



国家超级计算广州中心
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

XCPC-Template

CREATED BY

Luliet Lyan & Bleu Echo

NSCC-GZ

School of Computer Science & Engineering
Sun Yat-Sen University

Supervisor: Dr Dan Huang

Co-Supervisor: Dr Zhiguang Chen

Friday 18th July, 2025

Contents

0	Preface	4	4	Basic Math	16
0.1	Template	4	4.1	Prime Numbers	16
0.2	Operator Precedence	4	4.1.1	Judging Prime Numbers	16
0.3	Time Complexity	4	4.1.2	Prime Factorization	16
0.4	If <code><bits/stdc++.h></code> Failed	5	4.1.3	Euler's Sieve	16
1	Basic Algorithm	6	4.2	Divisor	16
1.1	Quick Sort	6	4.2.1	Find All Divisors	16
1.2	Binary Search	6	4.2.2	The Number of Divisors	16
1.3	Ternary Search	6	4.2.3	The Sum of Divisors	16
1.4	High Precision	6	4.2.4	Euclidean Algorithm	17
1.4.1	High Precision Add	6	4.3	Euler Function	17
1.4.2	High Precision Subsection	7	4.3.1	Simple Method	17
1.4.3	High Precision Multiply	7	4.3.2	Euler's Sieve Method	17
1.4.4	High Precision Divide	7	4.4	Exponentiating by Squaring	17
1.5	Prefix Sum & Difference Array	7	4.5	Extended Euclidean Algorithm	17
1.5.1	1D Prefix Sum	7	4.6	Chinese Remainder Theorem	17
1.5.2	2D Prefix Sum	8	4.7	Gauss-Jordan Elimination	18
1.5.3	1D Difference Array	8	4.7.1	Linear Equation Group	18
1.5.4	2D Difference Array	8	4.7.2	XOR Linear Equation Group	18
2	Basic Data Structures	9	4.8	Combinatorial Counting	18
2.1	Linked List	9	4.8.1	Recurrence Relation	18
2.1.1	Singly Linked List	9	4.8.2	Preprocessing & Inverse Element	18
2.1.2	Bidirectional Linked List	9	4.8.3	Lucas Theorem	19
2.2	Stack & Queue	9	4.8.4	Factorization Method	19
2.2.1	Monotonic Stack	9	4.8.5	Catalan Number	19
2.2.2	Monotonic Queue	9	4.9	Inclusion-Exclusion Principle	19
2.3	KMP	9	4.10	Game Theory	20
2.4	Trie	9	4.10.1	NIM Game	20
2.5	Disjoint-Set	10	5	Basic DP	21
2.6	Hash	10	5.1	Knapsack Problem	21
2.6.1	Simple Hash	10	5.1.1	01 Knapsack	21
2.6.2	String Hash	10	5.1.2	Complete Knapsack	21
2.7	STL	10	5.1.3	Mutiple Knapsack	21
3	Search & Graph Theory	12	5.1.4	Grouped Knapsack	21
3.1	Representation of Tree & Graph	12	5.2	Linear DP	21
3.1.1	Adjacency Matrix	12	5.2.1	LIS	21
3.1.2	Adjacency List	12	5.2.2	LCS	22
3.2	DFS & BFS	12	5.3	Interval DP	22
3.2.1	DFS	12	5.4	Counting DP	22
3.2.2	BFS	12	5.5	Digit DP	22
3.3	Topological Sort	12	5.6	State Compression DP	23
3.4	Shortest Path	12	5.7	Tree DP	23
3.4.1	Dijkstra	12	5.8	Memoized Search	24
3.4.2	Bellman-Ford	12	6	Advanced Basic	26
3.4.3	SPFA	13	6.1	Slow Multiplication	26
3.4.4	Detecting Negative Circle in SPFA	13	6.2	Sum of Geometric Series	26
3.4.5	Floyd	13	6.3	Sort	26
3.5	Minimum Spanning Tree	13	6.3.1	Card Balancing Problem	26
3.5.1	Prim	13	6.3.2	2D Card Balancing Problem	26
3.5.2	Kruskal	14	6.3.3	Dual Heaps	26
3.6	Bipartite Graph	14	6.4	RMQ	27
3.6.1	Coloring Method	14			
3.6.2	Hungarian Algorithm	15			

7	Advanced Data Structures	28	11	Advanced DP	46
7.1	Binary Indexed Tree	28	11.1	Advanced Linear DP	46
7.2	Segment Tree	28	11.1.1	Two-pass grid collection problem	46
7.2.1	Maintain the Maximum	28	11.2	Advanced LIS	46
7.2.2	Maintain the Maximum Subarray Sum	28	11.2.1	Longest Bitonic Subsequence . .	46
7.2.3	Maintain the GCD	29	11.2.2	MSIS	46
7.2.4	Optimize Range Updates	29	11.2.3	LCIS	46
7.3	Persistent Data Structure	30	11.3	Knapsack Problem	46
7.3.1	Persistent Trie	30	11.3.1	How To Initialize	46
7.3.2	Persistent Segment Tree	30	11.3.2	Multiple Knapsack Problem . . .	47
7.4	Treap	31	11.3.3	Two-Dimensional Cost Knapsack Problem	47
7.5	AC Automaton	32	11.3.4	Finding the Actual Solution Set .	47
8	Advanced Search	33	11.3.5	Maximum Linearly Independent Subset	48
8.1	Flood-Fill	33	11.3.6	Mixed Knapsack Problem	48
8.2	Multi-source BFS	33	11.3.7	Dependent Knapsack Problem . .	48
8.3	BFS with Deque	33	11.3.8	Number of Solutions	48
8.4	Bidirectional BFS	34	11.4	FSM	49
8.5	A*	34	11.4.1	Common FSM	49
8.6	DFS Connectivity Model	34	11.4.2	Linear DP + KMP	49
8.7	IDDFS	34	11.4.3	Linear DP + AC Automaton . . .	49
8.8	Bidirectional DFS	35	11.5	State Compression DP	50
8.9	IDA*	35	11.5.1	Bitmask DP & Rolling Array & Target-State Optimization	50
9	Advanced Graph Theory	36	11.5.2	Bitmask DP & Transfer Optimization	50
9.1	Detecting Negative Cycles	36	11.5.3	Bitmask DP & Repeated Coverage Problem	51
9.2	SPFA-SLF	36	11.5.4	Bitmask DP & MST	51
9.3	SPFA-Stack	36	11.6	Digit DP	52
9.4	SPFA & MIN & MAX	36	11.7	Queue Optimization for DP	52
9.5	Second Shortest Path	37			
9.6	Second Minimum Spanning Tree	37			
9.6.1	brute-force	37			
9.6.2	LCA	38			
9.7	Difference Constraints	39			
9.7.1	Maximum-Shortest Path	39			
9.7.2	Minimum-Longest Path	40			
9.8	LCA	40			
9.9	SCC	40			
9.10	DCC	41			
9.10.1	e-DCC	41			
9.10.2	v-DCC	41			
9.11	Bipartite Graph	41			
9.11.1	maximum matching	41			
9.11.2	minimum vertex cover	42			
9.11.3	maximum independent set	42			
9.11.4	minimum path cover	42			
9.12	Eulerian Circuit & Eulerian Path	43			
9.12.1	Eulerian Circuit	43			
9.12.2	Eulerian Path	43			
10	Advanced Math	45			
10.1	Euler's Totient Function	45			
10.1.1	GCD	45			
10.2	Matrix Multiplication	45			



Part I: Basic Template

CREATED BY

Luliet Lyan & Bleu Echo

NSCC-GZ

School of Computer Science & Engineering
Sun Yat-Sen University

Supervisor: Dr Dan Huang

Co-Supervisor: Dr Zhiguang Chen

0 ★ Preface

0.1 Template

```
1 #define itn int
2 #define nit int
3 #define nti int
4 #define tin int
5 #define tni int
6 #define retrun return
7 #define reutrn return
8 #define rutren return
9 #define fastin \
10 ios_base::sync_with_stdio(0); \
11 cin.tie(0), cout.tie(0);
12 #include <bits/stdc++.h>
13 using namespace std;
14 typedef long long LL;
15 typedef long double LD;
16 typedef pair<int, int> PII;
17 typedef pair<long long, long long> PLL;
18 typedef pair<double, double> PDD;
19 typedef vector<int> VI;
20 #ifndef ONLINE_JUDGE
21 #define dbg(args...) \
22 do \
23 { \
24     cout << "\033[32;1m" << #args << " -> \
25     "; \
26     err(args); \
27 } while (0)
28 #define dbg(...)
29 #endif
30 void err()
31 { cout << "\033[39;0m" << endl; }
32 template <template <typename...> class T,
33         typename t, typename... Args>
34 void err(T<t> a, Args... args)
35 {
36     for (auto x : a) cout << x << ' ';
37     err(args...);
38 }
39 template <typename T, typename... Args>
40 void err(T a, Args... args)
41 { cout << a << ' '; err(args...); }
42 const int INF = 0x3f3f3f3f;
43 const int mod = 1e9 + 7;
44 const double eps = 1e-6;
45 int main()
46 {
47     #ifndef ONLINE_JUDGE
48         freopen("test.in", "r", stdin);
49         freopen("test.out", "w", stdout);
50     #endif
51     fastin;
52     return 0;
53 }
```

0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about $10^7 \sim 10^8$ operations.
- Below is a guide on how to choose algorithms based on different input size ranges:
 1. $n \leq 30 \rightarrow$ Exponential complexity: DFS with pruning, State Compression DP
 2. $n \leq 100 \rightarrow O(n^3)$: Floyd, DP, Gaussian Elimination
 3. $n \leq 1000 \rightarrow O(n^2), O(n^2 \log n)$: DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
 4. $n \leq 10000 \rightarrow O(n^{\frac{3}{2}})$: Block Linked List, Mo's Algorithm
 5. $n \leq 100000 \rightarrow O(n \log n)$: sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
 6. $n \leq 1000000 \rightarrow O(n)$, or small-constant $O(n \log n)$: Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
 7. $n \leq 10000000 \rightarrow O(n)$: Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
 8. $n \leq 10^9 \rightarrow O(\sqrt{n})$: Primality Testing
 9. $n \leq 10^{18} \rightarrow O(\log n)$: GCD, Fast Exponentiation, Digit DP
 10. $n \leq 10^{1000} \rightarrow O((\log n)^2)$: Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
 11. $n \leq 10^{100000} \rightarrow O(\log k \cdot \log \log k)$, where k is the number of digits: Big Integer Add/Subtract, FFT/NTT

0.4 If <bits/stdc++.h> Failed

Replace it with:

```
1  #include <algorithm>
2  #include <bitset>
3  #include <complex>
4  #include <deque>
5  #include <exception>
6  #include <fstream>
7  #include <functional>
8  #include <iomanip>
9  #include <ios>
10 #include <iosfwd>
11 #include <iostream>
12 #include <istream>
13 #include <iterator>
14 #include <limits>
15 #include <list>
16 #include <locale>
17 #include <map>
18 #include <memory>
19 #include <numeric>
20 #include <ostream>
21 #include <queue>
22 #include <set>
23 #include <sstream>
24 #include <stack>
25 #include <stdexcept>
26 #include <streambuf>
27 #include <string>
28 #include <typeinfo>
29 #include <utility>
30 #include <valarray>
31 #include <vector>
32 #include <unordered_map>
33 #include <unordered_set>
```

1 ★ Basic Algorithm

1.1 Quick Sort

Sort the given array from index 1 to n.

```
1 void quick_sort(int l, int r)
2 {
3     if (l >= r) return;
4     int x = a[(l + r) >> 1], i = l - 1, j = r
        + 1;
5     while (i < j)
6     {
7         do i++; while (a[i] < x);
8         do j--; while (a[j] > x);
9         if (i < j) swap(a[i], a[j]);
10    }
11    quick_sort(l, j);
12    quick_sort(j + 1, r);
13    return;
14 }
```

1.2 Binary Search

```
1 // 区间 [l, r] 被划分成 [l, mid] 和 [mid + 1,
    r] 时使用
2 // 大于等于区间的最小值, check 应为 target <=
    a[mid]
3 int bsearch_1(int l, int r)
4 {
5     while (l < r)
6     {
7         int mid = l + r >> 1;
8         if (check(mid)) r = mid;
9         else l = mid + 1;
10    }
11    return l;
12 }
13 // 区间 [l, r] 被划分成 [l, mid - 1] 和 [mid,
    r] 时使用
14 // 小于等于区间的最大值, check 应为 target >=
    a[mid]
15 int bsearch_2(int l, int r)
16 {
17     while (l < r)
18     {
19         // 为什么要 l + r + 1: 因为 l 的更新条
            件是 mid 本身
20         // 当 r == l + 1 时 mid 向下取整必定取
            l, 有可能在满足 check(mid) 时导致无限循环
21         int mid = l + r + 1 >> 1;
22         if (check(mid)) l = mid;
23         else r = mid - 1;
24    }
25    return l;
26 }
27 // 浮点数二分
28 double bsearch_3(double l, double r)
29 {
30     // eps 表示精度, 取决于题目对精度的要求
31     const double eps = 1e-6;
32     while (r - l > eps)
```

```
33 {
34     double mid = (l + r) / 2;
35     if (check(mid)) r = mid;
36     else l = mid;
37 }
38 return l;
39 }
```

1.3 Ternary Search

```
1 // 整数三分
2 void tsearch_1(int l, int r)
3 {
4     while (l < r)
5     {
6         int lmid = l + (r - l) / 3, rmid = r -
            (r - l) / 3;
7         lans = cal(lmid), rans = cal(rmid);
8         if (lans <= rans) r = rmid - 1;
9         else l = lmid + 1;
10        if (lans <= rans) l = lmid + 1;
11        else r = rmid - 1;
12    }
13    // 求凹函数的极小值
14    cout << min(lans, rans) << endl;
15    // 求凸函数的极大值
16    cout << max(lans, rans) << endl;
17 }
18 // 浮点数三分
19 void tsearch_2(int l, int r)
20 {
21     const double eps = 1e-6;
22     while (r - l < eps)
23     {
24         double lmid = l + (r - l) / 3;
25         double rmid = r - (r - l) / 3;
26         lans = cal(lmid), rans = cal(rmid);
27         // 求凹函数的极小值
28         if (lans <= rans) r = rmid;
29         else l = lmid;
30         // 求凸函数的极大值
31         if (lans <= rans) l = lmid;
32         else r = rmid;
33    }
34 }
```

1.4 High Precision

1.4.1 High Precision Add

```
1 string s1, s2;
2 vector<int> a, b, c;
3 void add(vector<int> &a, vector<int> &b)
4 {
5     if (a.size() < b.size())
6     { add(b, a); return; }
7     int t = 0;
8     for (int i = 0; i < a.size(); i++)
9     {
10        t += a[i];
```

```

11     if (i < b.size()) t += b[i];
12     c.push_back(t % 10);
13     t /= 10;
14 }
15 while (t)
16     c.push_back(t % 10), t /= 10;
17 }
18 int main()
19 {
20     cin >> s1 >> s2;
21     for (int i = s1.size() - 1; i >= 0; i--)
22         a.push_back(s1[i] - '0');
23     for (int i = s2.size() - 1; i >= 0; i--)
24         b.push_back(s2[i] - '0');
25     add(a, b);
26     for (int i = c.size() - 1; i >= 0; i--)
27         cout << c[i];
28     return 0;
29 }

```

```

5 {
6     for (int i = 0, t = 0; i < a.size() || t; i++)
7     {
8         if (i < a.size()) t += a[i] * b;
9         c.push_back(t % 10);
10        t /= 10;
11    }
12    while (c.size() > 1 && c.back() == 0)
13        c.pop_back();
14 }
15 int main()
16 {
17     cin >> s1 >> b;
18     for (int i = s1.size() - 1; i >= 0; i--)
19         a.push_back(s1[i] - '0');
20     mul(a, b);
21     for (int i = c.size() - 1; i >= 0; i--)
22         cout << c[i];
23     return 0;
24 }

```

1.4.2 High Precision Subsection

```

1 vector<int> a, b, c;
2 string s1, s2;
3 void sub(vector<int> &a, vector<int> &b)
4 {
5     int t = 0;
6     for (int i = 0; i < a.size(); i++)
7     {
8         t = a[i] - t;
9         if (i < b.size()) t -= b[i];
10        c.push_back((t + 10) % 10);
11        if (t < 0) t = 1;
12        else t = 0;
13    }
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> s2;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     for (int i = s2.size() - 1; i >= 0; i--)
23         b.push_back(s2[i] - '0');
24     if (s1.size() < s2.size())
25         cout << '-', sub(b, a);
26     else if (s1.size() == s2.size() && s1 < s2)
27         cout << '-', sub(b, a);
28     else sub(a, b);
29     for (int i = c.size() - 1; i >= 0; i--)
30         cout << c[i];
31     return 0;
32 }

```

1.4.4 High Precision Divide

```

1 string s1, s2;
2 vector<int> a, c;
3 int b, r;
4 void divide(vector<int> &a, int b, int &r)
5 {
6     r = 0;
7     for (int i = a.size() - 1; i >= 0; i--)
8     {
9         r = r * 10 + a[i];
10        c.push_back(r / b);
11        r %= b;
12    }
13    reverse(c.begin(), c.end());
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> b;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     divide(a, b, r);
23     for (int i = c.size() - 1; i >= 0; i--)
24         cout << c[i];
25     cout << '\n' << r;
26     return 0;
27 }

```

1.4.3 High Precision Multiply

```

1 string s1, s2;
2 vector<int> a, c;
3 int b;
4 void mul(vector<int> &a, int b)

```

1.5 Prefix Sum & Difference Array

1.5.1 1D Prefix Sum

```

1 S[i] = a[1] + a[2] + ... a[i]
2 a[l] + ... + a[r] = S[r] - S[l - 1]

