



**国家超级计算广州中心**  
NATIONAL SUPERCOMPUTER CENTER IN GUANGZHOU

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# XCPC-Template

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# Part I: Basic Template

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## 0 ★ Preface

### 0.1 Template

```
1 #define itn int
2 #define nit int
3 #define nti int
4 #define tin int
5 #define tni int
6 #define retrun return
7 #define reutrn return
8 #define rutren return
9 #define fastin \
10     ios_base::sync_with_stdio(0); \
11     cin.tie(0), cout.tie(0);
12 #include <bits/stdc++.h>
13 using namespace std;
14 typedef long long LL;
15 typedef long double LD;
16 typedef pair<int, int> PII;
17 typedef pair<long long, long long> PLL;
18 typedef pair<double, double> PDD;
19 typedef vector<int> VI;
20 #ifndef ONLINE_JUDGE
21 #define dbg(args...) \
22     do \
23     { \
24         cout << "\033[32;1m" << #args << " -> \
25         "; \
26         err(args); \
27     } while (0)
28 #define dbg(...)
29 #endif
30 void err()
31 { cout << "\033[39;0m" << endl; }
32 template <template <typename...> class T,
33         typename t, typename... Args>
34 void err(T<t> a, Args... args)
35 {
36     for (auto x : a) cout << x << ' ';
37     err(args...);
38 }
39 template <typename T, typename... Args>
40 void err(T a, Args... args)
41 { cout << a << ' '; err(args...); }
42 const int INF = 0x3f3f3f3f;
43 const int mod = 1e9 + 7;
44 const double eps = 1e-6;
45 int main()
46 {
47     #ifndef ONLINE_JUDGE
48         freopen("test.in", "r", stdin);
49         freopen("test.out", "w", stdout);
50     #endif
51     fastin;
52     return 0;
53 }
```

### 0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

### 0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about  $10^7 \sim 10^8$  operations.
- Below is a guide on how to choose algorithms based on different input size ranges:
  1.  $n \leq 30 \rightarrow$  Exponential complexity: DFS with pruning, State Compression DP
  2.  $n \leq 100 \rightarrow O(n^3)$ : Floyd, DP, Gaussian Elimination
  3.  $n \leq 1000 \rightarrow O(n^2), O(n^2 \log n)$ : DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
  4.  $n \leq 10000 \rightarrow O(n^{\frac{3}{2}})$ : Block Linked List, Mo's Algorithm
  5.  $n \leq 100000 \rightarrow O(n \log n)$ : sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
  6.  $n \leq 1000000 \rightarrow O(n)$ , or small-constant  $O(n \log n)$ : Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
  7.  $n \leq 10000000 \rightarrow O(n)$ : Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
  8.  $n \leq 10^9 \rightarrow O(\sqrt{n})$ : Primality Testing
  9.  $n \leq 10^{18} \rightarrow O(\log n)$ : GCD, Fast Exponentiation, Digit DP
  10.  $n \leq 10^{1000} \rightarrow O((\log n)^2)$ : Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
  11.  $n \leq 10^{100000} \rightarrow O(\log k \cdot \log \log k)$ , where  $k$  is the number of digits: Big Integer Add/Subtract, FFT/NTT

## 0.4 If <bits/stdc++.h> Failed

Replace it with:

```
1  #include <algorithm>
2  #include <bitset>
3  #include <complex>
4  #include <deque>
5  #include <exception>
6  #include <fstream>
7  #include <functional>
8  #include <iomanip>
9  #include <ios>
10 #include <iosfwd>
11 #include <iostream>
12 #include <istream>
13 #include <iterator>
14 #include <limits>
15 #include <list>
16 #include <locale>
17 #include <map>
18 #include <memory>
19 #include <numeric>
20 #include <ostream>
21 #include <queue>
22 #include <set>
23 #include <sstream>
24 #include <stack>
25 #include <stdexcept>
26 #include <streambuf>
27 #include <string>
28 #include <typeinfo>
29 #include <utility>
30 #include <valarray>
31 #include <vector>
32 #include <unordered_map>
33 #include <unordered_set>
```

# 1 ★ Basic Algorithm

## 1.1 Quick Sort

Sort the given array from index 1 to n.

```
1 void quick_sort(int l, int r)
2 {
3     if (l >= r) return;
4     int x = a[(l + r) >> 1], i = l - 1, j = r
        + 1;
5     while (i < j)
6     {
7         do i++; while (a[i] < x);
8         do j--; while (a[j] > x);
9         if (i < j) swap(a[i], a[j]);
10    }
11    quick_sort(l, j);
12    quick_sort(j + 1, r);
13    return;
14 }
```

## 1.2 Binary Search

```
1 // 区间 [l, r] 被划分成 [l, mid] 和 [mid + 1,
    r] 时使用
2 // 大于等于区间的最小值, check 应为 target <=
    a[mid]
3 int bsearch_1(int l, int r)
4 {
5     while (l < r)
6     {
7         int mid = l + r >> 1;
8         if (check(mid)) r = mid;
9         else l = mid + 1;
10    }
11    return l;
12 }
13 // 区间 [l, r] 被划分成 [l, mid - 1] 和 [mid,
    r] 时使用
14 // 小于等于区间的最大值, check 应为 target >=
    a[mid]
15 int bsearch_2(int l, int r)
16 {
17     while (l < r)
18     {
19         // 为什么要 l + r + 1: 因为 l 的更新条
            件是 mid 本身
20         // 当 r == l + 1 时 mid 向下取整必定取
            l, 有可能在满足 check(mid) 时导致无限循环
21         int mid = l + r + 1 >> 1;
22         if (check(mid)) l = mid;
23         else r = mid - 1;
24    }
25    return l;
26 }
27 // 浮点数二分
28 double bsearch_3(double l, double r)
29 {
30     // eps 表示精度, 取决于题目对精度的要求
31     const double eps = 1e-6;
32     while (r - l > eps)
```

```
33 {
34     double mid = (l + r) / 2;
35     if (check(mid)) r = mid;
36     else l = mid;
37 }
38 return l;
39 }
```

## 1.3 Ternary Search

```
1 // 整数三分
2 void tsearch_1(int l, int r)
3 {
4     while (l < r)
5     {
6         int lmid = l + (r - l) / 3, rmid = r -
            (r - l) / 3;
7         lans = cal(lmid), rans = cal(rmid);
8         if (lans <= rans) r = rmid - 1;
9         else l = lmid + 1;
10        if (lans <= rans) l = lmid + 1;
11        else r = rmid - 1;
12    }
13    // 求凹函数的极小值
14    cout << min(lans, rans) << endl;
15    // 求凸函数的极大值
16    cout << max(lans, rans) << endl;
17 }
18 // 浮点数三分
19 void tsearch_2(int l, int r)
20 {
21     const double eps = 1e-6;
22     while (r - l < eps)
23     {
24         double lmid = l + (r - l) / 3;
25         double rmid = r - (r - l) / 3;
26         lans = cal(lmid), rans = cal(rmid);
27         // 求凹函数的极小值
28         if (lans <= rans) r = rmid;
29         else l = lmid;
30         // 求凸函数的极大值
31         if (lans <= rans) l = lmid;
32         else r = rmid;
33    }
34 }
```

## 1.4 High Precision

### 1.4.1 High Precision Add

```
1 string s1, s2;
2 vector<int> a, b, c;
3 void add(vector<int> &a, vector<int> &b)
4 {
5     if (a.size() < b.size())
6     { add(b, a); return; }
7     int t = 0;
8     for (int i = 0; i < a.size(); i++)
9     {
10        t += a[i];
```

```

11     if (i < b.size()) t += b[i];
12     c.push_back(t % 10);
13     t /= 10;
14 }
15 while (t)
16     c.push_back(t % 10), t /= 10;
17 }
18 int main()
19 {
20     cin >> s1 >> s2;
21     for (int i = s1.size() - 1; i >= 0; i--)
22         a.push_back(s1[i] - '0');
23     for (int i = s2.size() - 1; i >= 0; i--)
24         b.push_back(s2[i] - '0');
25     add(a, b);
26     for (int i = c.size() - 1; i >= 0; i--)
27         cout << c[i];
28     return 0;
29 }

```

```

5 {
6     for (int i = 0, t = 0; i < a.size() || t; i++)
7     {
8         if (i < a.size()) t += a[i] * b;
9         c.push_back(t % 10);
10        t /= 10;
11    }
12    while (c.size() > 1 && c.back() == 0)
13        c.pop_back();
14 }
15 int main()
16 {
17     cin >> s1 >> b;
18     for (int i = s1.size() - 1; i >= 0; i--)
19         a.push_back(s1[i] - '0');
20    mul(a, b);
21    for (int i = c.size() - 1; i >= 0; i--)
22        cout << c[i];
23    return 0;
24 }

```

## 1.4.2 High Precision Subsection

```

1 vector<int> a, b, c;
2 string s1, s2;
3 void sub(vector<int> &a, vector<int> &b)
4 {
5     int t = 0;
6     for (int i = 0; i < a.size(); i++)
7     {
8         t = a[i] - t;
9         if (i < b.size()) t -= b[i];
10        c.push_back((t + 10) % 10);
11        if (t < 0) t = 1;
12        else t = 0;
13    }
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> s2;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22     for (int i = s2.size() - 1; i >= 0; i--)
23         b.push_back(s2[i] - '0');
24     if (s1.size() < s2.size())
25         cout << '-', sub(b, a);
26     else if (s1.size() == s2.size() && s1 < s2)
27         cout << '-', sub(b, a);
28     else sub(a, b);
29     for (int i = c.size() - 1; i >= 0; i--)
30         cout << c[i];
31     return 0;
32 }

```

## 1.4.4 High Precision Divide

```

1 string s1, s2;
2 vector<int> a, c;
3 int b, r;
4 void divide(vector<int> &a, int b, int &r)
5 {
6     r = 0;
7     for (int i = a.size() - 1; i >= 0; i--)
8     {
9         r = r * 10 + a[i];
10        c.push_back(r / b);
11        r %= b;
12    }
13    reverse(c.begin(), c.end());
14    while (c.size() > 1 && c.back() == 0)
15        c.pop_back();
16 }
17 int main()
18 {
19     cin >> s1 >> b;
20     for (int i = s1.size() - 1; i >= 0; i--)
21         a.push_back(s1[i] - '0');
22    divide(a, b, r);
23    for (int i = c.size() - 1; i >= 0; i--)
24        cout << c[i];
25    cout << '\n' << r;
26    return 0;
27 }

```

## 1.4.3 High Precision Multiply

```

1 string s1, s2;
2 vector<int> a, c;
3 int b;
4 void mul(vector<int> &a, int b)

```

## 1.5 Prefix Sum & Difference Array

### 1.5.1 1D Prefix Sum

```

1 S[i] = a[1] + a[2] + ... a[i]
2 a[l] + ... + a[r] = S[r] - S[l - 1]

```



### 1.5.2 2D Prefix Sum

```
1 // S[i, j] = i 行 j 列左上部分所有元素和为:
2 s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1]
  + a[i][j]
3 // 以 (x1, y1) 为左上角, (x2, y2) 为右下角的
  子矩阵的和为:
4 S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1] +
  S[x1 - 1][y1 - 1]
```

```
23         insert(x1, y1, x2, y2, c);
24     }
25     // 其他过程略
26 }
```

### 1.5.3 1D Difference Array

```
1 const int N = 100010;
2 int n, m;
3 int a[N], b[N];
4 void insert(int l, int r, int c)
5 { b[l] += c; b[r + 1] -= c; }
6 int main()
7 {
8     cin >> n >> m;
9     for (int i = 1; i <= n; i++)
10         cin >> a[i];
11     for (int i = 1; i <= n; i++)
12         insert(i, i, a[i]);
13     while (m--)
14     {
15         int l, r, c;
16         cin >> l >> r >> c;
17         insert(l, r, c);
18     }
19     for (int i = 1; i <= n; i++)
20         b[i] += b[i - 1],
21         cout << b[i] << ' ';
22     return 0;
23 }
```

### 1.5.4 2D Difference Array

```
1 const int N = 1010;
2 int n, m, q, a[N][N], b[N][N];
3 void insert(int x1, int y1, int x2, int y2,
  int c)
4 {
5     b[x1][y1] += c;
6     b[x2 + 1][y2 + 1] += c;
7     b[x1][y2 + 1] -= c;
8     b[x2 + 1][y1] -= c;
9 }
10 int main()
11 {
12     cin >> n >> m >> q;
13     for (int i = 1; i <= n; i++)
14         for (int j = 1; j <= m; j++)
15             cin >> a[i][j];
16     for (int i = 1; i <= n; i++)
17         for (int j = 1; j <= m; j++)
18             insert(i, j, i, j, a[i][j]);
19     while (q--)
20     {
21         int x1, x2, y1, y2, c;
22         cin >> x1 >> y1 >> x2 >> y2 >> c;
```

## 2 ★ Basic Data Structures

### 2.1 Linked List

#### 2.1.1 Singly Linked List

```
1  const int N = 100010;
2  int n, h[N], e[N], ne[N], idx = 1;
3  void init() { ne[0] = -1; }
4  void insert(int k, int x) // 第 k 个节点后
    插入
5  { e[idx] = x, ne[idx] = ne[k], ne[k] = idx
    ++; }
6  void del(int k) // 第 k 个节点后删除
7  { ne[k] = ne[ne[k]]; }
```

