 遍历每一列
39             for (int i = 1; i <= m; i++)
40             {
41                 // 遍历当前列的每一种用二进制数字表示
42                 // 的摆放状态: 1 指横向摆放, 0 指空位
43                 for (int j = 0; j < 1 << n; j++)
44                 {
45                     // 遍历上一列的每一种用二进制数
46                     // 字表示的摆放状态: 1 指横向摆放, 0 指空位
47                     for (int k = 0; k < 1 << n; k
48                         ++
49                         {
50                         // 满足两个条件: 两列的摆放
51                         // 互不冲突; 两列摆放状态的结合状态是一个可取
52                         // 的状态则累加情况数
53                         if (!(j & k) && st[j | k])

```

```

40         f[i][j] += f[i - 1][k]
41     };
42     // 输出摆放好第 m 列且第 (m + 1) 列没有
43     // 任何方格的状态数
44     cout << f[m][0] << '\n';
45 }
46 }

```

## 5.7 Tree DP

```

1  // Don't use I/O functions from stdio.h!!!
2  #define itn int
3  #define nit int
4  #define nti int
5  #define tin int
6  #define tni int
7  #define retrun return
8  #define reutrn return
9  #define rutren return
10 #define INF 0x3f3f3f3f
11 #include <bits/stdc++.h>
12 using namespace std;
13 typedef pair<int, int> PII;
14 typedef long long LL;
15
16 const int N = 6010;
17
18 int n;
19 int e[N], ne[N], happy[N], h[N], idx;
20 int f[N][2];
21 bool has_father[N];
22 void add(int a, int b)
23 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
24 }
25 void dfs(int u)
26 {
27     f[u][1] = happy[u];
28     for (int i = h[u]; ~i; i = ne[i])
29     {
30         dfs(e[i]);
31         f[u][0] += max(f[e[i]][0], f[e[i]][1]);
32         f[u][1] += f[e[i]][0];
33     }
34 }
35 int main()
36 {
37     memset(h, -1, sizeof h);
38     cin >> n;
39     for (int i = 1; i <= n; i++) cin >> happy[
40         i];
41     for (int i = 0; i < n - 1; i++)
42     {
43         int a, b;
44         cin >> a >> b;
45         has_father[a] = true;
46         add(b, a);
47     }
48     int root = 1;
49     while (has_father[root]) root++;
50     dfs(root);
51     cout << max(f[root][0], f[root][1]);
52 }

```



## 5.8 Memoized Search

```
1  const int N = 310;
2  int n, m,
3  h[N][N], f[N][N],
4  dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1,
5      0};
6  int dp(int x, int y)
7  {
8      int &v = f[x][y];
9      if (v != -1) return v;
10     v = 1;
11     for (int i = 0; i < 4; i++)
12     {
13         int a = x + dx[i], b = y + dy[i];
14         if (a >= 1 && a <= n && b >= 1 && b <=
15             m && h[a][b] < h[x][y])
```

```
14         v = max(v, dp(a, b) + 1);
15     }
16     return v;
17 }
18 int main()
19 {
20     cin >> n >> m;
21     for (int i = 1; i <= n; i++)
22         for (int j = 1; j <= m; j++)
23             cin >> h[i][j];
24     memset(f, -1, sizeof f);
25     int res = 0;
26     for (int i = 1; i <= n; i++)
27         for (int j = 1; j <= m; j++)
28             res = max(res, dp(i, j));
29     cout << res;
30 }
```



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## Part II: Advanced Template

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## 6 ★ Advanced Basic

### 6.1 Slow Multiplication

```
1 LL mul(LL a, LL b, LL p)
2 {
3     LL ans = 0;
4     while (b)
5     {
6         if (b & 1) ans = (ans + a) % p;
7         a = a * 2 % p; b >>= 1;
8     }
9     return ans;
10 }
```

### 6.2 Sum of Geometric Series

```
1 const int mod = 9901;
2 int a, b;
3 int qmi(int a, int k)
4 {
5     int res = 1;
6     a %= mod;
7     while (k)
8     {
9         if (k & 1)
10             res = res * a % mod;
11         a = a * a % mod;
12         k >>= 1;
13     }
14     return res;
15 }
16 int sum(int p, int k)
17 {
18     if (k == 1) return 1;
19     if (k % 2 == 0)
20         return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
21     return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
22 }
23 int main()
24 {
25     // 以  $a^b$  约数之和为例求等比数列和
26     cin >> a >> b;
27     int res = 1;
28     for (int i = 2; i <= a / i; i++)
29         if (a % i == 0)
30         {
31             int s = 0;
32             while (a % i == 0) a /= i, s++;
33             res = res * sum(i, b * s + 1) % mod;
34         }
35     if (a > 1) res = res * sum(a, b + 1) % mod;
36 }
```

## 6.3 Sort

### 6.3.1 Card Balancing Problem

```
1 cin >> n;
2 for (int i = 1; i <= n; i++)
3     cin >> a[i], avg += a[i];
4 avg /= n;
5 for (int i = 1; i <= n; i++)
6     if (a[i] != avg)
7         a[i + 1] += a[i] - avg, ans++;
8 cout << ans;
```

### 6.3.2 2D Card Balancing Problem

```
1 const int N = 100010;
2 int row[N], col[N], c[N], s[N];
3 LL work(int n, int a[])
4 {
5     for (int i = 1; i <= n; i++)
6         s[i] = s[i - 1] + a[i];
7     if (s[n] % n) return -1;
8     int avg = s[n] / n;
9     c[1] = 0;
10    for (int i = 2; i <= n; i++)
11        c[i] = s[i - 1] - (i - 1) * avg;
12    sort(c + 1, c + n + 1);
13    LL res = 0;
14    for (int i = 1; i <= n; i++)
15        res += abs(c[i] - c[(n + 1) / 2]);
16    return res;
17 }
18 int main()
19 {
20     int n, m, cnt;
21     cin >> n >> m >> cnt;
22     while (cnt--)
23     {
24         int x, y;
25         cin >> x >> y;
26         row[x]++; col[y]++;
27     }
28     LL r = work(n, row);
29     LL c = work(m, col);
30     if (r != -1 && c != -1)
31         cout << "both " << r + c;
32     else if (r != -1)
33         cout << "row " << r;
34     else if (c != -1)
35         cout << "column " << c;
36     else cout << "impossible";
37 }
```

### 6.3.3 Dual Heaps

```
1 if (down.empty() || x <= down.top())
2     down.push(x);
3 else up.push(x);
4 if (down.size() > up.size() + 1)
5     up.push(down.top(), down.pop());
```

```

6  if (up.size() > down.size())
7      down.push(up.top()), up.pop();
8  if (i % 2)
9  {
10     cout << down.top() << ' ';
11     if (++cnt % 10 == 0) cout << '\n';
12 }

```

## 6.4 RMQ

```

1  const int N = 200010, M = 18;
2  int n, m, w[N], f[N][M];
3  void init()
4  {
5      for (int j = 0; j < M; j++)
6          for (int i = 1; i + (1 << j) - 1 <= n;
7              i++)
8              if (!j) f[i][j] = w[i];
9              else // 也可以是最小值
10                 f[i][j] = max(f[i][j - 1], f[i
11                     + (1 << j - 1)][j - 1]);
12 }
13 int query(int l, int r)
14 {
15     int len = r - l + 1;
16     int k = log(len) / log(2);
17     return max(f[l][k], f[r - (1 << k) + 1][k
18         ]);
19 }

```

## 7 ★ Advanced Data Structures

### 7.1 Binary Indexed Tree

```
1 // 支持区间修改、区间查询
2 // 利用变差分求二阶区间和
3 const int N = 100010;
4 int n, m, a[N];
5 LL tr1[N], tr2[N];
6 int lowbit(int x) { return x & -x; }
7 void add(LL tr[], LL x, LL c)
8 {
9     for (int i = x; i <= n; i += lowbit(i))
10         tr[i] += c;
11 }
12 LL sum(LL tr[], LL x)
13 {
14     LL res = 0;
15     for (int i = x; i; i -= lowbit(i))
16         res += tr[i];
17     return res;
18 }
19 LL prefix_sum(LL x)
20 { return sum(tr1, x) * (x + 1) - sum(tr2, x)
21   ; }
22 int main()
23 {
24     cin >> n >> m;
25     for (int i = 1; i <= n; i++)
26         cin >> a[i];
27     for (int i = 1; i <= n; i++)
28     {
29         int b = a[i] - a[i - 1];
30         add(tr1, i, b);
31         add(tr2, i, (LL)i * b);
32     }
33     while (m--)
34     {
35         char op[2];
36         int l, r, d;
37         cin >> op >> l >> r;
38         if (*op == 'Q')
39             cout << prefix_sum(r) - prefix_sum
40             (l - 1) << '\n';
41         else
42         {
43             cin >> d;
44             add(tr1, l, d), add(tr2, l, (LL)l
45             * d),
46             add(tr1, r + 1, -d),
47             add(tr2, r + 1, (LL)-(r + 1) * d);
48         }
49     }
50 }
```

## 7.2 Segment Tree

### 7.2.1 Maintain the Maximum

```
1 struct Node
2 { int l, r, v; } tr[N * 4];
```

```
3 void pushup(int u)
4 {
5     tr[u].v = max(tr[u << 1].v, tr[u << 1 |
6     1].v);
7 }
8 void build(int u, int l, int r)
9 {
10     tr[u] = {l, r};
11     if (l == r) return;
12     int mid = l + r >> 1;
13     build(u << 1, l, mid),
14     build(u << 1 | 1, mid + 1, r);
15 }
16 int query(int u, int l, int r)
17 {
18     if (tr[u].l >= l && tr[u].r <= r)
19         return tr[u].v;
20     int mid = tr[u].l + tr[u].r >> 1;
21     int v = 0;
22     if (l <= mid)
23         v = query(u << 1, l, r);
24     if (r > mid)
25         v = max(v, query(u << 1 | 1, l, r));
26     return v;
27 }
28 void modify(int u, int x, int v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31         tr[u].v = v;
32     else
33     {
34         int mid = tr[u].l + tr[u].r >> 1;
35         if (x <= mid)
36             modify(u << 1, x, v);
37         else
38             modify(u << 1 | 1, x, v);
39         pushup(u);
40     }
41 }
```

### 7.2.2 Maintain the Maximum Subarray Sum