```


1.5.2 2D Prefix Sum

```
1 // S[i, j] = i 行 j 列左上部分所有元素和为:
2 s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1]
  + a[i][j]
3 // 以 (x1, y1) 为左上角, (x2, y2) 为右下角的
  子矩阵的和为:
4 S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1] +
  S[x1 - 1][y1 - 1]
```

```
23         insert(x1, y1, x2, y2, c);
24     }
25     // 其他过程略
26 }
```

1.5.3 1D Difference Array

```
1 const int N = 100010;
2 int n, m;
3 int a[N], b[N];
4 void insert(int l, int r, int c)
5 { b[l] += c; b[r + 1] -= c; }
6 int main()
7 {
8     cin >> n >> m;
9     for (int i = 1; i <= n; i++)
10         cin >> a[i];
11     for (int i = 1; i <= n; i++)
12         insert(i, i, a[i]);
13     while (m--)
14     {
15         int l, r, c;
16         cin >> l >> r >> c;
17         insert(l, r, c);
18     }
19     for (int i = 1; i <= n; i++)
20         b[i] += b[i - 1],
21         cout << b[i] << ' ';
22     return 0;
23 }
```

1.5.4 2D Difference Array

```
1 const int N = 1010;
2 int n, m, q, a[N][N], b[N][N];
3 void insert(int x1, int y1, int x2, int y2,
4             int c)
5 {
6     b[x1][y1] += c;
7     b[x2 + 1][y2 + 1] += c;
8     b[x1][y2 + 1] -= c;
9     b[x2 + 1][y1] -= c;
10 }
11 int main()
12 {
13     cin >> n >> m >> q;
14     for (int i = 1; i <= n; i++)
15         for (int j = 1; j <= m; j++)
16             cin >> a[i][j];
17     for (int i = 1; i <= n; i++)
18         for (int j = 1; j <= m; j++)
19             insert(i, j, i, j, a[i][j]);
20     while (q--)
21     {
22         int x1, x2, y1, y2, c;
23         cin >> x1 >> y1 >> x2 >> y2 >> c;
```

2 ★ Basic Data Structures

2.1 Linked List

2.1.1 Singly Linked List

```
1  const int N = 100010;
2  int n, h[N], e[N], ne[N], idx = 1;
3  void init() { ne[0] = -1; }
4  void insert(int k, int x) // 第 k 个节点后
    插入
5  { e[idx] = x, ne[idx] = ne[k], ne[k] = idx
    ++; }
6  void del(int k) // 第 k 个节点后删除
7  { ne[k] = ne[ne[k]]; }
```

2.1.2 Bidirectional Linked List

```
1  const int N = 100010;
2  int n, r[N], l[N], e[N], idx = 2;
3  void init() { r[0] = 1; l[1] = 0; }
4  void insert(int k, int x) // 第 k 个节点后插
    入
5  {
6      e[idx] = x;
7      r[idx] = r[k];
8      l[idx] = k;
9      l[r[k]] = idx;
10     r[k] = idx++;
11 }
12 void remove(int k) // 删除 k 本身
13 { r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

2.2 Stack & Queue

2.2.1 Monotonic Stack

```
1  // 常见模型：找出每个数左边离它最近的比它大/小
    的数
2  int tt = 0;
3  for (int i = 1; i <= n; i++)
4  {
5      while (tt && check(stk[tt], i)) tt--;
6      stk[++tt] = i;
7  }
```

2.2.2 Monotonic Queue

```
1  // 常见模型：找出滑动窗口中的最大值/最小值
2  int hh = 0, tt = -1;
3  for (int i = 0; i < n; i++)
4  {
5      while (hh <= tt && check_out(q[hh]))
6          hh++; // 判断队头是否滑出窗口
7      while (hh <= tt && check(q[tt], i))
8          tt--;
```

```
9      q[++tt] = i;
10 }
```

2.3 KMP

```
1  const int N = 100010, M = 1000010;
2  int n, m;
3  char p[N], s[M];
4  void getNext(int ne[])
5  {
6      for (int i = 2, j = 0; i <= n; i++)
7      {
8          while (j && p[j + 1] != p[i])
9              j = ne[j];
10         if (p[j + 1] == p[i]) j++;
11         ne[i] = j;
12     }
13 }
14 int KMP()
15 {
16     int *ne = new int[n + 1];
17     getNext(ne);
18     for (int i = 1, j = 0; i <= m; i++)
19     {
20         while (j && p[j + 1] != s[i])
21             j = ne[j];
22         if (p[j + 1] == s[i]) j++;
23         if (j == n) cout << i - n << ' ';
24     }
25     return -1;
26 }
```

2.4 Trie

```
1  const int N = 100010;
2  int trie[N][26], cnt[N], idx = 0;
3  void insert(string &str) // 插入到 Trie
    数组
4  {
5      int p = 0;
6      for (auto c : str)
7      {
8          int u = c - 'a';
9          if (!trie[p][u])
10             trie[p][u] = ++idx;
11             p = trie[p][u];
12     }
13     cnt[p]++;
14 }
15 int query(string &str) // 查询字符串出现的
    次数
16 {
17     int p = 0;
18     for (auto c : str)
19     {
20         int u = c - 'a';
21         if (!trie[p][u]) return 0;
22         p = trie[p][u];
23     }
24     return cnt[p];
25 }
```

2.5 Disjoint-Set

```

1  const int N = 100010;
2  int n, m, p[N], Size[N], D[N];
3  void init()
4  {
5      for (int i = 1; i <= n; i++)
6          p[i] = i, Size[i] = 1, D[i] = 0;
7  }
8  int find(int x)
9  {
10     if (p[x] != x)
11     {
12         int u = find(p[x]);
13         D[x] += D[p[x]]; // 视具体情况计算
14         p[x] = u;
15     }
16     return p[x];
17 }
18 void merge(int a, int b, int distance)
19 {
20     int x = find(a), y = find(b);
21     if (x != y)
22     {
23         p[x] = y;
24         D[x] = distance; // 视具体情况计算
25         Size[y] += Size[x];
26     }
27 }
```

2.6 Hash

2.6.1 Simple Hash

```

1  // (1) 拉链法
2  int h[N], e[N], ne[N], idx;
3  void insert(int x)
4  {
5      int k = (x % N + N) % N;
6      e[idx] = x, ne[idx] = h[k], h[k] = idx++;
7  }
8  bool find(int x)
9  {
10     for (int i = h[(x % N + N) % N]; i != -1;
11         i = ne[i])
12         if (e[i] == x) return true;
13     return false;
14 }
15 // (2) 开放寻址法
16 int find(int x)
17 {
18     int t = (x % N + N) % N;
19     while (h[t] != null && h[t] != x)
20         t++; if (t == N) t = 0;
21     return t;
22 }
```

2.6.2 String Hash

```

1  typedef unsigned long long ULL;
2  ULL h[N], p[N];
3  void init()
4  {
5      p[0] = 1;
6      for (int i = 1; i <= n; i++) { h[i] = h[i - 1] * P + str[i]; p[i] = p[i - 1] * P; }
7  }
8  ULL get(int l, int r) { return h[r] - h[l - 1] * p[r - l + 1]; }
```

2.7 STL

```

1  // vector
2  size()      返回元素个数
3  empty()     返回是否为空
4  clear()     清空
5  front()/back()
6  push_back()/pop_back()
7  begin()/end()
8  []
9  支持比较运算, 按字典序
10 // pair<int, int>
11 first       第一个元素
12 second      第二个元素
13 支持比较运算, 以first为第一关键字, 以second为第二关键字 (字典序)
14 // string
15 size()/length() 返回字符串长度
16 empty()
17 clear()
18 substr(起始下标, (子串长度)) 返回子串
19 c_str() 返回字符串所在字符数组的起始地址
20 // queue
21 size()
22 empty()
23 push()      向队尾插入一个元素
24 front()     返回队头元素
25 back()     返回队尾元素
26 pop()      弹出队头元素
27 // priority_queue
28 size()
29 empty()
30 push()      插入一个元素
31 top()       返回堆顶元素
32 pop()       弹出堆顶元素
33 定义成小根堆的方式: priority_queue<int, vector<int>, greater<int>> q;
34 // stack
35 size()
36 empty()
37 push()      向栈顶插入一个元素
38 top()       返回栈顶元素
39 pop()       弹出栈顶元素
40 // deque
41 size()
42 empty()
43 clear()
44 front()/back()
45 push_back()/pop_back()
```

```

46 push_front()/pop_front()
47 begin()/end()
48 []
49 // set, map, multiset, multimap: 基于平衡二叉
    树 (红黑树) 动态维护有序序列
50 size()
51 empty()
52 clear()
53 begin()/end()
54 ++, -- 返回前驱和后继, 时间复杂度  $O(\log n)$ 
55 // set/multiset
56 insert() 插入一个数
57 find() 查找一个数
58 count() 返回某一个数的个数
59 erase()
60 (1) 输入是一个数x, 删除所有x,  $O(k + \log n)$ 
    (2) 输入一个迭代器, 删除这个迭代器
61 lower_bound()/upper_bound()
62 lower_bound(x) 返回大于等于x的最小的数的
    迭代器
63 upper_bound(x) 返回大于x的最小的数的迭
    代器
64 // map/multimap
65 insert() 插入的数是一个pair
66 erase() 输入的参数是pair或者迭代器
67 find()
68 [] 注意multimap不支持此操作。 时间复
    杂度是  $O(\log n)$ 
69 lower_bound()/upper_bound()
70 // unordered_set, unordered_map,
    unordered_multiset, unordered_multimap
71 增删改查的时间复杂度是  $O(1)$ 
72 不支持 lower_bound()/upper_bound(), 迭代器的
    ++, --
73 // bitset
74 bitset<10000> s;
75 ~, &, |, ^
76 >>, <<
77 ==, !=
78 []
79 count() 返回有多少个1
80 any() 判断是否至少有一个1
81 none() 判断是否全为0
82 set() 把所有位置成1
83 set(k, v) 将第k位变成v
84 reset() 把所有位变成0
85 flip() 等价于~
86 flip(k) 把第k位取反

```

3 ★ Search & Graph Theory

3.1 Representation of Tree & Graph

3.1.1 Adjacency Matrix

```
1 // g[a][b] = a->b
```

3.1.2 Adjacency List

```
1 int h[N], e[N], ne[N], idx;
2 void init() { memset(h, -1, sizeof h); }
3 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++; }
```

3.2 DFS & BFS

3.2.1 DFS

```
1 int dfs(int u)
2 {
3     st[u] = true; // 表示点 u 已经被遍历过
4     for (int i = h[u]; i != -1; i = ne[i])
5     { int j = e[i]; if (!st[j]) dfs(j); }
6 }
```

3.2.2 BFS

```
1 queue<int> q;
2 st[1] = true; q.push(1);
3 while (q.size())
4 {
5     int t = q.front(); q.pop();
6     for (int i = h[t]; i != -1; i = ne[i])
7     { if (!st[e[i]]) { st[e[i]] = true; q.
            push(e[i]); }
8 }
```

3.3 Topological Sort

```
1 const int N = 100010;
2 int e[2 * N], ne[2 * N], h[N], d[N], idx;
3 int n, m, q[N];
4 void init() { memset(h, -1, sizeof h); }
5 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++, d[b]++; }
6 bool topSort()
7 {
8     int hh = 0, tt = -1;
9     for (int i = 1; i <= n; i++)
10         if (!d[i]) q[++tt] = i;
11     while (hh <= tt)
```

```
12         for (int i = h[q[hh++]]; ~i; i = ne[i]
13             )
14             if (--d[e[i]] == 0) q[++tt] = e[i];
15     return tt == n - 1;
16 }
```

3.4 Shortest Path

3.4.1 Dijkstra

```
1 const int N = 1010;
2 int n, dist[N];
3 int h[N], w[N], e[N], ne[N], idx;
4 bool st[N];
5 void add(int a, int b, int c) { e[idx] = b,
    w[idx] = c, ne[idx] = h[a], h[a] = idx
    ++; }
6 int dijkstra() // 需要初始化 dist 与 h
7 {
8     dist[1] = 0;
9     priority_queue<PII, vector<PII>, greater<
        PII>> heap;
10    heap.push({0, 1});
11    while (heap.size())
12    {
13        auto t = heap.top();
14        heap.pop();
15        int ver = t.second, distance = t.first
            ;
16        if (st[ver]) continue;
17        st[ver] = true;
18        for (int i = h[ver]; i != -1; i = ne[i]
19            )
20            if (dist[e[i]] > distance + w[i])
21            {
22                dist[e[i]] = distance + w[i];
23                heap.push({dist[e[i]], e[i]});
24            }
25        if (dist[n] == 0x3f3f3f3f) return -1;
26        return dist[n];
27    }
```

3.4.2 Bellman-Ford

```
1 const int N = 100010;
2 int n, m, dist[N], backup[N];
3 struct Edge
4 {
5     int a, b, w;
6 } edges[N];
7 int bellman_ford()
8 {
9     memset(dist, 0x3f, sizeof dist);
10    dist[1] = 0;
11    for (int i = 0; i < n; i++)
12    {
13        memcpy(backup, dist, sizeof dist);
14        for (int j = 0; j < m; j++)
15        {
```

```

16         int a = edges[j].a, b = edges[j].b
           , w = edges[j].w;
17         dist[b] = min(dist[b], backup[a] +
18             w);
19     }
20     if (dist[n] > 0x3f3f3f3f / 2) return -1;
21     return dist[n];
22 }

```

3.4.3 SPFA

```

1  const int N = 100010;
2  int n, m, dist[N];
3  int e[2 * N], ne[2 * N], w[2 * N], h[N], idx
   ;
4  bool vis[N];
5  void spfa()    // 需要初始化 dist 与 h
6  {
7      queue<int> q;
8      q.push(1); vis[1] = true;
9      while (q.size())
10     {
11         int t = q.front();
12         q.pop();
13         vis[t] = false;
14         for (int i = h[t]; ~i; i = ne[i])
15             if (dist[e[i]] > dist[t] + w[i])
16             {
17                 dist[e[i]] = dist[t] + w[i];
18                 if (!vis[e[i]]) vis[e[i]] =
19                     true, q.push(j);
20             }
21     dist[n] > INF / 2 ? cout << "impossible" :
22         cout << dist[n];

```

3.4.4 Detecting Negative Circle in SPFA

```

1  void spfa()    // 只需要初始化 h
2  {
3      queue<int> q;
4      // 基于虚拟原点假设, 所有点放入队列
5      for (int i = 1; i <= n; i++) q.push(i), st
6          [i] = true;
7      while (q.size())
8      {
9          int t = q.front();
10         q.pop();
11         vis[t] = false;
12         for (int i = h[t]; ~i; i = ne[i])
13             if (dist[e[i]] > dist[t] + w[i])
14             {
15                 dist[e[i]] = dist[t] + w[i];
16                 // 新增
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n) return true
19                 if (!st[j]) q.push(j), st[j] =
20                     true;

```

```

19     }
20 }
21 return false;
22 }

```

3.4.5 Floyd

```

1  const int N = 210;
2  int g[N][N], n, m, k;
3  int main()
4  {
5      cin >> n >> m >> k;
6      memset(g, 0x3f, sizeof g);
7      for (int i = 1; i <= n; i++) g[i][i] = 0;
8      while (m--)
9      {
10         int a, b, c;
11         cin >> a >> b >> c;
12         g[a][b] = min(g[a][b], c);
13     }
14     for (int k = 1; k <= n; k++)
15         for (int i = 1; i <= n; i++)
16             for (int j = 1; j <= n; j++)
17                 g[i][j] = min(g[i][k] + g[k][j
18                     ], g[i][j]);
19     // 后续代码略
20     return 0;

```

3.5 Minimum Spanning Tree

3.5.1 Prim

```

1  const int N = 510;
2  int n, m, g[N][N], dist[N];
3  bool vis[N];
4  void prim()
5  {
6      int res = 0;
7      for (int i = 0; i < n; i++)
8      {
9          int t = -1;
10         for (int j = 1; j <= n; j++)
11             if (!vis[j] && (t == -1 || dist[j]
12                 < dist[t])) t = j;
13         if (i && dist[t] == INF) { res = INF;
14             break; }
15         if (i) res += dist[t];
16         vis[t] = true;
17         for (int j = 1; j <= n; j++) dist[j] =
18             min(dist[j], g[t][j]);
19     }
20     res == INF ? cout << "impossible" : cout
21         << res;
22 }
23 int main()
24 {
25     memset(g, 0x3f, sizeof g);
26     memset(dist, 0x3f, sizeof dist);
27     cin >> n >> m;
28     while (m--)

```

```

25 {
26     int a, b, c;
27     cin >> a >> b >> c;
28     g[a][b] = min(g[a][b], c);
29     g[b][a] = min(g[b][a], c);
30 }
31 prim();
32 return 0;
33 }

```

3.5.2 Kruskal

```

1  const int N = 100010;
2  int n, m;
3  int p[N];
4  struct Edge
5  {
6      int a, b, w;
7      bool operator<(const Edge &e) const {
8          return w < e.w; };
9  } edge[2 * N];
10 void init() { for (int i = 1; i <= n; i++) p[i] = i; }
11 int find(int x)
12 {
13     if (x != p[x]) p[x] = find(p[x]);
14     return p[x];
15 }
16 void merge(int x, int y) { p[find(x)] = find(y); }
17 void kruskal()
18 {
19     int res = 0, cnt = 0;
20     for (int i = 1; i <= m; i++)
21         if (find(edge[i].a) != find(edge[i].b))
22             {
23                 merge(edge[i].a, edge[i].b);
24                 res += edge[i].w;
25                 cnt++;
26             }
27     if (cnt < n - 1) res = INF;
28     res == INF ? cout << "impossible" : cout << res;
29 }
30 int main()
31 {
32     init();
33     cin >> n >> m;
34     for (int i = 1; i <= m; i++) cin >> edge[i].a >> edge[i].b >> edge[i].w;
35     sort(edge + 1, edge + m + 1);
36     kruskal();
37     return 0;
38 }

```