#### 2.1.2 Bidirectional Linked List

```
1  const int N = 100010;
2  int n, r[N], l[N], e[N], idx = 2;
3  void init() { r[0] = 1; l[1] = 0; }
4  void insert(int k, int x) // 第 k 个节点后插
    入
5  {
6      e[idx] = x;
7      r[idx] = r[k];
8      l[idx] = k;
9      l[r[k]] = idx;
10     r[k] = idx++;
11 }
12 void remove(int k) // 删除 k 本身
13 { r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

### 2.2 Stack & Queue

#### 2.2.1 Monotonic Stack

```
1  // 常见模型：找出每个数左边离它最近的比它大/小
    的数
2  int tt = 0;
3  for (int i = 1; i <= n; i++)
4  {
5      while (tt && check(stk[tt], i)) tt--;
6      stk[++tt] = i;
7  }
```

#### 2.2.2 Monotonic Queue

```
1  // 常见模型：找出滑动窗口中的最大值/最小值
2  int hh = 0, tt = -1;
3  for (int i = 0; i < n; i++)
4  {
5      while (hh <= tt && check_out(q[hh]))
6          hh++; // 判断队头是否滑出窗口
7      while (hh <= tt && check(q[tt], i))
8          tt--;
```

```
9      q[++tt] = i;
10 }
```

### 2.3 KMP

```
1  const int N = 100010, M = 1000010;
2  int n, m;
3  char p[N], s[M];
4  void getNext(int ne[])
5  {
6      for (int i = 2, j = 0; i <= n; i++)
7      {
8          while (j && p[j + 1] != p[i])
9              j = ne[j];
10         if (p[j + 1] == p[i]) j++;
11         ne[i] = j;
12     }
13 }
14 int KMP()
15 {
16     int *ne = new int[n + 1];
17     getNext(ne);
18     for (int i = 1, j = 0; i <= m; i++)
19     {
20         while (j && p[j + 1] != s[i])
21             j = ne[j];
22         if (p[j + 1] == s[i]) j++;
23         if (j == n) cout << i - n << ' ';
24     }
25     return -1;
26 }
```

### 2.4 Trie

```
1  const int N = 100010;
2  int trie[N][26], cnt[N], idx = 0;
3  void insert(string &str) // 插入到 Trie
    数组
4  {
5      int p = 0;
6      for (auto c : str)
7      {
8          int u = c - 'a';
9          if (!trie[p][u])
10             trie[p][u] = ++idx;
11             p = trie[p][u];
12     }
13     cnt[p]++;
14 }
15 int query(string &str) // 查询字符串出现
    的次数
16 {
17     int p = 0;
18     for (auto c : str)
19     {
20         int u = c - 'a';
21         if (!trie[p][u]) return 0;
22         p = trie[p][u];
23     }
24     return cnt[p];
```

## 2.5 Disjoint-Set

```

1  const int N = 100010;
2  int n, m, p[N], Size[N], D[N];
3  void init()
4  {
5      for (int i = 1; i <= n; i++)
6          p[i] = i, Size[i] = 1, D[i] = 0;
7  }
8  int find(int x)
9  {
10     if (p[x] != x)
11     {
12         int u = find(p[x]);
13         D[x] += D[p[x]]; // 视具体情况计算
14         p[x] = u;
15     }
16     return p[x];
17 }
18 void merge(int a, int b, int distance)
19 {
20     int x = find(a), y = find(b);
21     if (x != y)
22     {
23         p[x] = y;
24         D[x] = distance; // 视具体情况计算
25         Size[y] += Size[x];
26     }
27 }

```

## 2.6 Hash

### 2.6.1 Simple Hash

```

1  // (1) 拉链法
2  int h[N], e[N], ne[N], idx;
3  void insert(int x)
4  {
5      int k = (x % N + N) % N;
6      e[idx] = x, ne[idx] = h[k], h[k] = idx++;
7  }
8  bool find(int x)
9  {
10     for (int i = h[(x % N + N) % N]; i != -1;
11         i = ne[i])
12         if (e[i] == x) return true;
13     return false;
14 }
15 // (2) 开放寻址法
16 int find(int x)
17 {
18     int t = (x % N + N) % N;
19     while (h[t] != null && h[t] != x)
20         t++; if (t == N) t = 0;
21     return t;
22 }

```

### 2.6.2 String Hash

```

1  typedef unsigned long long ULL;
2  ULL h[N], p[N];
3  void init()
4  {
5      p[0] = 1;
6      for (int i = 1; i <= n; i++) { h[i] = h[i - 1] * P + str[i]; p[i] = p[i - 1] * P; }
7  }
8  ULL get(int l, int r) { return h[r] - h[l - 1] * p[r - l + 1]; }

```

## 2.7 STL

```

1  // vector
2  size()      返回元素个数
3  empty()     返回是否为空
4  clear()     清空
5  front()/back()
6  push_back()/pop_back()
7  begin()/end()
8  []
9  支持比较运算, 按字典序
10 // pair<int, int>
11 first       第一个元素
12 second      第二个元素
13 支持比较运算, 以first为第一关键字, 以second为第二关键字 (字典序)
14 // string
15 size()/length() 返回字符串长度
16 empty()
17 clear()
18 substr(起始下标, (子串长度)) 返回子串
19 c_str() 返回字符串所在字符数组的起始地址
20 // queue
21 size()
22 empty()
23 push()      向队尾插入一个元素
24 front()     返回队头元素
25 back()      返回队尾元素
26 pop()       弹出队头元素
27 // priority_queue
28 size()
29 empty()
30 push()      插入一个元素
31 top()       返回堆顶元素
32 pop()       弹出堆顶元素
33 定义成小根堆的方式: priority_queue<int, vector<int>, greater<int>> q;
34 // stack
35 size()
36 empty()
37 push()      向栈顶插入一个元素
38 top()       返回栈顶元素
39 pop()       弹出栈顶元素
40 // deque
41 size()
42 empty()
43 clear()
44 front()/back()
45 push_back()/pop_back()

```

```

46 push_front()/pop_front()
47 begin()/end()
48 []
49 // set, map, multiset, multimap: 基于平衡二叉
    树 (红黑树) 动态维护有序序列
50 size()
51 empty()
52 clear()
53 begin()/end()
54 ++, -- 返回前驱和后继, 时间复杂度  $O(\log n)$ 
55 // set/multiset
56 insert() 插入一个数
57 find() 查找一个数
58 count() 返回某一个数的个数
59 erase()
60 (1) 输入是一个数x, 删除所有x,  $O(k + \log n)$ 
    (2) 输入一个迭代器, 删除这个迭代器
61 lower_bound()/upper_bound()
62 lower_bound(x) 返回大于等于x的最小的数的
    迭代器
63 upper_bound(x) 返回大于x的最小的数的迭
    代器
64 // map/multimap
65 insert() 插入的数是一个pair
66 erase() 输入的参数是pair或者迭代器
67 find()
68 [] 注意multimap不支持此操作。 时间复
    杂度是  $O(\log n)$ 
69 lower_bound()/upper_bound()
70 // unordered_set, unordered_map,
    unordered_multiset, unordered_multimap
71 增删改查的时间复杂度是  $O(1)$ 
72 不支持 lower_bound()/upper_bound(), 迭代器的
    ++, --
73 // bitset
74 bitset<10000> s;
75 ~, &, |, ^
76 >>, <<
77 ==, !=
78 []
79 count() 返回有多少个1
80 any() 判断是否至少有一个1
81 none() 判断是否全为0
82 set() 把所有位置成1
83 set(k, v) 将第k位变成v
84 reset() 把所有位变成0
85 flip() 等价于~
86 flip(k) 把第k位取反

```

## 3 ★ Search & Graph Theory

### 3.1 Representation of Tree & Graph

#### 3.1.1 Adjacency Matrix

```
1 // g[a][b] = a->b
```

#### 3.1.2 Adjacency List

```
1 int h[N], e[N], ne[N], idx;
2 void init() { memset(h, -1, sizeof h); }
3 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++; }
```

## 3.2 DFS & BFS

#### 3.2.1 DFS

```
1 int dfs(int u)
2 {
3     st[u] = true; // 表示点 u 已经被遍历过
4     for (int i = h[u]; i != -1; i = ne[i])
5     { int j = e[i]; if (!st[j]) dfs(j); }
6 }
```

#### 3.2.2 BFS

```
1 queue<int> q;
2 st[1] = true; q.push(1);
3 while (q.size())
4 {
5     int t = q.front(); q.pop();
6     for (int i = h[t]; i != -1; i = ne[i])
7     { if (!st[e[i]]) { st[e[i]] = true; q.
            push(e[i]); }
8 }
```

## 3.3 Topological Sort

```
1 const int N = 100010;
2 int e[2 * N], ne[2 * N], h[N], d[N], idx;
3 int n, m, q[N];
4 void init() { memset(h, -1, sizeof h); }
5 void add(int a, int b) { e[idx] = b, ne[idx]
    = h[a], h[a] = idx++, d[b]++; }
6 bool topSort()
7 {
8     int hh = 0, tt = -1;
9     for (int i = 1; i <= n; i++)
10         if (!d[i]) q[++tt] = i;
11     while (hh <= tt)
```

```
12         for (int i = h[q[hh++]]; ~i; i = ne[i]
13             )
14             if (--d[e[i]] == 0) q[++tt] = e[i];
15     return tt == n - 1;
16 }
```

## 3.4 Shortest Path

#### 3.4.1 Dijkstra

```
1 const int N = 1010;
2 int n, dist[N];
3 int h[N], w[N], e[N], ne[N], idx;
4 bool st[N];
5 void add(int a, int b, int c) { e[idx] = b,
    w[idx] = c, ne[idx] = h[a], h[a] = idx
    ++; }
6 int dijkstra() // 需要初始化 dist 与 h
7 {
8     dist[1] = 0;
9     priority_queue<PII, vector<PII>, greater<
        PII>> heap;
10    heap.push({0, 1});
11    while (heap.size())
12    {
13        auto t = heap.top();
14        heap.pop();
15        int ver = t.second, distance = t.first;
16        if (st[ver]) continue;
17        st[ver] = true;
18        for (int i = h[ver]; i != -1; i = ne[i]
19            )
20            if (dist[e[i]] > distance + w[i])
21            {
22                dist[e[i]] = distance + w[i];
23                heap.push({dist[e[i]], e[i]});
24            }
25    }
26    if (dist[n] == 0x3f3f3f3f) return -1;
27    return dist[n];
28 }
```

#### 3.4.2 Bellman-Ford

```
1 const int N = 100010;
2 int n, m, dist[N], backup[N];
3 struct Edge
4 {
5     int a, b, w;
6 } edges[N];
7 int bellman_ford()
8 {
9     memset(dist, 0x3f, sizeof dist);
10    dist[1] = 0;
11    for (int i = 0; i < n; i++)
12    {
13        memcpy(backup, dist, sizeof dist);
14        for (int j = 0; j < m; j++)
15        {
```

```

16         int a = edges[j].a, b = edges[j].b
           , w = edges[j].w;
17         dist[b] = min(dist[b], backup[a] +
18             w);
19     }
20     if (dist[n] > 0x3f3f3f3f / 2) return -1;
21     return dist[n];
22 }

```

### 3.4.3 SPFA

```

1  const int N = 100010;
2  int n, m, dist[N];
3  int e[2 * N], ne[2 * N], w[2 * N], h[N], idx
   ;
4  bool vis[N];
5  void spfa()    // 需要初始化 dist 与 h
6  {
7      queue<int> q;
8      q.push(1); vis[1] = true;
9      while (q.size())
10     {
11         int t = q.front();
12         q.pop();
13         vis[t] = false;
14         for (int i = h[t]; ~i; i = ne[i])
15             if (dist[e[i]] > dist[t] + w[i])
16             {
17                 dist[e[i]] = dist[t] + w[i];
18                 if (!vis[e[i]]) vis[e[i]] =
19                     true, q.push(j);
20             }
21     dist[n] > INF / 2 ? cout << "impossible" :
22         cout << dist[n];

```

### 3.4.4 Detecting Negative Circle in SPFA

```

1  void spfa()    // 只需要初始化 h
2  {
3      queue<int> q;
4      // 基于虚拟原点假设, 所有点放入队列
5      for (int i = 1; i <= n; i++) q.push(i), st
6          [i] = true;
7      while (q.size())
8      {
9          int t = q.front();
10         q.pop();
11         vis[t] = false;
12         for (int i = h[t]; ~i; i = ne[i])
13             if (dist[e[i]] > dist[t] + w[i])
14             {
15                 dist[e[i]] = dist[t] + w[i];
16                 // 新增
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n) return true
19                 if (!st[j]) q.push(j), st[j] =
20                     true;

```

```

19     }
20 }
21 return false;
22 }

```

### 3.4.5 Floyd

```

1  const int N = 210;
2  int g[N][N], n, m, k;
3  int main()
4  {
5      cin >> n >> m >> k;
6      memset(g, 0x3f, sizeof g);
7      for (int i = 1; i <= n; i++) g[i][i] = 0;
8      while (m--)
9      {
10         int a, b, c;
11         cin >> a >> b >> c;
12         g[a][b] = min(g[a][b], c);
13     }
14     for (int k = 1; k <= n; k++)
15         for (int i = 1; i <= n; i++)
16             for (int j = 1; j <= n; j++)
17                 g[i][j] = min(g[i][k] + g[k][j]
18                     , g[i][j]);
19     // 后续代码略
20 }

```

## 3.5 Minimum Spanning Tree

### 3.5.1 Prim

