



**国家超级计算广州中心**  
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

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# XCPC-Template

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# Part I: Basic Template

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## 0 ★ Preface

### 0.1 Template

```
1 #define itn int
2 #define nit int
3 #define nti int
4 #define tin int
5 #define tni int
6 #define retrun return
7 #define reutrn return
8 #define rutren return
9 #define fastin \
10     ios_base::sync_with_stdio(0); \
11     cin.tie(0), cout.tie(0);
12 #include <bits/stdc++.h>
13 using namespace std;
14 typedef long long LL;
15 typedef long double LD;
16 typedef pair<int, int> PII;
17 typedef pair<long long, long long> PLL;
18 typedef pair<double, double> PDD;
19 typedef vector<int> VI;
20 #ifndef ONLINE_JUDGE
21 #define dbg(args...) \
22     do \
23     { \
24         cout << "\033[32;1m" << #args << " \
25         -> "; \
26         err(args); \
27     } while (0)
28 #define dbg(...)
29 #endif
30 void err()
31 { cout << "\033[39;0m" << endl; }
32 template <template <typename...> class T,
33         typename t, typename... Args>
34 void err(T<t> a, Args... args)
35 {
36     for (auto x : a) cout << x << ' ';
37     err(args...);
38 }
39 template <typename T, typename... Args>
40 void err(T a, Args... args)
41 { cout << a << ' '; err(args...); }
42 const int INF = 0x3f3f3f3f;
43 const int mod = 1e9 + 7;
44 const double eps = 1e-6;
45 int main()
46 {
47     #ifndef ONLINE_JUDGE
48         freopen("test.in", "r", stdin);
49         freopen("test.out", "w", stdout);
50     #endif
51     fastin;
52     return 0;
53 }
```

### 0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

### 0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about  $10^7 \sim 10^8$  operations.
- Below is a guide on how to choose algorithms based on different input size ranges:
  1.  $n \leq 30 \rightarrow$  Exponential complexity: DFS with pruning, State Compression DP
  2.  $n \leq 100 \rightarrow O(n^3)$ : Floyd, DP, Gaussian Elimination
  3.  $n \leq 1000 \rightarrow O(n^2), O(n^2 \log n)$ : DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
  4.  $n \leq 10000 \rightarrow O(n^3)$ : Block Linked List, Mo's Algorithm
  5.  $n \leq 100000 \rightarrow O(n \log n)$ : sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
  6.  $n \leq 1000000 \rightarrow O(n)$ , or small-constant  $O(n \log n)$ : Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
  7.  $n \leq 10000000 \rightarrow O(n)$ : Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
  8.  $n \leq 10^9 \rightarrow O(\sqrt{n})$ : Primality Testing
  9.  $n \leq 10^{18} \rightarrow O(\log n)$ : GCD, Fast Exponentiation, Digit DP
  10.  $n \leq 10^{1000} \rightarrow O((\log n)^2)$ : Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
  11.  $n \leq 10^{100000} \rightarrow O(\log k \cdot \log \log k)$ , where  $k$  is the number of digits: Big Integer Add/Subtract, FFT/NTT

## 0.4 If <bits/stdc++.h> Failed

Replace it with:

```
1  #include <algorithm>
2  #include <bitset>
3  #include <complex>
4  #include <deque>
5  #include <exception>
6  #include <fstream>
7  #include <functional>
8  #include <iomanip>
9  #include <ios>
10 #include <iosfwd>
11 #include <iostream>
12 #include <istream>
13 #include <iterator>
14 #include <limits>
15 #include <list>
16 #include <locale>
17 #include <map>
18 #include <memory>
19 #include <numeric>
20 #include <ostream>
21 #include <queue>
22 #include <set>
23 #include <sstream>
24 #include <stack>
25 #include <stdexcept>
26 #include <streambuf>
27 #include <string>
28 #include <typeinfo>
29 #include <utility>
30 #include <valarray>
31 #include <vector>
32 #include <unordered_map>
33 #include <unordered_set>
```

# 1 ★ Basic Algorithm

## 1.1 Quick Sort

Sort the given array from index 1 to n.

```
1 void quick_sort(int l, int r)
2 {
3     if (l >= r) return;
4     int x = a[(l + r) >> 1], i = l - 1, j
      = r + 1;
5     while (i < j)
6     {
7         do i++; while (a[i] < x);
8         do j--; while (a[j] > x);
9         if (i < j) swap(a[i], a[j]);
10    }
11    quick_sort(l, j);
12    quick_sort(j + 1, r);
13    return;
14 }
```

## 1.2 Binary Search

```
1 // 区间 [l, r] 被划分成 [l, mid] 和 [mid +
  1, r] 时使用
2 // 大于等于区间的最小值, check 应为 target
  <= a[mid]
3 int bsearch_1(int l, int r)
4 {
5     while (l < r)
6     {
7         int mid = l + r >> 1;
8         if (check(mid)) r = mid;
9         else l = mid + 1;
10    }
11    return l;
12 }
13 // 区间 [l, r] 被划分成 [l, mid - 1] 和 [
  mid, r] 时使用
14 // 小于等于区间的最大值, check 应为 target
  >= a[mid]
15 int bsearch_2(int l, int r)
16 {
17     while (l < r)
18     {
19         // 为什么要 l + r + 1: 因为 l 的更
          新条件是 mid 本身
20         // 当 r == l + 1 时 mid 向下取整必
          定取 l, 有可能在满足 check(mid) 时导致
          无限循环
21         int mid = l + r + 1 >> 1;
22         if (check(mid)) l = mid;
23         else r = mid - 1;
24    }
25    return l;
26 }
27 // 浮点数二分
28 double bsearch_3(double l, double r)
29 {
30     // eps 表示精度, 取决于题目对精度的要求
31     const double eps = 1e-6;
```

```
32     while (r - l > eps)
33     {
34         double mid = (l + r) / 2;
35         if (check(mid)) r = mid;
36         else l = mid;
37     }
38     return l;
39 }
```

## 1.3 Ternary Search

```
1 // 整数三分
2 void tsearch_1(int l, int r)
3 {
4     while (l < r)
5     {
6         int lmid = l + (r - l) / 3, rmid =
          r - (r - l) / 3;
7         lans = cal(lmid), rans = cal(rmid)
          ;
8         if (lans <= rans) r = rmid - 1;
9         else l = lmid + 1;
10        if (lans <= rans) l = lmid + 1;
11        else r = rmid - 1;
12    }
13    // 求凹函数的极小值
14    cout << min(lans, rans) << endl;
15    // 求凸函数的极大值
16    cout << max(lans, rans) << endl;
17 }
18 // 浮点数三分
19 void tsearch_2(int l, int r)
20 {
21     const double eps = 1e-6;
22     while (r - l < eps)
23     {
24         double lmid = l + (r - l) / 3;
25         double rmid = r - (r - l) / 3;
26         lans = cal(lmid), rans = cal(rmid)
          ;
27         // 求凹函数的极小值
28         if (lans <= rans) r = rmid;
29         else l = lmid;
30         // 求凸函数的极大值
31         if (lans <= rans) l = lmid;
32         else r = rmid;
33    }
34 }
```

## 1.4 High Precision

### 1.4.1 High Precision Add

```
1 string s1, s2;
2 vector<int> a, b, c;
3 void add(vector<int> &a, vector<int> &b)
4 {
5     if (a.size() < b.size())
6     { add(b, a); return; }
7     int t = 0;
```

```

8     for (int i = 0; i < a.size(); i++)
9     {
10         t += a[i];
11         if (i < b.size()) t += b[i];
12         c.push_back(t % 10);
13         t /= 10;
14     }
15     while (t)
16         c.push_back(t % 10), t /= 10;
17 }
18 int main()
19 {
20     cin >> s1 >> s2;
21     for (int i = s1.size() - 1; i >= 0; i--)
22         a.push_back(s1[i] - '0');
23     for (int i = s2.size() - 1; i >= 0; i--)
24         b.push_back(s2[i] - '0');
25     add(a, b);
26     for (int i = c.size() - 1; i >= 0; i--)
27         cout << c[i];
28     return 0;
29 }

```

### 1.4.2 High Precision Subsection

```

1 vector<int> a, b, c;
2 string s1, s2;
3 void sub(vector<int> &a, vector<int> &b)
4 {
5     int t = 0;
6     for (int i = 0; i < a.size(); i++)
7     {
8         t = a[i] - t;
9         if (i < b.size()) t -= b[i];
10        c.push_back((t + 10) % 10);
11        if (t < 0) t = 1;
12        else t = 0;
13    }
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> s2;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     for (int i = s2.size() - 1; i >= 0; i--)
23         b.push_back(s2[i] - '0');
24     if (s1.size() < s2.size())
25         cout << '-', sub(b, a);
26     else if (s1.size() == s2.size() && s1 < s2)
27         cout << '-', sub(b, a);
28     else sub(a, b);
29     for (int i = c.size() - 1; i >= 0; i--)
30         cout << c[i];
31     return 0;
32 }

```

### 1.4.3 High Precision Multiply

```

1 string s1, s2;
2 vector<int> a, c;
3 int b;
4 void mul(vector<int> &a, int b)
5 {
6     for (int i = 0, t = 0; i < a.size() || t; i++)
7     {
8         if (i < a.size()) t += a[i] * b;
9         c.push_back(t % 10);
10        t /= 10;
11    }
12    while (c.size() > 1 && c.back() == 0)
13        c.pop_back();
14 }
15 int main()
16 {
17     cin >> s1 >> b;
18     for (int i = s1.size() - 1; i >= 0; i--)
19         a.push_back(s1[i] - '0');
20     mul(a, b);
21     for (int i = c.size() - 1; i >= 0; i--)
22         cout << c[i];
23     return 0;
24 }

```

### 1.4.4 High Precision Divide

```

1 string s1, s2;
2 vector<int> a, c;
3 int b, r;
4 void divide(vector<int> &a, int b, int &r)
5 {
6     r = 0;
7     for (int i = a.size() - 1; i >= 0; i--)
8     {
9         r = r * 10 + a[i];
10        c.push_back(r / b);
11        r %= b;
12    }
13    reverse(c.begin(), c.end());
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> b;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     divide(a, b, r);
23     for (int i = c.size() - 1; i >= 0; i--)
24         cout << c[i];
25     cout << '\n' << r;
26     return 0;
27 }

```



## 1.5 Prefix Sum & Difference Array

### 1.5.1 1D Prefix Sum

```
1 S[i] = a[1] + a[2] + ... a[i]
2 a[l] + ... + a[r] = S[r] - S[l - 1]
```

### 1.5.2 2D Prefix Sum

```
1 // S[i, j] = i 行 j 列左上部分所有元素和为:
2 s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1] + a[i][j]
3 // 以 (x1, y1) 为左上角, (x2, y2) 为右下角
  的子矩阵的和为:
4 S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1]
  + S[x1 - 1][y1 - 1]
```

### 1.5.3 1D Difference Array

```
1 const int N = 100010;
2 int n, m;
3 int a[N], b[N];
4 void insert(int l, int r, int c)
5 { b[l] += c; b[r + 1] -= c; }
6 int main()
7 {
8     cin >> n >> m;
9     for (int i = 1; i <= n; i++)
10         cin >> a[i];
11     for (int i = 1; i <= n; i++)
12         insert(i, i, a[i]);
13     while (m--)
14     {
15         int l, r, c;
16         cin >> l >> r >> c;
17         insert(l, r, c);
18     }
19     for (int i = 1; i <= n; i++)
20         b[i] += b[i - 1],
21         cout << b[i] << ' ';
22     return 0;
23 }
```

### 1.5.4 2D Difference Array

```
1 const int N = 1010;
2 int n, m, q, a[N][N], b[N][N];
3 void insert(int x1, int y1, int x2, int y2
4 , int c)
5 {
6     b[x1][y1] += c;
7     b[x2 + 1][y2 + 1] += c;
8     b[x1][y2 + 1] -= c;
9     b[x2 + 1][y1] -= c;
10 }
11 int main()
12 {
```

```
12     cin >> n >> m >> q;
13     for (int i = 1; i <= n; i++)
14         for (int j = 1; j <= m; j++)
15             cin >> a[i][j];
16     for (int i = 1; i <= n; i++)
17         for (int j = 1; j <= m; j++)
18             insert(i, j, i, j, a[i][j]);
19     while (q--)
20     {
21         int x1, x2, y1, y2, c;
22         cin >> x1 >> y1 >> x2 >> y2 >> c;
23         insert(x1, y1, x2, y2, c);
24     }
25     // 其他过程略
26 }
```

## 2 ★ Basic Data Structures

### 2.1 Linked List

#### 2.1.1 Singly Linked List

```
1  const int N = 100010;
2  int n, h[N], e[N], ne[N], idx = 1;
3  void init() { ne[0] = -1; }
4  void insert(int k, int x) // 第 k 个节点
   后插入
5  { e[idx] = x, ne[idx] = ne[k], ne[k] = idx
   ++; }
6  void del(int k) // 第 k 个节点后删除
7  { ne[k] = ne[ne[k]]; }
```

#### 2.1.2 Bidirectional Linked List

```
1  const int N = 100010;
2  int n, r[N], l[N], e[N], idx = 2;
3  void init() { r[0] = 1; l[1] = 0; }
4  void insert(int k, int x) // 第 k 个节点后
   插入
5  {
6      e[idx] = x;
7      r[idx] = r[k];
8      l[idx] = k;
9      l[r[k]] = idx;
10     r[k] = idx++;
11 }
12 void remove(int k) // 删除 k 本身
13 { r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

## 2.2 Stack & Queue

### 2.2.1 Monotonic Stack

```
1  // 常见模型：找出每个数左边离它最近的比它大/
   小的数
2  int tt = 0;
3  for (int i = 1; i <= n; i++)
4  {
5      while (tt && check(stk[tt], i)) tt --
6      ;
7      stk[++tt] = i;
8  }
```

### 2.2.2 Monotonic Queue

```
1  // 常见模型：找出滑动窗口中的最大值/最小值
2  int hh = 0, tt = -1;
3  for (int i = 0; i < n; i++)
4  {
5      while (hh <= tt && check_out(q[hh]))
6          hh++; // 判断队头是否滑出窗口
7      while (hh <= tt && check(q[tt], i))
```

```
8          tt-- ;
9      q[++tt] = i;
10 }
```

### 2.3 KMP

```
1  const int N = 100010, M = 1000010;
2  int n, m;
3  char p[N], s[M];
4  void getNext(int ne[])
5  {
6      for (int i = 2, j = 0; i <= n; i++)
7      {
8          while (j && p[j + 1] != p[i])
9              j = ne[j];
10         if (p[j + 1] == p[i]) j++;
11         ne[i] = j;
12     }
13 }
14 int KMP()
15 {
16     int *ne = new int[n + 1];
17     getNext(ne);
18     for (int i = 1, j = 0; i <= m; i++)
19     {
20         while (j && p[j + 1] != s[i])
21             j = ne[j];
22         if (p[j + 1] == s[i]) j++;
23         if (j == n) cout << i - n << ' ';
24     }
25     return -1;
26 }
```

### 2.4 Trie

```
1  const int N = 100010;
2  int trie[N][26], cnt[N], idx = 0;
3  void insert(string &str) // 插入到 Trie
   数组
4  {
5      int p = 0;
6      for (auto c : str)
7      {
8          int u = c - 'a';
9          if (!trie[p][u])
10             trie[p][u] = ++idx;
11             p = trie[p][u];
12     }
13     cnt[p]++;
14 }
15 int query(string &str) // 查询字符串出
   现的次数
16 {
17     int p = 0;
18     for (auto c : str)
19     {
20         int u = c - 'a';
21         if (!trie[p][u]) return 0;
22         p = trie[p][u];
23     }
```