```

1  const int N = 100010, M = 200010;
2  int n, m;
3  int e[M], ne[M], h[N], color[N], idx;
4  bool dfs(int u, int c)
5  {
6      color[u] = c;
7      for (int i = h[u]; ~i; i = ne[i])
8          if (color[e[i]] == -1)
9              {
10                 if (!dfs(e[i], !c)) return false;
11             }
12         else if (color[e[i]] == c) return false;
13     return true;
14 }
15 bool check()
16 {
17     for (int i = 1; i <= n; i++)
18         if (color[i] == -1)
19             if (!dfs(i, 0)) return false;
20     return true;
21 }
22 int main()
23 {
24     // 注意另外初始化 h 与 color
25     cin >> n >> m;
26     while (m--)
27     {
28         int a, b;
29         cin >> a >> b;
30         add(a, b), add(b, a);
31     }
32     // 其余过程略
33 }

```

3.6 Bipartite Graph

3.6.1 Coloring Method

To check if a given graph is bipartite.

3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```
1  const int N = 510, M = 100010;
2  int n1, n2, m;
3  int e[M], ne[M], h[N], match[N], idx;
4  bool vis[N];
5  bool find(int x)
6  {
7      for (int i = h[x]; ~i; i = ne[i])
8          if (!vis[e[i]])
9              {
10                 vis[e[i]] = true;
11                 if (match[e[i]] == 0 || find(match[e[i]]))
12                     {
13                         match[e[i]] = x;
14                         return true;
15                     }
16             }
17      return false;
18  }
19  int main()
20  {
21      // 注意初始化 h
22      cin >> n1 >> n2 >> m;
23      while (m--)
24      {
25          int a, b;
26          cin >> a >> b;
27          add(a, b);
28      }
29      int res = 0;
30      for (int i = 1; i <= n1; i++)
31      {
32          memset(vis, false, sizeof vis);
33          if (find(i)) res++;
34      }
35      cout << res;
36      return 0;
37  }
```


4 ★ Basic Math

4.1 Prime Numbers

4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0) return false;
6     return true;
7 }
```

4.1.2 Prime Factorization

```
1 void divide(int x)
2 {
3     for (int i = 2; i <= x / i; i++)
4         if (x % i == 0)
5             { // 此条件成立时 i 一定是质数
6                 int s = 0;
7                 while (x % i == 0) x /= i, s++;
8                 cout << i << ' ' << s << '\n';
9             }
10    if (x > 1) cout << x << ' ' << 1 << '\n'
11 }
```

4.1.3 Euler's Sieve

```
1 int primes[N], cnt;
2 bool st[N];
3 void get_primes(int n)
4 {
5     for (int i = 2; i <= n; i++)
6     {
7         if (!st[i]) primes[cnt++] = i;
8         for (int j = 0; primes[j] <= n / i; j++)
9             {
10                st[primes[j] * i] = true;
11                if (i % primes[j] == 0) break;
12            }
13    }
14 }
```

4.2 Divisor

4.2.1 Find All Divisors

```
1 vector<int> get_divisors(int x)
2 {
3     vector<int> res;
4     for (int i = 1; i <= x / i; i++)
5         if (x % i == 0)
```

```
6         {
7             res.push_back(i);
8             if (i != x / i) res.push_back(x /
9             i);
10        }
11    sort(res.begin(), res.end());
12    return res;
13 }
```

4.2.2 The Number of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 int main()
4 {
5     cin >> n;
6     unordered_map<int, int> h;
7     while (n--)
8     {
9         int x;
10        cin >> x;
11        for (int i = 2; i <= x / i; i++)
12            while (x % i == 0) { h[i]++; x = x
13            / i; }
14        if (x > 1) h[x]++;
15    }
16    long long res = 1;
17    for (auto iter = h.begin(); iter != h.end()
18        ); iter++)
19        res = res * (iter->second + 1) % mod;
20    cout << res;
21    return 0;
22 }
```

4.2.3 The Sum of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 long long getSum(int x, int c)
4 {
5     long long s = 1;
6     while(c--) s = (s * x + 1) % mod;
7     return s;
8 }
9 int main()
10 {
11    cin >> n;
12    unordered_map<int, int> h;
13    while (n--)
14    {
15        int x;
16        cin >> x;
17        for (int i = 2; i <= x / i; i++)
18            while (x % i == 0) { h[i]++; x = x
19            / i; }
20        if (x > 1) h[x]++;
21    }
22    long long res = 1;
23    for (auto iter = h.begin(); iter != h.end()
24        ); iter++)
25        res = res * getSum(iter->first, iter->
26        second) % mod;
```

```

24     cout << res;
25     return 0;
26 }

```

4.2.4 Euclidean Algorithm

```

1  int gcd(int a, int b)
2  { return a % b == 0 ? b : gcd(b, a % b); }

```

4.3 Euler Function

4.3.1 Simple Method

```

1  int phi(int x)
2  {
3      int res = x;
4      for (int i = 2; i <= x / i; i++)
5          if (x % i == 0)
6              {
7                  res = res / i * (i - 1);
8                  while (x % i == 0) x /= i;
9              }
10     if (x > 1) res = res / x * (x - 1);
11     return res;
12 }

```

4.3.2 Euler's Sieve Method

```

1  const int N = 1000010;
2  int n, primes[N], phi[N], cnt;
3  bool st[N];
4  void getEuler()
5  {
6      phi[1] = 1;
7      for (int i = 2; i <= n; i++)
8          {
9              if (!st[i])
10                 {
11                     primes[cnt++] = i;
12                     // i 是质数, 它只会被本身整除, 所以
                     // 直接赋值 i - 1
13                     phi[i] = i - 1;
14                 }
15                 for (int j = 0; primes[j] <= n / i; j++)
16                     {
17                         st[i * primes[j]] = true;
18                         if (i % primes[j] == 0)
19                             {
20                                 // 如果 i % primes[j] == 0 成
                                 // 立表示 primes[j] 是 i 的最小质因子
                                 // 也是 primes[j] * i 的最小质
                                 // 因子
21                                 // 1 - 1 / primes[j] 这一项在
                                 // phi[i] 中计算过了, 只需将基数 N 修正为
                                 // primes[j] 倍
22                                 phi[primes[j] * i] = phi[i] *
23                                 primes[j];
24                                 break;

```

```

25         }
26         // 否则, primes[j] 不是 i 的质因
        // 子, 只是 primes[j] * i 的最小质因子
27         // 不仅需要将基数 N 修正为 primes[j]
        // 倍
28         // 还需要补上 1 - 1 / primes[j] 的
        // 分子项, 因此最终结果为 phi[i] * (primes[j]
        // - 1)
29         phi[primes[j] * i] = phi[i] * (
        primes[j] - 1);
30     }
31 }
32 }

```

4.4 Exponentiating by Squaring

```

1  LL qmi(int m, int k, int p)
2  {
3      LL res = 1 % p, t = m;
4      while (k)
5          {
6              if (k & 1) res = res * t % p;
7              t = t * t % p;
8              k >>= 1;
9          }
10     return res;
11 }

```

4.5 Extended Euclidean Algorithm

```

1  int exgcd(int a, int b, int &x, int &y)
2  {
3      if (!b)
4          {
5              x = 1;
6              y = 0;
7              return a;
8          }
9      int d = exgcd(b, a % b, y, x);
10     y -= (a / b) * x;
11     return d;
12 }

```

4.6 Chinese Remainder Theorem

```

1  LL exgcd(LL a, LL b, LL &x, LL &y)
2  {
3      if (!b) { x = 1, y = 0; return a; }
4      LL d = exgcd(b, a % b, y, x);
5      y -= a / b * x;
6      return d;
7  }
8  int main()
9  {

```

```

10 int n;
11 cin >> n;
12 LL x = 0, m1, a1;
13 cin >> m1 >> a1;
14 for (int i = 0; i < n - 1; i++)
15 {
16     LL m2, a2;
17     cin >> m2 >> a2;
18     LL k1, k2;
19     LL d = exgcd(m1, m2, k1, k2);
20     if ((a2 - a1) % d) { x = -1; break; }
21     k1 *= (a2 - a1) / d;
22     k1 = (k1 % (m2 / d) + m2 / d) % (m2 /
23 d);
24     x = k1 * m1 + a1;
25     LL m = abs(m1 / d * m2);
26     a1 = k1 * m1 + a1;
27     m1 = m;
28 }
29 if (x != -1)
30     x = (a1 % m1 + m1) % m1;
31 cout << x << '\n';
32 return 0;
33 }

```

4.7 Gauss-Jordan Elimination

4.7.1 Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++) // 找
            // 绝对值最大的行
8             if (fabs(a[i][c]) > fabs(a[t][c]))
9                 t = i;
10        if (fabs(a[t][c]) < eps) // 此
            // 时没必要对该列该行处理
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[t][i], a[r][i]); // 将
            // 绝对值最大的行换到最顶端
14        for (int i = n; i >= c; i--)
15            a[r][i] /= a[r][c]; // 将
            // 当前行的首位变成1
16        for (int i = r + 1; i < n; i++) // 用
            // 当前行将下面所有的列消成0
17            if (fabs(a[i][c]) > eps)
18                for (int j = n; j >= c; j--)
19                    a[i][j] -= a[r][j] * a[i][c];
20        r++;
21    }
22    if (r < n)
23    {
24        for (int i = r; i < n; i++)
25            if (fabs(a[i][n]) > eps)
26                return 2; // 无解
27        return 1; // 有无穷多组解
28    }
29 }

```

```

29 for (int i = n - 1; i >= 0; i--)
30     for (int j = i + 1; j < n; j++)
31         a[i][n] -= a[i][j] * a[j][n];
32 return 0; // 有解
33 }

```

4.7.2 XOR Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++)
8             if (a[i][c])
9                 t = i;
10        if (!a[t][c])
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[r][i], a[t][i]);
14        for (int i = r + 1; i < n; i++)
15            if (a[i][c])
16                for (int j = n; j >= c; j--)
17                    a[i][j] ^= a[r][j];
18        r++;
19    }
20    if (r < n)
21    {
22        for (int i = r; i < n; i++)
23            if (a[i][n])
24                return 2;
25        return 1;
26    }
27    for (int i = n - 1; i >= 0; i--)
28        for (int j = i + 1; j < n; j++)
29            a[i][n] ^= a[i][j] * a[j][n];
30    return 0;
31 }

```

4.8 Combinatorial Counting

4.8.1 Recurrence Relation

```

1 void init()
2 {
3     for (int i = 0; i < N; i++)
4         for (int j = 0; j <= i; j++)
5             if (!j) c[i][j] = 1;
6             else c[i][j] = (c[i - 1][j] + c[i - 1][j - 1]) % mod;
7 }

```

4.8.2 Preprocessing & Inverse Element

```

1 const int N = 100010, mod = 1e9 + 7;
2 int n, fact[N], infact[N];
3 int qmi(int a, int b, int p)
4 {

```

```

5   int res = 1;
6   while (b)
7   {
8       if (b & 1)
9           res = (LL)res * a % p;
10      a = (LL)a * a % p;
11      b >>= 1;
12  }
13  return res;
14  }
15  int main()
16  {
17      fact[0] = infact[0] = 1;
18      for (int i = 1; i < N; i++)
19      {
20          fact[i] = (LL)fact[i - 1] * i % mod;
21          infact[i] = (LL)infact[i - 1] * qmi(i,
22              mod - 2, mod) % mod;
23      }
24      // 此后 C(a, b) = (LL)fact[a] * infact[b]
25      // % mod * infact[a - b] % mod
26  }

```

4.8.3 Lucas Theorem

```

1  int qmi(int a, int k, int p)
2  {
3      int res = 1 % p;
4      while (k)
5      {
6          if (k & 1)
7              res = (LL)res * a % p;
8          a = (LL)a * a % p;
9          k >>= 1;
10     }
11     return res;
12 }
13 int C(int a, int b, int p)
14 {
15     if (a < b) return 0;
16     LL x = 1, y = 1;
17     // x = a * (a - 1) * (a - 2) * ... * (a -
18     //   b + 1) = a! / (a - b)! (mod p)
19     // y = 1 * 2 * ... * b = b! (mod p)
20     for (int i = a, j = 1; j <= b; i--, j++)
21     { x = (LL)x * i % p; y = (LL)y * j % p; }
22     return x * (LL)qmi(y, p - 2, p) % p;
23 }
24 int lucas(LL a, LL b, int p)
25 {
26     if (a < p && b < p)
27         return C(a, b, p);
28     return (LL)C(a % p, b % p, p) * lucas(a /
29         p, b / p, p) % p;
30 }

```

4.8.4 Factorization Method

```

1  const int N = 5010;
2  int n, primes[N], sum[N], cnt;
3  bool st[N];
4  void getPrimes(int n) { // 略 }

```

```

5  // 求 n! 中 p 的幂次
6  int get(int n, int p)
7  {
8      int res = 0;
9      while (n) { res += n / p; n /= p; }
10     return res;
11 }
12 void mul(vector<int> &a, int b) { // 高精度
13     // 乘, 略 }
14 int main()
15 {
16     int a, b;
17     cin >> a >> b;
18     getPrimes(a);
19     for (int i = 0; i < cnt; i++)
20     {
21         int p = primes[i];
22         sum[i] = get(a, p) - get(b, p) - get(a
23             - b, p);
24     }
25     vector<int> res;
26     res.push_back(1);
27     for (int i = 0; i < cnt; i++)
28     {
29         for (int j = 0; j < sum[i]; j++)
30             mul(res, primes[i]);
31     }
32     for (int i = res.size() - 1; i >= 0; i--)
33         cout << res[i];
34 }

```

4.8.5 Catalan Number

```

1  const int N = 100010, mod = 1e9 + 7;
2  int qmi(int a, int k, int p) { // 略 }
3  int main()
4  {
5      int n;
6      cin >> n;
7      int a = n * 2, b = n, res = 1;
8      for (int i = a; i > a - b; i--)
9          res = (LL)res * i % mod;
10     for (int i = 1; i <= b; i++)
11         res = (LL)res * qmi(i, mod - 2, mod) %
12         mod;
13     res = (LL)res * qmi(n + 1, mod - 2, mod) %
14     mod;
15 }

```

4.9 Inclusion-Exclusion Principle

```

1  const int N = 20;
2  int n, m, res = 0, p[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < m; i++)
7          cin >> p[i];
8      // 使用二进制数字表示数字选取情况
9      for (int i = 1; i < 1 << m; i++)
10     {

```

```

11     int t = 1, cnt = 0;
12     // 遍历每个被选取的质数
13     for (int j = 0; j < m; j++)
14         if (i >> j & 1)
15         {
16             cnt++;
17             // 一个质数能被选取的条件应该是
其累乘积不超过目标数字
18             if ((LL)t * p[j] > n)
19                 { t = -1; break; }
20             t *= p[j];
21         }
22     if (t != -1)
23         // 容斥原理公式中奇数个并集系数为 1
, 反之为 -1
24         if (cnt % 2) res += n / t;
25         else res -= n / t;
26     }
27     cout << res;
28 }

```

4.10 Game Theory

4.10.1 NIM Game

```

1  const int N = 110, M = 100010;
2  int k, n, s[N], f[M];
3  int sg(int x)
4  {
5      if (f[x] != -1) return f[x];
6      // 到达节点得 SG 函数集合
7      unordered_set<int> S;
8      // 能取走石子就说明能到达, 并且递归向下求解
9      for (int i = 0; i < k; i++)
10     {
11         int sum = s[i];
12         if (x >= sum) S.insert(sg(x - sum));
13     }
14     // SG 从小到大遍历并返回, 找到最小的、不包含
在 SG 函数集合中的自然数
15     for (int i = 0;; i++)
16         if (!S.count(i))
17             return f[x] = i;
18 }
19
20 int main()
21 {
22     cin >> k;
23     for (int i = 0; i < k; i++) cin >> s[i];
24     cin >> n;
25     memset(f, -1, sizeof f);
26     int res = 0;
27     // 每一堆石子都是一个入度为 0 的起始点
28     for (int i = 0; i < n; i++)
29     {
30         int x;
31         cin >> x;
32         res ^= sg(x);
33     }
34     res ? cout << "Yes" : cout << "No";
35     return 0;
36 }

```

5 ★ Basic DP

5.1 Knapsack Problem

5.1.1 01 Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = m; j >= v[i]; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

5.1.2 Complete Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = v[i]; j <= m; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

5.1.3 Mutiple Knapsack

```
1  const int N = 25010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         for (int k = 1; k <= s; k *= 2)
11         {
12             for (int j = m; j >= k * v; j--)
13                 f[j] = max(f[j], f[j - k * v] + k * w);
14             s -= k;
15         }
16         if (s)
17             for (int j = m; j >= s * v; j--)
18                 f[j] = max(f[j], f[j - s * v] + s * w);
19     }
```

```
20     cout << f[m];
21 }
```

5.1.4 Grouped Knapsack

```
1  const int N = 120;
2  int n, m, s[N], v[N][N], w[N][N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> s[i];
9          for (int j = 1; j <= s[i]; j++)
10             cin >> v[i][j] >> w[i][j];
11     }
12     for (int i = 1; i <= n; i++)
13         for (int j = m; j >= 0; j--)
14             for (int k = 1; k <= s[i]; k++)
15                 if (v[i][k] <= j)
16                     f[j] = max(f[j], f[j - v[i][k]] + w[i][k]);
17     cout << f[m];
18 }
```

5.2 Linear DP

5.2.1 LIS

Here is an $O(n^2)$ solution:

```
1  const int N = 1010;
2  int n, a[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[j] < a[i])
13                 f[i] = max(f[i], f[j] + 1);
14     }
15     int res = 0;
16     for (int i = 1; i <= n; i++)
17         res = max(res, f[i]);
18     cout << res;
19 }
```

Another is an $O(n \log n)$ solution:

```
1  const int N = 100010;
2  int n, a[N], q[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++) cin >> a[i];
7      int len = 0;
8      q[len] = -INF;
9      for (int i = 1; i <= n; i++)
```