```

1  const int N = 510;
2  int n, m, g[N][N], dist[N];
3  bool vis[N];
4  void prim()
5  {
6      int res = 0;
7      for (int i = 0; i < n; i++)
8      {
9          int t = -1;
10         for (int j = 1; j <= n; j++)
11             if (!vis[j] && (t == -1 || dist[j]
12                 < dist[t])) t = j;
13         if (i && dist[t] == INF) { res = INF;
14             break; }
15         if (i) res += dist[t];
16         vis[t] = true;
17         for (int j = 1; j <= n; j++) dist[j] =
18             min(dist[j], g[t][j]);
19     }
20     res == INF ? cout << "impossible" : cout
21         << res;
22 }
23 int main()
24 {
25     memset(g, 0x3f, sizeof g);
26     memset(dist, 0x3f, sizeof dist);
27     cin >> n >> m;
28     while (m--)

```

```

25 {
26     int a, b, c;
27     cin >> a >> b >> c;
28     g[a][b] = min(g[a][b], c);
29     g[b][a] = min(g[b][a], c);
30 }
31 prim();
32 return 0;
33 }

```

### 3.5.2 Kruskal

```

1  const int N = 100010;
2  int n, m;
3  int p[N];
4  struct Edge
5  {
6      int a, b, w;
7      bool operator<(const Edge &e) const {
8          return w < e.w; };
9  } edge[2 * N];
10 void init() { for (int i = 1; i <= n; i++) p[i] = i; }
11 int find(int x)
12 {
13     if (x != p[x]) p[x] = find(p[x]);
14     return p[x];
15 }
16 void merge(int x, int y) { p[find(x)] = find(y); }
17 void kruskal()
18 {
19     int res = 0, cnt = 0;
20     for (int i = 1; i <= m; i++)
21         if (find(edge[i].a) != find(edge[i].b))
22             {
23                 merge(edge[i].a, edge[i].b);
24                 res += edge[i].w;
25                 cnt++;
26             }
27     if (cnt < n - 1) res = INF;
28     res == INF ? cout << "impossible" : cout << res;
29 }
30 int main()
31 {
32     init();
33     cin >> n >> m;
34     for (int i = 1; i <= m; i++) cin >> edge[i].a >> edge[i].b >> edge[i].w;
35     sort(edge + 1, edge + m + 1);
36     kruskal();
37     return 0;
38 }

```

```

1  const int N = 100010, M = 200010;
2  int n, m;
3  int e[M], ne[M], h[N], color[N], idx;
4  bool dfs(int u, int c)
5  {
6      color[u] = c;
7      for (int i = h[u]; ~i; i = ne[i])
8          if (color[e[i]] == -1)
9              {
10                 if (!dfs(e[i], !c)) return false;
11             }
12         else if (color[e[i]] == c) return false;
13     return true;
14 }
15 bool check()
16 {
17     for (int i = 1; i <= n; i++)
18         if (color[i] == -1)
19             if (!dfs(i, 0)) return false;
20     return true;
21 }
22 int main()
23 {
24     // 注意另外初始化 h 与 color
25     cin >> n >> m;
26     while (m--)
27     {
28         int a, b;
29         cin >> a >> b;
30         add(a, b), add(b, a);
31     }
32     // 其余过程略
33 }

```

## 3.6 Bipartite Graph

### 3.6.1 Coloring Method

To check if a given graph is bipartite.

### 3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```
1  const int N = 510, M = 100010;
2  int n1, n2, m;
3  int e[M], ne[M], h[N], match[N], idx;
4  bool vis[N];
5  bool find(int x)
6  {
7      for (int i = h[x]; ~i; i = ne[i])
8          if (!vis[e[i]])
9              {
10                 vis[e[i]] = true;
11                 if (match[e[i]] == 0 || find(match[e[i]]))
12                     {
13                         match[e[i]] = x;
14                         return true;
15                     }
16             }
17      return false;
18  }
19  int main()
20  {
21      // 注意初始化 h
22      cin >> n1 >> n2 >> m;
23      while (m--)
24      {
25          int a, b;
26          cin >> a >> b;
27          add(a, b);
28      }
29      int res = 0;
30      for (int i = 1; i <= n1; i++)
31      {
32          memset(vis, false, sizeof vis);
33          if (find(i)) res++;
34      }
35      cout << res;
36      return 0;
37  }
```



## 4 ★ Basic Math

### 4.1 Prime Numbers

#### 4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
1 bool is_prime(int x)
2 {
3     if (x < 2) return false;
4     for (int i = 2; i <= x / i; i++)
5         if (x % i == 0) return false;
6     return true;
7 }
```

#### 4.1.2 Prime Factorization

```
1 void divide(int x)
2 {
3     for (int i = 2; i <= x / i; i++)
4         if (x % i == 0)
5             { // 此条件成立时 i 一定是质数
6                 int s = 0;
7                 while (x % i == 0) x /= i, s++;
8                 cout << i << ' ' << s << '\n';
9             }
10    if (x > 1) cout << x << ' ' << 1 << '\n'
11 }
```

#### 4.1.3 Euler's Sieve

```
1 int primes[N], cnt;
2 bool st[N];
3 void get_primes(int n)
4 {
5     for (int i = 2; i <= n; i++)
6     {
7         if (!st[i]) primes[cnt++] = i;
8         for (int j = 0; primes[j] <= n / i; j++)
9             {
10                st[primes[j] * i] = true;
11                if (i % primes[j] == 0) break;
12            }
13    }
14 }
```

## 4.2 Divisor

#### 4.2.1 Find All Divisors

```
1 vector<int> get_divisors(int x)
2 {
3     vector<int> res;
4     for (int i = 1; i <= x / i; i++)
5         if (x % i == 0)
```

```
6         {
7             res.push_back(i);
8             if (i != x / i) res.push_back(x /
9             i);
10        }
11    sort(res.begin(), res.end());
12    return res;
13 }
```

#### 4.2.2 The Number of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 int main()
4 {
5     cin >> n;
6     unordered_map<int, int> h;
7     while (n--)
8     {
9         int x;
10        cin >> x;
11        for (int i = 2; i <= x / i; i++)
12            while (x % i == 0) { h[i]++; x = x
13            / i; }
14        if (x > 1) h[x]++;
15    }
16    long long res = 1;
17    for (auto iter = h.begin(); iter != h.end()
18        ); iter++)
19        res = res * (iter->second + 1) % mod;
20    cout << res;
21    return 0;
22 }
```

#### 4.2.3 The Sum of Divisors

```
1 const int mod = 1e9 + 7;
2 int n;
3 long long getSum(int x, int c)
4 {
5     long long s = 1;
6     while(c--) s = (s * x + 1) % mod;
7     return s;
8 }
9 int main()
10 {
11    cin >> n;
12    unordered_map<int, int> h;
13    while (n--)
14    {
15        int x;
16        cin >> x;
17        for (int i = 2; i <= x / i; i++)
18            while (x % i == 0) { h[i]++; x = x
19            / i; }
20        if (x > 1) h[x]++;
21    }
22    long long res = 1;
23    for (auto iter = h.begin(); iter != h.end()
24        ); iter++)
25        res = res * getSum(iter->first, iter->
26        second) % mod;
27 }
```

```

24     cout << res;
25     return 0;
26 }

```

#### 4.2.4 Euclidean Algorithm

```

1  int gcd(int a, int b)
2  { return a % b == 0 ? b : gcd(b, a % b); }

```

### 4.3 Euler Function

#### 4.3.1 Simple Method

```

1  int phi(int x)
2  {
3      int res = x;
4      for (int i = 2; i <= x / i; i++)
5          if (x % i == 0)
6              {
7                  res = res / i * (i - 1);
8                  while (x % i == 0) x /= i;
9              }
10     if (x > 1) res = res / x * (x - 1);
11     return res;
12 }

```

#### 4.3.2 Euler's Sieve Method

```

1  const int N = 1000010;
2  int n, primes[N], phi[N], cnt;
3  bool st[N];
4  void getEuler()
5  {
6      phi[1] = 1;
7      for (int i = 2; i <= n; i++)
8          {
9              if (!st[i])
10                 {
11                     primes[cnt++] = i;
12                     // i 是质数, 它只会被本身整除, 所以
                     // 直接赋值 i - 1
13                     phi[i] = i - 1;
14                 }
15                 for (int j = 0; primes[j] <= n / i; j++)
16                     {
17                         st[i * primes[j]] = true;
18                         if (i % primes[j] == 0)
19                             {
20                                 // 如果 i % primes[j] == 0 成
                                 // 立表示 primes[j] 是 i 的最小质因子
                                 // 也是 primes[j] * i 的最小质
                                 // 因子
21                                 // 1 - 1 / primes[j] 这一项在
                                 // phi[i] 中计算过了, 只需将基数 N 修正为
                                 // primes[j] 倍
22                                 phi[primes[j] * i] = phi[i] *
23                                 primes[j];
24                                 break;

```

```

25         }
26         // 否则, primes[j] 不是 i 的质因
        // 子, 只是 primes[j] * i 的最小质因子
27         // 不仅需要将基数 N 修正为 primes[j]
        // 倍
28         // 还需要补上 1 - 1 / primes[j] 的
        // 分子项, 因此最终结果为 phi[i] * (primes[j]
        // - 1)
29         phi[primes[j] * i] = phi[i] * (
        primes[j] - 1);
30     }
31 }
32 }

```

### 4.4 Exponentiating by Squaring

```

1  LL qmi(int m, int k, int p)
2  {
3      LL res = 1 % p, t = m;
4      while (k)
5          {
6              if (k & 1) res = res * t % p;
7              t = t * t % p;
8              k >>= 1;
9          }
10     return res;
11 }

```

### 4.5 Extended Euclidean Algorithm

```

1  int exgcd(int a, int b, int &x, int &y)
2  {
3      if (!b)
4          {
5              x = 1;
6              y = 0;
7              return a;
8          }
9      int d = exgcd(b, a % b, y, x);
10     y -= (a / b) * x;
11     return d;
12 }

```

### 4.6 Chinese Remainder Theorem

```

1  LL exgcd(LL a, LL b, LL &x, LL &y)
2  {
3      if (!b) { x = 1, y = 0; return a; }
4      LL d = exgcd(b, a % b, y, x);
5      y -= a / b * x;
6      return d;
7  }
8  int main()
9  {

```

```

10 int n;
11 cin >> n;
12 LL x = 0, m1, a1;
13 cin >> m1 >> a1;
14 for (int i = 0; i < n - 1; i++)
15 {
16     LL m2, a2;
17     cin >> m2 >> a2;
18     LL k1, k2;
19     LL d = exgcd(m1, m2, k1, k2);
20     if ((a2 - a1) % d) { x = -1; break; }
21     k1 *= (a2 - a1) / d;
22     k1 = (k1 % (m2 / d) + m2 / d) % (m2 /
23 d);
24     x = k1 * m1 + a1;
25     LL m = abs(m1 / d * m2);
26     a1 = k1 * m1 + a1;
27     m1 = m;
28 }
29 if (x != -1)
30     x = (a1 % m1 + m1) % m1;
31 cout << x << '\n';
32 return 0;
33 }

```

## 4.7 Gauss-Jordan Elimination

### 4.7.1 Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++) // 找
            // 绝对值最大的行
8             if (fabs(a[i][c]) > fabs(a[t][c]))
9                 t = i;
10        if (fabs(a[t][c]) < eps) // 此
            // 时没必要对该列该行处理
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[t][i], a[r][i]); // 将
            // 绝对值最大的行换到最顶端
14        for (int i = n; i >= c; i--)
15            a[r][i] /= a[r][c]; // 将
            // 当前行的首位变成1
16        for (int i = r + 1; i < n; i++) // 用
            // 当前行将下面所有的列消成0
17            if (fabs(a[i][c]) > eps)
18                for (int j = n; j >= c; j--)
19                    a[i][j] -= a[r][j] * a[i][c];
20        r++;
21    }
22    if (r < n)
23    {
24        for (int i = r; i < n; i++)
25            if (fabs(a[i][n]) > eps)
26                return 2; // 无解
27        return 1; // 有无穷多组解
28    }
29 }

```

```

29 for (int i = n - 1; i >= 0; i--)
30     for (int j = i + 1; j < n; j++)
31         a[i][n] -= a[i][j] * a[j][n];
32 return 0; // 有解
33 }

```

### 4.7.2 XOR Linear Equation Group

```

1 int gauss()
2 {
3     int c, r;
4     for (c = 0, r = 0; c < n; c++)
5     {
6         int t = r;
7         for (int i = r; i < n; i++)
8             if (a[i][c])
9                 t = i;
10        if (!a[t][c])
11            continue;
12        for (int i = c; i <= n; i++)
13            swap(a[r][i], a[t][i]);
14        for (int i = r + 1; i < n; i++)
15            if (a[i][c])
16                for (int j = n; j >= c; j--)
17                    a[i][j] ^= a[r][j];
18        r++;
19    }
20    if (r < n)
21    {
22        for (int i = r; i < n; i++)
23            if (a[i][n])
24                return 2;
25        return 1;
26    }
27    for (int i = n - 1; i >= 0; i--)
28        for (int j = i + 1; j < n; j++)
29            a[i][n] ^= a[i][j] * a[j][n];
30    return 0;
31 }

```

## 4.8 Combinatorial Counting

### 4.8.1 Recurrence Relation

```

1 void init()
2 {
3     for (int i = 0; i < N; i++)
4         for (int j = 0; j <= i; j++)
5             if (!j) c[i][j] = 1;
6             else c[i][j] = (c[i - 1][j] + c[i
7 - 1][j - 1]) % mod;
8 }

```

### 4.8.2 Preprocessing & Inverse Element

```

1 const int N = 100010, mod = 1e9 + 7;
2 int n, fact[N], infact[N];
3 int qmi(int a, int b, int p)
4 {

```