```

24     return cnt[p];
25 }

```

## 2.5 Disjoint-Set

```

1  const int N = 100010;
2  int n, m, p[N], Size[N], D[N];
3  void init()
4  {
5      for (int i = 1; i <= n; i++)
6          p[i] = i, Size[i] = 1, D[i] = 0;
7  }
8  int find(int x)
9  {
10     if (p[x] != x)
11     {
12         int u = find(p[x]);
13         D[x] += D[p[x]]; // 视具体情况计算
14         p[x] = u;
15     }
16     return p[x];
17 }
18 void merge(int a, int b, int distance)
19 {
20     int x = find(a), y = find(b);
21     if (x != y)
22     {
23         p[x] = y;
24         D[x] = distance; // 视具体情况计算
25         Size[y] += Size[x];
26     }
27 }

```

## 2.6 Hash

### 2.6.1 Simple Hash

```

1  // (1) 拉链法
2  int h[N], e[N], ne[N], idx;
3  void insert(int x)
4  {
5      int k = (x % N + N) % N;
6      e[idx] = x, ne[idx] = h[k], h[k] = idx;
7  }
8  bool find(int x)
9  {
10     for (int i = h[(x % N + N) % N]; i != -1; i = ne[i])
11         if (e[i] == x) return true;
12     return false;
13 }
14 // (2) 开放寻址法
15 int find(int x)
16 {
17     int t = (x % N + N) % N;
18     while (h[t] != null && h[t] != x)
19         t++; if (t == N) t = 0; }
20     return t;
21 }

```

### 2.6.2 String Hash

```

1  typedef unsigned long long ULL;
2  ULL h[N], p[N];
3  void init()
4  {
5      p[0] = 1;
6      for (int i = 1; i <= n; i++) { h[i]
7          = h[i - 1] * P + str[i]; p[i] = p[i -
8          1] * P; }
9  }
10 ULL get(int l, int r) { return h[r] - h[l
11     - 1] * p[r - l + 1]; }

```

## 2.7 STL

```

1  // vector
2  size()      返回元素个数
3  empty()     返回是否为空
4  clear()     清空
5  front()/back()
6  push_back()/pop_back()
7  begin()/end()
8  []
9  支持比较运算, 按字典序
10 // pair<int, int>
11 first       第一个元素
12 second      第二个元素
13 支持比较运算, 以first为第一关键字, 以second
14             为第二关键字 (字典序)
15 // string
16 size()/length() 返回字符串长度
17 empty()
18 clear()
19 substr(起始下标, (子串长度)) 返回子串
20 c_str() 返回字符串所在字符数组的起始地址
21 // queue
22 size()
23 empty()
24 push()      向队尾插入一个元素
25 front()     返回队头元素
26 back()      返回队尾元素
27 pop()       弹出队头元素
28 // priority_queue
29 size()
30 empty()
31 push()      插入一个元素
32 top()       返回堆顶元素
33 pop()       弹出堆顶元素
34 定义成小根堆的方式: priority_queue<int,
35                     vector<int>, greater<int>> q;
36 // stack
37 size()
38 empty()
39 push()      向栈顶插入一个元素
40 top()       返回栈顶元素
41 pop()       弹出栈顶元素
42 // deque
43 size()
44 empty()
45 clear()
46 front()/back()
47 push_back()/pop_back()

```

```

46 push_front()/pop_front()
47 begin()/end()
48 []
49 // set, map, multiset, multimap: 基于平衡二
    叉树 (红黑树) 动态维护有序序列
50 size()
51 empty()
52 clear()
53 begin()/end()
54 ++, -- 返回前驱和后继, 时间复杂度  $O(\log n)$ 
55 // set/multiset
56     insert()  插入一个数
57     find()    查找一个数
58     count()   返回某一个数的个数
59     erase()
60         (1) 输入是一个数x, 删除所有x,  $O(k + \log n)$ 
61         (2) 输入一个迭代器, 删除这个迭代器
62     lower_bound()/upper_bound()
63         lower_bound(x) 返回大于等于x的最小的
        数的迭代器
64         upper_bound(x) 返回大于x的最小的数
        的迭代器
65 // map/multimap
66     insert()  插入的数是一个pair
67     erase()   输入的参数是pair或者迭代器
68     find()
69     []        注意multimap不支持此操作。时
        间复杂度是  $O(\log n)$ 
70     lower_bound()/upper_bound()
71 // unordered_set, unordered_map,
    unordered_multiset, unordered_multimap
72 增删改查的时间复杂度是  $O(1)$ 
73 不支持 lower_bound()/upper_bound(), 迭代器
    的++, --
74 // bitset
75 bitset<10000> s;
76 ~, &, |, ^
77 >>, <<
78 ==, !=
79 []
80 count()      返回有多少个1
81 any()        判断是否至少有一个1
82 none()       判断是否全为0
83 set()        把所有位置成1
84 set(k, v)    将第k位变成v
85 reset()      把所有位变成0
86 flip()       等价于~
87 flip(k)      把第k位取反

```

## 3 ★ Search & Graph Theory

### 3.1 Representation of Tree & Graph

#### 3.1.1 Adjacency Matrix

```
1 // g[a][b] = a->b
```

#### 3.1.2 Adjacency List

```
1 int h[N], e[N], ne[N], idx;
2 void init() { memset(h, -1, sizeof h); }
3 void add(int a, int b) { e[idx] = b, ne[
    idx] = h[a], h[a] = idx++ ; }
```

### 3.2 DFS & BFS

#### 3.2.1 DFS

```
1 int dfs(int u)
2 {
3     st[u] = true; // 表示点 u 已经被遍历过
4     for (int i = h[u]; i != -1; i = ne[i])
5     { int j = e[i]; if (!st[j]) dfs(j); }
6 }
```

#### 3.2.2 BFS

```
1 queue<int> q;
2 st[1] = true; q.push(1);
3 while (q.size())
4 {
5     int t = q.front(); q.pop();
6     for (int i = h[t]; i != -1; i = ne[i])
7         if (!st[e[i]]) { st[e[i]] = true;
8             q.push(e[i]); }
9 }
```

### 3.3 Topological Sort

```
1 const int N = 100010;
2 int e[2 * N], ne[2 * N], h[N], d[N], idx;
3 int n, m, q[N];
4 void init() { memset(h, -1, sizeof h); }
5 void add(int a, int b) { e[idx] = b, ne[
    idx] = h[a], h[a] = idx++, d[b]++; }
6 bool topSort()
7 {
8     int hh = 0, tt = -1;
9     for (int i = 1; i <= n; i++)
10         if (!d[i]) q[++tt] = i;
11     while (hh <= tt)
```

```
12         for (int i = h[q[hh++]]; ~i; i =
13             ne[i])
14             if (--d[e[i]] == 0) q[++tt] =
15                 e[i];
16     return tt == n - 1;
17 }
```

### 3.4 Shortest Path

#### 3.4.1 Dijkstra

```
1 const int N = 1010;
2 int n, dist[N];
3 int h[N], w[N], e[N], ne[N], idx;
4 bool st[N];
5 void add(int a, int b, int c) { e[idx] = b
    , w[idx] = c, ne[idx] = h[a], h[a] =
    idx++; }
6 int dijkstra() // 需要初始化 dist 与 h
7 {
8     dist[1] = 0;
9     priority_queue<PII, vector<PII>,
10         greater<PII>> heap;
11     heap.push({0, 1});
12     while (heap.size())
13     {
14         auto t = heap.top();
15         heap.pop();
16         int ver = t.second, distance = t.
17             first;
18         if (st[ver]) continue;
19         st[ver] = true;
20         for (int i = h[ver]; i != -1; i =
21             ne[i])
22             if (dist[e[i]] > distance + w[
23                 i])
24             {
25                 dist[e[i]] = distance + w[
26                     i];
27                 heap.push({dist[e[i]], e[i]
28                     });
29             }
30     }
31     if (dist[n] == 0x3f3f3f3f) return -1;
32     return dist[n];
33 }
```

#### 3.4.2 Bellman-Ford

```
1 const int N = 100010;
2 int n, m, dist[N], backup[N];
3 struct Edge
4 {
5     int a, b, w;
6 } edges[N];
7 int bellman_ford()
8 {
9     memset(dist, 0x3f, sizeof dist);
10    dist[1] = 0;
11    for (int i = 0; i < n; i++)
12    {
```

```

13     memcpy(backup, dist, sizeof dist);
14     for (int j = 0; j < m; j++)
15     {
16         int a = edges[j].a, b = edges[
17         j].b, w = edges[j].w;
18         dist[b] = min(dist[b], backup[
19         a] + w);
20     }
21     if (dist[n] > 0x3f3f3f3f / 2) return
22     -1;
23     return dist[n];
24 }

```

```

13     {
14         dist[e[i]] = dist[t] + w[i
15     ];
16         // 新增
17         cnt[j] = cnt[t] + 1;
18         if (cnt[j] >= n) return
19         true
20         if (!st[j]) q.push(j), st[
21         j] = true;
22     }
23     }
24     return false;
25 }

```

### 3.4.3 SPFA

```

1  const int N = 100010;
2  int n, m, dist[N];
3  int e[2 * N], ne[2 * N], w[2 * N], h[N],
4  idx;
5  bool vis[N];
6  void spfa()    // 需要初始化 dist 与 h
7  {
8      queue<int> q;
9      q.push(1); vis[1] = true;
10     while (q.size())
11     {
12         int t = q.front();
13         q.pop();
14         vis[t] = false;
15         for (int i = h[t]; ~i; i = ne[i])
16             if (dist[e[i]] > dist[t] + w[i
17             ])
18             {
19                 dist[e[i]] = dist[t] + w[i
20                 ];
21                 if (!vis[e[i]]) vis[e[i]]
22                 = true, q.push(j);
23             }
24     }
25     dist[n] > INF / 2 ? cout << "
26     impossible" : cout << dist[n];
27 }

```

### 3.4.5 Floyd

```

1  const int N = 210;
2  int g[N][N], n, m, k;
3  int main()
4  {
5      cin >> n >> m >> k;
6      memset(g, 0x3f, sizeof g);
7      for (int i = 1; i <= n; i++) g[i][i] =
8      0;
9      while (m--)
10     {
11         int a, b, c;
12         cin >> a >> b >> c;
13         g[a][b] = min(g[a][b], c);
14     }
15     for (int k = 1; k <= n; k++)
16         for (int i = 1; i <= n; i++)
17             for (int j = 1; j <= n; j++)
18                 g[i][j] = min(g[i][k] + g[
19                 k][j], g[i][j]);
20     // 后续代码略
21     return 0;
22 }

```

### 3.4.4 Detecting Negative Circle in SPFA

```

1  void spfa()    // 只需要初始化 h
2  {
3      queue<int> q;
4      // 基于虚拟原点假设, 所有点放入队列
5      for (int i = 1; i <= n; i++) q.push(i)
6      , st[i] = true;
7      while (q.size())
8      {
9          int t = q.front();
10         q.pop();
11         vis[t] = false;
12         for (int i = h[t]; ~i; i = ne[i])
13             if (dist[e[i]] > dist[t] + w[i
14             ])
15             {
16                 dist[e[i]] = dist[t] + w[i
17                 ];
18                 if (!vis[e[i]]) vis[e[i]]
19                 = true, q.push(j);
20             }
21     }
22 }

```

## 3.5 Minimum Spanning Tree

### 3.5.1 Prim

```

1  const int N = 510;
2  int n, m, g[N][N], dist[N];
3  bool vis[N];
4  void prim()
5  {
6      int res = 0;
7      for (int i = 0; i < n; i++)
8      {
9          int t = -1;
10         for (int j = 1; j <= n; j++)
11             if (!vis[j] && (t == -1 ||
12             dist[j] < dist[t])) t = j;
13         if (i && dist[t] == INF) { res =
14         INF; break; }
15         if (i) res += dist[t];
16         vis[t] = true;
17         for (int j = 1; j <= n; j++) dist[
18         j] = min(dist[j], g[t][j]);
19     }
20 }

```

```

16     }
17     res == INF ? cout << "impossible" :
        cout << res;
18 }
19 int main()
20 {
21     memset(g, 0x3f, sizeof g);
22     memset(dist, 0x3f, sizeof dist);
23     cin >> n >> m;
24     while (m--)
25     {
26         int a, b, c;
27         cin >> a >> b >> c;
28         g[a][b] = min(g[a][b], c);
29         g[b][a] = min(g[b][a], c);
30     }
31     prim();
32     return 0;
33 }

```

### 3.5.2 Kruskal

```

1  const int N = 100010;
2  int n, m;
3  int p[N];
4  struct Edge
5  {
6      int a, b, w;
7      bool operator<(const Edge &e) const {
9          return w < e.w; };
8  } edge[2 * N];
9  void init() { for (int i = 1; i <= n; i++)
10     p[i] = i; }
10 int find(int x)
11 {
12     if (x != p[x]) p[x] = find(p[x]);
13     return p[x];
14 }
15 void merge(int x, int y) { p[find(x)] =
16     find(y); }
16 void kruskal()
17 {
18     int res = 0, cnt = 0;
19     for (int i = 1; i <= m; i++)
20         if (find(edge[i].a) != find(edge[i]
21             .b))
22             {
23                 merge(edge[i].a, edge[i].b);
24                 res += edge[i].w;
25                 cnt++;
26             }
27     if (cnt < n - 1) res = INF;
28     res == INF ? cout << "impossible" :
29         cout << res;
30 }
31 int main()
32 {
33     init();
34     cin >> n >> m;
35     for (int i = 1; i <= m; i++) cin >>
36         edge[i].a >> edge[i].b >> edge[i].w;
37     sort(edge + 1, edge + m + 1);
38     kruskal();
39     return 0;

```

```

37 }

```

## 3.6 Bipartite Graph

### 3.6.1 Coloring Method

To check if a given graph is bipartite.