```

10 {
11     int l = 0, r = len;
12     while (l < r)
13     {
14         int mid = l + r + 1 >> 1;
15         if (q[mid] < a[i]) l = mid;
16         else r = mid - 1;
17     }
18     len = max(r + 1, len);
19     q[r + 1] = a[i];
20 }
21 cout << len;
22 }

```

5.2.2 LCS

```

1 const int N = 1010;
2 int n, m, f[N][N];
3 char a[N], b[N];
4 int main()
5 {
6     cin >> n >> m >> (a + 1) >> (b + 1);
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= m; j++)
9             {
10                 f[i][j] = max(f[i - 1][j], f[i][j
- 1]);
11                 if (a[i] == b[j])
12                     f[i][j] = max(f[i][j], f[i -
1][j - 1] + 1);
13             }
14     cout << f[n][m];
15 }

```

5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```

1 const int N = 310;
2 int n, s[N], f[N][N];
3 int main()
4 {
5     cin >> n;
6     for (int i = 1; i <= n; i++)
7         cin >> s[i], s[i] += s[i - 1];
8     for (int len = 2; len <= n; len++)
9         for (int i = 1; i + len - 1 <= n; i++)
10            {
11                int l = i, r = i + len - 1;
12                f[l][r] = INF;
13                for (int k = l; k < r; k++)
14                    f[l][r] = min(f[l][r], f[l][k]
+ f[k + 1][r] + s[r] - s[l - 1]);
15            }
16     cout << f[1][n];
17 }

```

5.4 Counting DP

```

1 const int N = 1010, M = 1e9 + 7;
2 int n, f[N][N];
3 int main()
4 {
5     cin >> n;
6     f[0][0] = 1;
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= i; j++)
9             f[i][j] = (f[i - 1][j - 1] + f[i -
j][j]) % M;
10    int ans = 0;
11    for (int i = 1; i <= n; i++)
12        ans = (ans + f[n][i]) % M;
13    cout << ans;
14 }

```

5.5 Digit DP

```

1 // 求数 n 的位数
2 int get(int n)
3 {
4     int res = 0;
5     while (n) n /= 10, res++;
6     return res;
7 }
8 int count(int n, int i)
9 {
10    int res = 0, dgt = get(n);
11    for (int j = 1; j <= dgt; j++)
12        {
13            // p 为当前遍历位次(第 j 位)的数大小
14            // <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
15            // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
16            int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
17            // 求要选的数在 i 的左边的数小于 l 的情况:
18            // 1)、当 i 不为 0 时 xxx : 0...0 ~ l - 1, 即 l * (右边的数的位数) == l * p 种选法
19            // 2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0...1 ~ l - 1, 即 (l - 1) * (右边的数的位数) == (l - 1) * p 种选法
20            if (i) res += l * p;
21            else res += (l - 1) * p;
22            // 求要选的数在 i 的左边的数等于 l 的情况: (即视频中的 xxx == l 时)
23            // 1)、i > dj 时 0 种选法
24            // 2)、i == dj 时 yyy : 0...0 ~ r 即 r + 1 种选法
25            // 3)、i < dj 时 yyy : 0...0 ~ 9...9 即 10^(右边的数的位数) == p 种选法
26            if (i == dj) res += r + 1;
27            if (i < dj) res += p;
28        }
29    return res;
30 }
31 int main()
32 {
33     int a, b;

```

```

33 while (cin >> a >> b, a)
34 {
35     if (a > b) swap(a, b);
36     for (int i = 0; i <= 9; ++i)
37         cout << count(b, i) - count(a - 1,
38             i) << ' ';
39     // 利用前缀和思想: [l, r] 的和 = s[r] -
    s[l - 1]
40     cout << '\n';
41 }

```

5.6 State Compression DP

```

1  const int N = 12, M = 1 << 12;
2  int n, m;
3  LL f[N][M];
4  bool st[M];
5  int main()
6  {
7      while (cin >> n >> m, n || m)
8      {
9          memset(f, 0, sizeof f);
10         for (int i = 0; i < 1 << n; i++)
11         {
12             st[i] = true;
13             // 统计连续 0 的个数, 若连续 0 为奇
            数个就不能正好放得下竖放的方格
14             int cnt = 0;
15             for (int j = 0; j < n && st[i]; j
                ++))
16                 if (i >> j & 1)
17                 {
18                     // 当前格子被使用
19                     // 如果连续 0 的数量为奇数
            个, 当前格子被使用的后果就是导致格子重合,
            所以不可取
20                     if (cnt & 1)
21                         st[i] = false;
22                     // 刷新状态
23                     cnt = 0;
24                 }
25                 else cnt++;
26             // 最后再判断一次, 防止漏判
27             if (cnt & 1)
28                 st[i] = false;
29         }
30         // 没有摆放任何棋子的状态默认只有 1 种取
            法
31         f[0][0] = 1;
32         // 遍历每一列
33         for (int i = 1; i <= m; i++)
34             // 遍历当前列的每一种用二进制数字表
            示的摆放状态: 1 指横向摆放, 0 指空位
35             for (int j = 0; j < 1 << n; j++)
36                 // 遍历上一列的每一种用二进制数
            字表示的摆放状态: 1 指横向摆放, 0 指空位
37                 for (int k = 0; k < 1 << n; k
                    ++))
38                     // 满足两个条件: 两列的摆放
            互不冲突; 两列摆放状态的结合状态是一个可取
            的状态则累加情况数
39                     if (!(j & k) && st[j | k])

```

```

40             f[i][j] += f[i - 1][k]
41             ];
42             // 输出摆放好第 m 列且第 (m + 1) 列没有
            任何方格的状态数
43             cout << f[m][0] << '\n';
44         }
45     }

```

5.7 Tree DP

```

1  // Don't use I/O functions from stdio.h!!!
2  #define itn int
3  #define nit int
4  #define nti int
5  #define tin int
6  #define tni int
7  #define retrun return
8  #define reutrn return
9  #define rutren return
10 #define INF 0x3f3f3f3f
11 #include <bits/stdc++.h>
12 using namespace std;
13 typedef pair<int, int> PII;
14 typedef long long LL;
15
16 const int N = 6010;
17
18 int n;
19 int e[N], ne[N], happy[N], h[N], idx;
20 int f[N][2];
21 bool has_father[N];
22 void add(int a, int b)
23 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
24 }
25 void dfs(int u)
26 {
27     f[u][1] = happy[u];
28     for (int i = h[u]; ~i; i = ne[i])
29         dfs(e[i]);
30     f[u][0] += max(f[e[i]][0], f[e[i]][1]);
31     f[u][1] += f[e[i]][0];
32 }
33
34 int main()
35 {
36     memset(h, -1, sizeof h);
37     cin >> n;
38     for (int i = 1; i <= n; i++) cin >> happy[
39         i];
40     for (int i = 0; i < n - 1; i++)
41     {
42         int a, b;
43         cin >> a >> b;
44         has_father[a] = true;
45         add(b, a);
46     }
47     int root = 1;
48     while (has_father[root]) root++;
49     dfs(root);
50     cout << max(f[root][0], f[root][1]);
51 }

```


5.8 Memoized Search

```
1  const int N = 310;
2  int n, m,
3  h[N][N], f[N][N],
4  dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1,
5      0};
6  int dp(int x, int y)
7  {
8      int &v = f[x][y];
9      if (v != -1) return v;
10     v = 1;
11     for (int i = 0; i < 4; i++)
12     {
13         int a = x + dx[i], b = y + dy[i];
14         if (a >= 1 && a <= n && b >= 1 && b <=
15             m && h[a][b] < h[x][y])
```

```
14         v = max(v, dp(a, b) + 1);
15     }
16     return v;
17 }
18 int main()
19 {
20     cin >> n >> m;
21     for (int i = 1; i <= n; i++)
22         for (int j = 1; j <= m; j++)
23             cin >> h[i][j];
24     memset(f, -1, sizeof f);
25     int res = 0;
26     for (int i = 1; i <= n; i++)
27         for (int j = 1; j <= m; j++)
28             res = max(res, dp(i, j));
29     cout << res;
30 }
```



国家超级计算广州中心
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

Part II: Advanced Template

CREATED BY

Luliet Lyan & Bleu Echo

NSCC-GZ

School of Computer Science & Engineering
Sun Yat-Sen University

Supervisor: Dr Dan Huang

Co-Supervisor: Dr Zhiguang Chen

6 ★ Advanced Basic

6.1 Slow Multiplication

```
1 LL mul(LL a, LL b, LL p)
2 {
3     LL ans = 0;
4     while (b)
5     {
6         if (b & 1) ans = (ans + a) % p;
7         a = a * 2 % p; b >>= 1;
8     }
9     return ans;
10 }
```

6.2 Sum of Geometric Series

```
1 const int mod = 9901;
2 int a, b;
3 int qmi(int a, int k)
4 {
5     int res = 1;
6     a %= mod;
7     while (k)
8     {
9         if (k & 1)
10             res = res * a % mod;
11         a = a * a % mod;
12         k >>= 1;
13     }
14     return res;
15 }
16 int sum(int p, int k)
17 {
18     if (k == 1) return 1;
19     if (k % 2 == 0)
20         return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
21     return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
22 }
23 int main()
24 {
25     // 以  $a^b$  约数之和为例求等比数列和
26     cin >> a >> b;
27     int res = 1;
28     for (int i = 2; i <= a / i; i++)
29         if (a % i == 0)
30         {
31             int s = 0;
32             while (a % i == 0) a /= i, s++;
33             res = res * sum(i, b * s + 1) % mod;
34         }
35     if (a > 1) res = res * sum(a, b + 1) % mod;
36 }
```

6.3 Sort

6.3.1 Card Balancing Problem

```
1 cin >> n;
2 for (int i = 1; i <= n; i++)
3     cin >> a[i], avg += a[i];
4 avg /= n;
5 for (int i = 1; i <= n; i++)
6     if (a[i] != avg)
7         a[i + 1] += a[i] - avg, ans++;
8 cout << ans;
```

6.3.2 2D Card Balancing Problem

```
1 const int N = 100010;
2 int row[N], col[N], c[N], s[N];
3 LL work(int n, int a[])
4 {
5     for (int i = 1; i <= n; i++)
6         s[i] = s[i - 1] + a[i];
7     if (s[n] % n) return -1;
8     int avg = s[n] / n;
9     c[1] = 0;
10    for (int i = 2; i <= n; i++)
11        c[i] = s[i - 1] - (i - 1) * avg;
12    sort(c + 1, c + n + 1);
13    LL res = 0;
14    for (int i = 1; i <= n; i++)
15        res += abs(c[i] - c[(n + 1) / 2]);
16    return res;
17 }
18 int main()
19 {
20     int n, m, cnt;
21     cin >> n >> m >> cnt;
22     while (cnt--)
23     {
24         int x, y;
25         cin >> x >> y;
26         row[x]++; col[y]++;
27     }
28     LL r = work(n, row);
29     LL c = work(m, col);
30     if (r != -1 && c != -1)
31         cout << "both " << r + c;
32     else if (r != -1)
33         cout << "row " << r;
34     else if (c != -1)
35         cout << "column " << c;
36     else cout << "impossible";
37 }
```

6.3.3 Dual Heaps

```
1 if (down.empty() || x <= down.top())
2     down.push(x);
3 else up.push(x);
4 if (down.size() > up.size() + 1)
5     up.push(down.top(), down.pop());
```

```

6  if (up.size() > down.size())
7      down.push(up.top()), up.pop();
8  if (i % 2)
9  {
10     cout << down.top() << ' ';
11     if (++cnt % 10 == 0) cout << '\n';
12 }

```

6.4 RMQ

```

1  const int N = 200010, M = 18;
2  int n, m, w[N], f[N][M];
3  void init()
4  {
5      for (int j = 0; j < M; j++)
6          for (int i = 1; i + (1 << j) - 1 <= n;
7              i++)
8              if (!j) f[i][j] = w[i];
9              else // 也可以是最小值
10                 f[i][j] = max(f[i][j - 1], f[i
11                     + (1 << j - 1)][j - 1]);
12 }
13 int query(int l, int r)
14 {
15     int len = r - l + 1;
16     int k = log(len) / log(2);
17     return max(f[l][k], f[r - (1 << k) + 1][k
18         ]);
19 }

```

7 ★ Advanced Data Structures

7.1 Binary Indexed Tree

```
1 // 支持区间修改、区间查询
2 // 利用变差分求二阶区间和
3 const int N = 100010;
4 int n, m, a[N];
5 LL tr1[N], tr2[N];
6 int lowbit(int x) { return x & -x; }
7 void add(LL tr[], LL x, LL c)
8 {
9     for (int i = x; i <= n; i += lowbit(i))
10         tr[i] += c;
11 }
12 LL sum(LL tr[], LL x)
13 {
14     LL res = 0;
15     for (int i = x; i; i -= lowbit(i))
16         res += tr[i];
17     return res;
18 }
19 LL prefix_sum(LL x)
20 { return sum(tr1, x) * (x + 1) - sum(tr2, x); }
21 int main()
22 {
23     cin >> n >> m;
24     for (int i = 1; i <= n; i++)
25         cin >> a[i];
26     for (int i = 1; i <= n; i++)
27     {
28         int b = a[i] - a[i - 1];
29         add(tr1, i, b);
30         add(tr2, i, (LL)i * b);
31     }
32     while (m--)
33     {
34         char op[2];
35         int l, r, d;
36         cin >> op >> l >> r;
37         if (*op == 'Q')
38             cout << prefix_sum(r) - prefix_sum(l - 1) << '\n';
39         else
40         {
41             cin >> d;
42             add(tr1, l, d), add(tr2, l, (LL)l
43             * d),
44             add(tr1, r + 1, -d),
45             add(tr2, r + 1, (LL)-(r + 1) * d);
46         }
47     }
```

7.2 Segment Tree

7.2.1 Maintain the Maximum

```
1 struct Node
2 { int l, r, v; } tr[N * 4];
```

```
3 void pushup(int u)
4 {
5     tr[u].v = max(tr[u << 1].v, tr[u << 1 |
6     1].v);
7 }
8 void build(int u, int l, int r)
9 {
10     tr[u] = {l, r};
11     if (l == r) return;
12     int mid = l + r >> 1;
13     build(u << 1, l, mid),
14     build(u << 1 | 1, mid + 1, r);
15 }
16 int query(int u, int l, int r)
17 {
18     if (tr[u].l >= l && tr[u].r <= r)
19         return tr[u].v;
20     int mid = tr[u].l + tr[u].r >> 1;
21     int v = 0;
22     if (l <= mid)
23         v = query(u << 1, l, r);
24     if (r > mid)
25         v = max(v, query(u << 1 | 1, l, r));
26     return v;
27 }
28 void modify(int u, int x, int v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31         tr[u].v = v;
32     else
33     {
34         int mid = tr[u].l + tr[u].r >> 1;
35         if (x <= mid)
36             modify(u << 1, x, v);
37         else
38             modify(u << 1 | 1, x, v);
39         pushup(u);
40     }
```

7.2.2 Maintain the Maximum Subarray Sum

```
1 struct Node
2 { int l, r, sum, lmax, rmax, tmax; } tr[N *
3     4];
4 void pushup(Node &u, Node &l, Node &r)
5 {
6     u.sum = l.sum + r.sum;
7     u.lmax = max(l.lmax, l.sum + r.lmax);
8     u.rmax = max(r.rmax, r.sum + l.rmax);
9     u.tmax = max(max(l.tmax, r.tmax), l.rmax +
10     r.lmax);
11 }
12 void pushup(int u)
13 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
14 void build(int u, int l, int r)
15 {
16     if (l == r)
17         tr[u] = {l, r, w[r], w[r], w[r], w[r]
18         ]};
19     else
20     {
```

```

18     tr[u] = {l, r};
19     int mid = l + r >> 1;
20     build(u << 1, l, mid),
21     build(u << 1 | 1, mid + 1, r);
22     pushup(u);
23 }
24 }
25 void modify(int u, int x, int v)
26 {
27     if (tr[u].l == x && tr[u].r == x)
28         tr[u] = {x, x, v, v, v, v};
29     else
30     {
31         int mid = tr[u].l + tr[u].r >> 1;
32         if (x <= mid)
33             modify(u << 1, x, v);
34         else
35             modify(u << 1 | 1, x, v);
36         pushup(u);
37     }
38 }
39 Node query(int u, int l, int r)
40 {
41     if (tr[u].l >= l && tr[u].r <= r)
42         return tr[u];
43     else
44     {
45         int mid = tr[u].l + tr[u].r >> 1;
46         if (r <= mid)
47             return query(u << 1, l, r);
48         else if (l > mid)
49             return query(u << 1 | 1, l, r);
50         else
51         {
52             auto left = query(u << 1, l, r);
53             auto right = query(u << 1 | 1, l,
54                               r);
55             Node res;
56             pushup(res, left, right);
57             return res;
58         }
59     }

```

7.2.3 Maintain the GCD

```

1  struct Node
2  { int l, r; LL sum, d; } tr[N * 4];
3  LL gcd(LL a, LL b)
4  { return b ? gcd(b, a % b) : a; }
5  void pushup(Node &u, Node &l, Node &r)
6  {
7      u.sum = l.sum + r.sum;
8      u.d = gcd(l.d, r.d);
9  }
10 void pushup(int u)
11 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
12 void build(int u, int l, int r)
13 {
14     if (l == r)
15     {
16         LL b = w[r] - w[r - 1];
17         tr[u] = {l, r, b, b};