```
1 struct Node
2 { int l, r, sum, lmax, rmax, tmax; } tr[N *
3   4];
4 void pushup(Node &u, Node &l, Node &r)
5 {
6     u.sum = l.sum + r.sum;
7     u.lmax = max(l.lmax, l.sum + r.lmax);
8     u.rmax = max(r.rmax, r.sum + l.rmax);
9     u.tmax = max(max(l.tmax, r.tmax), l.rmax +
10     r.lmax);
11 }
12 void pushup(int u)
13 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]);
14 }
15 void build(int u, int l, int r)
16 {
17     if (l == r)
18         tr[u] = {l, r, w[r], w[r], w[r], w[r]
19         };
20     else
21     {
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
```

```

18     tr[u] = {l, r};
19     int mid = l + r >> 1;
20     build(u << 1, l, mid),
21     build(u << 1 | 1, mid + 1, r);
22     pushup(u);
23 }
24 }
25 void modify(int u, int x, int v)
26 {
27     if (tr[u].l == x && tr[u].r == x)
28         tr[u] = {x, x, v, v, v, v};
29     else
30     {
31         int mid = tr[u].l + tr[u].r >> 1;
32         if (x <= mid)
33             modify(u << 1, x, v);
34         else
35             modify(u << 1 | 1, x, v);
36         pushup(u);
37     }
38 }
39 Node query(int u, int l, int r)
40 {
41     if (tr[u].l >= l && tr[u].r <= r)
42         return tr[u];
43     else
44     {
45         int mid = tr[u].l + tr[u].r >> 1;
46         if (r <= mid)
47             return query(u << 1, l, r);
48         else if (l > mid)
49             return query(u << 1 | 1, l, r);
50         else
51         {
52             auto left = query(u << 1, l, r);
53             auto right = query(u << 1 | 1, l,
54                               r);
55             Node res;
56             pushup(res, left, right);
57             return res;
58         }
59 }

```

### 7.2.3 Maintain the GCD

```

1  struct Node
2  { int l, r; LL sum, d; } tr[N * 4];
3  LL gcd(LL a, LL b)
4  { return b ? gcd(b, a % b) : a; }
5  void pushup(Node &u, Node &l, Node &r)
6  {
7      u.sum = l.sum + r.sum;
8      u.d = gcd(l.d, r.d);
9  }
10 void pushup(int u)
11 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
12 void build(int u, int l, int r)
13 {
14     if (l == r)
15     {
16         LL b = w[r] - w[r - 1];
17         tr[u] = {l, r, b, b};

```

```

18     }
19     else
20     {
21         tr[u].l = l, tr[u].r = r;
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
28 void modify(int u, int x, LL v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31     {
32         LL b = tr[u].sum + v;
33         tr[u] = {x, x, b, b};
34     }
35     else
36     {
37         int mid = tr[u].l + tr[u].r >> 1;
38         if (x <= mid)
39             modify(u << 1, x, v);
40         else
41             modify(u << 1 | 1, x, v);
42         pushup(u);
43     }
44 }
45 Node query(int u, int l, int r)
46 {
47     if (tr[u].l >= l && tr[u].r <= r)
48         return tr[u];
49     else
50     {
51         int mid = tr[u].l + tr[u].r >> 1;
52         if (r <= mid)
53             return query(u << 1, l, r);
54         else if (l > mid)
55             return query(u << 1 | 1, l, r);
56         else
57         {
58             auto left = query(u << 1, l, r);
59             auto right = query(u << 1 | 1, l,
60                               r);
61             Node res;
62             pushup(res, left, right);
63             return res;
64         }
65 }

```

### 7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```

1  struct Node
2  { int l, r; LL sum, add; } tr[N * 4];
3  void pushup(int u)
4  { tr[u].sum = tr[u << 1].sum + tr[u << 1 |
5    1].sum; }
6  void pushdown(int u)
7  {
8      auto &root = tr[u],
9      &left = tr[u << 1],

```

```

9      &right = tr[u << 1 | 1];
10  if (root.add)
11  {
12      left.add += root.add,
13      left.sum += (LL)(left.r - left.l + 1)
14      * root.add;
15      right.add += root.add,
16      right.sum += (LL)(right.r - right.l +
17      1) * root.add;
18      root.add = 0;
19  }
20  }
21  void build(int u, int l, int r)
22  {
23      if (l == r) tr[u] = {l, r, w[r], 0};
24      else
25      {
26          tr[u] = {l, r};
27          int mid = l + r >> 1;
28          build(u << 1, l, mid);
29          build(u << 1 | 1, mid + 1, r);
30          pushup(u);
31      }
32  }
33  void modify(int u, int l, int r, int d)
34  {
35      if (tr[u].l >= l && tr[u].r <= r)
36      {
37          tr[u].sum += (LL)(tr[u].r - tr[u].l +
38          1) * d;
39          tr[u].add += d;
40      }
41      else
42      {
43          pushdown(u);
44          int mid = tr[u].l + tr[u].r >> 1;
45          if (l <= mid)
46              modify(u << 1, l, r, d);
47          if (r > mid)
48              modify(u << 1 | 1, l, r, d);
49          pushup(u);
50      }
51  }
52  LL query(int u, int l, int r)
53  {
54      if (tr[u].l >= l && tr[u].r <= r)
55          return tr[u].sum;
56      pushdown(u);
57      int mid = tr[u].l + tr[u].r >> 1;
58      LL sum = 0;
59      if (l <= mid)
60          sum += query(u << 1, l, r);
61      if (r > mid)
62          sum += query(u << 1 | 1, l, r);
63      return sum;
64  }

```

## 7.3 Persistent Data Structure

### 7.3.1 Persistent Trie

```

1  const int N = 600010, M = N * 25;
2  int n, m, s[N], root[N], idx;

```

```

3  int trie[M][2], max_id[M];
4  void insert(int i, int k, int p, int q)
5  {
6      if (k < 0)
7      {
8          max_id[q] = i;
9          return;
10     }
11     int v = s[i] >> k & 1;
12     if (p)
13         trie[q][v ^ 1] = trie[p][v ^ 1];
14     trie[q][v] = ++idx;
15     insert(i, k - 1, trie[p][v], trie[q][v]);
16     max_id[q] = max(max_id[trie[q][0]], max_id
17     [trie[q][1]]);
18 }
19 int query(int root, int C, int L)
20 {
21     int p = root;
22     for (int i = 23; i >= 0; i--)
23     {
24         int v = C >> i & 1;
25         if (max_id[trie[p][v ^ 1]] >= L)
26             p = trie[p][v ^ 1];
27         else
28             p = trie[p][v];
29     }
30     return C ^ s[max_id[p]];
31 }
32 // insert(i, 23, root[i - 1], root[i]);
33 // query(root[r - 1], l - 1, x ^ s[n]);

```

### 7.3.2 Persistent Segment Tree

```

1  const int N = 100010, M = 10010;
2  int n, m, a[N], root[N], idx;
3  vector<int> nums;
4  struct Node
5  {
6      int l, r;
7      int cnt;
8  } tr[N * 4 + N * 17];
9  int find(int x)
10 {
11     return lower_bound(nums.begin(), nums.end
12     (), x) - nums.begin();
13 }
14 int build(int l, int r)
15 {
16     int p = ++idx;
17     if (l == r)
18         return p;
19     int mid = l + r >> 1;
20     tr[p].l = build(l, mid), tr[p].r = build(
21     mid + 1, r);
22     return p;
23 }
24 int insert(int p, int l, int r, int x)
25 {
26     int q = ++idx;
27     tr[q] = tr[p];
28     if (l == r)
29     {
30         tr[q].cnt++;
31     }
32     else
33     {
34         int mid = l + r >> 1;
35         if (x <= mid)
36             tr[q].l = insert(tr[p].l, l, mid, x);
37         else
38             tr[q].r = insert(tr[p].r, mid + 1, r, x);
39     }
40     return q;
41 }
42 int query(int p, int l, int r, int x)
43 {
44     if (l == r)
45         return tr[p].cnt;
46     int mid = l + r >> 1;
47     if (x <= mid)
48         return query(tr[p].l, l, mid, x);
49     else
50         return query(tr[p].r, mid + 1, r, x);
51 }

```

```

29     return q;
30 }
31 int mid = l + r >> 1;
32 if (x <= mid)
33     tr[q].l = insert(tr[p].l, l, mid, x);
34 else
35     tr[q].r = insert(tr[p].r, mid + 1, r,
36                     x);
37 tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].
38 cnt;
39 return q;
40 }
41 int query(int q, int p, int l, int r, int k)
42 {
43     if (l == r)
44         return r;
45     int cnt = tr[tr[q].l].cnt - tr[tr[p].l].
46 cnt;
47 int mid = l + r >> 1;
48 if (k <= cnt)
49     return query(tr[q].l, tr[p].l, l, mid,
50 k);
51 else
52     return query(tr[q].r, tr[p].r, mid +
53 1, r, k - cnt);
54 }