```

1  const int N = 100010, M = 200010;
2  int n, m;
3  int e[M], ne[M], h[N], color[N], idx;
4  bool dfs(int u, int c)
5  {
6      color[u] = c;
7      for (int i = h[u]; ~i; i = ne[i])
8          if (color[e[i]] == -1)
9              {
10                 if (!dfs(e[i], !c)) return false;
11             }
12     else if (color[e[i]] == c) return
13         false;
14     return true;
15 }
16 bool check()
17 {
18     for (int i = 1; i <= n; i++)
19         if (color[i] == -1)
20             if (!dfs(i, 0)) return false;
21     return true;
22 }
23 int main()
24 {
25     // 注意另外初始化 h 与 color
26     cin >> n >> m;
27     while (m--)
28     {
29         int a, b;
30         cin >> a >> b;
31         add(a, b), add(b, a);
32     }
33     // 其余过程略

```

### 3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```
1  const int N = 510, M = 100010;
2  int n1, n2, m;
3  int e[M], ne[M], h[N], match[N], idx;
4  bool vis[N];
5  bool find(int x)
6  {
7      for (int i = h[x]; ~i; i = ne[i])
8          if (!vis[e[i]])
9              {
10                 vis[e[i]] = true;
11                 if (match[e[i]] == 0 || find(match[e[i]]))
12                     {
13                         match[e[i]] = x;
14                         return true;
15                     }
16             }
17     return false;
18 }
19 int main()
20 {
21     // 注意初始化 h
22     cin >> n1 >> n2 >> m;
23     while (m--)
24     {
25         int a, b;
26         cin >> a >> b;
27         add(a, b);
28     }
29     int res = 0;
30     for (int i = 1; i <= n1; i++)
31     {
32         memset(vis, false, sizeof vis);
33         if (find(i)) res++;
34     }
35     cout << res;
36     return 0;
37 }
```



## 4 ★ Basic Math

### 4.1 Prime Numbers

#### 4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0) return false;
6     return true;
7 }
```

#### 4.1.2 Prime Factorization

```
1 void divide(int x)
2 {
3     for (int i = 2; i <= x / i; i++)
4         if (x % i == 0)
5             { // 此条件成立时 i 一定是质数
6                 int s = 0;
7                 while (x % i == 0) x /= i, s
8                     ++;
9                 cout << i << ' ' << s << '\n';
10            }
11     if (x > 1) cout << x << ' ' << 1 << '\n';
12 }
```

#### 4.1.3 Euler's Sieve

```
1 int primes[N], cnt;
2 bool st[N];
3 void get_primes(int n)
4 {
5     for (int i = 2; i <= n; i++)
6     {
7         if (!st[i]) primes[cnt++] = i;
8         for (int j = 0; primes[j] <= n / i; j++)
9             {
10                 st[primes[j] * i] = true;
11                 if (i % primes[j] == 0) break;
12             }
13     }
14 }
```

## 4.2 Divisor

### 4.2.1 Find All Divisors

```
1 vector<int> get_divisors(int x)
2 {
3     vector<int> res;
```

```
4     for (int i = 1; i <= x / i; i++)
5         if (x % i == 0)
6             {
7                 res.push_back(i);
8                 if (i != x / i) res.push_back(
9                     x / i);
10            }
11     sort(res.begin(), res.end());
12     return res;
```

### 4.2.2 The Number of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 int main()
4 {
5     cin >> n;
6     unordered_map<int, int> h;
7     while (n--)
8     {
9         int x;
10        cin >> x;
11        for (int i = 2; i <= x / i; i++)
12            while (x % i == 0) { h[i]++; x
13                = x / i; }
14        if (x > 1) h[x]++;
15    }
16    long long res = 1;
17    for (auto iter = h.begin(); iter != h.
18        end(); iter++)
19        res = res * (iter->second + 1) %
20        mod;
21    cout << res;
22    return 0;
23 }
```

### 4.2.3 The Sum of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 long long getSum(int x, int c)
4 {
5     long long s = 1;
6     while(c--) s = (s * x + 1) % mod;
7     return s;
8 }
9 int main()
10 {
11     cin >> n;
12     unordered_map<int, int> h;
13     while (n--)
14     {
15         int x;
16         cin >> x;
17         for (int i = 2; i <= x / i; i++)
18             while (x % i == 0) { h[i]++; x
19                 = x / i; }
20         if (x > 1) h[x]++;
21     }
22     long long res = 1;
```

```

22     for (auto iter = h.begin(); iter != h.
        end(); iter++)
23         res = res * getSum(iter->first,
            iter->second) % mod;
24     cout << res;
25     return 0;
26 }

```

#### 4.2.4 Euclidean Algorithm

```

1  int gcd(int a, int b)
2  { return a % b == 0 ? b : gcd(b, a % b); }

```

### 4.3 Euler Function

#### 4.3.1 Simple Method

```

1  int phi(int x)
2  {
3      int res = x;
4      for (int i = 2; i <= x / i; i++)
5          if (x % i == 0)
6              {
7                  res = res / i * (i - 1);
8                  while (x % i == 0) x /= i;
9              }
10     if (x > 1) res = res / x * (x - 1);
11     return res;
12 }

```

#### 4.3.2 Euler's Sieve Method

```

1  const int N = 1000010;
2  int n, primes[N], phi[N], cnt;
3  bool st[N];
4  void getEuler()
5  {
6      phi[1] = 1;
7      for (int i = 2; i <= n; i++)
8          {
9              if (!st[i])
10                 {
11                     primes[cnt++] = i;
12                     // i 是质数, 它只会被本身整除,
13                     // 所以直接赋值 i - 1
14                     phi[i] = i - 1;
15                     for (int j = 0; primes[j] <= n / i; j++)
16                         {
17                             st[i * primes[j]] = true;
18                             if (i % primes[j] == 0)
19                                 {
20                                     // 如果 i % primes[j] == 0
21                                     // 成立表示 primes[j] 是 i 的最小质因子
22                                     // 也是 primes[j] * i 的最
23                                     // 小质因子

```

```

22                                     // 1 - 1 / primes[j] 这一
23                                     // 项在 phi[i] 中计算过了, 只需将基数 N 修
24                                     // 正为 primes[j] 倍
25                                     phi[primes[j] * i] = phi[i]
26                                     * primes[j];
27                                     break;
28                                     }
29                                     // 否则, primes[j] 不是 i 的质
30                                     // 因子, 只是 primes[j] * i 的最小质因子
31                                     // 不仅需要将基数 N 修正为
32                                     // primes[j] 倍
33                                     // 还需要补上 1 - 1 / primes[j]
34                                     // 的分子项, 因此最终结果为 phi[i] * (
35                                     // primes[j] - 1)
36                                     phi[primes[j] * i] = phi[i] *
37                                     (primes[j] - 1);
38                                     }
39     }
40 }

```

### 4.4 Exponentiating by Squaring

```

1  LL qmi(int m, int k, int p)
2  {
3      LL res = 1 % p, t = m;
4      while (k)
5          {
6              if (k & 1) res = res * t % p;
7              t = t * t % p;
8              k >>= 1;
9          }
10     return res;
11 }

```

### 4.5 Extended Euclidean Algorithm

```

1  int exgcd(int a, int b, int &x, int &y)
2  {
3      if (!b)
4          {
5              x = 1;
6              y = 0;
7              return a;
8          }
9      int d = exgcd(b, a % b, y, x);
10     y -= (a / b) * x;
11     return d;
12 }

```

### 4.6 Chinese Remainder Theorem

```

1  LL exgcd(LL a, LL b, LL &x, LL &y)
2  {
3      if (!b) { x = 1, y = 0; return a; }

```

```

4    LL d = exgcd(b, a % b, y, x);
5    y -= a / b * x;
6    return d;
7 }
8 int main()
9 {
10     int n;
11     cin >> n;
12     LL x = 0, m1, a1;
13     cin >> m1 >> a1;
14     for (int i = 0; i < n - 1; i++)
15     {
16         LL m2, a2;
17         cin >> m2 >> a2;
18         LL k1, k2;
19         LL d = exgcd(m1, m2, k1, k2);
20         if ((a2 - a1) % d) { x = -1; break
; }
21         k1 *= (a2 - a1) / d;
22         k1 = (k1 % (m2 / d) + m2 / d) % (
m2 / d);
23         x = k1 * m1 + a1;
24         LL m = abs(m1 / d * m2);
25         a1 = k1 * m1 + a1;
26         m1 = m;
27     }
28     if (x != -1)
29         x = (a1 % m1 + m1) % m1;
30     cout << x << '\n';
31     return 0;
32 }

```

## 4.7 Gauss-Jordan Elimination

### 4.7.1 Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++) //
找绝对值最大的行
8             if (fabs(a[i][c]) > fabs(a[t][
c]))
9                 t = i;
10        if (fabs(a[t][c]) < eps) //
此时没必要对该列该行处理
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[t][i], a[r][i]); //
将绝对值最大的行换到最顶端
14        for (int i = n; i >= c; i--)
15            a[r][i] /= a[r][c]; //
将当前行的首位变成1
16        for (int i = r + 1; i < n; i++) //
用当前行将下面所有的列消成0
17            if (fabs(a[i][c]) > eps)
18                for (int j = n; j >= c; j
--))
19                    a[i][j] -= a[r][j] * a
[i][c];

```

```

20        r++;
21    }
22    if (r < n)
23    {
24        for (int i = r; i < n; i++)
25            if (fabs(a[i][n]) > eps)
26                return 2; // 无解
27        return 1; // 有无穷多组解
28    }
29    for (int i = n - 1; i >= 0; i--)
30        for (int j = i + 1; j < n; j++)
31            a[i][n] -= a[i][j] * a[j][n];
32    return 0; // 有解
33 }

```

### 4.7.2 XOR Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++)
8             if (a[i][c])
9                 t = i;
10        if (!a[t][c])
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[r][i], a[t][i]);
14        for (int i = r + 1; i < n; i++)
15            if (a[i][c])
16                for (int j = n; j >= c; j
--))
17                    a[i][j] ^= a[r][j];
18        r++;
19    }
20    if (r < n)
21    {
22        for (int i = r; i < n; i++)
23            if (a[i][n])
24                return 2;
25        return 1;
26    }
27    for (int i = n - 1; i >= 0; i--)
28        for (int j = i + 1; j < n; j++)
29            a[i][n] ^= a[i][j] * a[j][n];
30    return 0;
31 }

```

## 4.8 Combinatorial Counting

### 4.8.1 Recurrence Relation

```

1 void init()
2 {
3     for (int i = 0; i < N; i++)
4         for (int j = 0; j <= i; j++)
5             if (!j) c[i][j] = 1;
6             else c[i][j] = (c[i - 1][j] +
c[i - 1][j - 1]) % mod;

```

```
7 }
```

## 4.8.2 Preprocessing & Inverse Element

```
1 const int N = 100010, mod = 1e9 + 7;
2 int n, fact[N], infact[N];
3 int qmi(int a, int b, int p)
4 {
5     int res = 1;
6     while (b)
7     {
8         if (b & 1)
9             res = (LL)res * a % p;
10        a = (LL)a * a % p;
11        b >>= 1;
12    }
13    return res;
14 }
15 int main()
16 {
17     fact[0] = infact[0] = 1;
18     for (int i = 1; i < N; i++)
19     {
20         fact[i] = (LL)fact[i - 1] * i %
mod;
21         infact[i] = (LL)infact[i - 1] *
qmi(i, mod - 2, mod) % mod;
22     }
23     // 此后 C(a, b) = (LL)fact[a] * infact
[b] % mod * infact[a - b] % mod
24 }
```

## 4.8.3 Lucas Theorem

```
1 int qmi(int a, int k, int p)
2 {
3     int res = 1 % p;
4     while (k)
5     {
6         if (k & 1)
7             res = (LL)res * a % p;
8         a = (LL)a * a % p;
9         k >>= 1;
10    }
11    return res;
12 }
13 int C(int a, int b, int p)
14 {
15     if (a < b) return 0;
16     LL x = 1, y = 1;
17     // x = a * (a - 1) * (a - 2) * ... * (
a - b + 1) = a! / (a - b)! (mod p)
18     // y = 1 * 2 * ... * b = b! (mod p)
19     for (int i = a, j = 1; j <= b; i--, j
++)
20     { x = (LL)x * i % p; y = (LL)y * j % p
; }
21     return x * (LL)qmi(y, p - 2, p) % p;
22 }
23 int lucas(LL a, LL b, int p)
24 {
25     if (a < p && b < p)
```

```
26         return C(a, b, p);
27     return (LL)C(a % p, b % p, p) * lucas(
a / p, b / p, p) % p;
28 }
```

## 4.8.4 Factorization Method

```
1 const int N = 5010;
2 int n, primes[N], sum[N], cnt;
3 bool st[N];
4 void getPrimes(int n) { // 略 }
5 // 求 n! 中 p 的幂次
6 int get(int n, int p)
7 {
8     int res = 0;
9     while (n) { res += n / p; n /= p; }
10    return res;
11 }
12 void mul(vector<int> &a, int b) { // 高精
度乘, 略 }
13 int main()
14 {
15     int a, b;
16     cin >> a >> b;
17     getPrimes(a);
18     for (int i = 0; i < cnt; i++)
19     {
20         int p = primes[i];
21         sum[i] = get(a, p) - get(b, p) -
get(a - b, p);
22     }
23     vector<int> res;
24     res.push_back(1);
25     for (int i = 0; i < cnt; i++)
26         for (int j = 0; j < sum[i]; j++)
27             mul(res, primes[i]);
28     for (int i = res.size() - 1; i >= 0; i
--)
29         cout << res[i];
30 }
```

## 4.8.5 Catalan Number

```
1 const int N = 100010, mod = 1e9 + 7;
2 int qmi(int a, int k, int p) { // 略 }
3 int main()
4 {
5     int n;
6     cin >> n;
7     int a = n * 2, b = n, res = 1;
8     for (int i = a; i > a - b; i--)
9         res = (LL)res * i % mod;
10    for (int i = 1; i <= b; i++)
11        res = (LL)res * qmi(i, mod - 2,
mod) % mod;
12    res = (LL)res * qmi(n + 1, mod - 2,
mod) % mod;
13 }
```

## 4.9 Inclusion-Exclusion Principle

```
1  const int N = 20;
2  int n, m, res = 0, p[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < m; i++)
7          cin >> p[i];
8      // 使用二进制数字表示数字选取情况
9      for (int i = 1; i < 1 << m; i++)
10     {
11         int t = 1, cnt = 0;
12         // 遍历每个被选取的质数
13         for (int j = 0; j < m; j++)
14             if (i >> j & 1)
15             {
16                 cnt++;
17                 // 一个质数能被选取的条件应
18                 // 该是其累乘积不超过目标数字
19                 if ((LL)t * p[j] > n)
20                     { t = -1; break; }
21                 t *= p[j];
22             }
23             if (t != -1)
24                 // 容斥原理公式中奇数个并集系数
25                 // 为 1, 反之为 -1
26                 if (cnt % 2) res += n / t;
27                 else res -= n / t;
28     }
29     cout << res;
30 }
```

```
21 {
22     cin >> k;
23     for (int i = 0; i < k; i++) cin >> s[i];
24     cin >> n;
25     memset(f, -1, sizeof f);
26     int res = 0;
27     // 每一堆石子都是一个入度为 0 的起始点
28     for (int i = 0; i < n; i++)
29     {
30         int x;
31         cin >> x;
32         res ^= sg(x);
33     }
34     res ? cout << "Yes" : cout << "No";
35     return 0;
36 }
```

## 4.10 Game Theory

### 4.10.1 NIM Game

```
1  const int N = 110, M = 100010;
2  int k, n, s[N], f[M];
3  int sg(int x)
4  {
5      if (f[x] != -1) return f[x];
6      // 到达节点得 SG 函数集合
7      unordered_set<int> S;
8      // 能取走石子就说明能到达, 并且递归向下
9      // 求解
10     for (int i = 0; i < k; i++)
11     {
12         int sum = s[i];
13         if (x >= sum) S.insert(sg(x - sum));
14     }
15     // SG 从小到达遍历并返回, 找到最小的、不
16     // 包含在 SG 函数集合中的自然数
17     for (int i = 0;; i++)
18         if (!S.count(i))
19             return f[x] = i;
20 }
```

```
21 int main()
22 {
23     cin >> k;
24     for (int i = 0; i < k; i++) cin >> s[i];
25     cin >> n;
26     memset(f, -1, sizeof f);
27     int res = 0;
28     // 每一堆石子都是一个入度为 0 的起始点
29     for (int i = 0; i < n; i++)
30     {
31         int x;
32         cin >> x;
33         res ^= sg(x);
34     }
35     res ? cout << "Yes" : cout << "No";
36     return 0;
37 }
```

## 5 ★ Basic DP

### 5.1 Knapsack Problem

#### 5.1.1 01 Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = m; j >= v[i]; j--)
10             f[j] = max(f[j], f[j - v[i]] +
11                 w[i]);
12     cout << f[m];
13 }
```