```

```

18     }
19     else
20     {
21         tr[u].l = l, tr[u].r = r;
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
28 void modify(int u, int x, LL v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31     {
32         LL b = tr[u].sum + v;
33         tr[u] = {x, x, b, b};
34     }
35     else
36     {
37         int mid = tr[u].l + tr[u].r >> 1;
38         if (x <= mid)
39             modify(u << 1, x, v);
40         else
41             modify(u << 1 | 1, x, v);
42         pushup(u);
43     }
44 }
45 Node query(int u, int l, int r)
46 {
47     if (tr[u].l >= l && tr[u].r <= r)
48         return tr[u];
49     else
50     {
51         int mid = tr[u].l + tr[u].r >> 1;
52         if (r <= mid)
53             return query(u << 1, l, r);
54         else if (l > mid)
55             return query(u << 1 | 1, l, r);
56         else
57         {
58             auto left = query(u << 1, l, r);
59             auto right = query(u << 1 | 1, l,
60                               r);
61             Node res;
62             pushup(res, left, right);
63             return res;
64         }
65     }

```

7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```

1  struct Node
2  { int l, r; LL sum, add; } tr[N * 4];
3  void pushup(int u)
4  { tr[u].sum = tr[u << 1].sum + tr[u << 1 |
5    1].sum; }
6  void pushdown(int u)
7  {
8      auto &root = tr[u],
9      &left = tr[u << 1],

```

```

9      &right = tr[u << 1 | 1];
10  if (root.add)
11  {
12      left.add += root.add,
13      left.sum += (LL)(left.r - left.l + 1)
14      * root.add;
15      right.add += root.add,
16      right.sum += (LL)(right.r - right.l +
17      1) * root.add;
18      root.add = 0;
19  }
20  }
21  void build(int u, int l, int r)
22  {
23      if (l == r) tr[u] = {l, r, w[r], 0};
24      else
25      {
26          tr[u] = {l, r};
27          int mid = l + r >> 1;
28          build(u << 1, l, mid);
29          build(u << 1 | 1, mid + 1, r);
30          pushup(u);
31      }
32  }
33  void modify(int u, int l, int r, int d)
34  {
35      if (tr[u].l >= l && tr[u].r <= r)
36      {
37          tr[u].sum += (LL)(tr[u].r - tr[u].l +
38          1) * d;
39          tr[u].add += d;
40      }
41      else
42      {
43          pushdown(u);
44          int mid = tr[u].l + tr[u].r >> 1;
45          if (l <= mid)
46              modify(u << 1, l, r, d);
47          if (r > mid)
48              modify(u << 1 | 1, l, r, d);
49          pushup(u);
50      }
51  }
52  LL query(int u, int l, int r)
53  {
54      if (tr[u].l >= l && tr[u].r <= r)
55          return tr[u].sum;
56      pushdown(u);
57      int mid = tr[u].l + tr[u].r >> 1;
58      LL sum = 0;
59      if (l <= mid)
60          sum += query(u << 1, l, r);
61      if (r > mid)
62          sum += query(u << 1 | 1, l, r);
63      return sum;
64  }

```

7.3 Persistent Data Structure

7.3.1 Persistent Trie

```

1  const int N = 600010, M = N * 25;
2  int n, m, s[N], root[N], idx;

```

```

3  int trie[M][2], max_id[M];
4  void insert(int i, int k, int p, int q)
5  {
6      if (k < 0)
7      {
8          max_id[q] = i;
9          return;
10     }
11     int v = s[i] >> k & 1;
12     if (p)
13         trie[q][v ^ 1] = trie[p][v ^ 1];
14     trie[q][v] = ++idx;
15     insert(i, k - 1, trie[p][v], trie[q][v]);
16     max_id[q] = max(max_id[trie[q][0]], max_id
17     [trie[q][1]]);
18 }
19 int query(int root, int C, int L)
20 {
21     int p = root;
22     for (int i = 23; i >= 0; i--)
23     {
24         int v = C >> i & 1;
25         if (max_id[trie[p][v ^ 1]] >= L)
26             p = trie[p][v ^ 1];
27         else
28             p = trie[p][v];
29     }
30     return C ^ s[max_id[p]];
31 }
32 // insert(i, 23, root[i - 1], root[i]);
33 // query(root[r - 1], l - 1, x ^ s[n]);

```

7.3.2 Persistent Segment Tree

```

1  const int N = 100010, M = 10010;
2  int n, m, a[N], root[N], idx;
3  vector<int> nums;
4  struct Node
5  {
6      int l, r;
7      int cnt;
8  } tr[N * 4 + N * 17];
9  int find(int x)
10 {
11     return lower_bound(nums.begin(), nums.end
12     (), x) - nums.begin();
13 }
14 int build(int l, int r)
15 {
16     int p = ++idx;
17     if (l == r)
18         return p;
19     int mid = l + r >> 1;
20     tr[p].l = build(l, mid), tr[p].r = build(
21     mid + 1, r);
22     return p;
23 }
24 int insert(int p, int l, int r, int x)
25 {
26     int q = ++idx;
27     tr[q] = tr[p];
28     if (l == r)
29     {
30         tr[q].cnt++;
31     }
32     else
33     {
34         int mid = l + r >> 1;
35         if (x <= mid)
36             tr[q].l = insert(tr[p].l, l, mid, x);
37         else
38             tr[q].r = insert(tr[p].r, mid + 1, r, x);
39     }
40     return q;
41 }
42 int query(int p, int l, int r, int x)
43 {
44     if (l == r)
45         return tr[p].cnt;
46     int mid = l + r >> 1;
47     if (x <= mid)
48         return query(tr[p].l, l, mid, x);
49     else
50         return query(tr[p].r, mid + 1, r, x);
51 }

```

```

29     return q;
30 }
31 int mid = l + r >> 1;
32 if (x <= mid)
33     tr[q].l = insert(tr[p].l, l, mid, x);
34 else
35     tr[q].r = insert(tr[p].r, mid + 1, r,
36                     x);
37 tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].
38 cnt;
39 return q;
40 }
41 int query(int q, int p, int l, int r, int k)
42 {
43     if (l == r)
44         return r;
45     int cnt = tr[tr[q].l].cnt - tr[tr[p].l].
46 cnt;
47 int mid = l + r >> 1;
48 if (k <= cnt)
49     return query(tr[q].l, tr[p].l, l, mid,
50 k);
51 else
52     return query(tr[q].r, tr[p].r, mid +
53 1, r, k - cnt);
54 }

```

7.4 Treap

```

1  const int N = 100010, INF = 1e8;
2  int n, root, idx;
3  struct Node
4  { int l, r, key, val, cnt, size; } tr[N];
5  void pushup(int p)
6  {
7      tr[p].size = tr[tr[p].l].size +
8      tr[tr[p].r].size + tr[p].cnt;
9  }
10 int get_node(int key)
11 {
12     tr[++idx].key = key;
13     tr[idx].val = rand();
14     tr[idx].cnt = tr[idx].size = 1;
15     return idx;
16 }
17 void zig(int &p)
18 {
19     int q = tr[p].l;
20     tr[p].l = tr[q].r, tr[q].r = p, p = q;
21     pushup(tr[p].r), pushup(p);
22 }
23 void zag(int &p)
24 {
25     int q = tr[p].r;
26     tr[p].r = tr[q].l, tr[q].l = p, p = q;
27     pushup(tr[p].l), pushup(p);
28 }
29 void build()
30 {
31     get_node(-INF), get_node(INF);
32     root = 1, tr[1].r = 2;
33     pushup(root);
34     if (tr[1].val < tr[2].val) zag(root);

```

```

35 }
36 void insert(int &p, int key)
37 {
38     if (!p) p = get_node(key);
39     else if (tr[p].key == key) tr[p].cnt++;
40     else if (tr[p].key > key)
41     {
42         insert(tr[p].l, key);
43         if (tr[tr[p].l].val > tr[p].val)
44             zig(p);
45     }
46     else
47     {
48         insert(tr[p].r, key);
49         if (tr[tr[p].r].val > tr[p].val)
50             zag(p);
51     }
52     pushup(p);
53 }
54 void remove(int &p, int key)
55 {
56     if (!p) return;
57     if (tr[p].key == key)
58     {
59         if (tr[p].cnt > 1) tr[p].cnt--;
60         else if (tr[p].l || tr[p].r)
61         {
62             if (!tr[p].r || tr[tr[p].l].val >
63 tr[tr[p].r].val)
64             {
65                 zig(p);
66                 remove(tr[p].r, key);
67             }
68             else
69             {
70                 zag(p);
71                 remove(tr[p].l, key);
72             }
73         }
74         else p = 0;
75     }
76     else if (tr[p].key > key)
77         remove(tr[p].l, key);
78     else remove(tr[p].r, key);
79     pushup(p);
80 }
81 int get_rank_by_key(int p, int key)
82 {
83     if (!p) return 0;
84     if (tr[p].key == key)
85         return tr[tr[p].l].size + 1;
86     if (tr[p].key > key)
87         return get_rank_by_key(tr[p].l, key);
88     return tr[tr[p].l].size + tr[p].cnt +
89 get_rank_by_key(tr[p].r, key);
90 }
91 int get_key_by_rank(int p, int rank)
92 {
93     if (!p) return INF;
94     if (tr[tr[p].l].size >= rank)
95         return get_key_by_rank(tr[p].l, rank);
96     if (tr[tr[p].l].size + tr[p].cnt >= rank)
97         return tr[p].key;
98     return get_key_by_rank(tr[p].r, rank - tr[
99 tr[p].l].size - tr[p].cnt);
100 }

```



```

98 int get_prev(int p, int key)
99 {
100     if (!p) return -INF;
101     if (tr[p].key >= key)
102         return get_prev(tr[p].l, key);
103     return max(tr[p].key, get_prev(tr[p].r,
104                                   key));
105 }
106 int get_next(int p, int key)
107 {
108     if (!p) return INF;
109     if (tr[p].key <= key)
110         return get_next(tr[p].r, key);
111     return min(tr[p].key, get_next(tr[p].l,
112                                   key));
113 }

```

7.5 AC Automaton

```

1  const int N = 10010, M = 1000010, S = 55;
2  int n, tr[N * S][26], cnt[N * S], idx;
3  int q[N * S], ne[N * S];
4  char str[M];
5  void insert()
6  {
7      int p = 0;
8      for (int i = 0; str[i]; i++)
9      {
10         int t = str[i] - 'a';
11         if (!tr[p][t]) tr[p][t] = ++idx;
12         p = tr[p][t];
13     }
14     cnt[p]++;
15 }
16 void build()
17 {
18     int hh = 0, tt = -1;
19     for (int i = 0; i < 26; i++)
20         if (tr[0][i]) q[++tt] = tr[0][i];
21     while (hh <= tt)
22     {
23         int t = q[hh++];
24         for (int i = 0; i < 26; i++)
25         {
26             int p = tr[t][i];
27             if (!p) tr[t][i] = tr[ne[t]][i];
28             else
29             {
30                 ne[p] = tr[ne[t]][i];
31                 q[++tt] = p;
32             }
33         }
34     }
35 }

```

8 ★ Advanced Search

8.1 Flood-Fill

```
1  const int N = 1010, M = N * N;
2  int n, m;
3  char g[N][N];
4  PII q[M];
5  bool st[N][N];
6  void bfs(int sx, int sy)
7  {
8      int hh = 0, tt = 0;
9      q[0] = {sx, sy}; st[sx][sy] = true;
10     while (hh <= tt)
11     {
12         PII t = q[hh++];
13         for (int i = t.first - 1; i <= t.first + 1; i++)
14             for (int j = t.second - 1; j <= t.second + 1; j++)
15             {
16                 if (i == t.first && j == t.second)
17                     continue;
18                 if (i < 0 || i >= n || j < 0 || j >= m)
19                     continue;
20                 if (g[i][j] == '.' || st[i][j])
21                     continue;
22                 q[++tt] = {i, j};
23                 st[i][j] = true;
24             }
25     }
26 }
27 int main()
28 {
29     int cnt = 0;
30     for (int i = 0; i < n; i++)
31         for (int j = 0; j < m; j++)
32             if (g[i][j] == 'W' && !st[i][j])
33                 { bfs(i, j); cnt++; }
34 }
```

8.2 Multi-source BFS

```
1  const int N = 1010, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  PII q[M];
5  int dx[4] = {-1, 0, 1, 0},
6      dy[4] = {0, 1, 0, -1};
7  void bfs()
8  {
9      memset(dist, -1, sizeof dist);
10     int hh = 0, tt = -1;
11     for (int i = 1; i <= n; i++)
12         for (int j = 1; j <= m; j++)
13             if (g[i][j] == '1')
14             {
15                 dist[i][j] = 0;
```

```
16                 q[++tt] = {i, j};
17             }
18     while (hh <= tt)
19     {
20         auto t = q[hh++];
21         for (int i = 0; i < 4; i++)
22         {
23             int a = t.x + dx[i], b = t.y + dy[i];
24             if (a < 1 || a > n || b < 1 || b > m) continue;
25             if (dist[a][b] != -1) continue;
26             dist[a][b] = dist[t.x][t.y] + 1;
27             q[++tt] = {a, b};
28         }
29     }
30 }
```

8.3 BFS with Deque

```
1  const int N = 510, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  bool st[N][N];
5  int dx[4] = {-1, -1, 1, 1},
6      dy[4] = {-1, 1, 1, -1},
7      ix[4] = {-1, -1, 0, 0},
8      iy[4] = {-1, 0, 0, -1};
9  int bfs()
10 {
11     memset(dist, 0x3f, sizeof dist);
12     memset(st, 0, sizeof st);
13     dist[0][0] = 0;
14     deque<PII> q;
15     q.push_back({0, 0});
16     char cs[] = "\\ / \\/";
17     while (q.size())
18     {
19         PII t = q.front();
20         q.pop_front();
21         if (st[t.x][t.y]) continue;
22         st[t.x][t.y] = true;
23         for (int i = 0; i < 4; i++)
24         {
25             int a = t.x + dx[i], b = t.y + dy[i];
26             if (a < 0 || a > n || b < 0 || b > m) continue;
27             int ca = t.x + ix[i], cb = t.y + iy[i];
28             int d = dist[t.x][t.y] + (g[ca][cb] != cs[i]);
29             if (d < dist[a][b])
30             {
31                 dist[a][b] = d;
32                 if (g[ca][cb] != cs[i])
33                     q.push_back({a, b});
34                 else
35                     q.push_front({a, b});
36             }
37         }
38     }
39 }
40 return dist[n][m];
```

```
41 }
```

8.4 Bidirectional BFS

```
1 int bfs()
2 {
3     if (A == B) return 0;
4     queue<string> qa, qb;
5     unordered_map<string, int> da, db;
6     qa.push(A), qb.push(B);
7     da[A] = db[B] = 0;
8     int step = 0;
9     while (qa.size() && qb.size())
10    {
11        int t;
12        if (qa.size() < qb.size())
13            // PROCESS
14        else
15            // PROCESS
16        if (t <= 10) return t;
17        if (++step == 10) return -1;
18    }
19    return -1;
20 }
```

8.5 A*

```
1 const int N = 1010, M = 200010;
2 int n, m, S, T, K;
3 int h[N], rh[N], e[M], w[M], ne[M], idx;
4 int dist[N], cnt[N];
5 bool st[N];
6 void dijkstra()
7 {
8     priority_queue<PII, vector<PII>, greater<
9         PII>> heap;
10    heap.push({0, T});
11    memset(dist, 0x3f, sizeof dist);
12    dist[T] = 0;
13    while (heap.size())
14    {
15        auto t = heap.top();
16        heap.pop();
17        int ver = t.y;
18        if (st[ver]) continue;
19        st[ver] = true;
20        for (int i = rh[ver]; ~i; i = ne[i])
21        {
22            int j = e[i];
23            if (dist[j] > dist[ver] + w[i])
24            {
25                dist[j] = dist[ver] + w[i];
26                heap.push({dist[j], j});
27            }
28        }
29    }
30 }
31 int astar()
32 {
```

```
33     priority_queue<PIII, vector<PIII>, greater
34         <PIII>> heap;
35     heap.push({dist[S], {0, S}});
36     while (heap.size())
37     {
38         auto t = heap.top();
39         heap.pop();
40         int ver = t.y.y, distance = t.y.x;
41         cnt[ver]++;
42         if (cnt[T] == K) return distance;
43         for (int i = h[ver]; ~i; i = ne[i])
44         {
45             int j = e[i];
46             if (cnt[j] < K)
47                 heap.push({distance + w[i] +
48                     dist[j], {distance + w[i], j}});
49         }
50     }
51     return -1;
52 }
53 int main()
54 {
55     // PROCESS
56     dijkstra(); cout << astar();
57     // PROCESS
58 }
```

8.6 DFS Connectivity Model

```
1 char g[N][N];
2 int xa, ya, xb, yb;
3 int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0,
4     -1};
5 bool st[N][N];
6 bool dfs(int x, int y)
7 {
8     if (g[x][y] == '#') return false;
9     if (x == xb && y == yb) return true;
10    st[x][y] = true;
11    for (int i = 0; i < 4; i++)
12    {
13        int a = x + dx[i], b = y + dy[i];
14        if (a < 0 || a >= n || b < 0 || b >= n
15            ) continue;
16        if (st[a][b]) continue;
17        if (dfs(a, b)) return true;
18    }
19    return false;
20 }
```

8.7 IDDFS

```
1 const int N = 110;
2 int n, path[N];
3 bool dfs(int u, int k)
4 {
5     if (u == k)
6         return path[u - 1] == n;
7     bool st[N] = {0};
8     for (int i = u - 1; i >= 0; i--)
```

```

9     for (int j = i; j >= 0; j--)
10    {
11        int s = path[i] + path[j];
12        if (s > n || s <= path[u - 1] ||
13            st[s]) continue;
14        st[s] = true;
15        path[u] = s;
16        if (dfs(u + 1, k)) return true;
17    }
18    return false;
19 }

```

```

9 {
10     if (f() > maxn - depth) return false;
11     if (depth == maxn) return true;
12     for (int i = 0; i <= n; i++)
13     {
14         // OPERATION
15         if (IDAstar(depth + 1, maxn))
16             return true;
17         // OPERATION
18     }
19     return false;
20 }

```

8.8 Bidirectional DFS

```

1  const int N = 1 << 24;
2  int n, m, k, cnt = 0, ans;
3  int g[50], weights[N];
4  void dfs(int u, int s)
5  {
6      if (u == k)
7      {
8          weights[cnt++] = s;
9          return;
10     }
11     if ((LL)s + g[u] <= m)
12         dfs(u + 1, s + g[u]);
13     dfs(u + 1, s);
14 }
15 void dfs2(int u, int s)
16 {
17     if (u == n)
18     {
19         int l = 0, r = cnt - 1;
20         while (l < r)
21         {
22             int mid = l + r + 1 >> 1;
23             if (weights[mid] + (LL)s <= m)
24                 l = mid;
25             else r = mid - 1;
26         }
27         if (weights[l] + (LL)s <= m)
28             ans = max(ans, weights[l] + s);
29         return;
30     }
31     if ((LL)s + g[u] <= m)
32         dfs2(u + 1, s + g[u]);
33     dfs2(u + 1, s);
34 }

```

8.9 IDA*

```

1  const int N = 1e2;
2  int n, a[N];
3  string t;
4  int f()
5  {
6      // YOUR_F_FUNCTION
7  }
8  bool IDAstar(int depth, int maxn)

```

9 ★ Advanced Graph Theory

9.1 Detecting Negative Cycles

```
1  int n, m1, m2;
2  int h[N], e[M], w[M], ne[M], idx;
3  int dist[N], q[N], cnt[N];
4  bool st[N];
5  bool spfa()
6  {
7      memset(dist, 0, sizeof dist);
8      memset(cnt, 0, sizeof cnt);
9      memset(st, 0, sizeof st);
10     int hh = 0, tt = 0;
11     for (int i = 1; i <= n; i++)
12     {
13         q[tt++] = i;
14         st[i] = true;
15     }
16     while (hh != tt)
17     {
18         int t = q[hh++];
19         if (hh == N) hh = 0;
20         st[t] = false;
21         for (int i = h[t]; ~i; i = ne[i])
22         {
23             int j = e[i];
24             if (dist[j] > dist[t] + w[i])
25             {
26                 dist[j] = dist[t] + w[i];
27                 cnt[j] = cnt[t] + 1;
28                 if (cnt[j] >= n) return true;
29                 if (!st[j])
30                 {
31                     q[tt++] = j;
32                     if (tt == N) tt = 0;
33                     st[j] = true;
34                 }
35             }
36         }
37     }
38     return false;
39 }
```

9.2 SPFA-SLF

Using deque to solve SPFA question.