```

5   int res = 1;
6   while (b)
7   {
8       if (b & 1)
9           res = (LL)res * a % p;
10      a = (LL)a * a % p;
11      b >>= 1;
12  }
13  return res;
14  }
15  int main()
16  {
17      fact[0] = infact[0] = 1;
18      for (int i = 1; i < N; i++)
19      {
20          fact[i] = (LL)fact[i - 1] * i % mod;
21          infact[i] = (LL)infact[i - 1] * qmi(i,
22              mod - 2, mod) % mod;
23      }
24      // 此后 C(a, b) = (LL)fact[a] * infact[b]
25      // % mod * infact[a - b] % mod
26  }

```

### 4.8.3 Lucas Theorem

```

1  int qmi(int a, int k, int p)
2  {
3      int res = 1 % p;
4      while (k)
5      {
6          if (k & 1)
7              res = (LL)res * a % p;
8          a = (LL)a * a % p;
9          k >>= 1;
10     }
11     return res;
12 }
13 int C(int a, int b, int p)
14 {
15     if (a < b) return 0;
16     LL x = 1, y = 1;
17     // x = a * (a - 1) * (a - 2) * ... * (a - b + 1) = a! / (a - b)! (mod p)
18     // y = 1 * 2 * ... * b = b! (mod p)
19     for (int i = a, j = 1; j <= b; i--, j++)
20     { x = (LL)x * i % p; y = (LL)y * j % p; }
21     return x * (LL)qmi(y, p - 2, p) % p;
22 }
23 int lucas(LL a, LL b, int p)
24 {
25     if (a < p && b < p)
26         return C(a, b, p);
27     return (LL)C(a % p, b % p, p) * lucas(a / p, b / p, p) % p;
28 }

```

### 4.8.4 Factorization Method

```

1  const int N = 5010;
2  int n, primes[N], sum[N], cnt;
3  bool st[N];
4  void getPrimes(int n) { // 略 }

```

```

5  // 求 n! 中 p 的幂次
6  int get(int n, int p)
7  {
8      int res = 0;
9      while (n) { res += n / p; n /= p; }
10     return res;
11 }
12 void mul(vector<int> &a, int b) { // 高精度
13     // 乘, 略 }
14 int main()
15 {
16     int a, b;
17     cin >> a >> b;
18     getPrimes(a);
19     for (int i = 0; i < cnt; i++)
20     {
21         int p = primes[i];
22         sum[i] = get(a, p) - get(b, p) - get(a - b, p);
23     }
24     vector<int> res;
25     res.push_back(1);
26     for (int i = 0; i < cnt; i++)
27         for (int j = 0; j < sum[i]; j++)
28             mul(res, primes[i]);
29     for (int i = res.size() - 1; i >= 0; i--)
30         cout << res[i];
31 }

```

### 4.8.5 Catalan Number

```

1  const int N = 100010, mod = 1e9 + 7;
2  int qmi(int a, int k, int p) { // 略 }
3  int main()
4  {
5      int n;
6      cin >> n;
7      int a = n * 2, b = n, res = 1;
8      for (int i = a; i > a - b; i--)
9          res = (LL)res * i % mod;
10     for (int i = 1; i <= b; i++)
11         res = (LL)res * qmi(i, mod - 2, mod) % mod;
12     res = (LL)res * qmi(n + 1, mod - 2, mod) % mod;
13 }

```

When  $p$  is not prime or the range of number is too large:

```

1  const int N = 2000010;
2  int n, p, primes[N], cnt;
3  bool st[N];
4  void init(int n)
5  {
6      for (int i = 2; i <= n; i++)
7      {
8          if (!st[i]) primes[cnt++] = i;
9          for (int j = 0; primes[j] * i <= n; j++)
10             st[i * primes[j]] = true;
11             if (i % primes[j] == 0) break;
12         }
13     }
14 }

```

```

15 }
16 int qmi(int a, int k)
17 {
18     int res = 1;
19     while (k)
20     {
21         if (k & 1) res = (LL)res * a % p;
22         a = (LL)a * a % p;
23         k >>= 1;
24     }
25     return res;
26 }
27 int get(int n, int p)
28 {
29     int s = 0;
30     while (n)
31         s += n / p, n /= p;
32     return s;
33 }
34 int C(int a, int b)
35 {
36     int res = 1;
37     for (int i = 0; i < cnt; i++)
38     {
39         int s = get(a, primes[i]) - get(b,
40             primes[i]) - get(a - b, primes[i]);
41         res = (LL)res * qmi(primes[i], s) % p;
42     }
43     return res;
44 }
45 int main()
46 {
47     cin >> n >> p;
48     init(n << 1);
49     cout << (C(n << 1, n) - C(n << 1, n - 1) +
50         p) % p;
51 }

```

## 4.9 Inclusion-Exclusion Principle

```

1  const int N = 20;
2  int n, m, res = 0, p[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < m; i++)
7          cin >> p[i];
8      // 使用二进制数字表示数字选取情况
9      for (int i = 1; i < 1 << m; i++)
10     {
11         int t = 1, cnt = 0;
12         // 遍历每个被选取的质数
13         for (int j = 0; j < m; j++)
14             if (i >> j & 1)
15             {
16                 cnt++;
17                 // 一个质数能被选取的条件应该是
18                 // 其累乘积不超过目标数字
19                 if ((LL)t * p[j] > n)
20                     { t = -1; break; }
21                 t *= p[j];
22             }
23     }
24 }

```

```

22     if (t != -1)
23         // 容斥原理公式中奇数个并集系数为 1
24         // , 反之为 -1
25         if (cnt % 2) res += n / t;
26         else res -= n / t;
27     }
28     cout << res;
29 }

```

## 4.10 Game Theory

### 4.10.1 NIM Game

```

1  const int N = 110, M = 100010;
2  int k, n, s[N], f[M];
3  int sg(int x)
4  {
5      if (f[x] != -1) return f[x];
6      // 到达节点得 SG 函数集合
7      unordered_set<int> S;
8      // 能取走石子就说明能到达, 并且递归向下求解
9      for (int i = 0; i < k; i++)
10     {
11         int sum = s[i];
12         if (x >= sum) S.insert(sg(x - sum));
13     }
14     // SG 从小到大遍历并返回, 找到最小的、不包含
15     // 在 SG 函数集中的自然数
16     for (int i = 0; i < M; i++)
17         if (!S.count(i))
18             return f[x] = i;
19 }
20 int main()
21 {
22     cin >> k;
23     for (int i = 0; i < k; i++) cin >> s[i];
24     cin >> n;
25     memset(f, -1, sizeof f);
26     int res = 0;
27     // 每一堆石子都是一个入度为 0 的起始点
28     for (int i = 0; i < n; i++)
29     {
30         int x;
31         cin >> x;
32         res ^= sg(x);
33     }
34     res ? cout << "Yes" : cout << "No";
35     return 0;
36 }

```

## 5 ★ Basic DP

### 5.1 Knapsack Problem

#### 5.1.1 01 Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = m; j >= v[i]; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.2 Complete Knapsack

```
1  const int N = 1010;
2  int n, m, v[N], w[N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7          cin >> v[i] >> w[i];
8      for (int i = 1; i <= n; i++)
9          for (int j = v[i]; j <= m; j++)
10             f[j] = max(f[j], f[j - v[i]] + w[i]);
11     cout << f[m];
12 }
```

#### 5.1.3 Mutiple Knapsack

```
1  const int N = 25010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         for (int k = 1; k <= s; k *= 2)
11         {
12             for (int j = m; j >= k * v; j--)
13                 f[j] = max(f[j], f[j - k * v] + k * w);
14             s -= k;
15         }
16         if (s)
17             for (int j = m; j >= s * v; j--)
18                 f[j] = max(f[j], f[j - s * v] + s * w);
19     }
```

```
20     cout << f[m];
21 }
```

#### 5.1.4 Grouped Knapsack

```
1  const int N = 120;
2  int n, m, s[N], v[N][N], w[N][N], f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> s[i];
9          for (int j = 1; j <= s[i]; j++)
10             cin >> v[i][j] >> w[i][j];
11     }
12     for (int i = 1; i <= n; i++)
13         for (int j = m; j >= 0; j--)
14             for (int k = 1; k <= s[i]; k++)
15                 if (v[i][k] <= j)
16                     f[j] = max(f[j], f[j - v[i][k]] + w[i][k]);
17     cout << f[m];
18 }
```

## 5.2 Linear DP

### 5.2.1 LIS

Here is an  $O(n^2)$  solution:

```
1  const int N = 1010;
2  int n, a[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9      {
10         f[i] = 1;
11         for (int j = 1; j < i; j++)
12             if (a[j] < a[i])
13                 f[i] = max(f[i], f[j] + 1);
14     }
15     int res = 0;
16     for (int i = 1; i <= n; i++)
17         res = max(res, f[i]);
18     cout << res;
19 }
```

Another is an  $O(n \log n)$  solution:

```
1  const int N = 100010;
2  int n, a[N], q[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++) cin >> a[i];
7      int len = 0;
8      q[len] = -INF;
9      for (int i = 1; i <= n; i++)
```

```

10 {
11     int l = 0, r = len;
12     while (l < r)
13     {
14         int mid = l + r + 1 >> 1;
15         if (q[mid] < a[i]) l = mid;
16         else r = mid - 1;
17     }
18     len = max(r + 1, len);
19     q[r + 1] = a[i];
20 }
21 cout << len;
22 }

```

## 5.2.2 LCS

```

1 const int N = 1010;
2 int n, m, f[N][N];
3 char a[N], b[N];
4 int main()
5 {
6     cin >> n >> m >> (a + 1) >> (b + 1);
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= m; j++)
9             {
10                 f[i][j] = max(f[i - 1][j], f[i][j
11                     - 1]);
12                 if (a[i] == b[j])
13                     f[i][j] = max(f[i][j], f[i -
14                         1][j - 1] + 1);
15             }
16     cout << f[n][m];
17 }

```

## 5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```

1 const int N = 310;
2 int n, s[N], f[N][N];
3 int main()
4 {
5     cin >> n;
6     for (int i = 1; i <= n; i++)
7         cin >> s[i], s[i] += s[i - 1];
8     for (int len = 2; len <= n; len++)
9         for (int i = 1; i + len - 1 <= n; i++)
10            {
11                int l = i, r = i + len - 1;
12                f[l][r] = INF;
13                for (int k = l; k < r; k++)
14                    f[l][r] = min(f[l][r], f[l][k]
15                        + f[k + 1][r] + s[r] - s[l - 1]);
16            }
17     cout << f[1][n];
18 }

```

## 5.4 Counting DP

```

1 const int N = 1010, M = 1e9 + 7;
2 int n, f[N][N];
3 int main()
4 {
5     cin >> n;
6     f[0][0] = 1;
7     for (int i = 1; i <= n; i++)
8         for (int j = 1; j <= i; j++)
9             f[i][j] = (f[i - 1][j - 1] + f[i -
10                 j][j]) % M;
11     int ans = 0;
12     for (int i = 1; i <= n; i++)
13         ans = (ans + f[n][i]) % M;
14     cout << ans;
15 }

```

## 5.5 Digit DP

```

1 // 求数 n 的位数
2 int get(int n)
3 {
4     int res = 0;
5     while (n) n /= 10, res++;
6     return res;
7 }
8 int count(int n, int i)
9 {
10     int res = 0, dgt = get(n);
11     for (int j = 1; j <= dgt; j++)
12     {
13         // p 为当前遍历位次(第 j 位)的数大小
14         // <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
15         // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
16         int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
17         // 求要选的数在 i 的左边的数小于 l 的情况:
18         // 1)、当 i 不为 0 时 xxx : 0...0 ~ l - 1, 即 l * (右边的数的位数) == l * p 种选法
19         // 2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0...1 ~ l - 1, 即 (l - 1) * (右边的数的位数) == (l - 1) * p 种选法
20         if (i) res += l * p;
21         else res += (l - 1) * p;
22         // 求要选的数在 i 的左边的数等于 l 的情况: (即视频中的 xxx == l 时)
23         // 1)、i > dj 时 0 种选法
24         // 2)、i == dj 时 yyy : 0...0 ~ r 即 r + 1 种选法
25         // 3)、i < dj 时 yyy : 0...0 ~ 9...9 即 10^(右边的数的位数) == p 种选法
26         if (i == dj) res += r + 1;
27         if (i < dj) res += p;
28     }
29     return res;
30 }
31 int main()
32 {
33     int a, b;

```

```

33 while (cin >> a >> b, a)
34 {
35     if (a > b) swap(a, b);
36     for (int i = 0; i <= 9; ++i)
37         cout << count(b, i) - count(a - 1,
38             i) << ' ';
39     // 利用前缀和思想: [l, r] 的和 = s[r] -
    s[l - 1]
39     cout << '\n';
40 }
41 }