#### 5.1.2 Complete Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = v[i]; j <= m; j++)
10             f[j] = max(f[j], f[j - v[i]] +
11                 w[i]);
12     cout << f[m];
13 }
```

#### 5.1.3 Mutiple Knapsack

```
1  const int N = 25000;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      int cnt = 0;
7      for (int i = 1; i <= n; i++)
8      {
9          int a, b, s;
10         cin >> a >> b >> s;
11         int k = 1;
12         while (k <= s)
13         {
14             cnt++;
15             v[cnt] = a * k, w[cnt] = b * k
16             ;
17             s -= k, k *= 2;
18         }
19         if (s > 0)
20         {
21             cnt++;
22             v[cnt] = a * s, w[cnt] = b * s
23             ;
24         }
25     }
26     n = cnt;
27     for (int i = 1; i <= n; i++)
28         for (int j = m; j >= v[i]; j--)
29             f[j] = max(f[j], f[j - v[i]] +
30                 w[i]);
31     cout << f[m];
32 }
```

```
21         v[cnt] = a * s, w[cnt] = b * s
22         ;
23     }
24     n = cnt;
25     for (int i = 1; i <= n; i++)
26         for (int j = m; j >= v[i]; j--)
27             f[j] = max(f[j], f[j - v[i]] +
28                 w[i]);
29     cout << f[m];
30 }
```

#### 5.1.4 Grouped Knapsack

```
1  const int N = 120;
2  int n, m, s[N], v[N][N], w[N][N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> s[i];
9          for (int j = 1; j <= s[i]; j++)
10             cin >> v[i][j] >> w[i][j];
11     }
12     for (int i = 1; i <= n; i++)
13         for (int j = m; j >= 0; j--)
14             for (int k = 1; k <= s[i]; k
15                 ++
16                 )
17                 if (v[i][k] <= j)
18                     f[j] = max(f[j], f[j -
19                         v[i][k]] + w[i][k]);
20     cout << f[m];
21 }
```

## 5.2 Linear DP

### 5.2.1 LIS

Here is an  $O(n^2)$  solution:

```
1  const int N = 1010;
2  int n, a[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[j] < a[i])
13                 f[i] = max(f[i], f[j] + 1)
14         ;
15     }
16     int res = 0;
17     for (int i = 1; i <= n; i++)
18         res = max(res, f[i]);
19     cout << res;
20 }
```

Another is an  $O(n\log n)$  solution:

```
1  const int N = 100010;
2  int n, a[N], q[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++) cin >> a[i];
7      int len = 0;
8      q[len] = -INF;
9      for (int i = 1; i <= n; i++)
10     {
11         int l = 0, r = len;
12         while (l < r)
13         {
14             int mid = l + r + 1 >> 1;
15             if (q[mid] < a[i]) l = mid;
16             else r = mid - 1;
17         }
18         len = max(r + 1, len);
19         q[r + 1] = a[i];
20     }
21     cout << len;
22 }
```

## 5.2.2 LCS

```
1  const int N = 1010;
2  int n, m, f[N][N];
3  char a[N], b[N];
4  int main()
5  {
6      cin >> n >> m >> (a + 1) >> (b + 1);
7      for (int i = 1; i <= n; i++)
8          for (int j = 1; j <= m; j++)
9              {
10                 f[i][j] = max(f[i - 1][j], f[i][j - 1]);
11                 if (a[i] == b[j])
12                     f[i][j] = max(f[i][j], f[i - 1][j - 1] + 1);
13             }
14     cout << f[n][m];
15 }
```

## 5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```
1  const int N = 310;
2  int n, s[N], f[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> s[i], s[i] += s[i - 1];
8      for (int len = 2; len <= n; len++)
9          for (int i = 1; i + len - 1 <= n; i++)
10             {
```

```
11                 int l = i, r = i + len - 1;
12                 f[l][r] = INF;
13                 for (int k = l; k < r; k++)
14                     f[l][r] = min(f[l][r], f[l][k] + f[k + 1][r] + s[r] - s[l - 1]);
15             }
16     cout << f[1][n];
17 }
```

## 5.4 Counting DP

```
1  const int N = 1010, M = 1e9 + 7;
2  int n, f[N][N];
3  int main()
4  {
5      cin >> n;
6      f[0][0] = 1;
7      for (int i = 1; i <= n; i++)
8          for (int j = 1; j <= i; j++)
9              f[i][j] = (f[i - 1][j - 1] + f[i - j][j]) % M;
10     int ans = 0;
11     for (int i = 1; i <= n; i++)
12         ans = (ans + f[n][i]) % M;
13     cout << ans;
14 }
```

## 5.5 Digit DP

```
1  // 求数 n 的位数
2  int get(int n)
3  {
4      int res = 0;
5      while (n) n /= 10, res++;
6      return res;
7  }
8  int count(int n, int i)
9  {
10     int res = 0, dgt = get(n);
11     for (int j = 1; j <= dgt; j++)
12     {
13         // p 为当前遍历位次(第 j 位)的数大小 <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
14         // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
15         int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
16         // 求要选的数在 i 的左边的数小于 1 的情况:
17         // 1)、当 i 不为 0 时 xxx : 0...0 ~ 1 - 1, 即 1 * (右边的数的位数) == 1 * p 种选法
18         // 2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0...1 ~ 1 - 1, 即 (1 - 1) * (右边的数的位数) == (1 - 1) * p 种选法
19         if (i) res += 1 * p;
20         else res += (1 - 1) * p;
21         // 求要选的数在 i 的左边的数等于 1 的情况:(即视频中的 xxx == 1 时)
```

```

22         //      1)、i > dj 时 0 种选法
23         //      2)、i == dj 时 yyy : 0...0
        ~ r 即 r + 1 种选法
24         //      3)、i < dj 时 yyy : 0...0
        ~ 9...9 即 10^(右边的数的位数) == p 种
        选法 */
25         if (i == dj) res += r + 1;
26         if (i < dj) res += p;
27     }
28     return res;
29 }
30 int main()
31 {
32     int a, b;
33     while (cin >> a >> b, a)
34     {
35         if (a > b) swap(a, b);
36         for (int i = 0; i <= 9; ++i)
37             cout << count(b, i) - count(a
        - 1, i) << ' ';
38         // 利用前缀和思想: [l, r] 的和 = s[
        r] - s[l - 1]
39         cout << '\n';
40     }
41 }

```

```

31     f[0][0] = 1;
32     // 遍历每一列
33     for (int i = 1; i <= m; i++)
34         // 遍历当前列的每一种用二进制数
        字表示的摆放状态: 1 指横向摆放, 0 指空
        位
35         for (int j = 0; j < 1 << n; j
        ++))
36             // 遍历上一列的每一种用二进
        制数字表示的摆放状态: 1 指横向摆放, 0
        指空位
37             for (int k = 0; k < 1 << n
        ; k++)
38                 // 满足两个条件: 两列的
        摆放互不冲突; 两列摆放状态的结合状态是一
        个可取的状态则累加情况数
39                 if (!(j & k) && st[j |
        k])
40                     f[i][j] += f[i -
        1][k];
41                 // 输出摆放好第 m 列且第 (m + 1) 列
        没有任何方格的状态数
42                 cout << f[m][0] << '\n';
43             }
44 }

```

## 5.6 State Compression DP

```

1  const int N = 12, M = 1 << 12;
2  int n, m;
3  LL f[N][M];
4  bool st[M];
5  int main()
6  {
7      while (cin >> n >> m, n || m)
8      {
9          memset(f, 0, sizeof f);
10         for (int i = 0; i < 1 << n; i++)
11             {
12                 st[i] = true;
13                 // 统计连续 0 的个数, 若连续 0
                    为奇数个就不能正好放得下竖放的方格
14                 int cnt = 0;
15                 for (int j = 0; j < n && st[i
                    ]; j++)
16                     if (i >> j & 1)
17                         {
18                             // 当前格子被使用
19                             // 如果连续 0 的数量为
                    奇数个, 当前格子被使用的后果就是导致格子
                    重合, 所以不可取
20                             if (cnt & 1)
21                                 st[i] = false;
22                             // 刷新状态
23                             cnt = 0;
24                         }
25                     else cnt++;
26                 // 最后再判断一次, 防止漏判
27                 if (cnt & 1)
28                     st[i] = false;
29             }
30             // 没有摆放任何棋子的状态默认只有 1
                种取法

```

## 5.7 Tree DP

```

1  // Don't use I/O functions from stdio.h!!!
2  #define itn int
3  #define nit int
4  #define nti int
5  #define tin int
6  #define tni int
7  #define retrun return
8  #define reutrn return
9  #define rutren return
10 #define INF 0x3f3f3f3f
11 #include <bits/stdc++.h>
12 using namespace std;
13 typedef pair<int, int> PII;
14 typedef long long LL;
15
16 const int N = 6010;
17
18 int n;
19 int e[N], ne[N], happy[N], h[N], idx;
20 int f[N][2];
21 bool has_father[N];
22 void add(int a, int b)
23 { e[idx] = b, ne[idx] = h[a], h[a] = idx
    ++; }
24 void dfs(int u)
25 {
26     f[u][1] = happy[u];
27     for (int i = h[u]; ~i; i = ne[i])
28         dfs(e[i]);
29     f[u][0] += max(f[e[i]][0], f[e[i]
        ][1]);
30     f[u][1] += f[e[i]][0];
31 }
32 }
33 }

```



```

34 int main()
35 {
36     memset(h, -1, sizeof h);
37     cin >> n;
38     for (int i = 1; i <= n; i++) cin >>
happy[i];
39     for (int i = 0; i < n - 1; i++)
40     {
41         int a, b;
42         cin >> a >> b;
43         has_father[a] = true;
44         add(b, a);
45     }
46     int root = 1;
47     while (has_father[root]) root++;
48     dfs(root);
49     cout << max(f[root][0], f[root][1]);
50 }

```

## 5.8 Memoized Search

```

1  const int N = 310;
2  int n, m,
3  h[N][N], f[N][N],
4  dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1,
0};

```

```

5  int dp(int x, int y)
6  {
7      int &v = f[x][y];
8      if (v != -1) return v;
9      v = 1;
10     for (int i = 0; i < 4; i++)
11     {
12         int a = x + dx[i], b = y + dy[i];
13         if (a >= 1 && a <= n && b >= 1 &&
b <= m && h[a][b] < h[x][y])
14             v = max(v, dp(a, b) + 1);
15     }
16     return v;
17 }
18 int main()
19 {
20     cin >> n >> m;
21     for (int i = 1; i <= n; i++)
22         for (int j = 1; j <= m; j++)
23             cin >> h[i][j];
24     memset(f, -1, sizeof f);
25     int res = 0;
26     for (int i = 1; i <= n; i++)
27         for (int j = 1; j <= m; j++)
28             res = max(res, dp(i, j));
29     cout << res;
30 }

```



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## Part II: Advanced Template

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## 6 ★ Advanced Basic

### 6.1 Slow Multiplication

```
1 LL mul(LL a, LL b, LL p)
2 {
3     LL ans = 0;
4     while (b)
5     {
6         if (b & 1) ans = (ans + a) % p;
7         a = a * 2 % p; b >>= 1;
8     }
9     return ans;
10 }
```

### 6.2 Sum of Geometric Series

```
1 const int mod = 9901;
2 int a, b;
3 int qmi(int a, int k)
4 {
5     int res = 1;
6     a %= mod;
7     while (k)
8     {
9         if (k & 1)
10             res = res * a % mod;
11         a = a * a % mod;
12         k >>= 1;
13     }
14     return res;
15 }
16 int sum(int p, int k)
17 {
18     if (k == 1) return 1;
19     if (k % 2 == 0)
20         return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
21     return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
22 }
23 int main()
24 {
25     // 以  $a^b$  约数之和为例求等比数列和
26     cin >> a >> b;
27     int res = 1;
28     for (int i = 2; i <= a / i; i++)
29         if (a % i == 0)
30         {
31             int s = 0;
32             while (a % i == 0) a /= i, s++;
33             res = res * sum(i, b * s + 1) % mod;
34         }
35     if (a > 1) res = res * sum(a, b + 1) % mod;
36 }
```

## 6.3 Sort

### 6.3.1 Card Balancing Problem

```
1 cin >> n;
2 for (int i = 1; i <= n; i++)
3     cin >> a[i], avg += a[i];
4 avg /= n;
5 for (int i = 1; i <= n; i++)
6     if (a[i] != avg)
7         a[i + 1] += a[i] - avg, ans++;
8 cout << ans;
```

### 6.3.2 2D Card Balancing Problem

```
1 const int N = 100010;
2 int row[N], col[N], c[N], s[N];
3 LL work(int n, int a[])
4 {
5     for (int i = 1; i <= n; i++)
6         s[i] = s[i - 1] + a[i];
7     if (s[n] % n) return -1;
8     int avg = s[n] / n;
9     c[1] = 0;
10    for (int i = 2; i <= n; i++)
11        c[i] = s[i - 1] - (i - 1) * avg;
12    sort(c + 1, c + n + 1);
13    LL res = 0;
14    for (int i = 1; i <= n; i++)
15        res += abs(c[i] - c[(n + 1) / 2]);
16    return res;
17 }
18 int main()
19 {
20     int n, m, cnt;
21     cin >> n >> m >> cnt;
22     while (cnt--)
23     {
24         int x, y;
25         cin >> x >> y;
26         row[x]++; col[y]++;
27     }
28     LL r = work(n, row);
29     LL c = work(m, col);
30     if (r != -1 && c != -1)
31         cout << "both " << r + c;
32     else if (r != -1)
33         cout << "row " << r;
34     else if (c != -1)
35         cout << "column " << c;
36     else cout << "impossible";
37 }
```

### 6.3.3 Dual Heaps

```
1 if (down.empty() || x <= down.top())
2     down.push(x);
3 else up.push(x);
4 if (down.size() > up.size() + 1)
5     up.push(down.top(), down.pop());
```

```

6  if (up.size() > down.size())
7      down.push(up.top()), up.pop();
8  if (i % 2)
9  {
10     cout << down.top() << ' ';
11     if (++cnt % 10 == 0) cout << '\n';
12 }

```

## 6.4 RMQ

```

1  const int N = 200010, M = 18;
2  int n, m, w[N], f[N][M];
3  void init()
4  {
5      for (int j = 0; j < M; j++)
6          for (int i = 1; i + (1 << j) - 1
7              <= n; i++)
8              if (!j) f[i][j] = w[i];
9              else // 也可以是最小值
10                 f[i][j] = max(f[i][j - 1],
11                             f[i + (1 << j - 1)][j - 1]);
12 }
13 int query(int l, int r)
14 {
15     int len = r - l + 1;
16     int k = log(len) / log(2);
17     return max(f[l][k], f[r - (1 << k) +
18               1][k]);
19 }