```
1  void spfa()
2  {
3      memset(dist, 0x3f, sizeof dist);
4      memset(st, 0, sizeof st);
5      deque<int> q;
6      q.push_back(s);
7      st[s] = 1, dist[s] = 0;
8      while (q.size())
9      {
10         int t = q.front();
11         q.pop_front();
12         st[t] = 0;
13         for (int i = h[t]; ~i; i = ne[i])
14         {
```

```
15         int j = e[i];
16         if (dist[j] > dist[t] + w[i])
17         {
18             dist[j] = dist[t] + w[i];
19             if (!st[j])
20             {
21                 st[j] = true;
22                 if (q.size() && dist[j] < dist[q.
23                     front()])
24                     q.push_front(j);
25                 else
26                     q.push_back(j);
27             }
28         }
29     }
30 }
```

9.3 SPFA-Stack

```
1  bool spfa()
2  {
3      int hh = 0, tt = 1;
4      memset(dist, -0x3f, sizeof dist);
5      dist[0] = 0;
6      q[0] = 0;
7      while (hh != tt)
8      {
9          int t = q[--tt];
10         st[t] = false;
11         for (int i = h[t]; ~i; i = ne[i])
12         {
13             int j = e[i];
14             if (dist[j] < dist[t] + w[i])
15             {
16                 dist[j] = dist[t] + w[i];
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n + 1) return true;
19                 if (!st[j])
20                 {
21                     st[j] = true;
22                     q[tt++] = j;
23                 }
24             }
25         }
26     }
27     return false;
28 }
```

9.4 SPFA & MIN & MAX

Using SPFA to maintain the minimum and maximum. In this case we need **Original Graph** and **Reverse Graph**, in which we can use `type == 0` or `type == 1` to describe.

```
1  void spfa(int h[], int dist[], int type)
2  {
3      int hh = 0, tt = 1;
4      if (type == 0)
5      {
```

```

6  memset(dist, 0x3f, sizeof dmin);
7  dist[1] = w[1];
8  q[0] = 1;
9  }
10 else
11 {
12  memset(dist, -0x3f, sizeof dmax);
13  dist[n] = w[n];
14  q[0] = n;
15  }
16  while (hh != tt)
17  {
18  int t = q[hh++];
19  if (hh == N) hh = 0;
20  st[t] = false;
21  for (int i = h[t]; ~i; i = ne[i])
22  {
23  int j = e[i];
24  if (type == 0 && dist[j] > min(dist[t],
25  w[j]) || type == 1 && dist[j] < max(dist
26  [t], w[j]))
27  {
28  if (type == 0)
29  dist[j] = min(dist[t], w[j]);
30  else
31  dist[j] = max(dist[t], w[j]);
32  if (!st[j])
33  {
34  q[tt++] = j;
35  if (tt == N) tt = 0;
36  st[j] = true;
37  }
38  }
39  }

```

```

Ver>> heap;
24 heap.push({S, 0, 0});
25 while (heap.size())
26 {
27 Ver t = heap.top();
28 heap.pop();
29 int ver = t.id, type = t.type, distance =
30 t.dist, count = cnt[ver][type];
31 if (st[ver][type])
32 continue;
33 st[ver][type] = true;
34 for (int i = h[ver]; ~i; i = ne[i])
35 {
36 int j = e[i];
37 if (dist[j][0] > distance + w[i])
38 {
39 dist[j][1] = dist[j][0], cnt[j][1] =
40 cnt[j][0];
41 heap.push({j, 1, dist[j][1]});
42 dist[j][0] = distance + w[i], cnt[j]
43 [0] = count;
44 heap.push({j, 0, dist[j][0]});
45 }
46 else if (dist[j][0] == distance + w[i])
47 cnt[j][0] += count;
48 else if (dist[j][1] > distance + w[i])
49 {
50 dist[j][1] = distance + w[i], cnt[j]
51 [1] = count;
52 heap.push({j, 1, dist[j][1]});
53 }
54 else if (dist[j][1] == distance + w[i])
55 cnt[j][1] += count;
56 }
57 }
58 int res = cnt[T][0];
59 if (dist[T][0] + 1 == dist[T][1])
60 res += cnt[T][1];
61 return res;

```

9.5 Second Shortest Path

```

1  const int N = 1010, M = 20010;
2  struct Ver
3  {
4  int id, type, dist;
5  bool operator>(const Ver &W) const
6  {
7  return dist > W.dist;
8  }
9  };
10 int n, m, S, T, dist[N][2], cnt[N][2];
11 int h[N], e[M], w[M], ne[M], idx;
12 bool st[N][2];
13 void add(int a, int b, int c)
14 {
15  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
16  a] = idx++;
17 }
18 int dijkstra()
19 {
20  memset(st, 0, sizeof st);
21  memset(dist, 0x3f, sizeof dist);
22  memset(cnt, 0, sizeof cnt);
23  dist[S][0] = 0, cnt[S][0] = 1;
24  priority_queue<Ver, vector<Ver>, greater<

```

9.6 Second Minimum Spanning Tree

9.6.1 brute-force

```

1  const int N = 510, M = 10010;
2  int n, m, p[N], dist1[N][N], dist2[N][N];
3  int h[N], e[N * 2], w[N * 2], ne[N * 2], idx
4  ;
5  struct Edge
6  {
7  int a, b, w;
8  bool f;
9  bool operator<(const Edge &e) const
10 { return w < e.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 {
14  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
15  a] = idx++;
16 }
17 int find(int x)

```

```

16 {
17     if (p[x] != x) p[x] = find(p[x]);
18     return p[x];
19 }
20 void dfs(int u, int fa, int maxd1, int maxd2
    , int d1[], int d2[])
21 {
22     d1[u] = maxd1, d2[u] = maxd2;
23     for (int i = h[u]; ~i; i = ne[i])
24     {
25         int j = e[i];
26         if (j != fa)
27         {
28             int td1 = maxd1, td2 = maxd2;
29             if (w[i] > td1)
30                 td2 = td1, td1 = w[i];
31             else if (w[i] < td1 && w[i] > td2)
32                 td2 = w[i];
33             dfs(j, u, td1, td2, d1, d2);
34         }
35     }
36 }
37 int main()
38 {
39     cin >> n >> m;
40     memset(h, -1, sizeof h);
41     for (int i = 0; i < m; i++)
42         cin >> edge[i].a >> edge[i].b >> edge[i].w
43         ;
44     sort(edge, edge + m);
45     for (int i = 1; i <= n; i++) p[i] = i;
46     LL sum = 0;
47     for (int i = 0; i < m; i++)
48     {
49         int a = edge[i].a, b = edge[i].b, w = edge
            [i].w;
50         int pa = find(a), pb = find(b);
51         if (pa != pb)
52         {
53             p[pa] = pb;
54             sum += w;
55             add(a, b, w), add(b, a, w);
56             edge[i].f = true;
57         }
58     }
59     for (int i = 1; i <= n; i++)
60         dfs(i, -1, -1e9, -1e9, dist1[i], dist2[i])
61         ;
62     LL res = 1e18;
63     for (int i = 0; i < m; i++)
64     {
65         if (!edge[i].f)
66         {
67             int a = edge[i].a, b = edge[i].b, w =
                edge[i].w;
68             LL t;
69             if (w > dist1[a][b])
70                 t = sum + w - dist1[a][b];
71             else if (w > dist2[a][b])
72                 t = sum + w - dist2[a][b];
73             res = min(res, t);
74         }
75     }
76 }

```

9.6.2 LCA

```

1  const int N = 100010, M = 300010;
2  int n, m, p[N], q[N];
3  int h[N], e[M], w[M], ne[M], idx;
4  int depth[N], fa[N][17], d1[N][17], d2[N]
    ][17];
5  struct Edge
6  {
7      int a, b, w;
8      bool used;
9      bool operator<(const Edge &t) const
10     { return w < t.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 { e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
    a] = idx++; }
14 int find(int x)
15 {
16     if (p[x] != x) p[x] = find(p[x]);
17     return p[x];
18 }
19 LL kruskal()
20 {
21     for (int i = 1; i <= n; i++) p[i] = i;
22     sort(edge, edge + m);
23     LL res = 0;
24     for (int i = 0; i < m; i++)
25     {
26         int a = find(edge[i].a), b = find(edge[i].
            b), w = edge[i].w;
27         if (a != b)
28         {
29             p[a] = b; res += w;
30             edge[i].used = true;
31         }
32     }
33     return res;
34 }
35 void build()
36 {
37     memset(h, -1, sizeof h);
38     for (int i = 0; i < m; i++)
39         if (edge[i].used)
40         {
41             int a = edge[i].a, b = edge[i].b, w =
                edge[i].w;
42             add(a, b, w), add(b, a, w);
43         }
44 }
45 void bfs()
46 {
47     memset(depth, 0x3f, sizeof depth);
48     depth[0] = 0, depth[1] = 1, q[0] = 1;
49     int hh = 0, tt = 0;
50     while (hh <= tt)
51     {
52         int t = q[hh++];
53         for (int i = h[t]; ~i; i = ne[i])
54         {
55             int j = e[i];
56             if (depth[j] > depth[t] + 1)
57             {
58                 depth[j] = depth[t] + 1;
59                 q[++tt] = j;
60                 fa[j][0] = t;
61                 d1[j][0] = w[i], d2[j][0] = -INF;

```

```

62     for (int k = 1; k <= 16; k++)
63     {
64         int anc = fa[j][k - 1];
65         fa[j][k] = fa[anc][k - 1];
66         int distance[4] = {d1[j][k - 1],
67                             d2[j][k - 1],
68                             d1[anc][k - 1],
69                             d2[anc][k - 1]};
70         d1[j][k] = d2[j][k] = -INF;
71         for (int u = 0; u < 4; u++)
72         {
73             int d = distance[u];
74             if (d > d1[j][k])
75                 d2[j][k] = d1[j][k], d1[j][k] =
76                 d;
77             else if (d != d1[j][k] && d > d2[j]
78                 [k])
79                 d2[j][k] = d;
80         }
81     }
82 }
83 }
84 int lca(int a, int b, int w)
85 {
86     static int distance[N * 2];
87     int cnt = 0;
88     if (depth[a] < depth[b])
89         swap(a, b);
90     for (int k = 16; k >= 0; k--)
91     if (depth[fa[a][k]] >= depth[b])
92     {
93         distance[cnt++] = d1[a][k];
94         distance[cnt++] = d2[a][k];
95         a = fa[a][k];
96     }
97     if (a != b)
98     {
99         for (int k = 16; k >= 0; k--)
100         if (fa[a][k] != fa[b][k])
101         {
102             distance[cnt++] = d1[a][k];
103             distance[cnt++] = d2[a][k];
104             distance[cnt++] = d1[b][k];
105             distance[cnt++] = d2[b][k];
106             a = fa[a][k], b = fa[b][k];
107         }
108         distance[cnt++] = d1[a][0];
109         distance[cnt++] = d1[b][0];
110     }
111     int dist1 = -INF, dist2 = -INF;
112     for (int i = 0; i < cnt; i++)
113     {
114         int d = distance[i];
115         if (d > dist1)
116             dist2 = dist1, dist1 = d;
117         else if (d != dist1 && d > dist2)
118             dist2 = d;
119     }
120     if (w > dist1) return w - dist1;
121     if (w > dist2) return w - dist2;
122     return INF;
123 }
124 int main()
125 {

```

```

126     cin >> n >> m;
127     for (int i = 0; i < m; i++)
128     {
129         int a, b, c;
130         cin >> a >> b >> c;
131         edge[i] = {a, b, c};
132     }
133     LL sum = kruskal();
134     build();
135     bfs();
136     LL res = 1e18;
137     for (int i = 0; i < m; i++)
138     if (!edge[i].used)
139     {
140         int a = edge[i].a, b = edge[i].b, w =
141         edge[i].w;
142         res = min(res, sum + lca(a, b, w));
143     }
144     cout << res;
145 }

```

9.7 Difference Constraints

- size == N: Feasible Solution
- size == 1: Maximum/Minimum
- Maximum: Shortest Path
- Minimum: Longest Path

9.7.1 Maximum-Shortest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, 0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] > dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n) return true;
26                if (!st[j])
27                {
28                    st[j] = true;
29                    q[tt++] = j;
30                    if (tt == N) tt = 0;
31                }

```



```

32     }
33 }
34 }
35 return false;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_b <= x_a + k
40     // PROCESS
41 }

```

9.7.2 Minimum-Longest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, -0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] < dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n) return false;
26                if (!st[j])
27                {
28                    st[j] = true;
29                    q[tt++] = j;
30                    if (tt == N) tt = 0;
31                }
32            }
33        }
34    }
35    return true;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_a + k <= x_b
40     // PROCESS
41 }

```

9.8 LCA

```

1 int n, m, h[N], e[M], ne[M], idx;
2 int depth[N], fa[N][16], q[N];
3 void bfs(int root)
4 {

```

```

5     memset(depth, 0x3f, sizeof depth);
6     depth[0] = 0;
7     depth[root] = 1;
8     int hh = 0, tt = 0;
9     q[0] = root;
10    while (hh <= tt)
11    {
12        int t = q[hh++];
13        for (int i = h[t]; ~i; i = ne[i])
14        {
15            int j = e[i];
16            if (depth[j] > depth[t] + 1)
17            {
18                depth[j] = depth[t] + 1;
19                q[++tt] = j;
20                fa[j][0] = t;
21                for (int k = 1; k <= 15; k++)
22                    fa[j][k] = fa[fa[j][k-1]][k-1];
23            }
24        }
25    }
26 }
27 int lca(int a, int b)
28 {
29     if (depth[a] < depth[b]) swap(a, b);
30     for (int k = 15; k >= 0; k--)
31         if (depth[fa[a][k]] >= depth[b])
32             a = fa[a][k];
33     if (a == b) return a;
34     for (int k = 15; k >= 0; k--)
35         if (fa[a][k] != fa[b][k])
36         {
37             a = fa[a][k];
38             b = fa[b][k];
39         }
40     return fa[a][0];
41 }

```

9.9 SCC

```

1 void tarjan(int u)
2 {
3     dfn[u] = low[u] = ++timestamp;
4     stack[++top] = u, in_stk[u] = true;
5     for (int i = h[u]; ~i; i = ne[i])
6     {
7         int j = e[i];
8         if (!dfn[j])
9         {
10            tarjan(j);
11            low[u] = min(low[u], low[j]);
12        }
13        else if (in_stk[j])
14            low[u] = min(low[u], dfn[j]);
15    }
16    if (dfn[u] == low[u])
17    {
18        int y;
19        ++scc_cnt;
20        do
21        {
22            y = stk[top--];
23            in_stk[y] = false;

```

```

24     id[y] = scc_cnt;
25 } while (y != u);
26 }
27 }

```

9.10 DCC

9.10.1 e-DCC

```

1  const int N = 5010, M = 20010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, id[N], dcc_cnt, d[N];
5  bool is_bridge[M];
6  void tarjan(int u, int from)
7  {
8      dfn[u] = low[u] = ++timestamp;
9      stk[++top] = u;
10     for (int i = h[u]; ~i; i = ne[i])
11     {
12         int j = e[i];
13         if (!dfn[j])
14         {
15             tarjan(j, i);
16             low[u] = min(low[u], low[j]);
17             if (dfn[u] < low[j])
18                 is_bridge[i] = is_bridge[i ^ 1] = true;
19         }
20         else if (i != (from ^ 1))
21             low[u] = min(low[u], dfn[j]);
22     }
23     if (dfn[u] == low[u])
24     {
25         ++dcc_cnt;
26         int y;
27         do
28         {
29             y = stk[top--];
30             id[y] = dcc_cnt;
31         } while (y != u);
32     }
33 }

```

9.10.2 v-DCC