```

## 5.6 State Compression DP

```

1  const int N = 12, M = 1 << 12;
2  int n, m;
3  LL f[N][M];
4  bool st[M];
5  int main()
6  {
7      while (cin >> n >> m, n || m)
8      {
9          memset(f, 0, sizeof f);
10         for (int i = 0; i < 1 << n; i++)
11         {
12             st[i] = true;
13             // 统计连续 0 的个数, 若连续 0 为奇
            数个就不能正好放得下竖放的方格
14             int cnt = 0;
15             for (int j = 0; j < n && st[i]; j
16                 ++
17                 if (i >> j & 1)
18                 {
19                     // 当前格子被使用
20                     // 如果连续 0 的数量为奇数
21                     个, 当前格子被使用的后果就是导致格子重合,
22                     所以不可取
23                     if (cnt & 1)
24                         st[i] = false;
25                     // 刷新状态
26                     cnt = 0;
27                 }
28                 else cnt++;
29             // 最后再判断一次, 防止漏判
30             if (cnt & 1)
31                 st[i] = false;
32         }
33         // 没有摆放任何棋子的状态默认只有 1 种取
34         法
35         f[0][0] = 1;
36         // 遍历每一列
37         for (int i = 1; i <= m; i++)
38         {
39             // 遍历当前列的每一种用二进制数字表示
40             的摆放状态: 1 指横向摆放, 0 指空位
41             for (int j = 0; j < 1 << n; j++)
42             {
43                 // 遍历上一列的每一种用二进制数
44                 字表示的摆放状态: 1 指横向摆放, 0 指空位
45                 for (int k = 0; k < 1 << n; k
46                     ++
47                     // 满足两个条件: 两列的摆放
48                     互不冲突; 两列摆放状态的结合状态是一个可取
49                     的状态则累加情况数
50                     if (!(j & k) && st[j | k])

```

```

40         f[i][j] += f[i - 1][k
41         ];
42         // 输出摆放好第 m 列且第 (m + 1) 列没有
43         任何方格的状态数
44         cout << f[m][0] << '\n';
45     }
46 }

```

## 5.7 Tree DP

```

1  // Don't use I/O functions from stdio.h!!!
2  #define itn int
3  #define nit int
4  #define nti int
5  #define tin int
6  #define tni int
7  #define retrun return
8  #define reutrn return
9  #define rutren return
10 #define INF 0x3f3f3f3f
11 #include <bits/stdc++.h>
12 using namespace std;
13 typedef pair<int, int> PII;
14 typedef long long LL;
15
16 const int N = 6010;
17
18 int n;
19 int e[N], ne[N], happy[N], h[N], idx;
20 int f[N][2];
21 bool has_father[N];
22 void add(int a, int b)
23 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
24 }
25 void dfs(int u)
26 {
27     f[u][1] = happy[u];
28     for (int i = h[u]; ~i; i = ne[i])
29     {
30         dfs(e[i]);
31         f[u][0] += max(f[e[i]][0], f[e[i]][1]);
32         f[u][1] += f[e[i]][0];
33     }
34 }
35 int main()
36 {
37     memset(h, -1, sizeof h);
38     cin >> n;
39     for (int i = 1; i <= n; i++) cin >> happy[
40         i];
41     for (int i = 0; i < n - 1; i++)
42     {
43         int a, b;
44         cin >> a >> b;
45         has_father[a] = true;
46         add(b, a);
47     }
48     int root = 1;
49     while (has_father[root]) root++;
50     dfs(root);
51     cout << max(f[root][0], f[root][1]);
52 }

```



## 5.8 Memoized Search

```
1  const int N = 310;
2  int n, m,
3  h[N][N], f[N][N],
4  dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1,
5      0};
6  int dp(int x, int y)
7  {
8      int &v = f[x][y];
9      if (v != -1) return v;
10     v = 1;
11     for (int i = 0; i < 4; i++)
12     {
13         int a = x + dx[i], b = y + dy[i];
14         if (a >= 1 && a <= n && b >= 1 && b <=
15             m && h[a][b] < h[x][y])
```

```
14         v = max(v, dp(a, b) + 1);
15     }
16     return v;
17 }
18 int main()
19 {
20     cin >> n >> m;
21     for (int i = 1; i <= n; i++)
22         for (int j = 1; j <= m; j++)
23             cin >> h[i][j];
24     memset(f, -1, sizeof f);
25     int res = 0;
26     for (int i = 1; i <= n; i++)
27         for (int j = 1; j <= m; j++)
28             res = max(res, dp(i, j));
29     cout << res;
30 }
```



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## Part II: Advanced Template

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## 6 ★ Advanced Basic

### 6.1 Slow Multiplication

```
1 LL mul(LL a, LL b, LL p)
2 {
3     LL ans = 0;
4     while (b)
5     {
6         if (b & 1) ans = (ans + a) % p;
7         a = a * 2 % p; b >>= 1;
8     }
9     return ans;
10 }
```

### 6.2 Sum of Geometric Series

```
1 const int mod = 9901;
2 int a, b;
3 int qmi(int a, int k)
4 {
5     int res = 1;
6     a %= mod;
7     while (k)
8     {
9         if (k & 1)
10             res = res * a % mod;
11         a = a * a % mod;
12         k >>= 1;
13     }
14     return res;
15 }
16 int sum(int p, int k)
17 {
18     if (k == 1) return 1;
19     if (k % 2 == 0)
20         return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
21     return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
22 }
23 int main()
24 {
25     // 以  $a^b$  约数之和为例求等比数列和
26     cin >> a >> b;
27     int res = 1;
28     for (int i = 2; i <= a / i; i++)
29         if (a % i == 0)
30         {
31             int s = 0;
32             while (a % i == 0) a /= i, s++;
33             res = res * sum(i, b * s + 1) % mod;
34         }
35     if (a > 1) res = res * sum(a, b + 1) % mod;
36 }
```

## 6.3 Sort

### 6.3.1 Card Balancing Problem

```
1 cin >> n;
2 for (int i = 1; i <= n; i++)
3     cin >> a[i], avg += a[i];
4 avg /= n;
5 for (int i = 1; i <= n; i++)
6     if (a[i] != avg)
7         a[i + 1] += a[i] - avg, ans++;
8 cout << ans;
```

### 6.3.2 2D Card Balancing Problem

```
1 const int N = 100010;
2 int row[N], col[N], c[N], s[N];
3 LL work(int n, int a[])
4 {
5     for (int i = 1; i <= n; i++)
6         s[i] = s[i - 1] + a[i];
7     if (s[n] % n) return -1;
8     int avg = s[n] / n;
9     c[1] = 0;
10    for (int i = 2; i <= n; i++)
11        c[i] = s[i - 1] - (i - 1) * avg;
12    sort(c + 1, c + n + 1);
13    LL res = 0;
14    for (int i = 1; i <= n; i++)
15        res += abs(c[i] - c[(n + 1) / 2]);
16    return res;
17 }
18 int main()
19 {
20     int n, m, cnt;
21     cin >> n >> m >> cnt;
22     while (cnt--)
23     {
24         int x, y;
25         cin >> x >> y;
26         row[x]++; col[y]++;
27     }
28     LL r = work(n, row);
29     LL c = work(m, col);
30     if (r != -1 && c != -1)
31         cout << "both " << r + c;
32     else if (r != -1)
33         cout << "row " << r;
34     else if (c != -1)
35         cout << "column " << c;
36     else cout << "impossible";
37 }
```

### 6.3.3 Dual Heaps

```
1 if (down.empty() || x <= down.top())
2     down.push(x);
3 else up.push(x);
4 if (down.size() > up.size() + 1)
5     up.push(down.top(), down.pop());
```

```

6  if (up.size() > down.size())
7      down.push(up.top()), up.pop();
8  if (i % 2)
9  {
10     cout << down.top() << ' ';
11     if (++cnt % 10 == 0) cout << '\n';
12 }

```

## 6.4 RMQ

```

1  const int N = 200010, M = 18;
2  int n, m, w[N], f[N][M];
3  void init()
4  {
5      for (int j = 0; j < M; j++)
6          for (int i = 1; i + (1 << j) - 1 <= n;
7              i++)
8              if (!j) f[i][j] = w[i];
9              else // 也可以是最小值
10                 f[i][j] = max(f[i][j - 1], f[i
11                     + (1 << j - 1)][j - 1]);
12 }
13 int query(int l, int r)
14 {
15     int len = r - l + 1;
16     int k = log(len) / log(2);
17     return max(f[l][k], f[r - (1 << k) + 1][k
18         ]);
19 }

```

## 7 ★ Advanced Data Structures

### 7.1 Binary Indexed Tree

```
1 // 支持区间修改、区间查询
2 // 利用变差分求二阶区间和
3 const int N = 100010;
4 int n, m, a[N];
5 LL tr1[N], tr2[N];
6 int lowbit(int x) { return x & -x; }
7 void add(LL tr[], LL x, LL c)
8 {
9     for (int i = x; i <= n; i += lowbit(i))
10         tr[i] += c;
11 }
12 LL sum(LL tr[], LL x)
13 {
14     LL res = 0;
15     for (int i = x; i; i -= lowbit(i))
16         res += tr[i];
17     return res;
18 }
19 LL prefix_sum(LL x)
20 { return sum(tr1, x) * (x + 1) - sum(tr2, x); }
21 int main()
22 {
23     cin >> n >> m;
24     for (int i = 1; i <= n; i++)
25         cin >> a[i];
26     for (int i = 1; i <= n; i++)
27     {
28         int b = a[i] - a[i - 1];
29         add(tr1, i, b);
30         add(tr2, i, (LL)i * b);
31     }
32     while (m--)
33     {
34         char op[2];
35         int l, r, d;
36         cin >> op >> l >> r;
37         if (*op == 'Q')
38             cout << prefix_sum(r) - prefix_sum(l - 1) << '\n';
39         else
40         {
41             cin >> d;
42             add(tr1, l, d), add(tr2, l, (LL)l
43             * d),
44             add(tr1, r + 1, -d),
45             add(tr2, r + 1, (LL)-(r + 1) * d);
46         }
47     }
```

## 7.2 Segment Tree

### 7.2.1 Maintain the Maximum

```
1 struct Node
2 { int l, r, v; } tr[N * 4];
```

```
3 void pushup(int u)
4 {
5     tr[u].v = max(tr[u << 1].v, tr[u << 1 |
6     1].v);
7 }
8 void build(int u, int l, int r)
9 {
10     tr[u] = {l, r};
11     if (l == r) return;
12     int mid = l + r >> 1;
13     build(u << 1, l, mid),
14     build(u << 1 | 1, mid + 1, r);
15 }
16 int query(int u, int l, int r)
17 {
18     if (tr[u].l >= l && tr[u].r <= r)
19         return tr[u].v;
20     int mid = tr[u].l + tr[u].r >> 1;
21     int v = 0;
22     if (l <= mid)
23         v = query(u << 1, l, r);
24     if (r > mid)
25         v = max(v, query(u << 1 | 1, l, r));
26     return v;
27 }
28 void modify(int u, int x, int v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31         tr[u].v = v;
32     else
33     {
34         int mid = tr[u].l + tr[u].r >> 1;
35         if (x <= mid)
36             modify(u << 1, x, v);
37         else
38             modify(u << 1 | 1, x, v);
39         pushup(u);
40     }
```

### 7.2.2 Maintain the Maximum Subarray Sum

```
1 struct Node
2 { int l, r, sum, lmax, rmax, tmax; } tr[N *
3     4];
4 void pushup(Node &u, Node &l, Node &r)
5 {
6     u.sum = l.sum + r.sum;
7     u.lmax = max(l.lmax, l.sum + r.lmax);
8     u.rmax = max(r.rmax, r.sum + l.rmax);
9     u.tmax = max(max(l.tmax, r.tmax), l.rmax +
10     r.lmax);
11 }
12 void pushup(int u)
13 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
14 void build(int u, int l, int r)
15 {
16     if (l == r)
17         tr[u] = {l, r, w[r], w[r], w[r], w[r]
18         ]};
19     else
20     {
```

```

18     tr[u] = {l, r};
19     int mid = l + r >> 1;
20     build(u << 1, l, mid),
21     build(u << 1 | 1, mid + 1, r);
22     pushup(u);
23 }
24 }
25 void modify(int u, int x, int v)
26 {
27     if (tr[u].l == x && tr[u].r == x)
28         tr[u] = {x, x, v, v, v, v};
29     else
30     {
31         int mid = tr[u].l + tr[u].r >> 1;
32         if (x <= mid)
33             modify(u << 1, x, v);
34         else
35             modify(u << 1 | 1, x, v);
36         pushup(u);
37     }
38 }
39 Node query(int u, int l, int r)
40 {
41     if (tr[u].l >= l && tr[u].r <= r)
42         return tr[u];
43     else
44     {
45         int mid = tr[u].l + tr[u].r >> 1;
46         if (r <= mid)
47             return query(u << 1, l, r);
48         else if (l > mid)
49             return query(u << 1 | 1, l, r);
50         else
51         {
52             auto left = query(u << 1, l, r);
53             auto right = query(u << 1 | 1, l,
54                               r);
55             Node res;
56             pushup(res, left, right);
57             return res;
58         }
59     }

```

### 7.2.3 Maintain the GCD

```

1  struct Node
2  { int l, r; LL sum, d; } tr[N * 4];
3  LL gcd(LL a, LL b)
4  { return b ? gcd(b, a % b) : a; }
5  void pushup(Node &u, Node &l, Node &r)
6  {
7      u.sum = l.sum + r.sum;
8      u.d = gcd(l.d, r.d);
9  }
10 void pushup(int u)
11 { pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
12 void build(int u, int l, int r)
13 {
14     if (l == r)
15     {
16         LL b = w[r] - w[r - 1];
17         tr[u] = {l, r, b, b};

```

```

18     }
19     else
20     {
21         tr[u].l = l, tr[u].r = r;
22         int mid = l + r >> 1;
23         build(u << 1, l, mid),
24         build(u << 1 | 1, mid + 1, r);
25         pushup(u);
26     }
27 }
28 void modify(int u, int x, LL v)
29 {
30     if (tr[u].l == x && tr[u].r == x)
31     {
32         LL b = tr[u].sum + v;
33         tr[u] = {x, x, b, b};
34     }
35     else
36     {
37         int mid = tr[u].l + tr[u].r >> 1;
38         if (x <= mid)
39             modify(u << 1, x, v);
40         else
41             modify(u << 1 | 1, x, v);
42         pushup(u);
43     }
44 }
45 Node query(int u, int l, int r)
46 {
47     if (tr[u].l >= l && tr[u].r <= r)
48         return tr[u];
49     else
50     {
51         int mid = tr[u].l + tr[u].r >> 1;
52         if (r <= mid)
53             return query(u << 1, l, r);
54         else if (l > mid)
55             return query(u << 1 | 1, l, r);
56         else
57         {
58             auto left = query(u << 1, l, r);
59             auto right = query(u << 1 | 1, l,
60                               r);
61             Node res;
62             pushup(res, left, right);
63             return res;
64         }
65     }