```

## 7.4 Treap

```

1  const int N = 100010, INF = 1e8;
2  int n, root, idx;
3  struct Node
4  { int l, r, key, val, cnt, size; } tr[N];
5  void pushup(int p)
6  {
7      tr[p].size = tr[tr[p].l].size +
8      tr[tr[p].r].size + tr[p].cnt;
9  }
10 int get_node(int key)
11 {
12     tr[++idx].key = key;
13     tr[idx].val = rand();
14     tr[idx].cnt = tr[idx].size = 1;
15     return idx;
16 }
17 void zig(int &p)
18 {
19     int q = tr[p].l;
20     tr[p].l = tr[q].r, tr[q].r = p, p = q;
21     pushup(tr[p].r), pushup(p);
22 }
23 void zag(int &p)
24 {
25     int q = tr[p].r;
26     tr[p].r = tr[q].l, tr[q].l = p, p = q;
27     pushup(tr[p].l), pushup(p);
28 }
29 void build()
30 {
31     get_node(-INF), get_node(INF);
32     root = 1, tr[1].r = 2;
33     pushup(root);
34     if (tr[1].val < tr[2].val) zag(root);

```

```

35 }
36 void insert(int &p, int key)
37 {
38     if (!p) p = get_node(key);
39     else if (tr[p].key == key) tr[p].cnt++;
40     else if (tr[p].key > key)
41     {
42         insert(tr[p].l, key);
43         if (tr[tr[p].l].val > tr[p].val)
44             zig(p);
45     }
46     else
47     {
48         insert(tr[p].r, key);
49         if (tr[tr[p].r].val > tr[p].val)
50             zag(p);
51     }
52     pushup(p);
53 }
54 void remove(int &p, int key)
55 {
56     if (!p) return;
57     if (tr[p].key == key)
58     {
59         if (tr[p].cnt > 1) tr[p].cnt--;
60         else if (tr[p].l || tr[p].r)
61         {
62             if (!tr[p].r || tr[tr[p].l].val >
63 tr[tr[p].r].val)
64             {
65                 zig(p);
66                 remove(tr[p].r, key);
67             }
68             else
69             {
70                 zag(p);
71                 remove(tr[p].l, key);
72             }
73         }
74         else p = 0;
75     }
76     else if (tr[p].key > key)
77         remove(tr[p].l, key);
78     else remove(tr[p].r, key);
79     pushup(p);
80 }
81 int get_rank_by_key(int p, int key)
82 {
83     if (!p) return 0;
84     if (tr[p].key == key)
85         return tr[tr[p].l].size + 1;
86     if (tr[p].key > key)
87         return get_rank_by_key(tr[p].l, key);
88     return tr[tr[p].l].size + tr[p].cnt +
89 get_rank_by_key(tr[p].r, key);
90 }
91 int get_key_by_rank(int p, int rank)
92 {
93     if (!p) return INF;
94     if (tr[tr[p].l].size >= rank)
95         return get_key_by_rank(tr[p].l, rank);
96     if (tr[tr[p].l].size + tr[p].cnt >= rank)
97         return tr[p].key;
98     return get_key_by_rank(tr[p].r, rank - tr[
99 tr[p].l].size - tr[p].cnt);
100 }

```



```

98 int get_prev(int p, int key)
99 {
100     if (!p) return -INF;
101     if (tr[p].key >= key)
102         return get_prev(tr[p].l, key);
103     return max(tr[p].key, get_prev(tr[p].r,
104                                   key));
105 }
106 int get_next(int p, int key)
107 {
108     if (!p) return INF;
109     if (tr[p].key <= key)
110         return get_next(tr[p].r, key);
111     return min(tr[p].key, get_next(tr[p].l,
112                                   key));
113 }

```

## 7.5 AC Automaton

```

1  const int N = 10010, M = 1000010, S = 55;
2  int n, tr[N * S][26], cnt[N * S], idx;
3  int q[N * S], ne[N * S];
4  char str[M];
5  void insert()
6  {
7      int p = 0;
8      for (int i = 0; str[i]; i++)
9      {
10         int t = str[i] - 'a';
11         if (!tr[p][t]) tr[p][t] = ++idx;
12         p = tr[p][t];
13     }
14     cnt[p]++;
15 }
16 void build()
17 {
18     int hh = 0, tt = -1;
19     for (int i = 0; i < 26; i++)
20         if (tr[0][i]) q[++tt] = tr[0][i];
21     while (hh <= tt)
22     {
23         int t = q[hh++];
24         for (int i = 0; i < 26; i++)
25         {
26             int p = tr[t][i];
27             if (!p) tr[t][i] = tr[ne[t]][i];
28             else
29             {
30                 ne[p] = tr[ne[t]][i];
31                 q[++tt] = p;
32             }
33         }
34     }
35 }

```

## 8 ★ Advanced Search

### 8.1 Flood-Fill

```
1  const int N = 1010, M = N * N;
2  int n, m;
3  char g[N][N];
4  PII q[M];
5  bool st[N][N];
6  void bfs(int sx, int sy)
7  {
8      int hh = 0, tt = 0;
9      q[0] = {sx, sy}; st[sx][sy] = true;
10     while (hh <= tt)
11     {
12         PII t = q[hh++];
13         for (int i = t.first - 1; i <= t.first + 1; i++)
14             for (int j = t.second - 1; j <= t.second + 1; j++)
15             {
16                 if (i == t.first && j == t.second)
17                     continue;
18                 if (i < 0 || i >= n || j < 0 || j >= m)
19                     continue;
20                 if (g[i][j] == '.' || st[i][j])
21                     continue;
22                 q[++tt] = {i, j};
23                 st[i][j] = true;
24             }
25     }
26 }
27 int main()
28 {
29     int cnt = 0;
30     for (int i = 0; i < n; i++)
31         for (int j = 0; j < m; j++)
32             if (g[i][j] == 'W' && !st[i][j])
33                 { bfs(i, j); cnt++; }
34 }
```

### 8.2 Multi-source BFS

```
1  const int N = 1010, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  PII q[M];
5  int dx[4] = {-1, 0, 1, 0},
6      dy[4] = {0, 1, 0, -1};
7  void bfs()
8  {
9      memset(dist, -1, sizeof dist);
10     int hh = 0, tt = -1;
11     for (int i = 1; i <= n; i++)
12         for (int j = 1; j <= m; j++)
13             if (g[i][j] == '1')
14             {
15                 dist[i][j] = 0;
```

```
16                 q[++tt] = {i, j};
17             }
18     while (hh <= tt)
19     {
20         auto t = q[hh++];
21         for (int i = 0; i < 4; i++)
22         {
23             int a = t.x + dx[i], b = t.y + dy[i];
24             if (a < 1 || a > n || b < 1 || b > m) continue;
25             if (dist[a][b] != -1) continue;
26             dist[a][b] = dist[t.x][t.y] + 1;
27             q[++tt] = {a, b};
28         }
29     }
30 }
```

### 8.3 BFS with Deque

```
1  const int N = 510, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  bool st[N][N];
5  int dx[4] = {-1, -1, 1, 1},
6      dy[4] = {-1, 1, 1, -1},
7      ix[4] = {-1, -1, 0, 0},
8      iy[4] = {-1, 0, 0, -1};
9  int bfs()
10 {
11     memset(dist, 0x3f, sizeof dist);
12     memset(st, 0, sizeof st);
13     dist[0][0] = 0;
14     deque<PII> q;
15     q.push_back({0, 0});
16     char cs[] = "\\ / \\/";
17     while (q.size())
18     {
19         PII t = q.front();
20         q.pop_front();
21         if (st[t.x][t.y]) continue;
22         st[t.x][t.y] = true;
23         for (int i = 0; i < 4; i++)
24         {
25             int a = t.x + dx[i], b = t.y + dy[i];
26             if (a < 0 || a > n || b < 0 || b > m) continue;
27             int ca = t.x + ix[i], cb = t.y + iy[i];
28             int d = dist[t.x][t.y] + (g[ca][cb] != cs[i]);
29             if (d < dist[a][b])
30             {
31                 dist[a][b] = d;
32                 if (g[ca][cb] != cs[i])
33                     q.push_back({a, b});
34                 else
35                     q.push_front({a, b});
36             }
37         }
38     }
39 }
40 return dist[n][m];
```

```
41 }
```

## 8.4 Bidirectional BFS

```
1 int bfs()
2 {
3     if (A == B) return 0;
4     queue<string> qa, qb;
5     unordered_map<string, int> da, db;
6     qa.push(A), qb.push(B);
7     da[A] = db[B] = 0;
8     int step = 0;
9     while (qa.size() && qb.size())
10    {
11        int t;
12        if (qa.size() < qb.size())
13            // PROCESS
14        else
15            // PROCESS
16        if (t <= 10) return t;
17        if (++step == 10) return -1;
18    }
19    return -1;
20 }
```

## 8.5 A\*

```
1 const int N = 1010, M = 200010;
2 int n, m, S, T, K;
3 int h[N], rh[N], e[M], w[M], ne[M], idx;
4 int dist[N], cnt[N];
5 bool st[N];
6 void dijkstra()
7 {
8     priority_queue<PII, vector<PII>, greater<
9         PII>> heap;
10    heap.push({0, T});
11    memset(dist, 0x3f, sizeof dist);
12    dist[T] = 0;
13    while (heap.size())
14    {
15        auto t = heap.top();
16        heap.pop();
17        int ver = t.y;
18        if (st[ver]) continue;
19        st[ver] = true;
20        for (int i = rh[ver]; ~i; i = ne[i])
21        {
22            int j = e[i];
23            if (dist[j] > dist[ver] + w[i])
24            {
25                dist[j] = dist[ver] + w[i];
26                heap.push({dist[j], j});
27            }
28        }
29    }
30 }
31 int astar()
32 {
```

```
33     priority_queue<PIII, vector<PIII>, greater
34         <PIII>> heap;
35     heap.push({dist[S], {0, S}});
36     while (heap.size())
37     {
38         auto t = heap.top();
39         heap.pop();
40         int ver = t.y.y, distance = t.y.x;
41         cnt[ver]++;
42         if (cnt[T] == K) return distance;
43         for (int i = h[ver]; ~i; i = ne[i])
44         {
45             int j = e[i];
46             if (cnt[j] < K)
47                 heap.push({distance + w[i] +
48                     dist[j], {distance + w[i], j}});
49         }
50     }
51     return -1;
52 }
53 int main()
54 {
55     // PROCESS
56     dijkstra(); cout << astar();
57     // PROCESS
58 }
```