```

1  const int N = 1010, M = 1010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, dcc_cnt, root;
5  vector<int> dcc[N];
6  bool cut[N];
7  void init()
8  {
9      for (int i = 1; i <= dcc_cnt; i++)
10         dcc[i].clear();
11     idx = n = timestamp = top = dcc_cnt = 0;
12     memset(h, -1, sizeof h);
13     memset(dfn, 0, sizeof dfn);
14     memset(cut, 0, sizeof cut);
15 }

```

```

16 void tarjan(int u)
17 {
18     dfn[u] = low[u] = ++timestamp;
19     stk[++top] = u;
20     if (u == root && h[u] == -1)
21     {
22         dcc_cnt++;
23         dcc[dcc_cnt].push_back(u);
24         return;
25     }
26     int cnt = 0;
27     for (int i = h[u]; ~i; i = ne[i])
28     {
29         int j = e[i];
30         if (!dfn[j])
31         {
32             tarjan(j);
33             low[u] = min(low[u], low[j]);
34             if (dfn[u] <= low[j])
35             {
36                 cnt++;
37                 if (u != root || cnt > 1)
38                     cut[u] = true;
39                 ++dcc_cnt;
40                 int y;
41                 do
42                 {
43                     y = stk[top--];
44                     dcc[dcc_cnt].push_back(y);
45                 } while (y != j);
46                 dcc[dcc_cnt].push_back(u);
47             }
48         }
49         else
50             low[u] = min(low[u], dfn[j]);
51     }
52 }

```

9.11 Bipartite Graph

The maximum matching
(by the Hungarian algorithm) =
the minimum vertex cover =
total number of vertices -
maximum independent set =
total number of vertices -
minimum path cover.

9.11.1 maximum matching

```

1  const int N = 110;
2  int n, m;
3  int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
4  PII match[N][N];
5  bool g[N][N], st[N][N];
6  bool find(int x, int y)
7  {
8      for (int i = 0; i < 4; i++)
9      {
10         int a = x + dx[i], b = y + dy[i];
11         if (a && a <= n && b && b <= n && !g[a][b] && !st[a][b])

```

```

12 {
13     st[a][b] = true;
14     PII t = match[a][b];
15     if (t.x == -1 || find(t.x, t.y))
16     {
17         match[a][b] = {x, y};
18         return true;
19     }
20 }
21 }
22 return false;
23 }
24 int main()
25 {
26     // PROCESS
27     memset(match, -1, sizeof match);
28     int res = 0;
29     for (int i = 1; i <= n; i++)
30     for (int j = 1; j <= n; j++)
31         if ((i + j) % 2 && !g[i][j])
32         {
33             memset(st, 0, sizeof st);
34             if (find(i, j))
35                 res++;
36         }
37     // PROCESS
38 }

```

9.11.2 minimum vertex cover

```

1  const int N = 110;
2  int n, m, k, match[N];
3  bool g[N][N], st[N];
4  bool find(int x)
5  {
6      for (int i = 0; i < m; i++)
7          if (!st[i] && g[x][i])
8          {
9              st[i] = true;
10             if (match[i] == -1 || find(match[i]))
11             {
12                 match[i] = x;
13                 return true;
14             }
15         }
16     return false;
17 }
18 int main()
19 {
20     while (cin >> n, n)
21     {
22         cin >> m >> k;
23         memset(g, 0, sizeof g);
24         memset(match, -1, sizeof match);
25         while (k--)
26         {
27             int t, a, b;
28             cin >> t >> a >> b;
29             if (!a || !b) continue;
30             g[a][b] = true;
31         }
32         int res = 0;
33         for (int i = 0; i < n; i++)
34         {

```

```

35             memset(st, 0, sizeof st);
36             if (find(i)) res++;
37         }
38         cout << res << '\n';
39     }
40 }

```

9.11.3 maximum independent set

```

1  const int N = 110;
2  int n, m, k;
3  PII match[N][N];
4  bool g[N][N], st[N][N];
5  int dx[8] = {-2, -1, 1, 2, 2, 1, -1, -2};
6  int dy[8] = {1, 2, 2, 1, -1, -2, -2, -1};
7  bool find(int x, int y)
8  {
9      for (int i = 0; i < 8; i++)
10     {
11         int a = x + dx[i], b = y + dy[i];
12         if (a < 1 || a > n || b < 1 || b > m)
13             continue;
14         if (g[a][b]) continue;
15         if (st[a][b]) continue;
16         st[a][b] = true;
17         PII t = match[a][b];
18         if (t.x == 0 || find(t.x, t.y))
19         {
20             match[a][b] = {x, y};
21             return true;
22         }
23     }
24     return false;
25 }
26 int main()
27 {
28     // PROCESS
29     int res = 0;
30     for (int i = 1; i <= n; i++)
31     for (int j = 1; j <= m; j++)
32     {
33         if (g[i][j] || (i + j) % 2)
34             continue;
35         memset(st, 0, sizeof st);
36         if (find(i, j)) res++;
37     }
38     cout << n * m - k - res << '\n';
39 }

```

9.11.4 minimum path cover

- Only for DAG.
- If you need to compute the **minimum path cover with repeated nodes**, you need to perform transitive closure as shown in the following code.

```

1  const int N = 210, M = 30010;
2  int n, m, match[N];
3  bool d[N][N], st[N];
4  bool find(int x)

```

```

5 {
6     for (int i = 1; i <= n; i++)
7         if (d[x][i] && !st[i])
8         {
9             st[i] = true;
10            int t = match[i];
11            if (t == 0 || find(t))
12            {
13                match[i] = x;
14                return true;
15            }
16        }
17    return false;
18 }
19 int main()
20 {
21     // 传递闭包
22     for (int k = 1; k <= n; k++)
23         for (int i = 1; i <= n; i++)
24             for (int j = 1; j <= n; j++)
25                 d[i][j] |= d[i][k] & d[k][j];
26     int res = 0;
27     for (int i = 1; i <= n; i++)
28     {
29         memset(st, 0, sizeof st);
30         if (find(i)) res++;
31     }
32     cout << n - res;
33 }

```

```

21     t = i / 2 + 1;
22     if (i & 1) t = -t;
23 }
24 else t = i + 1;
25 int j = e[i];
26 i = ne[i];
27 dfs(j);
28 ans[++cnt] = t;
29 }
30 }
31 int main()
32 {
33     cin >> type >> n >> m;
34     memset(h, -1, sizeof h);
35     for (int i = 0; i < m; i++)
36     {
37         int a, b;
38         cin >> a >> b;
39         add(a, b);
40         if (type == 1) add(b, a);
41         din[b]++, dout[a]++;
42     }
43     if (type == 1)
44     {
45         for (int i = 1; i <= n; i++)
46             if (din[i] + dout[i] & 1)
47             {
48                 cout << "NO\n";
49                 return 0;
50             }
51     }
52     else
53     {
54         for (int i = 1; i <= n; i++)
55             if (din[i] != dout[i])
56             {
57                 cout << "NO\n";
58                 return 0;
59             }
60     }
61     for (int i = 1; i <= n; i++)
62         if (h[i] != -1)
63         {
64             dfs(i);
65             break;
66         }
67 }

```

9.12 Eulerian Circuit & Eulerian Path

9.12.1 Eulerian Circuit

- **Undirected Graph:** If and only if it is connected and every vertex has even degree.
- **Directed Graph:** If and only if it is strongly connected and each vertex has equal in-degree and out-degree.

```

1 int type, n, m;
2 int h[N], e[M], ne[M], idx;
3 bool used[M];
4 int ans[M], cn, din[N], dout[N];
5 void add(int a, int b)
6 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
7 }
8 void dfs(int u)
9 {
10     for (int &i = h[u]; ~i; i = ne[i])
11     {
12         if (used[i])
13             i = ne[i];
14         continue;
15     }
16     used[i] = true;
17     if (type == 1) used[i ^ 1] = true;
18     int t;
19     if (type == 1)
20     {

```

9.12.2 Eulerian Path

Undirected Graph

If and only if it is connected (ignoring isolated vertices) and has exactly 0 or 2 vertices with odd degree.

```

1 const int N = 510;
2 int n = 500, m, g[N][N];
3 int ans[1100], cnt, d[N];
4 void dfs(int u)
5 {
6     for (int i = 1; i <= n; i++)
7         if (g[u][i])
8         {
9             g[u][i]--, g[i][u]--;
10            dfs(i);

```

```

11     }
12     ans[++cnt] = u;
13 }
14 int main()
15 {
16     cin >> m;
17     while (m--)
18     {
19         int a, b;
20         cin >> a >> b;
21         g[a][b]++, g[b][a]++;
22         d[a]++, d[b]++;
23     }
24     int start = 1;
25     while (!d[start])
26         ++start;
27     for (int i = 1; i <= 500; i++)
28         if (d[i] % 2)
29         {
30             start = i;
31             break;
32         }
33     dfs(start);
34 }

```

Directed Graph

If and only if it is connected in terms of non-zero degree vertices, and

- At most one vertex has
(out-degree) — (in-degree) = 1
- At most one vertex has
(in-degree) — (out-degree) = 1
- All other vertices have
equal in-degree and out-degree

```

27     dout[a]++, din[b]++;
28     p[find(a)] = find(b);
29 }
30 int start = 0, end = 0;
31 bool success = true;
32 for (int i = 0; i < 26; i++)
33     if (din[i] != dout[i])
34     {
35         if (din[i] == dout[i] + 1) end++;
36         else if (din[i] + 1 == dout[i])
37             start++;
38         else
39         {
40             success = false;
41             break;
42         }
43     }
44 if (success && (!(start && !end || start
45 == 1 && end == 1))
46     success = false;
47 int rep = -1;
48 for (int i = 0; i < 26; i++)
49     if (st[i])
50     {
51         if (rep == -1) rep = find(i);
52         else if (rep != find(i))
53         {
54             success = false;
55             break;
56         }
57     }
58 return 0;
59 }

```

```

1  const int N = 30;
2  int n, p[N], din[N], dout[N];
3  bool st[N];
4  int find(int x)
5  {
6      if (x != p[x]) p[x] = find(p[x]);
7      return p[x];
8  }
9  int main()
10 {
11     char str[1010];
12     int T;
13     cin >> T;
14     while (T--)
15     {
16         cin >> n;
17         memset(din, 0, sizeof din);
18         memset(dout, 0, sizeof dout);
19         memset(st, 0, sizeof st);
20         for (int i = 0; i < 26; i++) p[i] = i;
21         for (int i = 0; i < n; i++)
22         {
23             cin >> str;
24             int a = str[0] - 'a',
25                 b = str[strlen(str) - 1] - 'a';
26             st[a] = st[b] = true;

```

10 ★ Advanced Math

10.1 Euler's Totient Function

10.1.1 GCD

```
1  const int N = 1e7 + 10;
2  int primes[N], cnt, phi[N];
3  bool st[N];
4  LL s[N];
5  void init(int n)
6  {
7      for (int i = 2; i <= n; i++)
8      {
9          if (!st[i])
10             {
11                 primes[cnt++] = i;
12                 phi[i] = i - 1;
13             }
14             for (int j = 0; primes[j] * i <= n; j++)
15                 {
16                     st[primes[j] * i] = true;
17                     if (i % primes[j] == 0)
18                         {
19                             phi[i * primes[j]] = phi[i] *
primes[j];
20                             break;
21                         }
22                     phi[i * primes[j]] = phi[i] * (
primes[j] - 1);
23                 }
24             }
25     for (int i = 1; i <= n; i++)
26         s[i] = s[i - 1] + phi[i];
27 }
28 int main()
29 {
30     int n; cin >> n;
31     init(n);
32     LL res = 0;
33     for (int i = 0; i < cnt; i++)
34     {
35         int p = primes[i];
36         res += s[n / p] * 2 + 1;
37     }
38 }
```

```
11 void mul(int c[][N], int a[][N], int b[][N])
12 {
13     int temp[N][N] = {0};
14     for (int i = 0; i < N; i++)
15         for (int j = 0; j < N; j++)
16             for (int k = 0; k < N; k++)
17                 temp[i][j] = (temp[i][j] + (LL
)a[i][k] * b[k][j]) % m;
18     memcpy(c, temp, sizeof temp);
19 }
20 int main()
21 {
22     while (n)
23     {
24         if (n & 1) mul(f1, f1, a);
25         mul(a, a, a); n >>= 1;
26     }
27 }
```

10.2 Matrix Multiplication

```
1  const int N = 3;
2  int n, m;
3  void mul(int c[], int a[], int b[][N])
4  {
5      int temp[N] = {0};
6      for (int i = 0; i < N; i++)
7          for (int j = 0; j < N; j++)
8              temp[i] = (temp[i] + (LL)a[j] * b[
j][i]) % m;
9      memcpy(c, temp, sizeof temp);
10 }
```

11 ★ Advanced DP

11.1 Advanced Linear DP

11.1.1 Two-pass grid collection problem

In this case we run DP on two different roads at the same time:

```
1  const int N = 15;
2  int n, w[N][N], f[N * 2][N][N];
3  int main()
4  {
5      cin >> n;
6      // INPUT w[N][N]
7      for (int k = 2; k <= n * 2; k++)
8          for (int i1 = 1; i1 <= k; i1++)
9              for (int i2 = 1; i2 <= k; i2++)
10                 {
11                     int j1 = k - i1, j2 = k - i2;
12                     int t = w[i1][j1];
13                     if (i1 != i2) t += w[i2][j2];
14                     int &x = f[k][i1][i2];
15                     x = max(x, f[k - 1][i1 - 1][i2 - 1] +
16                             t);
17                     x = max(x, f[k - 1][i1 - 1][i2] + t);
18                     x = max(x, f[k - 1][i1][i2 - 1] + t);
19                     x = max(x, f[k - 1][i1][i2] + t);
20                 }
21     cout << f[n * 2][n][n] << '\n';
22     return 0;
23 }
```

11.2 Advanced LIS

11.2.1 Longest Bitonic Subsequence

```
1  const int N = 1010;
2  int n, a[N], f[N], g[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          {
10             f[i] = 1;
11             for (int j = 1; j < i; j++)
12                 if (a[i] > a[j])
13                     f[i] = max(f[i], f[j] + 1);
14             }
15             g[i] = 1;
16             for (int j = n; j > i; j--)
17                 if (a[i] > a[j])
18                     g[i] = max(g[i], g[j] + 1);
19             }
20     int ans = 0;
21     for (int i = 1; i <= n; i++)
22         ans = max(ans, f[i] + g[i] - 1);
23     cout << ans << '\n';
24 }
```

```
26     return 0;
27 }
```

11.2.2 MSIS

MSIS means Maximum Sum Increasing Subsequence

```
1  const int N = 1010;
2  int n, w[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 0; i < n; i++) cin >> w[i];
7      int res = 0;
8      for (int i = 0; i < n; i++)
9          {
10             f[i] = w[i];
11             for (int j = 0; j < i; j++)
12                 if (w[i] > w[j])
13                     f[i] = max(f[i], f[j] + w[i]);
14             res = max(res, f[i]);
15         }
16     cout << res;
17 }
```

11.2.3 LCIS

LCIS means Longest Common Increasing Subsequence

```
1  const int N = 3010;
2  int n, a[N], b[N], f[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          cin >> b[i];
10     for (int i = 1; i <= n; i++)
11         {
12             int maxv = 1;
13             for (int j = 1; j <= n; j++)
14                 {
15                     f[i][j] = f[i - 1][j];
16                     if (a[i] == b[j])
17                         f[i][j] = max(f[i][j], maxv);
18                     if (a[i] > b[j])
19                         maxv = max(maxv, f[i - 1][j] +
20                                     1);
21                 }
22             int res = 0;
23             for (int i = 1; i <= n; i++)
24                 res = max(res, f[n][i]);
25             cout << res;
26         }
```

11.3 Knapsack Problem

11.3.1 How To Initialize

Initialization for Counting the Number of Solutions:

- **2D Case:**

- When volume is at most j : $f[0][i] = 1$ for $0 \leq i \leq m$, others are 0
- When volume is exactly j : $f[0][0] = 1$, others are 0
- When volume is at least j : $f[0][0] = 1$, others are 0

- **1D Case:**

- When volume is at most j : $f[i] = 1$ for $0 \leq i \leq m$
- When volume is exactly j : $f[0] = 1$, others are 0
- When volume is at least j : $f[0] = 1$, others are 0

```

13     int hh = 0, tt = -1;
14     for (int j = r; j <= m; j += v)
15     {
16         while (hh <= tt && j - s * v > q[hh])
17             hh++;
18         while (hh <= tt && g[q[tt]] + (j - q[tt]) / v * w <= g[j])
19             tt--;
20         q[++tt] = j;
21         f[j] = g[q[hh]] + (j - q[hh]) / v * w;
22     }
23 }
24 }
25 cout << f[m];
26 }

```

Initialization for Finding Maximum or Minimum Value:

- **2D Case:**

- When volume is at most j : $f[i][k] = 0$ for $0 \leq i \leq n, 0 \leq k \leq m$ (only for maximizing value)
- When volume is exactly j :
 - * For minimizing value: $f[0][0] = 0$, others are INF
 - * For maximizing value: $f[0][0] = 0$, others are -INF
- When volume is at least j : $f[0][0] = 0$, others are INF (only for minimizing value)

- **1D Case:**

- When volume is at most j : $f[i] = 0$ for $0 \leq i \leq m$ (only for maximizing value)
- When volume is exactly j :
 - * For minimizing value: $f[0] = 0$, others are INF
 - * For maximizing value: $f[0] = 0$, others are -INF
- When volume is at least j : $f[0] = 0$, others are INF (only for minimizing value)