```

### 7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```

1  struct Node
2  { int l, r; LL sum, add; } tr[N * 4];
3  void pushup(int u)
4  { tr[u].sum = tr[u << 1].sum + tr[u << 1 |
5    1].sum; }
6  void pushdown(int u)
7  {
8      auto &root = tr[u],
9      &left = tr[u << 1],

```

```

9      &right = tr[u << 1 | 1];
10  if (root.add)
11  {
12      left.add += root.add,
13      left.sum += (LL)(left.r - left.l + 1)
14      * root.add;
15      right.add += root.add,
16      right.sum += (LL)(right.r - right.l +
17      1) * root.add;
18      root.add = 0;
19  }
20  }
21  void build(int u, int l, int r)
22  {
23      if (l == r) tr[u] = {l, r, w[r], 0};
24      else
25      {
26          tr[u] = {l, r};
27          int mid = l + r >> 1;
28          build(u << 1, l, mid);
29          build(u << 1 | 1, mid + 1, r);
30          pushup(u);
31      }
32  }
33  void modify(int u, int l, int r, int d)
34  {
35      if (tr[u].l >= l && tr[u].r <= r)
36      {
37          tr[u].sum += (LL)(tr[u].r - tr[u].l +
38          1) * d;
39          tr[u].add += d;
40      }
41      else
42      {
43          pushdown(u);
44          int mid = tr[u].l + tr[u].r >> 1;
45          if (l <= mid)
46              modify(u << 1, l, r, d);
47          if (r > mid)
48              modify(u << 1 | 1, l, r, d);
49          pushup(u);
50      }
51  }
52  LL query(int u, int l, int r)
53  {
54      if (tr[u].l >= l && tr[u].r <= r)
55          return tr[u].sum;
56      pushdown(u);
57      int mid = tr[u].l + tr[u].r >> 1;
58      LL sum = 0;
59      if (l <= mid)
60          sum += query(u << 1, l, r);
61      if (r > mid)
62          sum += query(u << 1 | 1, l, r);
63      return sum;
64  }

```

## 7.3 Persistent Data Structure

### 7.3.1 Persistent Trie

```

1  const int N = 600010, M = N * 25;
2  int n, m, s[N], root[N], idx;

```

```

3  int trie[M][2], max_id[M];
4  void insert(int i, int k, int p, int q)
5  {
6      if (k < 0)
7      {
8          max_id[q] = i;
9          return;
10     }
11     int v = s[i] >> k & 1;
12     if (p)
13         trie[q][v ^ 1] = trie[p][v ^ 1];
14     trie[q][v] = ++idx;
15     insert(i, k - 1, trie[p][v], trie[q][v]);
16     max_id[q] = max(max_id[trie[q][0]], max_id
17     [trie[q][1]]);
18 }
19 int query(int root, int C, int L)
20 {
21     int p = root;
22     for (int i = 23; i >= 0; i--)
23     {
24         int v = C >> i & 1;
25         if (max_id[trie[p][v ^ 1]] >= L)
26             p = trie[p][v ^ 1];
27         else
28             p = trie[p][v];
29     }
30     return C ^ s[max_id[p]];
31 }
32 // insert(i, 23, root[i - 1], root[i]);
33 // query(root[r - 1], l - 1, x ^ s[n]);

```

### 7.3.2 Persistent Segment Tree

```

1  const int N = 100010, M = 10010;
2  int n, m, a[N], root[N], idx;
3  vector<int> nums;
4  struct Node
5  {
6      int l, r;
7      int cnt;
8  } tr[N * 4 + N * 17];
9  int find(int x)
10 {
11     return lower_bound(nums.begin(), nums.end
12     (), x) - nums.begin();
13 }
14 int build(int l, int r)
15 {
16     int p = ++idx;
17     if (l == r)
18         return p;
19     int mid = l + r >> 1;
20     tr[p].l = build(l, mid), tr[p].r = build(
21     mid + 1, r);
22     return p;
23 }
24 int insert(int p, int l, int r, int x)
25 {
26     int q = ++idx;
27     tr[q] = tr[p];
28     if (l == r)
29     {
30         tr[q].cnt++;
31     }
32     else
33     {
34         int mid = l + r >> 1;
35         if (x <= mid)
36             tr[q].l = insert(tr[p].l, l, mid, x);
37         else
38             tr[q].r = insert(tr[p].r, mid + 1, r, x);
39     }
40     return q;
41 }
42 int query(int p, int l, int r, int x)
43 {
44     if (l == r)
45         return tr[p].cnt;
46     int mid = l + r >> 1;
47     if (x <= mid)
48         return query(tr[p].l, l, mid, x);
49     else
50         return query(tr[p].r, mid + 1, r, x);
51 }

```

```

29     return q;
30 }
31 int mid = l + r >> 1;
32 if (x <= mid)
33     tr[q].l = insert(tr[p].l, l, mid, x);
34 else
35     tr[q].r = insert(tr[p].r, mid + 1, r,
36                     x);
37 tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].
38 cnt;
39 return q;
40 }
41 int query(int q, int p, int l, int r, int k)
42 {
43     if (l == r)
44         return r;
45     int cnt = tr[tr[q].l].cnt - tr[tr[p].l].
46 cnt;
47 int mid = l + r >> 1;
48 if (k <= cnt)
49     return query(tr[q].l, tr[p].l, l, mid,
50 k);
51 else
52     return query(tr[q].r, tr[p].r, mid +
53 1, r, k - cnt);
54 }

```

## 7.4 Treap

```

1  const int N = 100010, INF = 1e8;
2  int n, root, idx;
3  struct Node
4  { int l, r, key, val, cnt, size; } tr[N];
5  void pushup(int p)
6  {
7      tr[p].size = tr[tr[p].l].size +
8      tr[tr[p].r].size + tr[p].cnt;
9  }
10 int get_node(int key)
11 {
12     tr[++idx].key = key;
13     tr[idx].val = rand();
14     tr[idx].cnt = tr[idx].size = 1;
15     return idx;
16 }
17 void zig(int &p)
18 {
19     int q = tr[p].l;
20     tr[p].l = tr[q].r, tr[q].r = p, p = q;
21     pushup(tr[p].r), pushup(p);
22 }
23 void zag(int &p)
24 {
25     int q = tr[p].r;
26     tr[p].r = tr[q].l, tr[q].l = p, p = q;
27     pushup(tr[p].l), pushup(p);
28 }
29 void build()
30 {
31     get_node(-INF), get_node(INF);
32     root = 1, tr[1].r = 2;
33     pushup(root);
34     if (tr[1].val < tr[2].val) zag(root);

```

```

35 }
36 void insert(int &p, int key)
37 {
38     if (!p) p = get_node(key);
39     else if (tr[p].key == key) tr[p].cnt++;
40     else if (tr[p].key > key)
41     {
42         insert(tr[p].l, key);
43         if (tr[tr[p].l].val > tr[p].val)
44             zig(p);
45     }
46     else
47     {
48         insert(tr[p].r, key);
49         if (tr[tr[p].r].val > tr[p].val)
50             zag(p);
51     }
52     pushup(p);
53 }
54 void remove(int &p, int key)
55 {
56     if (!p) return;
57     if (tr[p].key == key)
58     {
59         if (tr[p].cnt > 1) tr[p].cnt--;
60         else if (tr[p].l || tr[p].r)
61         {
62             if (!tr[p].r || tr[tr[p].l].val >
63 tr[tr[p].r].val)
64             {
65                 zig(p);
66                 remove(tr[p].r, key);
67             }
68             else
69             {
70                 zag(p);
71                 remove(tr[p].l, key);
72             }
73         }
74         else p = 0;
75     }
76     else if (tr[p].key > key)
77         remove(tr[p].l, key);
78     else remove(tr[p].r, key);
79     pushup(p);
80 }
81 int get_rank_by_key(int p, int key)
82 {
83     if (!p) return 0;
84     if (tr[p].key == key)
85         return tr[tr[p].l].size + 1;
86     if (tr[p].key > key)
87         return get_rank_by_key(tr[p].l, key);
88     return tr[tr[p].l].size + tr[p].cnt +
89 get_rank_by_key(tr[p].r, key);
90 }
91 int get_key_by_rank(int p, int rank)
92 {
93     if (!p) return INF;
94     if (tr[tr[p].l].size >= rank)
95         return get_key_by_rank(tr[p].l, rank);
96     if (tr[tr[p].l].size + tr[p].cnt >= rank)
97         return tr[p].key;
98     return get_key_by_rank(tr[p].r, rank - tr[
99 tr[p].l].size - tr[p].cnt);
100 }

```



```

98 int get_prev(int p, int key)
99 {
100     if (!p) return -INF;
101     if (tr[p].key >= key)
102         return get_prev(tr[p].l, key);
103     return max(tr[p].key, get_prev(tr[p].r,
104                                     key));
105 }
106 int get_next(int p, int key)
107 {
108     if (!p) return INF;
109     if (tr[p].key <= key)
110         return get_next(tr[p].r, key);
111     return min(tr[p].key, get_next(tr[p].l,
112                                     key));
113 }

```

## 7.5 AC Automaton

```

1  const int N = 10010, M = 1000010, S = 55;
2  int n, tr[N * S][26], cnt[N * S], idx;
3  int q[N * S], ne[N * S];
4  char str[M];
5  void insert()
6  {
7      int p = 0;
8      for (int i = 0; str[i]; i++)
9      {
10         int t = str[i] - 'a';
11         if (!tr[p][t]) tr[p][t] = ++idx;
12         p = tr[p][t];
13     }
14     cnt[p]++;
15 }
16 void build()
17 {
18     int hh = 0, tt = -1;
19     for (int i = 0; i < 26; i++)
20         if (tr[0][i]) q[++tt] = tr[0][i];
21     while (hh <= tt)
22     {
23         int t = q[hh++];
24         for (int i = 0; i < 26; i++)
25         {
26             int p = tr[t][i];
27             if (!p) tr[t][i] = tr[ne[t]][i];
28             else
29             {
30                 ne[p] = tr[ne[t]][i];
31                 q[++tt] = p;
32             }
33         }
34     }
35 }

```

## 8 ★ Advanced Search

### 8.1 Flood-Fill

```
1  const int N = 1010, M = N * N;
2  int n, m;
3  char g[N][N];
4  PII q[M];
5  bool st[N][N];
6  void bfs(int sx, int sy)
7  {
8      int hh = 0, tt = 0;
9      q[0] = {sx, sy}; st[sx][sy] = true;
10     while (hh <= tt)
11     {
12         PII t = q[hh++];
13         for (int i = t.first - 1; i <= t.first + 1; i++)
14             for (int j = t.second - 1; j <= t.second + 1; j++)
15             {
16                 if (i == t.first && j == t.second)
17                     continue;
18                 if (i < 0 || i >= n || j < 0 || j >= m)
19                     continue;
20                 if (g[i][j] == '.' || st[i][j])
21                     continue;
22                 q[++tt] = {i, j};
23                 st[i][j] = true;
24             }
25     }
26 }
27 int main()
28 {
29     int cnt = 0;
30     for (int i = 0; i < n; i++)
31         for (int j = 0; j < m; j++)
32             if (g[i][j] == 'W' && !st[i][j])
33                 { bfs(i, j); cnt++; }
34 }
```

### 8.2 Multi-source BFS

```
1  const int N = 1010, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  PII q[M];
5  int dx[4] = {-1, 0, 1, 0},
6      dy[4] = {0, 1, 0, -1};
7  void bfs()
8  {
9      memset(dist, -1, sizeof dist);
10     int hh = 0, tt = -1;
11     for (int i = 1; i <= n; i++)
12         for (int j = 1; j <= m; j++)
13             if (g[i][j] == '1')
14             {
15                 dist[i][j] = 0;
```

```
16                 q[++tt] = {i, j};
17             }
18     while (hh <= tt)
19     {
20         auto t = q[hh++];
21         for (int i = 0; i < 4; i++)
22         {
23             int a = t.x + dx[i], b = t.y + dy[i];
24             if (a < 1 || a > n || b < 1 || b > m) continue;
25             if (dist[a][b] != -1) continue;
26             dist[a][b] = dist[t.x][t.y] + 1;
27             q[++tt] = {a, b};
28         }
29     }
30 }
```

### 8.3 BFS with Deque

```
1  const int N = 510, M = N * N;
2  int n, m, dist[N][N];
3  char g[N][N];
4  bool st[N][N];
5  int dx[4] = {-1, -1, 1, 1},
6      dy[4] = {-1, 1, 1, -1},
7      ix[4] = {-1, -1, 0, 0},
8      iy[4] = {-1, 0, 0, -1};
9  int bfs()
10 {
11     memset(dist, 0x3f, sizeof dist);
12     memset(st, 0, sizeof st);
13     dist[0][0] = 0;
14     deque<PII> q;
15     q.push_back({0, 0});
16     char cs[] = "\\ / \\/";
17     while (q.size())
18     {
19         PII t = q.front();
20         q.pop_front();
21         if (st[t.x][t.y]) continue;
22         st[t.x][t.y] = true;
23         for (int i = 0; i < 4; i++)
24         {
25             int a = t.x + dx[i], b = t.y + dy[i];
26             if (a < 0 || a > n || b < 0 || b > m) continue;
27             int ca = t.x + ix[i], cb = t.y + iy[i];
28             int d = dist[t.x][t.y] + (g[ca][cb] != cs[i]);
29             if (d < dist[a][b])
30             {
31                 dist[a][b] = d;
32                 if (g[ca][cb] != cs[i])
33                     q.push_back({a, b});
34                 else
35                     q.push_front({a, b});
36             }
37         }
38     }
39 }
40 return dist[n][m];
```