## 8.6 DFS Connectivity Model

```
1 char g[N][N];
2 int xa, ya, xb, yb;
3 int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0,
4     -1};
5 bool st[N][N];
6 bool dfs(int x, int y)
7 {
8     if (g[x][y] == '#') return false;
9     if (x == xb && y == yb) return true;
10    st[x][y] = true;
11    for (int i = 0; i < 4; i++)
12    {
13        int a = x + dx[i], b = y + dy[i];
14        if (a < 0 || a >= n || b < 0 || b >= n
15            ) continue;
16        if (st[a][b]) continue;
17        if (dfs(a, b)) return true;
18    }
19    return false;
20 }
```

## 8.7 IDDFS

```
1 const int N = 110;
2 int n, path[N];
3 bool dfs(int u, int k)
4 {
5     if (u == k)
6         return path[u - 1] == n;
7     bool st[N] = {0};
8     for (int i = u - 1; i >= 0; i--)
```

```

9     for (int j = i; j >= 0; j--)
10    {
11        int s = path[i] + path[j];
12        if (s > n || s <= path[u - 1] ||
13            st[s]) continue;
14        st[s] = true;
15        path[u] = s;
16        if (dfs(u + 1, k)) return true;
17    }
18    return false;
19 }

```

```

9 {
10     if (f() > maxn - depth) return false;
11     if (depth == maxn) return true;
12     for (int i = 0; i <= n; i++)
13     {
14         // OPERATION
15         if (IDAstar(depth + 1, maxn))
16             return true;
17         // OPERATION
18     }
19     return false;
20 }

```

## 8.8 Bidirectional DFS

```

1  const int N = 1 << 24;
2  int n, m, k, cnt = 0, ans;
3  int g[50], weights[N];
4  void dfs(int u, int s)
5  {
6      if (u == k)
7      {
8          weights[cnt++] = s;
9          return;
10     }
11     if ((LL)s + g[u] <= m)
12         dfs(u + 1, s + g[u]);
13     dfs(u + 1, s);
14 }
15 void dfs2(int u, int s)
16 {
17     if (u == n)
18     {
19         int l = 0, r = cnt - 1;
20         while (l < r)
21         {
22             int mid = l + r + 1 >> 1;
23             if (weights[mid] + (LL)s <= m)
24                 l = mid;
25             else r = mid - 1;
26         }
27         if (weights[l] + (LL)s <= m)
28             ans = max(ans, weights[l] + s);
29         return;
30     }
31     if ((LL)s + g[u] <= m)
32         dfs2(u + 1, s + g[u]);
33     dfs2(u + 1, s);
34 }

```

## 8.9 IDA\*

```

1  const int N = 1e2;
2  int n, a[N];
3  string t;
4  int f()
5  {
6      // YOUR_F_FUNCTION
7  }
8  bool IDAstar(int depth, int maxn)

```

## 9 ★ Advanced Graph Theory

### 9.1 Detecting Negative Cycles

```
1  int n, m1, m2;
2  int h[N], e[M], w[M], ne[M], idx;
3  int dist[N], q[N], cnt[N];
4  bool st[N];
5  bool spfa()
6  {
7      memset(dist, 0, sizeof dist);
8      memset(cnt, 0, sizeof cnt);
9      memset(st, 0, sizeof st);
10     int hh = 0, tt = 0;
11     for (int i = 1; i <= n; i++)
12     {
13         q[tt++] = i;
14         st[i] = true;
15     }
16     while (hh != tt)
17     {
18         int t = q[hh++];
19         if (hh == N) hh = 0;
20         st[t] = false;
21         for (int i = h[t]; ~i; i = ne[i])
22         {
23             int j = e[i];
24             if (dist[j] > dist[t] + w[i])
25             {
26                 dist[j] = dist[t] + w[i];
27                 cnt[j] = cnt[t] + 1;
28                 if (cnt[j] >= n) return true;
29                 if (!st[j])
30                 {
31                     q[tt++] = j;
32                     if (tt == N) tt = 0;
33                     st[j] = true;
34                 }
35             }
36         }
37     }
38     return false;
39 }
```

### 9.2 SPFA-SLF

Using deque to solve SPFA question.

```
1  void spfa()
2  {
3      memset(dist, 0x3f, sizeof dist);
4      memset(st, 0, sizeof st);
5      deque<int> q;
6      q.push_back(s);
7      st[s] = 1, dist[s] = 0;
8      while (q.size())
9      {
10         int t = q.front();
11         q.pop_front();
12         st[t] = 0;
13         for (int i = h[t]; ~i; i = ne[i])
14         {
```

```
15         int j = e[i];
16         if (dist[j] > dist[t] + w[i])
17         {
18             dist[j] = dist[t] + w[i];
19             if (!st[j])
20             {
21                 st[j] = true;
22                 if (q.size() && dist[j] < dist[q.
23                     front()])
24                     q.push_front(j);
25                 else
26                     q.push_back(j);
27             }
28         }
29     }
30 }
```

### 9.3 SPFA-Stack

```
1  bool spfa()
2  {
3      int hh = 0, tt = 1;
4      memset(dist, -0x3f, sizeof dist);
5      dist[0] = 0;
6      q[0] = 0;
7      while (hh != tt)
8      {
9          int t = q[--tt];
10         st[t] = false;
11         for (int i = h[t]; ~i; i = ne[i])
12         {
13             int j = e[i];
14             if (dist[j] < dist[t] + w[i])
15             {
16                 dist[j] = dist[t] + w[i];
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n + 1) return true;
19                 if (!st[j])
20                 {
21                     st[j] = true;
22                     q[tt++] = j;
23                 }
24             }
25         }
26     }
27     return false;
28 }
```

### 9.4 SPFA & MIN & MAX

Using SPFA to maintain the minimum and maximum. In this case we need **Original Graph** and **Reverse Graph**, in which we can use `type == 0` or `type == 1` to describe.

```
1  void spfa(int h[], int dist[], int type)
2  {
3      int hh = 0, tt = 1;
4      if (type == 0)
5      {
```

```

6   memset(dist, 0x3f, sizeof dmin);
7   dist[1] = w[1];
8   q[0] = 1;
9   }
10  else
11  {
12      memset(dist, -0x3f, sizeof dmax);
13      dist[n] = w[n];
14      q[0] = n;
15      }
16      while (hh != tt)
17      {
18          int t = q[hh++];
19          if (hh == N) hh = 0;
20          st[t] = false;
21          for (int i = h[t]; ~i; i = ne[i])
22          {
23              int j = e[i];
24              if (type == 0 && dist[j] > min(dist[t],
25                  w[j]) || type == 1 && dist[j] < max(dist
26                  [t], w[j]))
27              {
28                  if (type == 0)
29                      dist[j] = min(dist[t], w[j]);
30                  else
31                      dist[j] = max(dist[t], w[j]);
32                  if (!st[j])
33                  {
34                      q[tt++] = j;
35                      if (tt == N) tt = 0;
36                      st[j] = true;
37                  }
38              }
39          }

```

```

Ver>> heap;
24  heap.push({S, 0, 0});
25  while (heap.size())
26  {
27      Ver t = heap.top();
28      heap.pop();
29      int ver = t.id, type = t.type, distance =
30          t.dist, count = cnt[ver][type];
31      if (st[ver][type])
32          continue;
33      st[ver][type] = true;
34      for (int i = h[ver]; ~i; i = ne[i])
35      {
36          int j = e[i];
37          if (dist[j][0] > distance + w[i])
38          {
39              dist[j][1] = dist[j][0], cnt[j][1] =
40                  cnt[j][0];
41              heap.push({j, 1, dist[j][1]});
42              dist[j][0] = distance + w[i], cnt[j]
43                  [0] = count;
44              heap.push({j, 0, dist[j][0]});
45          }
46          else if (dist[j][0] == distance + w[i])
47              cnt[j][0] += count;
48          else if (dist[j][1] > distance + w[i])
49          {
50              dist[j][1] = distance + w[i], cnt[j]
51                  [1] = count;
52              heap.push({j, 1, dist[j][1]});
53          }
54          else if (dist[j][1] == distance + w[i])
55              cnt[j][1] += count;
56      }
57      int res = cnt[T][0];
58      if (dist[T][0] + 1 == dist[T][1])
59          res += cnt[T][1];
60      return res;

```

## 9.5 Second Shortest Path

```

1   const int N = 1010, M = 20010;
2   struct Ver
3   {
4       int id, type, dist;
5       bool operator>(const Ver &W) const
6       {
7           return dist > W.dist;
8       }
9   };
10  int n, m, S, T, dist[N][2], cnt[N][2];
11  int h[N], e[M], w[M], ne[M], idx;
12  bool st[N][2];
13  void add(int a, int b, int c)
14  {
15      e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
16          a] = idx++;
17  }
18  int dijkstra()
19  {
20      memset(st, 0, sizeof st);
21      memset(dist, 0x3f, sizeof dist);
22      memset(cnt, 0, sizeof cnt);
23      dist[S][0] = 0, cnt[S][0] = 1;
24      priority_queue<Ver, vector<Ver>, greater<

```

## 9.6 Second Minimum Spanning Tree

### 9.6.1 brute-force

```

1   const int N = 510, M = 10010;
2   int n, m, p[N], dist1[N][N], dist2[N][N];
3   int h[N], e[N * 2], w[N * 2], ne[N * 2], idx
4       ;
5   struct Edge
6   {
7       int a, b, w;
8       bool f;
9       bool operator<(const Edge &e) const
10      { return w < e.w; }
11  } edge[M];
12  void add(int a, int b, int c)
13  {
14      e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
15          a] = idx++;
16  }
17  int find(int x)