```

## 7 ★ Advanced Data Structures

### 7.1 Binary Indexed Tree

```
1 // 支持区间修改、区间查询
2 // 利用变差分求二阶区间和
3 const int N = 100010;
4 int n, m, a[N];
5 LL tr1[N], tr2[N];
6 int lowbit(int x) { return x & -x; }
7 void add(LL tr[], LL x, LL c)
8 {
9     for (int i = x; i <= n; i += lowbit(i))
10         tr[i] += c;
11 }
12 LL sum(LL tr[], LL x)
13 {
14     LL res = 0;
15     for (int i = x; i; i -= lowbit(i))
16         res += tr[i];
17     return res;
18 }
19 LL prefix_sum(LL x)
20 { return sum(tr1, x) * (x + 1) - sum(tr2, x); }
21 int main()
22 {
23     cin >> n >> m;
24     for (int i = 1; i <= n; i++)
25         cin >> a[i];
26     for (int i = 1; i <= n; i++)
27     {
28         int b = a[i] - a[i - 1];
29         add(tr1, i, b);
30         add(tr2, i, (LL)i * b);
31     }
32     while (m--)
33     {
34         char op[2];
35         int l, r, d;
36         cin >> op >> l >> r;
37         if (*op == 'Q')
38             cout << prefix_sum(r) -
39             prefix_sum(l - 1) << '\n';
40         else
41         {
42             cin >> d;
43             add(tr1, l, d), add(tr2, l, (LL)l * d);
44             add(tr1, r + 1, -d),
45             add(tr2, r + 1, (LL)-(r + 1) * d);
46         }
47     }
```

## 7.2 Segment Tree

### 7.2.1 Maintain the Maximum

```
1 struct Node
2 { int l, r, v; } tr[N * 4];
3 void pushup(int u)
4 {
5     tr[u].v = max(tr[u << 1].v, tr[u << 1
6     | 1].v);
7 }
8 void build(int u, int l, int r)
9 {
10     tr[u] = {l, r};
11     if (l == r) return;
12     int mid = l + r >> 1;
13     build(u << 1, l, mid),
14     build(u << 1 | 1, mid + 1, r);
15 }
16 int query(int u, int l, int r)
17 {
18     if (tr[u].l >= l && tr[u].r <= r)
19         return tr[u].v;
20     int mid = tr[u].l + tr[u].r >> 1;
21     int v = 0;
22     if (l <= mid)
23         v = query(u << 1, l, r);
24     if (r > mid)
25         v = max(v, query(u << 1 | 1, l, r));
26     return v;
27 }
28 void modify(int u, int x, int v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31         tr[u].v = v;
32     else
33     {
34         int mid = tr[u].l + tr[u].r >> 1;
35         if (x <= mid)
36             modify(u << 1, x, v);
37         else
38             modify(u << 1 | 1, x, v);
39         pushup(u);
40     }
```

### 7.2.2 Maintain the Maximum Subarray Sum

```
1 struct Node
2 { int l, r, sum, lmax, rmax, tmax; } tr[N
3     * 4];
4 void pushup(Node &u, Node &l, Node &r)
5 {
6     u.sum = l.sum + r.sum;
7     u.lmax = max(l.lmax, l.sum + r.lmax);
8     u.rmax = max(r.rmax, r.sum + l.rmax);
9     u.tmax = max(max(l.tmax, r.tmax), l.
10     rmax + r.lmax);
11 }
12 void pushup(int u)
13 { pushup(tr[u], tr[u << 1], tr[u << 1
14     | 1]); }
15 void build(int u, int l, int r)
16 {
17     if (l == r)
```

```

15     tr[u] = {l, r, w[r], w[r], w[r], w
[r]};
16     else
17     {
18         tr[u] = {l, r};
19         int mid = l + r >> 1;
20         build(u << 1, l, mid),
21         build(u << 1 | 1, mid + 1, r);
22         pushup(u);
23     }
24 }
25 void modify(int u, int x, int v)
26 {
27     if (tr[u].l == x && tr[u].r == x)
28         tr[u] = {x, x, v, v, v, v};
29     else
30     {
31         int mid = tr[u].l + tr[u].r >> 1;
32         if (x <= mid)
33             modify(u << 1, x, v);
34         else
35             modify(u << 1 | 1, x, v);
36         pushup(u);
37     }
38 }
39 Node query(int u, int l, int r)
40 {
41     if (tr[u].l >= l && tr[u].r <= r)
42         return tr[u];
43     else
44     {
45         int mid = tr[u].l + tr[u].r >> 1;
46         if (r <= mid)
47             return query(u << 1, l, r);
48         else if (l > mid)
49             return query(u << 1 | 1, l, r)
50         ;
51         else
52         {
53             auto left = query(u << 1, l, r
54             );
55             auto right = query(u << 1 | 1,
56             l, r);
57             Node res;
58             pushup(res, left, right);
59             return res;
60         }
61     }
62 }

```

### 7.2.3 Maintain the GCD

```

1 struct Node
2 { int l, r; LL sum, d; } tr[N * 4];
3 LL gcd(LL a, LL b)
4 { return b ? gcd(b, a % b) : a; }
5 void pushup(Node &u, Node &l, Node &r)
6 {
7     u.sum = l.sum + r.sum;
8     u.d = gcd(l.d, r.d);
9 }
10 void pushup(int u)
11 { pushup(tr[u], tr[u << 1], tr[u << 1 |
12     1]); }

```

```

12 void build(int u, int l, int r)
13 {
14     if (l == r)
15     {
16         LL b = w[r] - w[r - 1];
17         tr[u] = {l, r, b, b};
18     }
19     else
20     {
21         tr[u].l = l, tr[u].r = r;
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
28 void modify(int u, int x, LL v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31     {
32         LL b = tr[u].sum + v;
33         tr[u] = {x, x, b, b};
34     }
35     else
36     {
37         int mid = tr[u].l + tr[u].r >> 1;
38         if (x <= mid)
39             modify(u << 1, x, v);
40         else
41             modify(u << 1 | 1, x, v);
42         pushup(u);
43     }
44 }
45 Node query(int u, int l, int r)
46 {
47     if (tr[u].l >= l && tr[u].r <= r)
48         return tr[u];
49     else
50     {
51         int mid = tr[u].l + tr[u].r >> 1;
52         if (r <= mid)
53             return query(u << 1, l, r);
54         else if (l > mid)
55             return query(u << 1 | 1, l, r)
56         ;
57         else
58         {
59             auto left = query(u << 1, l, r
60             );
61             auto right = query(u << 1 | 1,
62             l, r);
63             Node res;
64             pushup(res, left, right);
65             return res;
66         }
67     }
68 }

```

### 7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```

1 struct Node

```

```

2 { int l, r; LL sum, add; } tr[N * 4];
3 void pushup(int u)
4 { tr[u].sum = tr[u << 1].sum + tr[u << 1 |
  1].sum; }
5 void pushdown(int u)
6 {
7     auto &root = tr[u],
8         &left = tr[u << 1],
9         &right = tr[u << 1 | 1];
10    if (root.add)
11    {
12        left.add += root.add,
13        left.sum += (LL)(left.r - left.l +
14        1) * root.add;
15        right.add += root.add,
16        right.sum += (LL)(right.r - right.
17        l + 1) * root.add;
18        root.add = 0;
19    }
20    void build(int u, int l, int r)
21    {
22        if (l == r) tr[u] = {l, r, w[r], 0};
23        else
24        {
25            tr[u] = {l, r};
26            int mid = l + r >> 1;
27            build(u << 1, l, mid);
28            build(u << 1 | 1, mid + 1, r);
29            pushup(u);
30        }
31    }
32    void modify(int u, int l, int r, int d)
33    {
34        if (tr[u].l >= l && tr[u].r <= r)
35        {
36            tr[u].sum += (LL)(tr[u].r - tr[u].
37            l + 1) * d;
38            tr[u].add += d;
39        }
40        else
41        {
42            pushdown(u);
43            int mid = tr[u].l + tr[u].r >> 1;
44            if (l <= mid)
45                modify(u << 1, l, r, d);
46            if (r > mid)
47                modify(u << 1 | 1, l, r, d);
48            pushup(u);
49        }
50    }
51    LL query(int u, int l, int r)
52    {
53        if (tr[u].l >= l && tr[u].r <= r)
54            return tr[u].sum;
55        pushdown(u);
56        int mid = tr[u].l + tr[u].r >> 1;
57        LL sum = 0;
58        if (l <= mid)
59            sum += query(u << 1, l, r);
60        if (r > mid)
61            sum += query(u << 1 | 1, l, r);
62        return sum;
63    }

```

## 7.3 Persistent Data Structure

### 7.3.1 Persistent Trie

```

1 const int N = 600010, M = N * 25;
2 int n, m, s[N], root[N], idx;
3 int trie[M][2], max_id[M];
4 void insert(int i, int k, int p, int q)
5 {
6     if (k < 0)
7     {
8         max_id[q] = i;
9         return;
10    }
11    int v = s[i] >> k & 1;
12    if (p)
13        trie[q][v ^ 1] = trie[p][v ^ 1];
14    trie[q][v] = ++idx;
15    insert(i, k - 1, trie[p][v], trie[q][v
16    ]);
17    max_id[q] = max(max_id[trie[q][0]],
18    max_id[trie[q][1]]);
19 }
20 int query(int root, int C, int L)
21 {
22     int p = root;
23     for (int i = 23; i >= 0; i--)
24     {
25         int v = C >> i & 1;
26         if (max_id[trie[p][v ^ 1]] >= L)
27             p = trie[p][v ^ 1];
28         else
29             p = trie[p][v];
30     }
31     return C ^ s[max_id[p]];
32 }
33 // insert(i, 23, root[i - 1], root[i]);
34 // query(root[r - 1], l - 1, x ^ s[n]);

```

### 7.3.2 Persistent Segment Tree

```

1 const int N = 100010, M = 10010;
2 int n, m, a[N], root[N], idx;
3 vector<int> nums;
4 struct Node
5 {
6     int l, r;
7     int cnt;
8 } tr[N * 4 + N * 17];
9 int find(int x)
10 {
11     return lower_bound(nums.begin(), nums.
12     end(), x) - nums.begin();
13 }
14 int build(int l, int r)
15 {
16     int p = ++idx;
17     if (l == r)
18         return p;
19     int mid = l + r >> 1;
20     tr[p].l = build(l, mid), tr[p].r =
21     build(mid + 1, r);
22     return p;
23 }

```

```

21 }
22 int insert(int p, int l, int r, int x)
23 {
24     int q = ++idx;
25     tr[q] = tr[p];
26     if (l == r)
27     {
28         tr[q].cnt++;
29         return q;
30     }
31     int mid = l + r >> 1;
32     if (x <= mid)
33         tr[q].l = insert(tr[p].l, l, mid,
34             x);
35     else
36         tr[q].r = insert(tr[p].r, mid + 1,
37             r, x);
38     tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].cnt;
39     return q;
40 }
41 int query(int q, int p, int l, int r, int k)
42 {
43     if (l == r)
44         return r;
45     int cnt = tr[tr[q].l].cnt - tr[tr[p].l].cnt;
46     int mid = l + r >> 1;
47     if (k <= cnt)
48         return query(tr[q].l, tr[p].l, l, mid, k);
49     else
50         return query(tr[q].r, tr[p].r, mid + 1, r, k - cnt);
51 }

```

## 7.4 Treap

```

1  const int N = 100010, INF = 1e8;
2  int n, root, idx;
3  struct Node
4  { int l, r, key, val, cnt, size; } tr[N];
5  void pushup(int p)
6  {
7      tr[p].size = tr[tr[p].l].size +
8          tr[tr[p].r].size + tr[p].cnt;
9  }
10 int get_node(int key)
11 {
12     tr[++idx].key = key;
13     tr[idx].val = rand();
14     tr[idx].cnt = tr[idx].size = 1;
15     return idx;
16 }
17 void zig(int &p)
18 {
19     int q = tr[p].l;
20     tr[p].l = tr[q].r, tr[q].r = p, p = q;
21     pushup(tr[p].r), pushup(p);
22 }
23 void zag(int &p)

```

```

24 {
25     int q = tr[p].r;
26     tr[p].r = tr[q].l, tr[q].l = p, p = q;
27     pushup(tr[p].l), pushup(p);
28 }
29 void build()
30 {
31     get_node(-INF), get_node(INF);
32     root = 1, tr[1].r = 2;
33     pushup(root);
34     if (tr[1].val < tr[2].val) zag(root);
35 }
36 void insert(int &p, int key)
37 {
38     if (!p) p = get_node(key);
39     else if (tr[p].key == key) tr[p].cnt++;
40     else if (tr[p].key > key)
41     {
42         insert(tr[p].l, key);
43         if (tr[tr[p].l].val > tr[p].val)
44             zig(p);
45     }
46     else
47     {
48         insert(tr[p].r, key);
49         if (tr[tr[p].r].val > tr[p].val)
50             zag(p);
51     }
52     pushup(p);
53 }
54 void remove(int &p, int key)
55 {
56     if (!p) return;
57     if (tr[p].key == key)
58     {
59         if (tr[p].cnt > 1) tr[p].cnt--;
60         else if (tr[p].l || tr[p].r)
61         {
62             if (!tr[p].r || tr[tr[p].l].val > tr[tr[p].r].val)
63             {
64                 zig(p);
65                 remove(tr[p].r, key);
66             }
67             else
68             {
69                 zag(p);
70                 remove(tr[p].l, key);
71             }
72         }
73         else p = 0;
74     }
75     else if (tr[p].key > key)
76         remove(tr[p].l, key);
77     else remove(tr[p].r, key);
78     pushup(p);
79 }
80 int get_rank_by_key(int p, int key)
81 {
82     if (!p) return 0;
83     if (tr[p].key == key)
84         return tr[tr[p].l].size + 1;
85     if (tr[p].key > key)
86         return get_rank_by_key(tr[p].l, key);

```



```

87     return tr[tr[p].l].size + tr[p].cnt +
        get_rank_by_key(tr[p].r, key);
88 }
89 int get_key_by_rank(int p, int rank)
90 {
91     if (!p) reutrn INF;
92     if (tr[tr[p].l].size >= rank)
93         reutrn get_key_by_rank(tr[p].l,
            rank);
94     if (tr[tr[p].l].size + tr[p].cnt >=
        rank)
95         reutrn tr[p].key;
96     return get_key_by_rank(tr[p].r, rank -
        tr[tr[p].l].size - tr[p].cnt);
97 }
98 int get_prev(int p, int key)
99 {
100     if (!p) return -INF;
101     if (tr[p].key >= key)
102         reutrn get_prev(tr[p].l, key);
103     return max(tr[p].key, get_prev(tr[p].r
        , key));
104 }
105 int get_next(int p, int key)
106 {
107     if (!p) reutrn INF;
108     if (tr[p].key <= key)
109         return get_next(tr[p].r, key);
110     return min(tr[p].key, get_next(tr[p].l
        , key));
111 }

```

```

29         {
30             ne[p] = tr[ne[t]][i];
31             q[++tt] = p;
32         }
33     }
34 }
35 }

```

## 7.5 AC Automaton

```

1  const int N = 10010, M = 1000010, S = 55;
2  int n, tr[N * S][26], cnt[N * S], idx;
3  int q[N * S], ne[N * S];
4  char str[M];
5  void insert()
6  {
7      int p = 0;
8      for (int i = 0; str[i]; i++)
9      {
10         int t = str[i] - 'a';
11         if (!tr[p][t]) tr[p][t] = ++idx;
12         p = tr[p][t];
13     }
14     cnt[p]++;
15 }
16 void build()
17 {
18     int hh = 0, tt = -1;
19     for (int i = 0; i < 26; i++)
20         if (tr[0][i]) q[++tt] = tr[0][i];
21     while (hh <= tt)
22     {
23         int t = q[hh++];
24         for (int i = 0; i < 26; i++)
25         {
26             int p = tr[t][i];
27             if (!p) tr[t][i] = tr[ne[t]][i];
28             else

```

## 8 ★ Advanced Search

### 8.1 Flood-Fill