```
41 }
```

## 8.4 Bidirectional BFS

```
1 int bfs()
2 {
3     if (A == B) return 0;
4     queue<string> qa, qb;
5     unordered_map<string, int> da, db;
6     qa.push(A), qb.push(B);
7     da[A] = db[B] = 0;
8     int step = 0;
9     while (qa.size() && qb.size())
10    {
11        int t;
12        if (qa.size() < qb.size())
13            // PROCESS
14        else
15            // PROCESS
16        if (t <= 10) return t;
17        if (++step == 10) return -1;
18    }
19    return -1;
20 }
```

## 8.5 A\*

```
1 const int N = 1010, M = 200010;
2 int n, m, S, T, K;
3 int h[N], rh[N], e[M], w[M], ne[M], idx;
4 int dist[N], cnt[N];
5 bool st[N];
6 void dijkstra()
7 {
8     priority_queue<PII, vector<PII>, greater<
9         PII>> heap;
10    heap.push({0, T});
11    memset(dist, 0x3f, sizeof dist);
12    dist[T] = 0;
13    while (heap.size())
14    {
15        auto t = heap.top();
16        heap.pop();
17        int ver = t.y;
18        if (st[ver]) continue;
19        st[ver] = true;
20        for (int i = rh[ver]; ~i; i = ne[i])
21        {
22            int j = e[i];
23            if (dist[j] > dist[ver] + w[i])
24            {
25                dist[j] = dist[ver] + w[i];
26                heap.push({dist[j], j});
27            }
28        }
29    }
30 }
31 int astar()
32 {
```

```
33     priority_queue<PIII, vector<PIII>, greater
34         <PIII>> heap;
35     heap.push({dist[S], {0, S}});
36     while (heap.size())
37     {
38         auto t = heap.top();
39         heap.pop();
40         int ver = t.y.y, distance = t.y.x;
41         cnt[ver]++;
42         if (cnt[T] == K) return distance;
43         for (int i = h[ver]; ~i; i = ne[i])
44         {
45             int j = e[i];
46             if (cnt[j] < K)
47                 heap.push({distance + w[i] +
48                     dist[j], {distance + w[i], j}});
49         }
50     }
51     return -1;
52 }
53 int main()
54 {
55     // PROCESS
56     dijkstra(); cout << astar();
57     // PROCESS
58 }
```

## 8.6 DFS Connectivity Model

```
1 char g[N][N];
2 int xa, ya, xb, yb;
3 int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0,
4     -1};
5 bool st[N][N];
6 bool dfs(int x, int y)
7 {
8     if (g[x][y] == '#') return false;
9     if (x == xb && y == yb) return true;
10    st[x][y] = true;
11    for (int i = 0; i < 4; i++)
12    {
13        int a = x + dx[i], b = y + dy[i];
14        if (a < 0 || a >= n || b < 0 || b >= n
15            ) continue;
16        if (st[a][b]) continue;
17        if (dfs(a, b)) return true;
18    }
19    return false;
20 }
```

## 8.7 IDDFS

```
1 const int N = 110;
2 int n, path[N];
3 bool dfs(int u, int k)
4 {
5     if (u == k)
6         return path[u - 1] == n;
7     bool st[N] = {0};
8     for (int i = u - 1; i >= 0; i--)
```

```

9     for (int j = i; j >= 0; j--)
10    {
11        int s = path[i] + path[j];
12        if (s > n || s <= path[u - 1] ||
13            st[s]) continue;
14        st[s] = true;
15        path[u] = s;
16        if (dfs(u + 1, k)) return true;
17    }
18    return false;
19 }

```

```

9 {
10     if (f() > maxn - depth) return false;
11     if (depth == maxn) return true;
12     for (int i = 0; i <= n; i++)
13     {
14         // OPERATION
15         if (IDAstar(depth + 1, maxn))
16             return true;
17         // OPERATION
18     }
19     return false;
20 }

```

## 8.8 Bidirectional DFS

```

1  const int N = 1 << 24;
2  int n, m, k, cnt = 0, ans;
3  int g[50], weights[N];
4  void dfs(int u, int s)
5  {
6      if (u == k)
7      {
8          weights[cnt++] = s;
9          return;
10     }
11     if ((LL)s + g[u] <= m)
12         dfs(u + 1, s + g[u]);
13     dfs(u + 1, s);
14 }
15 void dfs2(int u, int s)
16 {
17     if (u == n)
18     {
19         int l = 0, r = cnt - 1;
20         while (l < r)
21         {
22             int mid = l + r + 1 >> 1;
23             if (weights[mid] + (LL)s <= m)
24                 l = mid;
25             else r = mid - 1;
26         }
27         if (weights[l] + (LL)s <= m)
28             ans = max(ans, weights[l] + s);
29         return;
30     }
31     if ((LL)s + g[u] <= m)
32         dfs2(u + 1, s + g[u]);
33     dfs2(u + 1, s);
34 }

```

## 8.9 IDA\*

```

1  const int N = 1e2;
2  int n, a[N];
3  string t;
4  int f()
5  {
6      // YOUR_F_FUNCTION
7  }
8  bool IDAstar(int depth, int maxn)

```

## 9 ★ Advanced Graph Theory

### 9.1 Detecting Negative Cycles

```
1  int n, m1, m2;
2  int h[N], e[M], w[M], ne[M], idx;
3  int dist[N], q[N], cnt[N];
4  bool st[N];
5  bool spfa()
6  {
7      memset(dist, 0, sizeof dist);
8      memset(cnt, 0, sizeof cnt);
9      memset(st, 0, sizeof st);
10     int hh = 0, tt = 0;
11     for (int i = 1; i <= n; i++)
12     {
13         q[tt++] = i;
14         st[i] = true;
15     }
16     while (hh != tt)
17     {
18         int t = q[hh++];
19         if (hh == N) hh = 0;
20         st[t] = false;
21         for (int i = h[t]; ~i; i = ne[i])
22         {
23             int j = e[i];
24             if (dist[j] > dist[t] + w[i])
25             {
26                 dist[j] = dist[t] + w[i];
27                 cnt[j] = cnt[t] + 1;
28                 if (cnt[j] >= n) return true;
29                 if (!st[j])
30                 {
31                     q[tt++] = j;
32                     if (tt == N) tt = 0;
33                     st[j] = true;
34                 }
35             }
36         }
37     }
38     return false;
39 }
```

### 9.2 SPFA-SLF

Using deque to solve SPFA question.

```
1  void spfa()
2  {
3      memset(dist, 0x3f, sizeof dist);
4      memset(st, 0, sizeof st);
5      deque<int> q;
6      q.push_back(s);
7      st[s] = 1, dist[s] = 0;
8      while (q.size())
9      {
10         int t = q.front();
11         q.pop_front();
12         st[t] = 0;
13         for (int i = h[t]; ~i; i = ne[i])
14         {
```

```
15         int j = e[i];
16         if (dist[j] > dist[t] + w[i])
17         {
18             dist[j] = dist[t] + w[i];
19             if (!st[j])
20             {
21                 st[j] = true;
22                 if (q.size() && dist[j] < dist[q.
23                     front()])
24                     q.push_front(j);
25                 else
26                     q.push_back(j);
27             }
28         }
29     }
30 }
```

### 9.3 SPFA-Stack

```
1  bool spfa()
2  {
3      int hh = 0, tt = 1;
4      memset(dist, -0x3f, sizeof dist);
5      dist[0] = 0;
6      q[0] = 0;
7      while (hh != tt)
8      {
9          int t = q[--tt];
10         st[t] = false;
11         for (int i = h[t]; ~i; i = ne[i])
12         {
13             int j = e[i];
14             if (dist[j] < dist[t] + w[i])
15             {
16                 dist[j] = dist[t] + w[i];
17                 cnt[j] = cnt[t] + 1;
18                 if (cnt[j] >= n + 1) return true;
19                 if (!st[j])
20                 {
21                     st[j] = true;
22                     q[tt++] = j;
23                 }
24             }
25         }
26     }
27     return false;
28 }
```

### 9.4 SPFA & MIN & MAX

Using SPFA to maintain the minimum and maximum. In this case we need **Original Graph** and **Reverse Graph**, in which we can use `type == 0` or `type == 1` to describe.

```
1  void spfa(int h[], int dist[], int type)
2  {
3      int hh = 0, tt = 1;
4      if (type == 0)
5      {
```

```

6  memset(dist, 0x3f, sizeof dmin);
7  dist[1] = w[1];
8  q[0] = 1;
9  }
10 else
11 {
12  memset(dist, -0x3f, sizeof dmax);
13  dist[n] = w[n];
14  q[0] = n;
15  }
16  while (hh != tt)
17  {
18  int t = q[hh++];
19  if (hh == N) hh = 0;
20  st[t] = false;
21  for (int i = h[t]; ~i; i = ne[i])
22  {
23  int j = e[i];
24  if (type == 0 && dist[j] > min(dist[t],
25  w[j]) || type == 1 && dist[j] < max(dist
26  [t], w[j]))
27  {
28  if (type == 0)
29  dist[j] = min(dist[t], w[j]);
30  else
31  dist[j] = max(dist[t], w[j]);
32  if (!st[j])
33  {
34  q[tt++] = j;
35  if (tt == N) tt = 0;
36  st[j] = true;
37  }
38  }
39  }

```

```

Ver>> heap;
24 heap.push({S, 0, 0});
25 while (heap.size())
26 {
27 Ver t = heap.top();
28 heap.pop();
29 int ver = t.id, type = t.type, distance =
30 t.dist, count = cnt[ver][type];
31 if (st[ver][type])
32 continue;
33 st[ver][type] = true;
34 for (int i = h[ver]; ~i; i = ne[i])
35 {
36 int j = e[i];
37 if (dist[j][0] > distance + w[i])
38 {
39 dist[j][1] = dist[j][0], cnt[j][1] =
40 cnt[j][0];
41 heap.push({j, 1, dist[j][1]});
42 dist[j][0] = distance + w[i], cnt[j]
43 [0] = count;
44 heap.push({j, 0, dist[j][0]});
45 }
46 else if (dist[j][0] == distance + w[i])
47 cnt[j][0] += count;
48 else if (dist[j][1] > distance + w[i])
49 {
50 dist[j][1] = distance + w[i], cnt[j]
51 [1] = count;
52 heap.push({j, 1, dist[j][1]});
53 }
54 else if (dist[j][1] == distance + w[i])
55 cnt[j][1] += count;
56 }
57 }
58 int res = cnt[T][0];
59 if (dist[T][0] + 1 == dist[T][1])
60 res += cnt[T][1];
61 return res;

```

## 9.5 Second Shortest Path

```

1  const int N = 1010, M = 20010;
2  struct Ver
3  {
4  int id, type, dist;
5  bool operator>(const Ver &W) const
6  {
7  return dist > W.dist;
8  }
9  };
10 int n, m, S, T, dist[N][2], cnt[N][2];
11 int h[N], e[M], w[M], ne[M], idx;
12 bool st[N][2];
13 void add(int a, int b, int c)
14 {
15  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
16  a] = idx++;
17 }
18 int dijkstra()
19 {
20  memset(st, 0, sizeof st);
21  memset(dist, 0x3f, sizeof dist);
22  memset(cnt, 0, sizeof cnt);
23  dist[S][0] = 0, cnt[S][0] = 1;
24  priority_queue<Ver, vector<Ver>, greater<

```

## 9.6 Second Minimum Spanning Tree

### 9.6.1 brute-force

```

1  const int N = 510, M = 10010;
2  int n, m, p[N], dist1[N][N], dist2[N][N];
3  int h[N], e[N * 2], w[N * 2], ne[N * 2], idx
4  ;
5  struct Edge
6  {
7  int a, b, w;
8  bool f;
9  bool operator<(const Edge &e) const
10 { return w < e.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 {
14  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
15  a] = idx++;
16 }
17 int find(int x)

```

```

16 {
17     if (p[x] != x) p[x] = find(p[x]);
18     return p[x];
19 }
20 void dfs(int u, int fa, int maxd1, int maxd2
    , int d1[], int d2[])
21 {
22     d1[u] = maxd1, d2[u] = maxd2;
23     for (int i = h[u]; ~i; i = ne[i])
24     {
25         int j = e[i];
26         if (j != fa)
27         {
28             int td1 = maxd1, td2 = maxd2;
29             if (w[i] > td1)
30                 td2 = td1, td1 = w[i];
31             else if (w[i] < td1 && w[i] > td2)
32                 td2 = w[i];
33             dfs(j, u, td1, td2, d1, d2);
34         }
35     }
36 }
37 int main()
38 {
39     cin >> n >> m;
40     memset(h, -1, sizeof h);
41     for (int i = 0; i < m; i++)
42         cin >> edge[i].a >> edge[i].b >> edge[i].w
43         ;
44     sort(edge, edge + m);
45     for (int i = 1; i <= n; i++) p[i] = i;
46     LL sum = 0;
47     for (int i = 0; i < m; i++)
48     {
49         int a = edge[i].a, b = edge[i].b, w = edge
            [i].w;
50         int pa = find(a), pb = find(b);
51         if (pa != pb)
52         {
53             p[pa] = pb;
54             sum += w;
55             add(a, b, w), add(b, a, w);
56             edge[i].f = true;
57         }
58     }
59     for (int i = 1; i <= n; i++)
60         dfs(i, -1, -1e9, -1e9, dist1[i], dist2[i])
61         ;
62     LL res = 1e18;
63     for (int i = 0; i < m; i++)
64     {
65         if (!edge[i].f)
66         {
67             int a = edge[i].a, b = edge[i].b, w =
                edge[i].w;
68             LL t;
69             if (w > dist1[a][b])
70                 t = sum + w - dist1[a][b];
71             else if (w > dist2[a][b])
72                 t = sum + w - dist2[a][b];
73             res = min(res, t);
74         }
75     }
76 }

```

## 9.6.2 LCA