```

```

16 {
17     if (p[x] != x) p[x] = find(p[x]);
18     return p[x];
19 }
20 void dfs(int u, int fa, int maxd1, int maxd2
    , int d1[], int d2[])
21 {
22     d1[u] = maxd1, d2[u] = maxd2;
23     for (int i = h[u]; ~i; i = ne[i])
24     {
25         int j = e[i];
26         if (j != fa)
27         {
28             int td1 = maxd1, td2 = maxd2;
29             if (w[i] > td1)
30                 td2 = td1, td1 = w[i];
31             else if (w[i] < td1 && w[i] > td2)
32                 td2 = w[i];
33             dfs(j, u, td1, td2, d1, d2);
34         }
35     }
36 }
37 int main()
38 {
39     cin >> n >> m;
40     memset(h, -1, sizeof h);
41     for (int i = 0; i < m; i++)
42         cin >> edge[i].a >> edge[i].b >> edge[i].w
43         ;
44     sort(edge, edge + m);
45     for (int i = 1; i <= n; i++) p[i] = i;
46     LL sum = 0;
47     for (int i = 0; i < m; i++)
48     {
49         int a = edge[i].a, b = edge[i].b, w = edge
            [i].w;
50         int pa = find(a), pb = find(b);
51         if (pa != pb)
52         {
53             p[pa] = pb;
54             sum += w;
55             add(a, b, w), add(b, a, w);
56             edge[i].f = true;
57         }
58     }
59     for (int i = 1; i <= n; i++)
60         dfs(i, -1, -1e9, -1e9, dist1[i], dist2[i])
61         ;
62     LL res = 1e18;
63     for (int i = 0; i < m; i++)
64     {
65         if (!edge[i].f)
66         {
67             int a = edge[i].a, b = edge[i].b, w =
                edge[i].w;
68             LL t;
69             if (w > dist1[a][b])
70                 t = sum + w - dist1[a][b];
71             else if (w > dist2[a][b])
72                 t = sum + w - dist2[a][b];
73             res = min(res, t);
74         }
75     }
76 }

```

## 9.6.2 LCA

```

1     const int N = 100010, M = 300010;
2     int n, m, p[N], q[N];
3     int h[N], e[M], w[M], ne[M], idx;
4     int depth[N], fa[N][17], d1[N][17], d2[N]
        [17];
5     struct Edge
6     {
7         int a, b, w;
8         bool used;
9         bool operator<(const Edge &t) const
10         { return w < t.w; }
11     } edge[M];
12     void add(int a, int b, int c)
13     { e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
        a] = idx++; }
14     int find(int x)
15     {
16         if (p[x] != x) p[x] = find(p[x]);
17         return p[x];
18     }
19     LL kruskal()
20     {
21         for (int i = 1; i <= n; i++) p[i] = i;
22         sort(edge, edge + m);
23         LL res = 0;
24         for (int i = 0; i < m; i++)
25         {
26             int a = find(edge[i].a), b = find(edge[i].
                b), w = edge[i].w;
27             if (a != b)
28             {
29                 p[a] = b; res += w;
30                 edge[i].used = true;
31             }
32         }
33         return res;
34     }
35     void build()
36     {
37         memset(h, -1, sizeof h);
38         for (int i = 0; i < m; i++)
39             if (edge[i].used)
40             {
41                 int a = edge[i].a, b = edge[i].b, w =
                    edge[i].w;
42                 add(a, b, w), add(b, a, w);
43             }
44     }
45     void bfs()
46     {
47         memset(depth, 0x3f, sizeof depth);
48         depth[0] = 0, depth[1] = 1, q[0] = 1;
49         int hh = 0, tt = 0;
50         while (hh <= tt)
51         {
52             int t = q[hh++];
53             for (int i = h[t]; ~i; i = ne[i])
54             {
55                 int j = e[i];
56                 if (depth[j] > depth[t] + 1)
57                 {
58                     depth[j] = depth[t] + 1;
59                     q[++tt] = j;
60                     fa[j][0] = t;
61                     d1[j][0] = w[i], d2[j][0] = -INF;

```

```

62     for (int k = 1; k <= 16; k++)
63     {
64         int anc = fa[j][k - 1];
65         fa[j][k] = fa[anc][k - 1];
66         int distance[4] = {d1[j][k - 1],
67                             d2[j][k - 1],
68                             d1[anc][k - 1],
69                             d2[anc][k - 1]};
70         d1[j][k] = d2[j][k] = -INF;
71         for (int u = 0; u < 4; u++)
72         {
73             int d = distance[u];
74             if (d > d1[j][k])
75                 d2[j][k] = d1[j][k], d1[j][k] =
76                 d;
77             else if (d != d1[j][k] && d > d2[j]
78                 [k])
79                 d2[j][k] = d;
80         }
81     }
82 }
83 }
84 int lca(int a, int b, int w)
85 {
86     static int distance[N * 2];
87     int cnt = 0;
88     if (depth[a] < depth[b])
89         swap(a, b);
90     for (int k = 16; k >= 0; k--)
91     if (depth[fa[a][k]] >= depth[b])
92     {
93         distance[cnt++] = d1[a][k];
94         distance[cnt++] = d2[a][k];
95         a = fa[a][k];
96     }
97     if (a != b)
98     {
99         for (int k = 16; k >= 0; k--)
100         if (fa[a][k] != fa[b][k])
101         {
102             distance[cnt++] = d1[a][k];
103             distance[cnt++] = d2[a][k];
104             distance[cnt++] = d1[b][k];
105             distance[cnt++] = d2[b][k];
106             a = fa[a][k], b = fa[b][k];
107         }
108         distance[cnt++] = d1[a][0];
109         distance[cnt++] = d1[b][0];
110     }
111     int dist1 = -INF, dist2 = -INF;
112     for (int i = 0; i < cnt; i++)
113     {
114         int d = distance[i];
115         if (d > dist1)
116             dist2 = dist1, dist1 = d;
117         else if (d != dist1 && d > dist2)
118             dist2 = d;
119     }
120     if (w > dist1) return w - dist1;
121     if (w > dist2) return w - dist2;
122     return INF;
123 }
124 int main()
125 {

```

```

126     cin >> n >> m;
127     for (int i = 0; i < m; i++)
128     {
129         int a, b, c;
130         cin >> a >> b >> c;
131         edge[i] = {a, b, c};
132     }
133     LL sum = kruskal();
134     build();
135     bfs();
136     LL res = 1e18;
137     for (int i = 0; i < m; i++)
138     if (!edge[i].used)
139     {
140         int a = edge[i].a, b = edge[i].b, w =
141         edge[i].w;
142         res = min(res, sum + lca(a, b, w));
143     }
144     cout << res;
145 }

```

## 9.7 Difference Constraints

- size == N: Feasible Solution
- size == 1: Maximum/Minimum
- Maximum: Shortest Path
- Minimum: Longest Path

### 9.7.1 Maximum-Shortest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, 0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] > dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n) return true;
26                if (!st[j])
27                {
28                    st[j] = true;
29                    q[tt++] = j;
30                    if (tt == N) tt = 0;
31                }

```



```

32     }
33 }
34 }
35 return false;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_b <= x_a + k
40     // PROCESS
41 }

```

## 9.7.2 Minimum-Longest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, -0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] < dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n) return false;
26                if (!st[j])
27                {
28                    st[j] = true;
29                    q[tt++] = j;
30                    if (tt == N) tt = 0;
31                }
32            }
33        }
34    }
35    return true;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_a + k <= x_b
40     // PROCESS
41 }

```

## 9.8 LCA

```

1 int n, m, h[N], e[M], ne[M], idx;
2 int depth[N], fa[N][16], q[N];
3 void bfs(int root)
4 {

```

```

5     memset(depth, 0x3f, sizeof depth);
6     depth[0] = 0;
7     depth[root] = 1;
8     int hh = 0, tt = 0;
9     q[0] = root;
10    while (hh <= tt)
11    {
12        int t = q[hh++];
13        for (int i = h[t]; ~i; i = ne[i])
14        {
15            int j = e[i];
16            if (depth[j] > depth[t] + 1)
17            {
18                depth[j] = depth[t] + 1;
19                q[++tt] = j;
20                fa[j][0] = t;
21                for (int k = 1; k <= 15; k++)
22                    fa[j][k] = fa[fa[j][k-1]][k-1];
23            }
24        }
25    }
26 }
27 int lca(int a, int b)
28 {
29     if (depth[a] < depth[b]) swap(a, b);
30     for (int k = 15; k >= 0; k--)
31         if (depth[fa[a][k]] >= depth[b])
32             a = fa[a][k];
33     if (a == b) return a;
34     for (int k = 15; k >= 0; k--)
35         if (fa[a][k] != fa[b][k])
36         {
37             a = fa[a][k];
38             b = fa[b][k];
39         }
40     return fa[a][0];
41 }

```

## 9.9 SCC

```

1 void tarjan(int u)
2 {
3     dfn[u] = low[u] = ++timestamp;
4     stack[++top] = u, in_stk[u] = true;
5     for (int i = h[u]; ~i; i = ne[i])
6     {
7         int j = e[i];
8         if (!dfn[j])
9         {
10            tarjan(j);
11            low[u] = min(low[u], low[j]);
12        }
13        else if (in_stk[j])
14            low[u] = min(low[u], dfn[j]);
15    }
16    if (dfn[u] == low[u])
17    {
18        int y;
19        ++scc_cnt;
20        do
21        {
22            y = stk[top--];
23            in_stk[y] = false;

```

```

24     id[y] = scc_cnt;
25 } while (y != u);
26 }
27 }

```

## 9.10 DCC

### 9.10.1 e-DCC

```

1  const int N = 5010, M = 20010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, id[N], dcc_cnt, d[N];
5  bool is_bridge[M];
6  void tarjan(int u, int from)
7  {
8      dfn[u] = low[u] = ++timestamp;
9      stk[++top] = u;
10     for (int i = h[u]; ~i; i = ne[i])
11     {
12         int j = e[i];
13         if (!dfn[j])
14         {
15             tarjan(j, i);
16             low[u] = min(low[u], low[j]);
17             if (dfn[u] < low[j])
18                 is_bridge[i] = is_bridge[i ^ 1] = true;
19         }
20         else if (i != (from ^ 1))
21             low[u] = min(low[u], dfn[j]);
22     }
23     if (dfn[u] == low[u])
24     {
25         ++dcc_cnt;
26         int y;
27         do
28         {
29             y = stk[top--];
30             id[y] = dcc_cnt;
31         } while (y != u);
32     }
33 }