```
1  const int N = 1010, M = N * N;
2  int n, m;
3  char g[N][N];
4  PII q[M];
5  bool st[N][N];
6  void bfs(int sx, int sy)
7  {
8      int hh = 0, tt = 0;
9      q[0] = {sx, sy}; st[sx][sy] = true;
10     while (hh <= tt)
11     {
12         PII t = q[hh++];
13         for (int i = t.first - 1; i <= t.first + 1; i++)
14             for (int j = t.second - 1; j <= t.second + 1; j++)
15             {
16                 if (i == t.first && j == t.second)
17                     continue;
18                 if (i < 0 || i >= n || j < 0 || j >= m)
19                     continue;
20                 if (g[i][j] == '.' || st[i][j])
21                     continue;
22                 q[++tt] = {i, j};
23                 st[i][j] = true;
24             }
25     }
26 }
27 int main()
28 {
29     int cnt = 0;
30     for (int i = 0; i < n; i++)
31         for (int j = 0; j < m; j++)
32             if (g[i][j] == 'W' && !st[i][j])
33                 { bfs(i, j); cnt++; }
34 }
```

### 8.2 Multi-source BFS

```
1  const int N = 1010, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  PII q[M];
5  int dx[4] = {-1, 0, 1, 0},
6      dy[4] = {0, 1, 0, -1};
7  void bfs()
8  {
9      memset(dist, -1, sizeof dist);
10     int hh = 0, tt = -1;
11     for (int i = 1; i <= n; i++)
12         for (int j = 1; j <= m; j++)
13             if (g[i][j] == '1')
14                 {
```

```
15                 dist[i][j] = 0;
16                 q[++tt] = {i, j};
17             }
18     while (hh <= tt)
19     {
20         auto t = q[hh++];
21         for (int i = 0; i < 4; i++)
22         {
23             int a = t.x + dx[i], b = t.y + dy[i];
24             if (a < 1 || a > n || b < 1 || b > m) continue;
25             if (dist[a][b] != -1) continue;
26             dist[a][b] = dist[t.x][t.y] + 1;
27             q[++tt] = {a, b};
28         }
29     }
30 }
```

### 8.3 BFS with Deque

```
1  const int N = 510, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  bool st[N][N];
5  int dx[4] = {-1, -1, 1, 1},
6      dy[4] = {-1, 1, 1, -1},
7      ix[4] = {-1, -1, 0, 0},
8      iy[4] = {-1, 0, 0, -1};
9  int bfs()
10 {
11     memset(dist, 0x3f, sizeof dist);
12     memset(st, 0, sizeof st);
13     dist[0][0] = 0;
14     deque<PII> q;
15     q.push_back({0, 0});
16     char cs[] = "\\/\\";
17     while (q.size())
18     {
19         PII t = q.front();
20         q.pop_front();
21         if (st[t.x][t.y]) continue;
22         st[t.x][t.y] = true;
23         for (int i = 0; i < 4; i++)
24         {
25             int a = t.x + dx[i], b = t.y + dy[i];
26             if (a < 0 || a > n || b < 0 || b > m) continue;
27             int ca = t.x + ix[i], cb = t.y + iy[i];
28             int d = dist[t.x][t.y] + (g[ca][cb] != cs[i]);
29             if (d < dist[a][b])
30             {
31                 dist[a][b] = d;
32                 if (g[ca][cb] != cs[i])
33                     q.push_back({a, b});
34                 else
35                     q.push_front({a, b});
36             }
37         }
```

```

38     }
39 }
40 return dist[n][m];
41 }

```

## 8.4 Bidirectional BFS

```

1 int bfs()
2 {
3     if (A == B) return 0;
4     queue<string> qa, qb;
5     unordered_map<string, int> da, db;
6     qa.push(A), qb.push(B);
7     da[A] = db[B] = 0;
8     int step = 0;
9     while (qa.size() && qb.size())
10    {
11        int t;
12        if (qa.size() < qb.size())
13            // PROCESS
14        else
15            // PROCESS
16        if (t <= 10) return t;
17        if (++step == 10) return -1;
18    }
19    return -1;
20 }

```

## 8.5 A\*

```

1 const int N = 1010, M = 200010;
2 int n, m, S, T, K;
3 int h[N], rh[N], e[M], w[M], ne[M], idx;
4 int dist[N], cnt[N];
5 bool st[N];
6 void dijkstra()
7 {
8     priority_queue<PII, vector<PII>,
9     greater<PII>> heap;
10    heap.push({0, T});
11    memset(dist, 0x3f, sizeof dist);
12    dist[T] = 0;
13    while (heap.size())
14    {
15        auto t = heap.top();
16        heap.pop();
17        int ver = t.y;
18        if (st[ver]) continue;
19        st[ver] = true;
20        for (int i = rh[ver]; ~i; i = ne[i])
21        {
22            int j = e[i];
23            if (dist[j] > dist[ver] + w[i])
24            {
25                dist[j] = dist[ver] + w[i];
26            }
27        }
28    }
29 }

```

```

27     }
28 }
29 }
30
31 int astar()
32 {
33     priority_queue<PIII, vector<PIII>,
34     greater<PIII>> heap;
35     heap.push({dist[S], {0, S}});
36     while (heap.size())
37     {
38         auto t = heap.top();
39         heap.pop();
40         int ver = t.y.y, distance = t.y.x;
41         cnt[ver]++;
42         if (cnt[T] == K) return distance;
43         for (int i = h[ver]; ~i; i = ne[i])
44         {
45             int j = e[i];
46             if (cnt[j] < K)
47                 heap.push({distance + w[i]
48                 + dist[j], {distance + w[i], j}});
49         }
50     }
51     return -1;
52 }
53 int main()
54 {
55     // PROCESS
56     dijkstra(); cout << astar();
57     // PROCESS
58 }

```

## 8.6 DFS Connectivity Model

```

1 char g[N][N];
2 int xa, ya, xb, yb;
3 int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
4 bool st[N][N];
5 bool dfs(int x, int y)
6 {
7     if (g[x][y] == '#') return false;
8     if (x == xb && y == yb) return true;
9     st[x][y] = true;
10    for (int i = 0; i < 4; i++)
11    {
12        int a = x + dx[i], b = y + dy[i];
13        if (a < 0 || a >= n || b < 0 || b
14        >= n) continue;
15        if (st[a][b]) continue;
16        if (dfs(a, b)) return true;
17    }
18    return false;
19 }

```

## 8.7 Iterative Deepening

```

1 const int N = 110;

```

```

2  int n, path[N];
3  bool dfs(int u, int k)
4  {
5      if (u == k)
6          return path[u - 1] == n;
7      bool st[N] = {0};
8      for (int i = u - 1; i >= 0; i--)
9          for (int j = i; j >= 0; j--)
10             {
11                 int s = path[i] + path[j];
12                 if (s > n || s <= path[u - 1]
13                     || st[s]) continue;
14                 st[s] = true;
15                 path[u] = s;
16                 if (dfs(u + 1, k)) return true
17             };
18     return false;
19 }

```

## 8.8 Bidirectional DFS

```

1  const int N = 1 << 24;
2  int n, m, k, cnt = 0, ans;
3  int g[50], weights[N];
4  void dfs(int u, int s)
5  {
6      if (u == k)
7      {
8          weights[cnt++] = s;
9          return;
10     }
11     if ((LL)s + g[u] <= m)
12         dfs(u + 1, s + g[u]);
13     dfs(u + 1, s);
14 }
15 void dfs2(int u, int s)
16 {
17     if (u == n)
18     {
19         int l = 0, r = cnt - 1;
20         while (l < r)
21         {
22             int mid = l + r + 1 >> 1;
23             if (weights[mid] + (LL)s <= m)
24                 l = mid;
25             else r = mid - 1;
26         }
27         if (weights[l] + (LL)s <= m)
28             ans = max(ans, weights[l] + s);
29         return;
30     }
31     if ((LL)s + g[u] <= m)
32         dfs2(u + 1, s + g[u]);
33     dfs2(u + 1, s);
34 }

```

## 8.9 IDA\*

```

1  const int N = 1e2;
2  int n, a[N];
3  string t;
4  int f()
5  {
6      int tot = 0;
7      for (int i = 1; i < n; i++)
8          if (t[i - 1] != t[i] - 1)
9              tot++;
10     return (tot + 2) / 3;
11 }
12 bool IDAstar(int u, int maxn)
13 {
14     if (f() > maxn - u) return false;
15     if (u == maxn) return true;
16     string temp = t;
17     for (int len = 1; len <= n - 1; len++)
18     {
19         for (int l = 0; l <= n - len; l++)
20         {
21             int r = l + len - 1;
22             string substr = temp.substr(l,
23                 r - l + 1);
24             for (int k = r + 1; k < n; k
25                 ++)
26             {
27                 t.insert(k + 1, substr);
28                 t.erase(l, r - l + 1);
29                 if (IDAstar(u + 1, maxn))
30                     return true;
31                 t = temp;
32             }
33         }
34     }
35     return false;
36 }

```

## 9 ★ Advanced Graph Theory

### 9.1 Detecting Negative Cycles

```
1  int n, m1, m2;
2  int h[N], e[M], w[M], ne[M], idx;
3  int dist[N], q[N], cnt[N];
4  bool st[N];
5  bool spfa()
6  {
7      memset(dist, 0, sizeof dist);
8      memset(cnt, 0, sizeof cnt);
9      memset(st, 0, sizeof st);
10     int hh = 0, tt = 0;
11     for (int i = 1; i <= n; i++)
12     { q[tt++] = i; st[i] = true; }
13     while (hh != tt)
14     {
15         int t = q[hh++];
16         if (hh == N) hh = 0;
17         st[t] = false;
18         for (int i = h[t]; ~i; i = ne[i])
19         {
20             int j = e[i];
21             if (dist[j] > dist[t] + w[i])
22             {
23                 dist[j] = dist[t] + w[i];
24                 cnt[j] = cnt[t] + 1;
25                 if (cnt[j] >= n)
26                     return true;
27                 if (!st[j])
28                 {
29                     q[tt++] = j;
30                     if (tt == N) tt = 0;
31                     st[j] = true;
32                 }
33             }
34         }
35     }
36     return false;
37 }
```

### 9.2 SPFA-SLF

Using deque to solve SPFA question.

```
1  void spfa()
2  {
3      memset(dist, 0x3f, sizeof dist);
4      memset(st, 0, sizeof st);
5      deque<int> q;
6      q.push_back(s);
7      st[s] = 1, dist[s] = 0;
8      while (q.size())
9      {
10         int t = q.front();
11         q.pop_front();
12         st[t] = 0;
13         for (int i = h[t]; ~i; i = ne[i])
14         {
15             int j = e[i];
16             if (dist[j] > dist[t] + w[i])
```

```
17         {
18             dist[j] = dist[t] + w[i];
19             if (!st[j])
20             {
21                 st[j] = true;
22                 if (q.size() && dist[j]
23                     ] < dist[q.front()])
24                     q.push_front(j);
25                 else q.push_back(j);
26             }
27         }
28     }
29 }
```

### 9.3 SPFA-Stack

```
1  bool spfa()
2  {
3      int hh = 0, tt = 1;
4      memset(dist, -0x3f, sizeof dist);
5      dist[0] = 0; q[0] = 0;
6      while (hh != tt)
7      {
8          int t = q[--tt];
9          st[t] = false;
10         for(int i = h[t]; ~i; i = ne[i])
11         {
12             int j = e[i];
13             if (dist[j] < dist[t] + w[i])
14             {
15                 dist[j] = dist[t] + w[i];
16                 cnt[j] = cnt[t] + 1;
17                 if (cnt[j] >= n + 1)
18                     return true;
19                 if (!st[j])
20                 {
21                     st[j] = true; q[tt++]
22                     = j;
23                 }
24             }
25         }
26     }
```

### 9.4 SPFA & MIN & MAX

Using SPFA to maintain the minimum and maximum. In this case we need **Original Graph** and **Reverse Graph**, in which we can use `type == 0` or `type == 1` to describe.

```
1  void spfa(int h[], int dist[], int type)
2  {
3      int hh = 0, tt = 1;
4      if (type == 0)
5      {
6          memset(dist, 0x3f, sizeof dmin);
7          dist[1] = w[1]; q[0] = 1;
8      }
```

```

9     else
10    {
11        memset(dist, -0x3f, sizeof dmax);
12        dist[n] = w[n]; q[0] = n;
13    }
14    while (hh != tt)
15    {
16        int t = q[hh++];
17        if (hh == N) hh = 0;
18        st[t] = false;
19        for (int i = h[t]; ~i; i = ne[i])
20        {
21            int j = e[i];
22            if (type == 0 && dist[j] > min
23                (dist[t], w[j]) || type == 1 && dist[j]
24                < max(dist[t], w[j]))
25            {
26                if (type == 0)
27                    dist[j] = min(dist[t],
28                        w[j]);
29                else
30                    dist[j] = max(dist[t],
31                        w[j]);
32                if (!st[j])
33                {
34                    q[tt++] = j;
35                    if (tt == N) tt = 0;
36                    st[j] = true;
37                }
38            }
39        }
40    }

```

## 9.5 Second Shortest Path

```

1  const int N = 1010, M = 20010;
2  struct Ver
3  {
4      int id, type, dist;
5      bool operator>(const Ver &W) const
6      { return dist > W.dist; }
7  };
8  int n, m, S, T, dist[N][2], cnt[N][2];
9  int h[N], e[M], w[M], ne[M], idx;
10 bool st[N][2];
11 void add(int a, int b, int c)
12 {
13     e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++;
14 }
15 int dijkstra()
16 {
17     memset(st, 0, sizeof st);
18     memset(dist, 0x3f, sizeof dist);
19     memset(cnt, 0, sizeof cnt);
20     dist[S][0] = 0, cnt[S][0] = 1;
21     priority_queue<Ver, vector<Ver>,
22         greater<Ver>> heap;
23     heap.push({S, 0, 0});
24     while (heap.size())
25     {
26         Ver t = heap.top();

```

```

26         heap.pop();
27         int ver = t.id, type = t.type,
28         distance = t.dist, count = cnt[ver][
29         type];
30         if (st[ver][type]) continue;
31         st[ver][type] = true;
32         for (int i = h[ver]; ~i; i = ne[i]
33         )
34         {
35             int j = e[i];
36             if (dist[j][0] > distance + w[
37             i])
38             {
39                 dist[j][1] = dist[j][0],
40                 cnt[j][1] = cnt[j][0];
41                 heap.push({j, 1, dist[j]
42                 [1]});
43                 dist[j][0] = distance + w[
44                 i], cnt[j][0] = count;
45                 heap.push({j, 0, dist[j]
46                 [0]});
47             }
48             else if (dist[j][0] ==
49                 distance + w[i])
50                 cnt[j][0] += count;
51             else if (dist[j][1] > distance
52                 + w[i])
53             {
54                 dist[j][1] = distance + w[
55                 i], cnt[j][1] = count;
56                 heap.push({j, 1, dist[j]
57                 [1]});
58             }
59             else if (dist[j][1] ==
60                 distance + w[i])
61                 cnt[j][1] += count;
62         }
63     }
64     int res = cnt[T][0];
65     if (dist[T][0] + 1 == dist[T][1])
66         res += cnt[T][1];
67     return res;
68 }

```

## 9.6 Second Minimum Spanning Tree

### 9.6.1 brute-force