```

1  const int N = 110;
2  int n, V, M, f[N][N];
3  int main()
4  {
5      cin >> n >> V >> M;
6      for (int i = 0; i < n; i++)
7      {
8          int v, m, w;
9          cin >> v >> m >> w;
10         for (int j = V; j >= v; j--)
11             for (int k = M; k >= m; k--)
12                 f[j][k] = max(f[j][k], f[j - v][k - m] + w);
13     }
14     cout << f[V][M] << '\n';
15 }

```

11.3.2 Multiple Knapsack Problem

```

1  const int M = 20010;
2  int n, m, v, w, s;
3  int f[M], g[M], q[M];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8      {
9          cin >> v >> w >> s;
10         memcpy(g, f, sizeof g);
11         for (int r = 0; r < v; r++)
12         {

```

11.3.3 Two-Dimensional Cost Knapsack Problem

11.3.4 Finding the Actual Solution Set

```

1  const int N = 1010;
2  int n, m;
3  int v[N], w[N], f[N][N];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8          cin >> v[i] >> w[i];
9      for (int i = n; i >= 1; i--)
10         for (int j = 0; j <= m; j++)
11         {
12             f[i][j] = f[i + 1][j];
13             if (j >= v[i])
14                 f[i][j] = max(f[i][j], f[i + 1][j - v[i]] + w[i]);
15         }
16     int j = m;
17     for (int i = 1; i <= n; i++)
18         if (j >= v[i] && f[i][j] == f[i + 1][j - v[i]] + w[i])
19             {

```



```

20         cout << i << ' ';
21         j -= v[i];
22     }
23 }

```

11.3.5 Maximum Linearly Independent Subset

```

1  const int N = 110, M = 25010;
2  int n, v[N];
3  bool f[M];
4  int main()
5  {
6      int T; cin >> T;
7      while (T--)
8      {
9          cin >> n;
10         for (int i = 1; i <= n; ++i)
11             cin >> v[i];
12         sort(v + 1, v + n + 1);
13         int m = v[n], res = 0;
14         memset(f, 0, sizeof f);
15         f[0] = true;
16         for (int i = 1; i <= n; ++i)
17         {
18             if (f[v[i]]) continue;
19             res++;
20             for (int j = v[i]; j <= m; ++j)
21                 f[j] |= f[j - v[i]];
22         }
23         cout << res << '\n';
24     }
25 }

```

11.3.6 Mixed Knapsack Problem

```

1  const int N = 1010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         if (!s)
11         {
12             for (int j = v; j <= m; j++)
13                 f[j] = max(f[j], f[j - v] + w);
14         }
15         else
16         {
17             if (s == -1) s = 1;
18             for (int k = 1; k <= s; k *= 2)
19             {
20                 for (int j = m; j >= k * v; j
21                     --)
22                     f[j] = max(f[j], f[j - k *
23                         v] + k * w);
24                 s -= k;
25             }
26         }
27     }
28 }

```

```

23     }
24     if (s)
25     {
26         for (int j = m; j >= s * v; j
27             --)
28             f[j] = max(f[j], f[j - s *
29                 v] + s * w);
30     }
31     cout << f[m] << '\n';
32 }

```

11.3.7 Dependent Knapsack Problem

```

1  const int N = 110;
2  int n, m, root;
3  int h[N], e[N], ne[N], idx;
4  int v[N], w[N], [N][N];
5  void add(int a, int b)
6  {
7      e[idx] = b, ne[idx] = h[a], h[a] = idx++;
8  }
9  void dfs(int u)
10 {
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         int son = e[i];
14         dfs(son);
15         for (int j = m - v[u]; j >= 0; --j)
16             for (int k = 0; k <= j; ++k)
17                 f[u][j] = max(f[u][j], f[u][j
18                     - k] + f[son][k]);
19     }
20     for (int j = m; j >= v[u]; --j)
21         f[u][j] = f[u][j - v[u]] + w[u];
22     for (int j = 0; j < v[u]; ++j)
23         f[u][j] = 0;
24 }
25 int main()
26 {
27     memset(h, -1, sizeof h);
28     cin >> n >> m;
29     for (int i = 1; i <= n; ++i)
30     {
31         int p;
32         cin >> v[i] >> w[i] >> p;
33         if (p == -1) root = i;
34         else add(p, i);
35     }
36     dfs(root);
37     cout << f[root][m] << '\n';
38 }

```

11.3.8 Number of Solutions

```

1  const int N = 1010, mod = 1e9 + 7;
2  int n, m;
3  int w[N], v[N], f[N], g[N];
4  int main()
5  {
6      cin >> n >> m;

```

```

7   for (int i = 1; i <= n; ++i)
8       cin >> v[i] >> w[i];
9   g[0] = 1;
10  for (int i = 1; i <= n; ++i)
11  {
12      for (int j = m; j >= v[i]; --j)
13      {
14          int temp = max(f[j], f[j - v[i]] +
15                          w[i]), c = 0;
16          if (temp == f[j])
17              c = (c + g[j]) % mod;
18          if (temp == f[j - v[i]] + w[i])
19              c = (c + g[j - v[i]]) % mod;
20          f[j] = temp, g[j] = c;
21      }
22      int res = 0;
23      for (int j = 0; j <= m; ++j)
24          if (f[j] == f[m])
25              res = (res + g[j]) % mod;
26      cout << res << '\n';
27  }

```

```

13     ne[i] = j;
14 }
15 f[0][0] = 1;
16 for (int i = 0; i < n; i++)
17     for (int j = 0; j < m; j++)
18         for (char ch = 'a'; ch <= 'z'; ch++)
19         {
20             int ptr = j;
21             while (ptr && s[ptr + 1] != ch)
22                 ptr = ne[ptr];
23             if (s[ptr + 1] == ch) ptr++;
24             f[i + 1][ptr] = (f[i + 1][ptr] + f[i
25 ] [j]) % mod;
26         }
27 int res = 0;
28 for (int j = 0; j < m; j++)
29     res = (res + f[n][j]) % mod;
30 cout << res;
31 }

```

11.4.3 Linear DP + AC Automaton

11.4 FSM

11.4.1 Common FSM

```

1   const int N = 100010;
2   int n, w[N], f[N][2];
3   int main()
4   {
5       int T; cin >> T;
6       while (T--)
7       {
8           cin >> n;
9           for (int i = 1; i <= n; i++)
10              cin >> w[i];
11          for (int i = 1; i <= n; i++)
12          {
13              // YOUR_FSM_RULES
14              // f[i][0] =
15              // f[i][1] =
16          }
17          cout << max(f[n][0], f[n][1]) << '\n';
18      }
19  }

```

11.4.2 Linear DP + KMP

```

1   const int N = 55, mod = 1e9 + 7
2   int n, m, f[N][N], ne[N];
3   char s[N];
4   int main()
5   {
6       cin >> n >> s + 1;
7       m = strlen(s + 1);
8       for (int i = 2, j = 0; i <= m; i++)
9       {
10          while (j && s[i] != s[j + 1])
11              j = ne[j];
12          if (s[j + 1] == s[i]) j++;

```

```

1   const int N = 1010, INF = 0x3f3f3f3f;
2   int n, m, T = 1, f[N][N];
3   int tr[N][4], dar[N], idx;
4   int q[N], ne[N];
5   char str[N];
6   int get(char c)
7   {
8       if (c == 'A') return 0;
9       if (c == 'T') return 1;
10      if (c == 'G') return 2;
11      return 3;
12  }
13 void insert()
14 {
15     int p = 0;
16     for (int i = 0; str[i]; i++)
17     {
18         int t = get(str[i]);
19         if (tr[p][t] == 0)
20             tr[p][t] = ++idx;
21         p = tr[p][t];
22     }
23     dar[p] = 1;
24 }
25 void build()
26 {
27     int hh = 0, tt = -1;
28     for (int i = 0; i < 4; i++)
29         if (tr[0][i])
30             q[++tt] = tr[0][i];
31     while (hh <= tt)
32     {
33         int t = q[hh++];
34         for (int i = 0; i < 4; i++)
35         {
36             int p = tr[t][i];
37             if (!p)
38                 tr[t][i] = tr[ne[t]][i];
39             else
40             {
41                 ne[p] = tr[ne[t]][i];
42                 q[++tt] = p;

```

```

43     dar[p] |= dar[ne[p]];
44 }
45 }
46 }
47 }
48 int main()
49 {
50     int T = 1;
51     while (cin >> n, n)
52     {
53         memset(tr, 0, sizeof tr);
54         memset(dar, 0, sizeof dar);
55         memset(ne, 0, sizeof ne);
56         idx = 0;
57         for (int i = 0; i < n; i++)
58         {
59             cin >> str;
60             insert();
61         }
62         build();
63         cin >> (str + 1);
64         m = strlen(str + 1);
65         memset(f, 0x3f, sizeof f);
66         f[0][0] = 0;
67         for (int i = 0; i < m; i++)
68             for (int j = 0; j <= idx; j++)
69                 for (int k = 0; k < 4; k++)
70                 {
71                     int t = get(str[i + 1]) != k;
72                     int p = tr[j][k];
73                     if (!dar[p])
74                         f[i + 1][p] = min(f[i + 1][p], f[i][j] + t);
75                 }
76         int res = INF;
77         for (int i = 0; i <= idx; i++)
78             res = min(res, f[m][i]);
79         if (res == INF) res = -1;
80         cout << "Case " << T++
81              << ": " << res << '\n';
82     }
83 }

```

11.5 State Compression DP

11.5.1 Bitmask DP & Rolling Array & Target-State Optimization

```

1  const int N = 12, M = 1 << 10, K = 110;
2  int n, m, cnt[M];
3  vector<int> state, head[M];
4  LL f[N][K][M];
5  bool check(int state)
6  {
7      for (int i = 0; i < n; i++)
8          if ((state >> i & 1) && (state >> i + 1 & 1))
9              return false;
10     return true;
11 }
12 int count(int state)
13 {
14     int res = 0;

```

```

15     for (int i = 0; i < n; i++)
16         res += state >> i & 1;
17     return res;
18 }
19 int main()
20 {
21     cin >> n >> m;
22     for (int i = 0; i < 1 << n; i++)
23         if (check(i))
24         {
25             state.push_back(i);
26             cnt[i] = count(i);
27         }
28     for (int i = 0; i < state.size(); i++)
29         for (int j = 0; j < state.size(); j++)
30         {
31             int a = state[i], b = state[j];
32             if ((a & b) == 0 && check(a | b))
33                 head[i].push_back(j);
34         }
35     f[0][0][0] = 1;
36     for (int i = 1; i <= n + 1; i++)
37         for (int j = 0; j <= m; j++)
38             for (int a = 0; a < state.size(); a++)
39                 for (int b : head[a])
40                 {
41                     int c = cnt[state[a]];
42                     if (j >= c)
43                         f[i][j][a] += f[i - 1][j - c][b];
44                 }
45     cout << f[n + 1][m][0];
46 }

```

11.5.2 Bitmask DP & Transfer Optimization

```

1  const int N = 10, M = 1 << 10;
2  int n, m, g[1010], cnt[M];
3  int f[2][M][M];
4  vector<int> state;
5  bool check(int state)
6  {
7      for (int i = 0; i < m; i++)
8          if ((state >> i & 1) && ((state >> i + 1 & 1) || (state >> i + 2 & 1)))
9              return false;
10     return true;
11 }
12 int count(int state)
13 {
14     int res = 0;
15     for (int i = 0; i < m; i++)
16         if (state >> i & 1) res++;
17     return res;
18 }
19 int main()
20 {
21     cin >> n >> m;
22     for (int i = 1; i <= n; i++)
23         for (int j = 0; j < m; j++)
24         {
25             char c;
26             cin >> c;

```

```

27     g[i] += (c == 'H') << j;
28 }
29 for (int i = 0; i < 1 << m; i++)
30     if (check(i))
31     {
32         state.push_back(i);
33         cnt[i] = count(i);
34     }
35 for (int i = 1; i <= n; i++)
36     for (int j = 0; j < state.size(); j++)
37         for (int k = 0; k < state.size(); k++)
38             for (int u = 0; u < state.size(); u
39                 ++){
40                 int a = state[j], b = state[k], c
41                 = state[u];
42                 if (a & b | a & c | b & c)
43                     continue;
44                 if (g[i] & b | g[i - 1] & a)
45                     continue;
46                 f[i & 1][j][k] = max(f[i & 1][j][k]
47                     , f[i - 1 & 1][u][j] + cnt[b]);
48             }
49 int res = 0;
50 for (int i = 0; i < state.size(); i++)
51     for (int j = 0; j < state.size(); j++)
52         res = max(res, f[n & 1][i][j]);
53 cout << res;
54 }

```

```

31     }
32 }
33 }
34 }
35 void solve()
36 {
37     cin >> n >> m;
38     for (int i = 0; i < n; i++)
39         cin >> ver[i].x >> ver[i].y;
40     init_path();
41     memset(f, 0x3f, sizeof f);
42     f[0] = 0;
43     for (int cur_st = 0; cur_st + 1 < 1 << n;
44         cur_st++){
45         {
46             int t = -1;
47             for (int i = 0; i < n; i++)
48                 if (!(cur_st >> i & 1)) t = i;
49             for (int i = 0; i < n; i++)
50                 {
51                     int ne_st = path[t][i] | cur_st;
52                     f[ne_st] = min(f[ne_st], f[cur_st] +
53                         1);
54                 }
55             }
56         cout << f[(1 << n) - 1] << '\n';
57     }
58 }
59 int main()
60 {
61     cin >> T;
62     while (T--) solve();
63 }

```

11.5.3 Bitmask DP & Repeated Coverage Problem

```

1  const double eps = 1e-8;
2  const int N = 20, M = 1 << N;
3  int n, m, T, path[N][N], f[M]
4  PDD ver[N];
5  int cmp_lf(double a, double b)
6  {
7      if (fabs(a - b) < eps) return 0;
8      if (a > b) return 1;
9      return -1;
10 }
11 void init_path()
12 {
13     memset(path, 0, sizeof path);
14     for (int i = 0; i < n; i++)
15     {
16         path[i][i] = 1 << i;
17         for (int j = 0; j < n; j++)
18         {
19             double x1 = ver[i].x, y1 = ver[i].y;
20             double x2 = ver[j].x, y2 = ver[j].y;
21             if (!cmp_lf(x1, x2)) continue;
22             double a = (y1 / x1 - y2 / x2) / (x1 -
23                 x2);
24             double b = (y1 / x1) - a * x1;
25             if (cmp_lf(a, 0.0) >= 0)
26                 continue;
27             for (int k = 0; k < n; k++)
28             {
29                 double x = ver[k].x, y = ver[k].y;
30                 if (!cmp_lf(y, a * x * x + b * x))
31                     path[i][j] += 1 << k;
32             }
33         }
34     }
35 }

```

11.5.4 Bitmask DP & MST

```

1  const int N = 12, M = 1 << N;
2  int n, m, d[N][N], f[M][N], g[M];
3  int main()
4  {
5      cin >> n >> m;
6      memset(d, 0x3f, sizeof d);
7      for (int i = 0; i < n; i++)
8          d[i][i] = 0;
9      while (m--)
10     {
11         int a, b, c;
12         cin >> a >> b >> c;
13         a--, b--;
14         d[a][b] = d[b][a] = min(d[a][b], c);
15     }
16     for (int i = 1; i < 1 << n; i++)
17         for (int j = 0; j < n; j++)
18             if (i >> j & 1)
19                 for (int k = 0; k < n; k++)
20                     if (d[j][k] != INF)
21                         g[i] |= 1 << k;
22     memset(f, 0x3f, sizeof f);
23     for (int i = 0; i < n; i++)
24         f[1 << i][0] = 0;
25     for (int i = 1; i < 1 << n; i++)
26         for (int j = (i - 1); j; j = (j - 1) & i)
27             if ((g[j] & i) == i)
28                 {
29                     f[i] = min(f[i], f[j] + 1);
30                 }
31     }
32 }

```

```

29     int remain = i ^ j;
30     int cost = 0;
31     for (int k = 0; k < n; k++)
32         if (remain >> k & 1)
33         {
34             int t = INF;
35             for (int u = 0; u < n; u++)
36                 if (j >> u & 1)
37                     t = min(t, d[k][u]);
38             cost += t;
39         }
40     for (int k = 1; k < n; k++)
41         f[i][k] = min(f[i][k], f[j][k - 1]
42             + cost * k);
43     int res = INF;
44     for (int i = 0; i < n; i++)
45         res = min(res, f[(1 << n) - 1][i]);
46     cout << res;
47 }

```

```

        a[pos]);
13     }
14     return op ? res : f[pos][st] = res;
15 }
16 int calc(int x)
17 {
18     al = 0;
19     memset(f, -1, sizeof f);
20     while (x) a[++al] = x % b, x /= b;
21     return dp(al, 0, 1);
22 }
23 int main()
24 {
25     cin >> l >> r >> k >> b;
26     cout << calc(r) - calc(l - 1) << '\n';
27 }

```

11.6 Digit DP

```

1  const int N = 35;
2  int l, r, k, b, a[N], al, f[N][N];
3  int dp(int pos, int st, int op)
4  {
5      if (!pos) return st == k;
6      if (!op && ~f[pos][st])
7          return f[pos][st];
8      int res = 0, maxx = op ? min(a[pos], 1) :
9          1;
10     for (int i = 0; i <= maxx; i++)
11     {
12         if (st + i > k) continue;
13         res += dp(pos - 1, st + i, op && i ==

```

```

1  int n, m, s[300010], q[300010];
2  int main()
3  {
4      cin >> n >> m;
5      for (int i = 1; i <= n; i++)
6          cin >> s[i], s[i] += s[i - 1];
7      int res = INT_MIN, hh = 0, tt = 0;
8      for (int i = 1; i <= n; i++)
9      {
10         if (q[hh] < i - m) hh++;
11         res = max(res, s[i] - s[q[hh]]);
12         while (hh <= tt && s[q[tt]] >= s[i])
13             tt--;
14         q[++tt] = i;
15     }

```

11.7 Queue Optimization for DP