```

### 9.10.2 v-DCC

```

1  const int N = 1010, M = 1010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, dcc_cnt, root;
5  vector<int> dcc[N];
6  bool cut[N];
7  void init()
8  {
9      for (int i = 1; i <= dcc_cnt; i++)
10         dcc[i].clear();
11     idx = n = timestamp = top = dcc_cnt = 0;
12     memset(h, -1, sizeof h);
13     memset(dfn, 0, sizeof dfn);
14     memset(cut, 0, sizeof cut);
15 }

```

```

16 void tarjan(int u)
17 {
18     dfn[u] = low[u] = ++timestamp;
19     stk[++top] = u;
20     if (u == root && h[u] == -1)
21     {
22         dcc_cnt++;
23         dcc[dcc_cnt].push_back(u);
24         return;
25     }
26     int cnt = 0;
27     for (int i = h[u]; ~i; i = ne[i])
28     {
29         int j = e[i];
30         if (!dfn[j])
31         {
32             tarjan(j);
33             low[u] = min(low[u], low[j]);
34             if (dfn[u] <= low[j])
35             {
36                 cnt++;
37                 if (u != root || cnt > 1)
38                     cut[u] = true;
39                 ++dcc_cnt;
40                 int y;
41                 do
42                 {
43                     y = stk[top--];
44                     dcc[dcc_cnt].push_back(y);
45                 } while (y != j);
46                 dcc[dcc_cnt].push_back(u);
47             }
48         }
49         else
50             low[u] = min(low[u], dfn[j]);
51     }
52 }

```

## 9.11 Bipartite Graph

The maximum matching  
(by the Hungarian algorithm) =  
the minimum vertex cover =  
total number of vertices -  
maximum independent set =  
total number of vertices -  
minimum path cover.

### 9.11.1 maximum matching

```

1  const int N = 110;
2  int n, m;
3  int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
4  PII match[N][N];
5  bool g[N][N], st[N][N];
6  bool find(int x, int y)
7  {
8      for (int i = 0; i < 4; i++)
9      {
10         int a = x + dx[i], b = y + dy[i];
11         if (a && a <= n && b && b <= n && !g[a][b] && !st[a][b])

```

```

12 {
13     st[a][b] = true;
14     PII t = match[a][b];
15     if (t.x == -1 || find(t.x, t.y))
16     {
17         match[a][b] = {x, y};
18         return true;
19     }
20 }
21 }
22 return false;
23 }
24 int main()
25 {
26     // PROCESS
27     memset(match, -1, sizeof match);
28     int res = 0;
29     for (int i = 1; i <= n; i++)
30     for (int j = 1; j <= n; j++)
31         if ((i + j) % 2 && !g[i][j])
32         {
33             memset(st, 0, sizeof st);
34             if (find(i, j))
35                 res++;
36         }
37     // PROCESS
38 }

```

### 9.11.2 minimum vertex cover

```

1  const int N = 110;
2  int n, m, k, match[N];
3  bool g[N][N], st[N];
4  bool find(int x)
5  {
6      for (int i = 0; i < m; i++)
7          if (!st[i] && g[x][i])
8          {
9              st[i] = true;
10             if (match[i] == -1 || find(match[i]))
11             {
12                 match[i] = x;
13                 return true;
14             }
15         }
16     return false;
17 }
18 int main()
19 {
20     while (cin >> n, n)
21     {
22         cin >> m >> k;
23         memset(g, 0, sizeof g);
24         memset(match, -1, sizeof match);
25         while (k--)
26         {
27             int t, a, b;
28             cin >> t >> a >> b;
29             if (!a || !b) continue;
30             g[a][b] = true;
31         }
32         int res = 0;
33         for (int i = 0; i < n; i++)
34         {

```

```

35             memset(st, 0, sizeof st);
36             if (find(i)) res++;
37         }
38         cout << res << '\n';
39     }
40 }

```

### 9.11.3 maximum independent set

```

1  const int N = 110;
2  int n, m, k;
3  PII match[N][N];
4  bool g[N][N], st[N][N];
5  int dx[8] = {-2, -1, 1, 2, 2, 1, -1, -2};
6  int dy[8] = {1, 2, 2, 1, -1, -2, -2, -1};
7  bool find(int x, int y)
8  {
9      for (int i = 0; i < 8; i++)
10     {
11         int a = x + dx[i], b = y + dy[i];
12         if (a < 1 || a > n || b < 1 || b > m)
13             continue;
14         if (g[a][b]) continue;
15         if (st[a][b]) continue;
16         st[a][b] = true;
17         PII t = match[a][b];
18         if (t.x == 0 || find(t.x, t.y))
19         {
20             match[a][b] = {x, y};
21             return true;
22         }
23     }
24     return false;
25 }
26 int main()
27 {
28     // PROCESS
29     int res = 0;
30     for (int i = 1; i <= n; i++)
31     for (int j = 1; j <= m; j++)
32     {
33         if (g[i][j] || (i + j) % 2)
34             continue;
35         memset(st, 0, sizeof st);
36         if (find(i, j)) res++;
37     }
38     cout << n * m - k - res << '\n';
39 }

```

### 9.11.4 minimum path cover

- Only for DAG.
- If you need to compute the **minimum path cover with repeated nodes**, you need to perform transitive closure as shown in the following code.

```

1  const int N = 210, M = 30010;
2  int n, m, match[N];
3  bool d[N][N], st[N];
4  bool find(int x)

```

```

5 {
6     for (int i = 1; i <= n; i++)
7         if (d[x][i] && !st[i])
8         {
9             st[i] = true;
10            int t = match[i];
11            if (t == 0 || find(t))
12            {
13                match[i] = x;
14                return true;
15            }
16        }
17    return false;
18 }
19 int main()
20 {
21     // 传递闭包
22     for (int k = 1; k <= n; k++)
23         for (int i = 1; i <= n; i++)
24             for (int j = 1; j <= n; j++)
25                 d[i][j] |= d[i][k] & d[k][j];
26     int res = 0;
27     for (int i = 1; i <= n; i++)
28     {
29         memset(st, 0, sizeof st);
30         if (find(i)) res++;
31     }
32     cout << n - res;
33 }

```

```

21     t = i / 2 + 1;
22     if (i & 1) t = -t;
23 }
24 else t = i + 1;
25 int j = e[i];
26 i = ne[i];
27 dfs(j);
28 ans[++cnt] = t;
29 }
30 }
31 int main()
32 {
33     cin >> type >> n >> m;
34     memset(h, -1, sizeof h);
35     for (int i = 0; i < m; i++)
36     {
37         int a, b;
38         cin >> a >> b;
39         add(a, b);
40         if (type == 1) add(b, a);
41         din[b]++, dout[a]++;
42     }
43     if (type == 1)
44     {
45         for (int i = 1; i <= n; i++)
46             if (din[i] + dout[i] & 1)
47             {
48                 cout << "NO\n";
49                 return 0;
50             }
51     }
52     else
53     {
54         for (int i = 1; i <= n; i++)
55             if (din[i] != dout[i])
56             {
57                 cout << "NO\n";
58                 return 0;
59             }
60     }
61     for (int i = 1; i <= n; i++)
62         if (h[i] != -1)
63         {
64             dfs(i);
65             break;
66         }
67 }

```

## 9.12 Eulerian Circuit & Eulerian Path

### 9.12.1 Eulerian Circuit

- **Undirected Graph:** If and only if it is connected and every vertex has even degree.
- **Directed Graph:** If and only if it is strongly connected and each vertex has equal in-degree and out-degree.

```

1 int type, n, m;
2 int h[N], e[M], ne[M], idx;
3 bool used[M];
4 int ans[M], cn, din[N], dout[N];
5 void add(int a, int b)
6 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
7 }
8 void dfs(int u)
9 {
10     for (int &i = h[u]; ~i; i = ne[i])
11     {
12         if (used[i])
13             i = ne[i];
14         continue;
15     }
16     used[i] = true;
17     if (type == 1) used[i ^ 1] = true;
18     int t;
19     if (type == 1)
20     {

```

### 9.12.2 Eulerian Path

#### Undirected Graph

If and only if it is connected (ignoring isolated vertices) and has exactly 0 or 2 vertices with odd degree.

```

1 const int N = 510;
2 int n = 500, m, g[N][N];
3 int ans[1100], cnt, d[N];
4 void dfs(int u)
5 {
6     for (int i = 1; i <= n; i++)
7         if (g[u][i])
8         {
9             g[u][i]--, g[i][u]--;
10            dfs(i);

```

```

11     }
12     ans[++cnt] = u;
13 }
14 int main()
15 {
16     cin >> m;
17     while (m--)
18     {
19         int a, b;
20         cin >> a >> b;
21         g[a][b]++, g[b][a]++;
22         d[a]++, d[b]++;
23     }
24     int start = 1;
25     while (!d[start])
26         ++start;
27     for (int i = 1; i <= 500; i++)
28         if (d[i] % 2)
29         {
30             start = i;
31             break;
32         }
33     dfs(start);
34 }