```

1  const int N = 510, M = 10010;
2  int n, m, p[N], dist1[N][N], dist2[N][N];
3  int h[N], e[N * 2], w[N * 2], ne[N * 2],
4  idx;
5  struct Edge
6  {
7      int a, b, w;
8      bool f;
9      bool operator<(const Edge &e) const
10     { return w < e.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 {

```

```

13     e[idx] = b, w[idx] = c, ne[idx] = h[a],
        h[a] = idx++;
14 }
15 int find(int x)
16 {
17     if (p[x] != x) p[x] = find(p[x]);
18     return p[x];
19 }
20 void dfs(int u, int fa, int maxd1, int
        maxd2, int d1[], int d2[])
21 {
22     d1[u] = maxd1, d2[u] = maxd2;
23     for (int i = h[u]; ~i; i = ne[i])
24     {
25         int j = e[i];
26         if (j != fa)
27         {
28             int td1 = maxd1, td2 = maxd2;
29             if (w[i] > td1)
30                 td2 = td1, td1 = w[i];
31             else if (w[i] < td1 && w[i] > td2)
32                 td2 = w[i];
33             dfs(j, u, td1, td2, d1, d2);
34         }
35     }
36 }
37 int main()
38 {
39     cin >> n >> m;
40     memset(h, -1, sizeof h);
41     for (int i = 0; i < m; i++)
42         cin >> edge[i].a >> edge[i].b >> edge[
            i].w;
43     sort(edge, edge + m);
44     for (int i = 1; i <= n; i++) p[i] = i;
45     LL sum = 0;
46     for (int i = 0; i < m; i++)
47     {
48         int a = edge[i].a, b = edge[i].b, w =
            edge[i].w;
49         int pa = find(a), pb = find(b);
50         if (pa != pb)
51         {
52             p[pa] = pb;
53             sum += w;
54             add(a, b, w), add(b, a, w);
55             edge[i].f = true;
56         }
57     }
58     for (int i = 1; i <= n; i++)
59         dfs(i, -1, -1e9, -1e9, dist1[i], dist2
            [i]);
60     LL res = 1e18;
61     for (int i = 0; i < m; i++)
62         if (!edge[i].f)
63         {
64             int a = edge[i].a, b = edge[i].b, w
                = edge[i].w;
65             LL t;
66             if (w > dist1[a][b])
67                 t = sum + w - dist1[a][b];
68             else if (w > dist2[a][b])
69                 t = sum + w - dist2[a][b];
70             res = min(res, t);
71         }
72 }

```

## 9.6.2 LCA

```

1  const int N = 100010, M = 300010;
2  int n, m, p[N], q[N];
3  int h[N], e[M], w[M], ne[M], idx;
4  int depth[N], fa[N][17], d1[N][17], d2[N]
        [17];
5  struct Edge
6  {
7      int a, b, w;
8      bool used;
9      bool operator<(const Edge &t) const
10     { return w < t.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 {
14     e[idx] = b, w[idx] = c, ne[idx] = h[a],
        h[a] = idx++;
15 }
16 int find(int x)
17 {
18     if (p[x] != x) p[x] = find(p[x]);
19     return p[x];
20 }
21 LL kruskal()
22 {
23     for (int i = 1; i <= n; i++) p[i] = i;
24     sort(edge, edge + m);
25     LL res = 0;
26     for (int i = 0; i < m; i++)
27     {
28         int a = find(edge[i].a), b = find(edge
            [i].b), w = edge[i].w;
29         if (a != b)
30         {
31             p[a] = b; res += w;
32             edge[i].used = true;
33         }
34     }
35     return res;
36 }
37 void build()
38 {
39     memset(h, -1, sizeof h);
40     for (int i = 0; i < m; i++)
41         if (edge[i].used)
42         {
43             int a = edge[i].a, b = edge[i].b, w
                = edge[i].w;
44             add(a, b, w), add(b, a, w);
45         }
46 }
47 void bfs()
48 {
49     memset(depth, 0x3f, sizeof depth);
50     depth[0] = 0, depth[1] = 1, q[0] = 1;
51     int hh = 0, tt = 0;
52     while (hh <= tt)
53     {
54         int t = q[hh++];
55         for (int i = h[t]; ~i; i = ne[i])
56         {
57             int j = e[i];
58             if (depth[j] > depth[t] + 1)
59                 {

```

```

60     depth[j] = depth[t] + 1;
61     q[++tt] = j;
62     fa[j][0] = t;
63     d1[j][0] = w[i], d2[j][0] = -INF;
64     for (int k = 1; k <= 16; k++)
65     {
66         int anc = fa[j][k - 1];
67         fa[j][k] = fa[anc][k - 1];
68         int distance[4] = {d1[j][k - 1],
69                             d2[j][k - 1],
70                             d1[anc][k -
71                             1],
72                             d2[anc][k -
73                             1]};
74         d1[j][k] = d2[j][k] = -INF;
75         for (int u = 0; u < 4; u++)
76         {
77             int d = distance[u];
78             if (d > d1[j][k])
79                 d2[j][k] = d1[j][k], d1[j][k]
80                 = d;
81             else if (d != d1[j][k] && d >
82                 d2[j][k])
83                 d2[j][k] = d;
84         }
85     }
86 int lca(int a, int b, int w)
87 {
88     static int distance[N * 2];
89     int cnt = 0;
90     if (depth[a] < depth[b])
91         swap(a, b);
92     for (int k = 16; k >= 0; k--)
93         if (depth[fa[a][k]] >= depth[b])
94         {
95             distance[cnt++] = d1[a][k];
96             distance[cnt++] = d2[a][k];
97             a = fa[a][k];
98         }
99     if (a != b)
100     {
101         for (int k = 16; k >= 0; k--)
102             if (fa[a][k] != fa[b][k])
103             {
104                 distance[cnt++] = d1[a][k];
105                 distance[cnt++] = d2[a][k];
106                 distance[cnt++] = d1[b][k];
107                 distance[cnt++] = d2[b][k];
108                 a = fa[a][k], b = fa[b][k];
109             }
110         distance[cnt++] = d1[a][0];
111         distance[cnt++] = d1[b][0];
112     }
113     int dist1 = -INF, dist2 = -INF;
114     for (int i = 0; i < cnt; i++)
115     {
116         int d = distance[i];
117         if (d > dist1)
118             dist2 = dist1, dist1 = d;
119         else if (d != dist1 && d > dist2)
120             dist2 = d;
121     }

```

```

122     if (w > dist1) return w - dist1;
123     if (w > dist2) return w - dist2;
124     return INF;
125 }
126 int main()
127 {
128     cin >> n >> m;
129     for (int i = 0; i < m; i++)
130     {
131         int a, b, c;
132         cin >> a >> b >> c;
133         edge[i] = {a, b, c};
134     }
135     LL sum = kruskal();
136     build();
137     bfs();
138     LL res = 1e18;
139     for (int i = 0; i < m; i++)
140         if (!edge[i].used)
141         {
142             int a = edge[i].a, b = edge[i].b, w
143             = edge[i].w;
144             res = min(res, sum + lca(a, b, w));
145         }
146     cout << res;

```

## 9.7 Difference Constraints

- size == N: Feasible Solution
- size == 1: Maximum/Minimum
- Maximum: Shortest Path
- Minimum: Longest Path

### 9.7.1 Maximum-Shortest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, 0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] > dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n)

```



```

26         return true;
27     if (!st[j])
28     {
29         st[j] = true;
30         q[tt++] = j;
31         if (tt == N) tt = 0;
32     }
33 }
34 }
35 }
36 return false;
37 }
38 int main()
39 {
40     // add(a, b, k) means x_b <= x_a + k
41     // PROCESS
42 }

```

### 9.7.2 Minimum-Longest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, -0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] < dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n)
26                    return false;
27            }
28            if (!st[j])
29            {
30                st[j] = true;
31                q[tt++] = j;
32                if (tt == N) tt = 0;
33            }
34        }
35    }
36    return true;
37 }
38 int main()
39 {
40     // add(a, b, k) means x_a + k <= x_b
41     // PROCESS
42 }

```

## 9.8 LCA

```

1 int n, m, h[N], e[M], ne[M], idx;
2 int depth[N], fa[N][16], q[N];
3 void bfs(int root)
4 {
5     memset(depth, 0x3f, sizeof depth);
6     depth[0] = 0;
7     depth[root] = 1;
8     int hh = 0, tt = 0;
9     q[0] = root;
10    while (hh <= tt)
11    {
12        int t = q[hh++];
13        for (int i = h[t]; ~i; i = ne[i])
14        {
15            int j = e[i];
16            if (depth[j] > depth[t] + 1)
17            {
18                depth[j] = depth[t] + 1;
19                q[++tt] = j;
20                fa[j][0] = t;
21                for (int k = 1; k <= 15; k
22                    ++))
23                    fa[j][k] = fa[fa[j][k - 1]][k - 1];
24            }
25        }
26    }
27 int lca(int a, int b)
28 {
29     if (depth[a] < depth[b])
30         swap(a, b);
31     for (int k = 15; k >= 0; k--)
32         if (depth[fa[a][k]] >= depth[b])
33             a = fa[a][k];
34     if (a == b)
35         return a;
36     for (int k = 15; k >= 0; k--)
37         if (fa[a][k] != fa[b][k])
38         {
39             a = fa[a][k];
40             b = fa[b][k];
41         }
42     return fa[a][0];
43 }

```

## 9.9 SCC

```

1 void tarjan(int u)
2 {
3     dfn[u] = low[u] = ++timestamp;
4     stack[++top] = u, in_stk[u] = true;
5     for (int i = h[u]; ~i; i = ne[i])
6     {
7         int j = e[i];
8         if (!dfn[j])
9         {
10            tarjan(j);
11            low[u] = min(low[u], low[j]);
12        }

```

```

13         else if(in_stk[j])
14             low[u] = min(low[u], dfn[j]);
15     }
16     if(dfn[u] == low[u])
17     {
18         int y;
19         ++scc_cnt;
20         do
21         {
22             y = stk[top--];
23             in_stk[y] = false;
24             id[y] = scc_cnt;
25         } while(y != u);
26     }
27 }

```

## 9.10 DCC

### 9.10.1 e-DCC

```

1  const int N = 5010, M = 20010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, id[N], dcc_cnt, d[N];
5  bool is_bridge[M];
6  void tarjan(int u, int from)
7  {
8      dfn[u] = low[u] = ++timestamp;
9      stk[++top] = u;
10     for (int i = h[u]; ~i; i = ne[i])
11     {
12         int j = e[i];
13         if (!dfn[j])
14         {
15             tarjan(j, i);
16             low[u] = min(low[u], low[j]);
17             if (dfn[u] < low[j])
18                 is_bridge[i] = is_bridge[i ^ 1] =
19                 true;
20         }
21         else if (i != (from ^ 1))
22             low[u] = min(low[u], dfn[j]);
23     }
24     if (dfn[u] == low[u])
25     {
26         ++dcc_cnt;
27         int y;
28         do
29         {
30             y = stk[top--];
31             id[y] = dcc_cnt;
32         } while (y != u);
33     }

```

### 9.10.2 v-DCC

```

1  const int N = 1010, M = 1010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, dcc_cnt, root;

```

```

5  vector<int> dcc[N];
6  bool cut[N];
7  void init()
8  {
9      for (int i = 1; i <= dcc_cnt; i++)
10         dcc[i].clear();
11     idx = n = timestamp = top = dcc_cnt = 0;
12     memset(h, -1, sizeof h);
13     memset(dfn, 0, sizeof dfn);
14     memset(cut, 0, sizeof cut);
15 }
16 void tarjan(int u)
17 {
18     dfn[u] = low[u] = ++timestamp;
19     stk[++top] = u;
20     if (u == root && h[u] == -1)
21     {
22         dcc_cnt++;
23         dcc[dcc_cnt].push_back(u);
24         return;
25     }
26     int cnt = 0;
27     for (int i = h[u]; ~i; i = ne[i])
28     {
29         int j = e[i];
30         if (!dfn[j])
31         {
32             tarjan(j);
33             low[u] = min(low[u], low[j]);
34             if (dfn[u] <= low[j])
35             {
36                 cnt++;
37                 if (u != root || cnt > 1)
38                     cut[u] = true;
39                 ++dcc_cnt;
40                 int y;
41                 do
42                 {
43                     y = stk[top--];
44                     dcc[dcc_cnt].push_back(y);
45                 } while (y != j);
46                 dcc[dcc_cnt].push_back(u);
47             }
48         }
49         else
50             low[u] = min(low[u], dfn[j]);
51     }
52 }

```

## 9.11 Bipartite Graph

The maximum matching  
(by the Hungarian algorithm) =  
the minimum vertex cover =  
total number of vertices -  
maximum independent set =  
total number of vertices -  
minimum path cover.

### 9.11.1 maximum matching

```

1  const int N = 110;

```

```

2  int n, m;
3  int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
4  PII match[N][N];
5  bool g[N][N], st[N][N];
6  bool find(int x, int y)
7  {
8      for (int i = 0; i < 4; i++)
9      {
10         int a = x + dx[i], b = y + dy[i];
11         if (a && a <= n && b && b <= n &&
12             !g[a][b] && !st[a][b])
13             {
14                 st[a][b] = true;
15                 PII t = match[a][b];
16                 if (t.x == -1 || find(t.x, t.y))
17                     {
18                         match[a][b] = {x, y};
19                         return true;
20                     }
21             }
22         return false;
23     }
24 int main()
25 {
26     // PROCESS
27     memset(match, -1, sizeof match);
28     int res = 0;
29     for (int i = 1; i <= n; i++)
30         for (int j = 1; j <= n; j++)
31             if ((i + j) % 2 && !g[i][j])
32                 {
33                     memset(st, 0, sizeof st);
34                     if (find(i, j)) res++;
35                 }
36     // PROCESS
37 }

```

### 9.11.2 minimum vertex cover

```

1  const int N = 110;
2  int n, m, k, match[N];
3  bool g[N][N], st[N];
4  bool find(int x)
5  {
6      for (int i = 0; i < m; i++)
7          if (!st[i] && g[x][i])
8              {
9                  st[i] = true;
10                 if (match[i] == -1 || find(
11                     match[i]))
12                     {
13                         match[i] = x;
14                         return true;
15                     }
16             }
17     return false;
18 int main()
19 {
20     while (cin >> n, n)
21     {

```

```

22         cin >> m >> k;
23         memset(g, 0, sizeof g);
24         memset(match, -1, sizeof match);
25         while (k--)
26             {
27                 int t, a, b;
28                 cin >> t >> a >> b;
29                 if (!a || !b) continue;
30                 g[a][b] = true;
31             }
32         int res = 0;
33         for (int i = 0; i < n; i++)
34             {
35                 memset(st, 0, sizeof st);
36                 if (find(i)) res++;
37             }
38         cout << res << '\n';
39     }
40 }

```

### 9.11.3 maximum independent set

```

1  const int N = 110;
2  int n, m, k;
3  PII match[N][N];
4  bool g[N][N], st[N][N];
5  int dx[8] = {-2, -1, 1, 2, 2, 1, -1, -2};
6  int dy[8] = {1, 2, 2, 1, -1, -2, -2, -1};
7  bool find(int x, int y)
8  {
9      for (int i = 0; i < 8; i++)
10         {
11             int a = x + dx[i], b = y + dy[i];
12             if (a < 1 || a > n || b < 1 || b >
13                 m)
14                 continue;
15             if (g[a][b]) continue;
16             if (st[a][b]) continue;
17             st[a][b] = true;
18             PII t = match[a][b];
19             if (t.x == 0 || find(t.x, t.y))
20                 {
21                     match[a][b] = {x, y};
22                     return true;
23                 }
24         }
25     return false;
26 int main()
27 {
28     // PROCESS
29     int res = 0;
30     for (int i = 1; i <= n; i++)
31         for (int j = 1; j <= m; j++)
32             {
33                 if (g[i][j] || (i + j) % 2)
34                     continue;
35                 memset(st, 0, sizeof st);
36                 if (find(i, j)) res++;
37             }
38     cout << n * m - k - res << '\n';
39 }

```

### 9.11.4 minimum path cover

```
1  const int N = 210, M = 30010;
2  int n, m, match[N];
3  bool d[N][N], st[N];
4  bool find(int x)
5  {
6      for (int i = 1; i <= n; i++)
7          if (d[x][i] && !st[i])
8          {
9              st[i] = true;
10             int t = match[i];
11             if (t == 0 || find(t))
12             {
13                 match[i] = x;
14                 return true;
15             }
16         }
17     return false;
18 }
19 int main()
20 {
21     // 传递闭包
22     for (int k = 1; k <= n; k++)
23         for (int i = 1; i <= n; i++)
24             for (int j = 1; j <= n; j++)
25                 d[i][j] |= d[i][k] & d[k][j];
26     int res = 0;
27     for (int i = 1; i <= n; i++)
28     {
29         memset(st, 0, sizeof st);
30         if (find(i)) res++;
31     }
32     cout << n - res;
33 }
```

## 9.12 Eulerian Circuit & Eulerian Path

### 9.12.1 Eulerian Circuit

```
1  int type, n, m;
2  int h[N], e[M], ne[M], idx;
3  bool used[M];
4  int ans[M], cn, din[N], dout[N];
5  void add(int a, int b)
6  {
7      e[idx] = b, ne[idx] = h[a], h[a] = idx++;
8  }
9  void dfs(int u)
10 {
11     for (int &i = h[u]; ~i;)
12     {
13         if (used[i])
14             { i = ne[i]; continue; }
15         used[i] = true;
16         if (type == 1) used[i ^ 1] = true;
17         int t;
18         if (type == 1)
19         {
```

```
20             t = i / 2 + 1;
21             if (i & 1) t = -t;
22         }
23         else t = i + 1;
24         int j = e[i];
25         i = ne[i];
26         dfs(j);
27         ans[++cnt] = t;
28     }
29 }
30 int main()
31 {
32     cin >> type >> n >> m;
33     memset(h, -1, sizeof h);
34     for (int i = 0; i < m; i++)
35     {
36         int a, b;
37         cin >> a >> b;
38         add(a, b);
39         if (type == 1) add(b, a);
40         din[b]++, dout[a]++;
41     }
42     if (type == 1)
43     {
44         for (int i = 1; i <= n; i++)
45             if (din[i] + dout[i] & 1)
46             {
47                 cout << "NO\n";
48                 return 0;
49             }
50     }
51     else
52     {
53         for (int i = 1; i <= n; i++)
54             if (din[i] != dout[i])
55             {
56                 cout << "NO\n";
57                 return 0;
58             }
59     }
60     for (int i = 1; i <= n; i++)
61         if (h[i] != -1) { dfs(i); break; }
62 }
```

### 9.12.2 Eulerian Path

```
1  const int N = 510;
2  int n = 500, m, g[N][N];
3  int ans[1100], cnt, d[N];
4  void dfs(int u)
5  {
6      for (int i = 1; i <= n; i++)
7          if (g[u][i])
8          {
9              g[u][i]--, g[i][u]--;
10             dfs(i);
11         }
12     ans[++cnt] = u;
13 }
14 int main()
15 {
16     cin >> m;
17     while (m--)
18     {
```

```
19     int a, b;
20     cin >> a >> b;
21     g[a][b]++, g[b][a]++;
22     d[a]++, d[b]++;
23 }
24 int start = 1;
25 while (!d[start]) ++start;
26 for (int i = 1; i <= 500; i++)
27     if (d[i] % 2)
28         { start = i; break; }
29 dfs(start);
30 }
```

## 10 ★ Advanced Math

### 10.1 Euler's Totient Function

#### 10.1.1 GCD

```
1  const int N = 1e7 + 10;
2  int primes[N], cnt, phi[N];
3  bool st[N];
4  LL s[N];
5  void init(int n)
6  {
7      for (int i = 2; i <= n; i++)
8      {
9          if (!st[i])
10             {
11                 primes[cnt++] = i;
12                 phi[i] = i - 1;
13             }
14             for (int j = 0; primes[j] * i <= n; j++)
15             {
16                 st[primes[j] * i] = true;
17                 if (i % primes[j] == 0)
18                 {
19                     phi[i * primes[j]] = phi[i] * primes[j];
20                     break;
21                 }
22                 phi[i * primes[j]] = phi[i] * (primes[j] - 1);
23             }
24             }
25     for (int i = 1; i <= n; i++)
26         s[i] = s[i - 1] + phi[i];
27 }
28 int main()
29 {
30     int n; cin >> n;
31     init(n);
32     LL res = 0;
33     for (int i = 0; i < cnt; i++)
34     {
35         int p = primes[i];
36         res += s[n / p] * 2 + 1;
37     }
38 }
```

```
11 void mul(int c[][N], int a[][N], int b[][N])
12 {
13     int temp[N][N] = {0};
14     for (int i = 0; i < N; i++)
15         for (int j = 0; j < N; j++)
16             for (int k = 0; k < N; k++)
17                 temp[i][j] = (temp[i][j] +
18                     (LL)a[i][k] * b[k][j]) % m;
19     memcpy(c, temp, sizeof temp);
20 }
21 int main()
22 {
23     while (n)
24     {
25         if (n & 1) mul(f1, f1, a);
26         mul(a, a, a); n >>= 1;
27     }
```

### 10.2 Matrix Multiplication

```
1  const int N = 3;
2  int n, m;
3  void mul(int c[], int a[], int b[][N])
4  {
5      int temp[N] = {0};
6      for (int i = 0; i < N; i++)
7          for (int j = 0; j < N; j++)
8              temp[i] = (temp[i] + (LL)a[j]
9                  * b[j][i]) % m;
10     memcpy(c, temp, sizeof temp);
11 }
```

## 11 ★ Advanced DP

### 11.1 Advanced LIS

#### 11.1.1 MSIS

MSIS means Maximum Sum Increasing Subsequence

```
1  const int N = 1010;
2  int n, w[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 0; i < n; i++) cin >> w[i];
7      int res = 0;
8      for (int i = 0; i < n; i++)
9      {
10         f[i] = w[i];
11         for (int j = 0; j < i; j++)
12             if (w[i] > w[j])
13                 f[i] = max(f[i], f[j] + w[i]);
14         res = max(res, f[i]);
15     }
16     cout << res;
17 }
```

#### 11.1.2 LCIS

LCIS means Longest Common Increasing Subsequence

```
1  const int N = 3010;
2  int n, a[N], b[N], f[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          cin >> b[i];
10     for (int i = 1; i <= n; i++)
11     {
12         int maxv = 1;
13         for (int j = 1; j <= n; j++)
14         {
15             f[i][j] = f[i - 1][j];
16             if (a[i] == b[j])
17                 f[i][j] = max(f[i][j],
18                                 maxv);
19             if (a[i] > b[j])
20                 maxv = max(maxv, f[i - 1][j] + 1);
21         }
22         int res = 0;
23         for (int i = 1; i <= n; i++)
24             res = max(res, f[n][i]);
25         cout << res;
26     }
```

## 11.2 Knapsack Problem

### 11.2.1 Multiple Knapsack Problem

```
1  const int N = 20010;
2  int n, m, f[N], g[N], q[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         memcpy(g, f, sizeof f);
11         for (int j = 0; j < v; j++)
12         {
13             int hh = 0, tt = -1;
14             for (int k = j; k <= m; k += v
15                 )
16                 {
17                     if (hh <= tt && q[hh] < k
18                         - s * v)
19                         hh++;
20                     while (hh <= tt && g[q[tt]
21                         ] - (q[tt] - j) / v * w <= g[k] - (k
22                         - j) / v * w)
23                         tt--;
24                     q[++tt] = k;
25                     f[k] = g[q[hh]] + (k - q[
26                         hh]) / v * w;
27                 }
28             }
29         }
30     }
31     cout << f[m] << '\n';
32 }
```

### 11.2.2 Two-Dimensional Cost Knapsack Problem

```
1  const int N = 110;
2  int n, V, M, f[N][N];
3  int main()
4  {
5      cin >> n >> V >> M;
6      for (int i = 0; i < n; i++)
7      {
8          int v, m, w;
9          cin >> v >> m >> w;
10         for (int j = V; j >= v; j--)
11             for (int k = M; k >= m; k--)
12                 f[j][k] = max(f[j][k], f[j
13                     - v][k - m] + w);
14     }
15     cout << f[V][M] << '\n';
16 }
```

### 11.2.3 Finding the Actual Solution Set

```
1  const int N = 1010;
2  int n, m;
```

```

3  int v[N], w[N], f[N][N];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8          cin >> v[i] >> w[i];
9      for (int i = n; i >= 1; i--)
10         for (int j = 0; j <= m; j++)
11             {
12                 f[i][j] = f[i + 1][j];
13                 if (j >= v[i])
14                     f[i][j] = max(f[i][j], f[i
+ 1][j - v[i]] + w[i]);
15             }
16         int j = m;
17         for (int i = 1; i <= n; i++)
18             if (j >= v[i] && f[i][j] == f[i +
1][j - v[i]] + w[i])
19                 {
20                     cout << i << ' ';
21                     j -= v[i];
22                 }
23 }

```

#### 11.2.4 Maximum Linearly Independent Subset

```

1  const int N = 110, M = 25010;
2  int n, v[N];
3  bool f[M];
4  int main()
5  {
6      int T; cin >> T;
7      while (T--)
8      {
9          cin >> n;
10         for (int i = 1; i <= n; ++i)
11             cin >> v[i];
12         sort(v + 1, v + n + 1);
13         int m = v[n], res = 0;
14         memset(f, 0, sizeof f);
15         f[0] = true; // 状态的初值
16         for (int i = 1; i <= n; ++i)
17             {
18                 if (f[v[i]]) continue;
19                 res++;
20                 for (int j = v[i]; j <= m; ++j
)
21                     f[j] |= f[j - v[i]];
22             }
23         cout << res << '\n';
24     }
25 }

```

#### 11.2.5 Mixed Knapsack Problem

```

1  const int N = 1010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;

```

```

6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         if (!s)
11             {
12                 for (int j = v; j <= m; j++)
13                     f[j] = max(f[j], f[j - v]
+ w);
14             }
15         else
16             {
17                 if (s == -1)
18                     s = 1;
19                 for (int k = 1; k <= s; k *=
2)
20                 {
21                     for (int j = m; j >= k * v
; j--)
22                         f[j] = max(f[j], f[j -
k * v] + k * w);
23                     s -= k;
24                 }
25                 if (s)
26                 {
27                     for (int j = m; j >= s * v
; j--)
28                         f[j] = max(f[j], f[j -
s * v] + s * w);
29                 }
30             }
31         cout << f[m] << '\n';
32     }
33 }

```

#### 11.2.6 Dependent Knapsack Problem

```

1  const int N = 110;
2  int n, m, root;
3  int h[N], e[N], ne[N], idx;
4  int v[N], w[N], f[N][N];
5  void add(int a, int b)
6  {
7      e[idx] = b, ne[idx] = h[a], h[a] = idx
++;
8  }
9  void dfs(int u)
10 {
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         int son = e[i];
14         dfs(son);
15         for (int j = m - v[u]; j >= 0; --j
)
16             for (int k = 0; k <= j; ++k)
17                 f[u][j] = max(f[u][j], f[u
][j - k] + f[son][k]);
18     }
19     for (int j = m; j >= v[u]; --j)
20         f[u][j] = f[u][j - v[u]] + w[u];
21     for (int j = 0; j < v[u]; ++j)
22         f[u][j] = 0;
23 }
24 int main()

```



```

25 {
26     memset(h, -1, sizeof h);
27     cin >> n >> m;
28     for (int i = 1; i <= n; ++i)
29     {
30         int p;
31         cin >> v[i] >> w[i] >> p;
32         if (p == -1) root = i;
33         else add(p, i);
34     }
35     dfs(root);
36     cout << f[root][m] << '\n';
37 }

```

### 11.2.7 Number of Solutions

```

1  const int N = 1010, mod = 1e9 + 7;
2  int n, m;
3  int w[N], v[N], f[N], g[N];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; ++i)
8          cin >> v[i] >> w[i];
9      g[0] = 1;
10     for (int i = 1; i <= n; ++i)
11     {
12         for (int j = m; j >= v[i]; --j)
13         {
14             int temp = max(f[j], f[j - v[i]] + w[i]), c = 0;
15             if (temp == f[j])
16                 c = (c + g[j]) % mod;
17             if (temp == f[j - v[i]] + w[i])
18                 c = (c + g[j - v[i]]) % mod;
19             f[j] = temp, g[j] = c;
20         }
21     }
22     int res = 0;
23     for (int j = 0; j <= m; ++j)
24         if (f[j] == f[m])
25             res = (res + g[j]) % mod;
26     cout << res << '\n';
27 }

```

## 11.3 FSM

```

1  const int N = 100010;
2  int n, w[N], f[N][2];
3  int main()
4  {
5      int T; cin >> T;
6      while (T--)
7      {
8          cin >> n;
9          for (int i = 1; i <= n; ++i)
10             cin >> w[i];
11         for (int i = 1; i <= n; ++i)
12         {

```

```

13             // YOUR_FSM_RULES
14             // f[i][0] =
15             // f[i][1] =
16         }
17         cout << max(f[n][0], f[n][1]) << '\n';
18     }
19 }

```

## 11.4 Digit DP

```

1  const int N = 35;
2  int l, r, k, b, a[N], al, f[N][N];
3  int dp(int pos, int st, int op)
4  {
5      if (!pos) return st == k;
6      if (!op && ~f[pos][st])
7          return f[pos][st];
8      int res = 0, maxx = op ? min(a[pos], 1) : 1;
9      for (int i = 0; i <= maxx; ++i)
10     {
11         if (st + i > k) continue;
12         res += dp(pos - 1, st + i, op && i == a[pos]);
13     }
14     return op ? res : f[pos][st] = res;
15 }
16 int calc(int x)
17 {
18     al = 0;
19     memset(f, -1, sizeof f);
20     while (x) a[++al] = x % b, x /= b;
21     return dp(al, 0, 1);
22 }
23 int main()
24 {
25     cin >> l >> r >> k >> b;
26     cout << calc(r) - calc(l - 1) << '\n';
27 }

```

## 11.5 Queue Optimization for DP

```

1  int n, m, s[300010], q[300010];
2  int main()
3  {
4      cin >> n >> m;
5      for (int i = 1; i <= n; ++i)
6          cin >> s[i], s[i] += s[i - 1];
7      int res = INT_MIN, hh = 0, tt = 0;
8      for (int i = 1; i <= n; ++i)
9      {
10         if (q[hh] < i - m) hh++;
11         res = max(res, s[i] - s[q[hh]]);
12         while (hh <= tt && s[q[tt]] >= s[i]) tt--;
13         q[++tt] = i;
14     }
15 }

```