```

## Directed Graph

If and only if it is connected in terms of non-zero degree vertices, and

- At most one vertex has  
(out-degree) — (in-degree) = 1
- At most one vertex has  
(in-degree) — (out-degree) = 1
- All other vertices have  
equal in-degree and out-degree

```

27     dout[a]++, din[b]++;
28     p[find(a)] = find(b);
29 }
30 int start = 0, end = 0;
31 bool success = true;
32 for (int i = 0; i < 26; i++)
33     if (din[i] != dout[i])
34     {
35         if (din[i] == dout[i] + 1) end++;
36         else if (din[i] + 1 == dout[i])
37             start++;
38         else
39         {
40             success = false;
41             break;
42         }
43     }
44 if (success && (!(start && !end || start
45 == 1 && end == 1))
46     success = false;
47 int rep = -1;
48 for (int i = 0; i < 26; i++)
49     if (st[i])
50     {
51         if (rep == -1) rep = find(i);
52         else if (rep != find(i))
53         {
54             success = false;
55             break;
56         }
57     }
58 return 0;
59 }

```

```

1  const int N = 30;
2  int n, p[N], din[N], dout[N];
3  bool st[N];
4  int find(int x)
5  {
6      if (x != p[x]) p[x] = find(p[x]);
7      return p[x];
8  }
9  int main()
10 {
11     char str[1010];
12     int T;
13     cin >> T;
14     while (T--)
15     {
16         cin >> n;
17         memset(din, 0, sizeof din);
18         memset(dout, 0, sizeof dout);
19         memset(st, 0, sizeof st);
20         for (int i = 0; i < 26; i++) p[i] = i;
21         for (int i = 0; i < n; i++)
22         {
23             cin >> str;
24             int a = str[0] - 'a',
25                 b = str[strlen(str) - 1] - 'a';
26             st[a] = st[b] = true;

```

## 10 ★ Advanced Math

### 10.1 Euler's Totient Function

#### 10.1.1 GCD

```
1  const int N = 1e7 + 10;
2  int primes[N], cnt, phi[N];
3  bool st[N];
4  LL s[N];
5  void init(int n)
6  {
7      for (int i = 2; i <= n; i++)
8      {
9          if (!st[i])
10             {
11                 primes[cnt++] = i;
12                 phi[i] = i - 1;
13             }
14             for (int j = 0; primes[j] * i <= n; j++)
15                 {
16                     st[primes[j] * i] = true;
17                     if (i % primes[j] == 0)
18                         {
19                             phi[i * primes[j]] = phi[i] *
primes[j];
20                             break;
21                         }
22                     phi[i * primes[j]] = phi[i] * (
primes[j] - 1);
23                 }
24             }
25     for (int i = 1; i <= n; i++)
26         s[i] = s[i - 1] + phi[i];
27 }
28 int main()
29 {
30     int n; cin >> n;
31     init(n);
32     LL res = 0;
33     for (int i = 0; i < cnt; i++)
34     {
35         int p = primes[i];
36         res += s[n / p] * 2 + 1;
37     }
38 }
```

```
11 void mul(int c[][N], int a[][N], int b[][N])
12 {
13     int temp[N][N] = {0};
14     for (int i = 0; i < N; i++)
15         for (int j = 0; j < N; j++)
16             for (int k = 0; k < N; k++)
17                 temp[i][j] = (temp[i][j] + (LL
)a[i][k] * b[k][j]) % m;
18     memcpy(c, temp, sizeof temp);
19 }
20 int main()
21 {
22     while (n)
23     {
24         if (n & 1) mul(f1, f1, a);
25         mul(a, a, a); n >>= 1;
26     }
27 }
```

### 10.2 Matrix Multiplication

```
1  const int N = 3;
2  int n, m;
3  void mul(int c[], int a[], int b[][N])
4  {
5      int temp[N] = {0};
6      for (int i = 0; i < N; i++)
7          for (int j = 0; j < N; j++)
8              temp[i] = (temp[i] + (LL)a[j] * b[
j][i]) % m;
9      memcpy(c, temp, sizeof temp);
10 }
```

## 11 ★ Advanced DP

### 11.1 Advanced Linear DP

#### 11.1.1 Two-pass grid collection problem

In this case we run DP on two different roads at the same time:

```
1  const int N = 15;
2  int n, w[N][N], f[N * 2][N][N];
3  int main()
4  {
5      cin >> n;
6      // INPUT w[N][N]
7      for (int k = 2; k <= n * 2; k++)
8          for (int i1 = 1; i1 <= k; i1++)
9              for (int i2 = 1; i2 <= k; i2++)
10                 {
11                     int j1 = k - i1, j2 = k - i2;
12                     int t = w[i1][j1];
13                     if (i1 != i2) t += w[i2][j2];
14                     int &x = f[k][i1][i2];
15                     x = max(x, f[k - 1][i1 - 1][i2 - 1] +
16                             t);
17                     x = max(x, f[k - 1][i1 - 1][i2] + t);
18                     x = max(x, f[k - 1][i1][i2 - 1] + t);
19                     x = max(x, f[k - 1][i1][i2] + t);
20                 }
21     cout << f[n * 2][n][n] << '\n';
22     return 0;
23 }
```

## 11.2 Advanced LIS

### 11.2.1 Longest Bitonic Subsequence

```
1  const int N = 1010;
2  int n, a[N], f[N], g[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[i] > a[j])
13                 f[i] = max(f[i], f[j] + 1);
14     }
15     for (int i = n; i >= 1; i--)
16     {
17         g[i] = 1;
18         for (int j = n; j > i; j--)
19             if (a[i] > a[j])
20                 g[i] = max(g[i], g[j] + 1);
21     }
22     int ans = 0;
23     for (int i = 1; i <= n; i++)
24         ans = max(ans, g[i] + f[i] - 1);
25     cout << ans << '\n';
26 }
```

```
26     return 0;
27 }
```

#### 11.2.2 MSIS

MSIS means Maximum Sum Increasing Subsequence

```
1  const int N = 1010;
2  int n, w[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 0; i < n; i++) cin >> w[i];
7      int res = 0;
8      for (int i = 0; i < n; i++)
9      {
10         f[i] = w[i];
11         for (int j = 0; j < i; j++)
12             if (w[i] > w[j])
13                 f[i] = max(f[i], f[j] + w[i]);
14         res = max(res, f[i]);
15     }
16     cout << res;
17 }
```

#### 11.2.3 LCIS

LCIS means Longest Common Increasing Subsequence

```
1  const int N = 3010;
2  int n, a[N], b[N], f[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          cin >> b[i];
10     for (int i = 1; i <= n; i++)
11     {
12         int maxv = 1;
13         for (int j = 1; j <= n; j++)
14         {
15             f[i][j] = f[i - 1][j];
16             if (a[i] == b[j])
17                 f[i][j] = max(f[i][j], maxv);
18             if (a[i] > b[j])
19                 maxv = max(maxv, f[i - 1][j] +
20                             1);
21         }
22     }
23     int res = 0;
24     for (int i = 1; i <= n; i++)
25         res = max(res, f[n][i]);
26     cout << res;
27 }
```

## 11.3 Knapsack Problem

### 11.3.1 How To Initialize

Initialization for Counting the Number of Solutions:

- **2D Case:**

- When volume is at most  $j$ :  $f[0][i] = 1$  for  $0 \leq i \leq m$ , others are 0
- When volume is exactly  $j$ :  $f[0][0] = 1$ , others are 0
- When volume is at least  $j$ :  $f[0][0] = 1$ , others are 0

- **1D Case:**

- When volume is at most  $j$ :  $f[i] = 1$  for  $0 \leq i \leq m$
- When volume is exactly  $j$ :  $f[0] = 1$ , others are 0
- When volume is at least  $j$ :  $f[0] = 1$ , others are 0

```

13     int hh = 0, tt = -1;
14     for (int j = r; j <= m; j += v)
15     {
16         while (hh <= tt && j - s * v > q[hh])
17             hh++;
18         while (hh <= tt && g[q[tt]] + (j - q[tt]) / v * w <= g[j])
19             tt--;
20         q[++tt] = j;
21         f[j] = g[q[hh]] + (j - q[hh]) / v * w;
22     }
23 }
24 }
25 cout << f[m];
26 }

```

Initialization for Finding Maximum or Minimum Value:

- **2D Case:**

- When volume is at most  $j$ :  $f[i][k] = 0$  for  $0 \leq i \leq n, 0 \leq k \leq m$  (only for maximizing value)
- When volume is exactly  $j$ :
  - \* For minimizing value:  $f[0][0] = 0$ , others are INF
  - \* For maximizing value:  $f[0][0] = 0$ , others are -INF
- When volume is at least  $j$ :  $f[0][0] = 0$ , others are INF (only for minimizing value)

- **1D Case:**

- When volume is at most  $j$ :  $f[i] = 0$  for  $0 \leq i \leq m$  (only for maximizing value)
- When volume is exactly  $j$ :
  - \* For minimizing value:  $f[0] = 0$ , others are INF
  - \* For maximizing value:  $f[0] = 0$ , others are -INF
- When volume is at least  $j$ :  $f[0] = 0$ , others are INF (only for minimizing value)

```

1  const int N = 110;
2  int n, V, M, f[N][N];
3  int main()
4  {
5      cin >> n >> V >> M;
6      for (int i = 0; i < n; i++)
7      {
8          int v, m, w;
9          cin >> v >> m >> w;
10         for (int j = V; j >= v; j--)
11             for (int k = M; k >= m; k--)
12                 f[j][k] = max(f[j][k], f[j - v][k - m] + w);
13     }
14     cout << f[V][M] << '\n';
15 }

```

### 11.3.2 Multiple Knapsack Problem

```

1  const int M = 20010;
2  int n, m, v, w, s;
3  int f[M], g[M], q[M];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8      {
9          cin >> v >> w >> s;
10         memcpy(g, f, sizeof g);
11         for (int r = 0; r < v; r++)
12         {

```

### 11.3.3 Two-Dimensional Cost Knapsack Problem

### 11.3.4 Finding the Actual Solution Set

```

1  const int N = 1010;
2  int n, m;
3  int v[N], w[N], f[N][N];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8          cin >> v[i] >> w[i];
9      for (int i = n; i >= 1; i--)
10         for (int j = 0; j <= m; j++)
11         {
12             f[i][j] = f[i + 1][j];
13             if (j >= v[i])
14                 f[i][j] = max(f[i][j], f[i + 1][j - v[i]] + w[i]);
15         }
16     int j = m;
17     for (int i = 1; i <= n; i++)
18         if (j >= v[i] && f[i][j] == f[i + 1][j - v[i]] + w[i])
19             {

```



```

20         cout << i << ' ';
21         j -= v[i];
22     }
23 }

```

### 11.3.5 Maximum Linearly Independent Subset

```

1  const int N = 110, M = 25010;
2  int n, v[N];
3  bool f[M];
4  int main()
5  {
6      int T; cin >> T;
7      while (T--)
8      {
9          cin >> n;
10         for (int i = 1; i <= n; ++i)
11             cin >> v[i];
12         sort(v + 1, v + n + 1);
13         int m = v[n], res = 0;
14         memset(f, 0, sizeof f);
15         f[0] = true;
16         for (int i = 1; i <= n; ++i)
17         {
18             if (f[v[i]]) continue;
19             res++;
20             for (int j = v[i]; j <= m; ++j)
21                 f[j] |= f[j - v[i]];
22         }
23         cout << res << '\n';
24     }
25 }

```

### 11.3.6 Mixed Knapsack Problem

```

1  const int N = 1010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         if (!s)
11         {
12             for (int j = v; j <= m; j++)
13                 f[j] = max(f[j], f[j - v] + w);
14         }
15         else
16         {
17             if (s == -1) s = 1;
18             for (int k = 1; k <= s; k *= 2)
19             {
20                 for (int j = m; j >= k * v; j
21                     --)
22                     f[j] = max(f[j], f[j - k *
23                         v] + k * w);
24                 s -= k;
25             }
26         }
27     }
28 }

```

```

23     }
24     if (s)
25     {
26         for (int j = m; j >= s * v; j
27             --)
28             f[j] = max(f[j], f[j - s *
29                 v] + s * w);
30     }
31     cout << f[m] << '\n';
32 }

```

### 11.3.7 Dependent Knapsack Problem

```

1  const int N = 110;
2  int n, m, root;
3  int h[N], e[N], ne[N], idx;
4  int v[N], w[N], [N][N];
5  void add(int a, int b)
6  {
7      e[idx] = b, ne[idx] = h[a], h[a] = idx++;
8  }
9  void dfs(int u)
10 {
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         int son = e[i];
14         dfs(son);
15         for (int j = m - v[u]; j >= 0; --j)
16             for (int k = 0; k <= j; ++k)
17                 f[u][j] = max(f[u][j], f[u][j
18                     - k] + f[son][k]);
19     }
20     for (int j = m; j >= v[u]; --j)
21         f[u][j] = f[u][j - v[u]] + w[u];
22     for (int j = 0; j < v[u]; ++j)
23         f[u][j] = 0;
24 }
25 int main()
26 {
27     memset(h, -1, sizeof h);
28     cin >> n >> m;
29     for (int i = 1; i <= n; ++i)
30     {
31         int p;
32         cin >> v[i] >> w[i] >> p;
33         if (p == -1) root = i;
34         else add(p, i);
35     }
36     dfs(root);
37     cout << f[root][m] << '\n';
38 }

```

### 11.3.8 Number of Solutions

```

1  const int N = 1010, mod = 1e9 + 7;
2  int n, m;
3  int w[N], v[N], f[N], g[N];
4  int main()
5  {
6      cin >> n >> m;

```

```

7   for (int i = 1; i <= n; ++i)
8       cin >> v[i] >> w[i];
9   g[0] = 1;
10  for (int i = 1; i <= n; ++i)
11  {
12      for (int j = m; j >= v[i]; --j)
13      {
14          int temp = max(f[j], f[j - v[i]] +
15                          w[i]), c = 0;
16          if (temp == f[j])
17              c = (c + g[j]) % mod;
18          if (temp == f[j - v[i]] + w[i])
19              c = (c + g[j - v[i]]) % mod;
20          f[j] = temp, g[j] = c;
21      }
22      int res = 0;
23      for (int j = 0; j <= m; ++j)
24          if (f[j] == f[m])
25              res = (res + g[j]) % mod;
26      cout << res << '\n';
27  }

```

```

13     ne[i] = j;
14 }
15 f[0][0] = 1;
16 for (int i = 0; i < n; i++)
17     for (int j = 0; j < m; j++)
18         for (char ch = 'a'; ch <= 'z'; ch++)
19         {
20             int ptr = j;
21             while (ptr && s[ptr + 1] != ch)
22                 ptr = ne[ptr];
23             if (s[ptr + 1] == ch) ptr++;
24             f[i + 1][ptr] = (f[i + 1][ptr] + f[i
25 ] [j]) % mod;
26         }
27 int res = 0;
28 for (int j = 0; j < m; j++)
29     res = (res + f[n][j]) % mod;
30 cout << res;
31 }

```

### 11.4.3 Linear DP + AC Automaton

## 11.4 FSM

### 11.4.1 Common FSM

```

1   const int N = 100010;
2   int n, w[N], f[N][2];
3   int main()
4   {
5       int T; cin >> T;
6       while (T--)
7       {
8           cin >> n;
9           for (int i = 1; i <= n; i++)
10              cin >> w[i];
11          for (int i = 1; i <= n; i++)
12          {
13              // YOUR_FSM_RULES
14              // f[i][0] =
15              // f[i][1] =
16          }
17          cout << max(f[n][0], f[n][1]) << '\n';
18      }
19  }

```

### 11.4.2 Linear DP + KMP

```

1   const int N = 55, mod = 1e9 + 7
2   int n, m, f[N][N], ne[N];
3   char s[N];
4   int main()
5   {
6       cin >> n >> s + 1;
7       m = strlen(s + 1);
8       for (int i = 2, j = 0; i <= m; i++)
9       {
10          while (j && s[i] != s[j + 1])
11              j = ne[j];
12          if (s[j + 1] == s[i]) j++;

```

```

1   const int N = 1010, INF = 0x3f3f3f3f;
2   int n, m, T = 1, f[N][N];
3   int tr[N][4], dar[N], idx;
4   int q[N], ne[N];
5   char str[N];
6   int get(char c)
7   {
8       if (c == 'A') return 0;
9       if (c == 'T') return 1;
10      if (c == 'G') return 2;
11      return 3;
12  }
13 void insert()
14 {
15     int p = 0;
16     for (int i = 0; str[i]; i++)
17     {
18         int t = get(str[i]);
19         if (tr[p][t] == 0)
20             tr[p][t] = ++idx;
21         p = tr[p][t];
22     }
23     dar[p] = 1;
24 }
25 void build()
26 {
27     int hh = 0, tt = -1;
28     for (int i = 0; i < 4; i++)
29         if (tr[0][i])
30             q[++tt] = tr[0][i];
31     while (hh <= tt)
32     {
33         int t = q[hh++];
34         for (int i = 0; i < 4; i++)
35         {
36             int p = tr[t][i];
37             if (!p)
38                 tr[t][i] = tr[ne[t]][i];
39             else
40             {
41                 ne[p] = tr[ne[t]][i];
42                 q[++tt] = p;

```

```

43     dar[p] |= dar[ne[p]];
44 }
45 }
46 }
47 }
48 int main()
49 {
50     int T = 1;
51     while (cin >> n, n)
52     {
53         memset(tr, 0, sizeof tr);
54         memset(dar, 0, sizeof dar);
55         memset(ne, 0, sizeof ne);
56         idx = 0;
57         for (int i = 0; i < n; i++)
58         {
59             cin >> str;
60             insert();
61         }
62         build();
63         cin >> (str + 1);
64         m = strlen(str + 1);
65         memset(f, 0x3f, sizeof f);
66         f[0][0] = 0;
67         for (int i = 0; i < m; i++)
68             for (int j = 0; j <= idx; j++)
69                 for (int k = 0; k < 4; k++)
70                 {
71                     int t = get(str[i + 1]) != k;
72                     int p = tr[j][k];
73                     if (!dar[p])
74                         f[i + 1][p] = min(f[i + 1][p], f[i][j] + t);
75                 }
76         int res = INF;
77         for (int i = 0; i <= idx; i++)
78             res = min(res, f[m][i]);
79         if (res == INF) res = -1;
80         cout << "Case " << T++
81              << ": " << res << '\n';
82     }
83 }

```

## 11.5 Digit DP

```

1  const int N = 35;
2  int l, r, k, b, a[N], al, f[N][N];

```

```

3  int dp(int pos, int st, int op)
4  {
5      if (!pos) return st == k;
6      if (!op && ~f[pos][st])
7          return f[pos][st];
8      int res = 0, maxx = op ? min(a[pos], 1) : 1;
9      for (int i = 0; i <= maxx; i++)
10     {
11         if (st + i > k) continue;
12         res += dp(pos - 1, st + i, op && i == a[pos]);
13     }
14     return op ? res : f[pos][st] = res;
15 }
16 int calc(int x)
17 {
18     al = 0;
19     memset(f, -1, sizeof f);
20     while (x) a[++al] = x % b, x /= b;
21     return dp(al, 0, 1);
22 }
23 int main()
24 {
25     cin >> l >> r >> k >> b;
26     cout << calc(r) - calc(l - 1) << '\n';
27 }

```

## 11.6 Queue Optimization for DP

```

1  int n, m, s[300010], q[300010];
2  int main()
3  {
4      cin >> n >> m;
5      for (int i = 1; i <= n; i++)
6          cin >> s[i], s[i] += s[i - 1];
7      int res = INT_MIN, hh = 0, tt = 0;
8      for (int i = 1; i <= n; i++)
9      {
10         if (q[hh] < i - m) hh++;
11         res = max(res, s[i] - s[q[hh]]);
12         while (hh <= tt && s[q[tt]] >= s[i])
13             tt--;
14         q[++tt] = i;
15     }

```