```

1  const int N = 100010, M = 300010;
2  int n, m, p[N], q[N];
3  int h[N], e[M], w[M], ne[M], idx;
4  int depth[N], fa[N][17], d1[N][17], d2[N]
    [17];
5  struct Edge
6  {
7      int a, b, w;
8      bool used;
9      bool operator<(const Edge &t) const
10     { return w < t.w; }
11 } edge[M];
12 void add(int a, int b, int c)
13 { e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
    a] = idx++; }
14 int find(int x)
15 {
16     if (p[x] != x) p[x] = find(p[x]);
17     return p[x];
18 }
19 LL kruskal()
20 {
21     for (int i = 1; i <= n; i++) p[i] = i;
22     sort(edge, edge + m);
23     LL res = 0;
24     for (int i = 0; i < m; i++)
25     {
26         int a = find(edge[i].a), b = find(edge[i].
            b), w = edge[i].w;
27         if (a != b)
28         {
29             p[a] = b; res += w;
30             edge[i].used = true;
31         }
32     }
33     return res;
34 }
35 void build()
36 {
37     memset(h, -1, sizeof h);
38     for (int i = 0; i < m; i++)
39         if (edge[i].used)
40         {
41             int a = edge[i].a, b = edge[i].b, w =
                edge[i].w;
42             add(a, b, w), add(b, a, w);
43         }
44 }
45 void bfs()
46 {
47     memset(depth, 0x3f, sizeof depth);
48     depth[0] = 0, depth[1] = 1, q[0] = 1;
49     int hh = 0, tt = 0;
50     while (hh <= tt)
51     {
52         int t = q[hh++];
53         for (int i = h[t]; ~i; i = ne[i])
54         {
55             int j = e[i];
56             if (depth[j] > depth[t] + 1)
57             {
58                 depth[j] = depth[t] + 1;
59                 q[++tt] = j;
60                 fa[j][0] = t;
61                 d1[j][0] = w[i], d2[j][0] = -INF;

```

```

62     for (int k = 1; k <= 16; k++)
63     {
64         int anc = fa[j][k - 1];
65         fa[j][k] = fa[anc][k - 1];
66         int distance[4] = {d1[j][k - 1],
67                             d2[j][k - 1],
68                             d1[anc][k - 1],
69                             d2[anc][k - 1]};
70         d1[j][k] = d2[j][k] = -INF;
71         for (int u = 0; u < 4; u++)
72         {
73             int d = distance[u];
74             if (d > d1[j][k])
75                 d2[j][k] = d1[j][k], d1[j][k] =
76                 d;
77             else if (d != d1[j][k] && d > d2[j]
78                 [k])
79                 d2[j][k] = d;
80         }
81     }
82 }
83 }
84 int lca(int a, int b, int w)
85 {
86     static int distance[N * 2];
87     int cnt = 0;
88     if (depth[a] < depth[b])
89         swap(a, b);
90     for (int k = 16; k >= 0; k--)
91     if (depth[fa[a][k]] >= depth[b])
92     {
93         distance[cnt++] = d1[a][k];
94         distance[cnt++] = d2[a][k];
95         a = fa[a][k];
96     }
97     if (a != b)
98     {
99         for (int k = 16; k >= 0; k--)
100         if (fa[a][k] != fa[b][k])
101         {
102             distance[cnt++] = d1[a][k];
103             distance[cnt++] = d2[a][k];
104             distance[cnt++] = d1[b][k];
105             distance[cnt++] = d2[b][k];
106             a = fa[a][k], b = fa[b][k];
107         }
108         distance[cnt++] = d1[a][0];
109         distance[cnt++] = d1[b][0];
110     }
111     int dist1 = -INF, dist2 = -INF;
112     for (int i = 0; i < cnt; i++)
113     {
114         int d = distance[i];
115         if (d > dist1)
116             dist2 = dist1, dist1 = d;
117         else if (d != dist1 && d > dist2)
118             dist2 = d;
119     }
120     if (w > dist1) return w - dist1;
121     if (w > dist2) return w - dist2;
122     return INF;
123 }
124 int main()
125 {

```

```

126     cin >> n >> m;
127     for (int i = 0; i < m; i++)
128     {
129         int a, b, c;
130         cin >> a >> b >> c;
131         edge[i] = {a, b, c};
132     }
133     LL sum = kruskal();
134     build();
135     bfs();
136     LL res = 1e18;
137     for (int i = 0; i < m; i++)
138     if (!edge[i].used)
139     {
140         int a = edge[i].a, b = edge[i].b, w =
141         edge[i].w;
142         res = min(res, sum + lca(a, b, w));
143     }
144     cout << res;
145 }

```

## 9.7 Difference Constraints

- **size == N:** Feasible Solution
- **size == 1:** Maximum/Minimum
- **Maximum:** Shortest Path
- **Minimum:** Longest Path

### 9.7.1 Maximum-Shortest Path

```

1  bool spfa(int size)
2  {
3      int hh = 0, tt = 0;
4      memset(dist, 0x3f, sizeof dist);
5      memset(st, 0, sizeof st);
6      memset(cnt, 0, sizeof cnt);
7      for (int i = 1; i <= size; i++)
8      {
9          q[tt++] = i;
10         dist[i] = 0;
11         st[i] = true;
12     }
13     while (hh != tt)
14     {
15         int t = q[hh++];
16         if (hh == N) hh = 0;
17         st[t] = false;
18         for (int i = h[t]; ~i; i = ne[i])
19         {
20             int j = e[i];
21             if (dist[j] > dist[t] + w[i])
22             {
23                 dist[j] = dist[t] + w[i];
24                 cnt[j] = cnt[t] + 1;
25                 if (cnt[j] >= n) return true;
26                 if (!st[j])
27                 {
28                     st[j] = true;
29                     q[tt++] = j;
30                     if (tt == N) tt = 0;
31                 }

```



```

32     }
33 }
34 }
35 return false;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_b <= x_a + k
40     // PROCESS
41 }

```

## 9.7.2 Minimum-Longest Path

```

1 bool spfa(int size)
2 {
3     int hh = 0, tt = 0;
4     memset(dist, -0x3f, sizeof dist);
5     memset(st, 0, sizeof st);
6     memset(cnt, 0, sizeof cnt);
7     for (int i = 1; i <= size; i++)
8     {
9         q[tt++] = i;
10        dist[i] = 0;
11        st[i] = true;
12    }
13    while (hh != tt)
14    {
15        int t = q[hh++];
16        if (hh == N) hh = 0;
17        st[t] = false;
18        for (int i = h[t]; ~i; i = ne[i])
19        {
20            int j = e[i];
21            if (dist[j] < dist[t] + w[i])
22            {
23                dist[j] = dist[t] + w[i];
24                cnt[j] = cnt[t] + 1;
25                if (cnt[j] >= n) return false;
26                if (!st[j])
27                {
28                    st[j] = true;
29                    q[tt++] = j;
30                    if (tt == N) tt = 0;
31                }
32            }
33        }
34    }
35    return true;
36 }
37 int main()
38 {
39     // add(a, b, k) means x_a + k <= x_b
40     // PROCESS
41 }

```

## 9.8 LCA

```

1 int n, m, h[N], e[M], ne[M], idx;
2 int depth[N], fa[N][16], q[N];
3 void bfs(int root)
4 {

```

```

5     memset(depth, 0x3f, sizeof depth);
6     depth[0] = 0;
7     depth[root] = 1;
8     int hh = 0, tt = 0;
9     q[0] = root;
10    while (hh <= tt)
11    {
12        int t = q[hh++];
13        for (int i = h[t]; ~i; i = ne[i])
14        {
15            int j = e[i];
16            if (depth[j] > depth[t] + 1)
17            {
18                depth[j] = depth[t] + 1;
19                q[++tt] = j;
20                fa[j][0] = t;
21                for (int k = 1; k <= 15; k++)
22                    fa[j][k] = fa[fa[j][k-1]][k-1];
23            }
24        }
25    }
26 }
27 int lca(int a, int b)
28 {
29     if (depth[a] < depth[b]) swap(a, b);
30     for (int k = 15; k >= 0; k--)
31         if (depth[fa[a][k]] >= depth[b])
32             a = fa[a][k];
33     if (a == b) return a;
34     for (int k = 15; k >= 0; k--)
35         if (fa[a][k] != fa[b][k])
36         {
37             a = fa[a][k];
38             b = fa[b][k];
39         }
40     return fa[a][0];
41 }

```

## 9.9 SCC

```

1 void tarjan(int u)
2 {
3     dfn[u] = low[u] = ++timestamp;
4     stack[++top] = u, in_stk[u] = true;
5     for (int i = h[u]; ~i; i = ne[i])
6     {
7         int j = e[i];
8         if (!dfn[j])
9         {
10            tarjan(j);
11            low[u] = min(low[u], low[j]);
12        }
13        else if (in_stk[j])
14            low[u] = min(low[u], dfn[j]);
15    }
16    if (dfn[u] == low[u])
17    {
18        int y;
19        ++scc_cnt;
20        do
21        {
22            y = stk[top--];
23            in_stk[y] = false;

```

```

24     id[y] = scc_cnt;
25 } while (y != u);
26 }
27 }

```

## 9.10 DCC

### 9.10.1 e-DCC

```

1  const int N = 5010, M = 20010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, id[N], dcc_cnt, d[N];
5  bool is_bridge[M];
6  void tarjan(int u, int from)
7  {
8      dfn[u] = low[u] = ++timestamp;
9      stk[++top] = u;
10     for (int i = h[u]; ~i; i = ne[i])
11     {
12         int j = e[i];
13         if (!dfn[j])
14         {
15             tarjan(j, i);
16             low[u] = min(low[u], low[j]);
17             if (dfn[u] < low[j])
18                 is_bridge[i] = is_bridge[i ^ 1] = true;
19         }
20         else if (i != (from ^ 1))
21             low[u] = min(low[u], dfn[j]);
22     }
23     if (dfn[u] == low[u])
24     {
25         ++dcc_cnt;
26         int y;
27         do
28         {
29             y = stk[top--];
30             id[y] = dcc_cnt;
31         } while (y != u);
32     }
33 }

```

### 9.10.2 v-DCC

```

1  const int N = 1010, M = 1010;
2  int n, m, h[N], e[M], ne[M], idx;
3  int dfn[N], low[N], timestamp;
4  int stk[N], top, dcc_cnt, root;
5  vector<int> dcc[N];
6  bool cut[N];
7  void init()
8  {
9      for (int i = 1; i <= dcc_cnt; i++)
10         dcc[i].clear();
11     idx = n = timestamp = top = dcc_cnt = 0;
12     memset(h, -1, sizeof h);
13     memset(dfn, 0, sizeof dfn);
14     memset(cut, 0, sizeof cut);
15 }

```

```

16 void tarjan(int u)
17 {
18     dfn[u] = low[u] = ++timestamp;
19     stk[++top] = u;
20     if (u == root && h[u] == -1)
21     {
22         dcc_cnt++;
23         dcc[dcc_cnt].push_back(u);
24         return;
25     }
26     int cnt = 0;
27     for (int i = h[u]; ~i; i = ne[i])
28     {
29         int j = e[i];
30         if (!dfn[j])
31         {
32             tarjan(j);
33             low[u] = min(low[u], low[j]);
34             if (dfn[u] <= low[j])
35             {
36                 cnt++;
37                 if (u != root || cnt > 1)
38                     cut[u] = true;
39                 ++dcc_cnt;
40                 int y;
41                 do
42                 {
43                     y = stk[top--];
44                     dcc[dcc_cnt].push_back(y);
45                 } while (y != j);
46                 dcc[dcc_cnt].push_back(u);
47             }
48         }
49         else
50             low[u] = min(low[u], dfn[j]);
51     }
52 }

```

## 9.11 Bipartite Graph

The maximum matching  
(by the Hungarian algorithm) =  
the minimum vertex cover =  
total number of vertices -  
maximum independent set =  
total number of vertices -  
minimum path cover.

### 9.11.1 maximum matching

```

1  const int N = 110;
2  int n, m;
3  int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
4  PII match[N][N];
5  bool g[N][N], st[N][N];
6  bool find(int x, int y)
7  {
8      for (int i = 0; i < 4; i++)
9      {
10         int a = x + dx[i], b = y + dy[i];
11         if (a && a <= n && b && b <= n && !g[a][b] && !st[a][b])

```

```

12 {
13     st[a][b] = true;
14     PII t = match[a][b];
15     if (t.x == -1 || find(t.x, t.y))
16     {
17         match[a][b] = {x, y};
18         return true;
19     }
20 }
21 }
22 return false;
23 }
24 int main()
25 {
26     // PROCESS
27     memset(match, -1, sizeof match);
28     int res = 0;
29     for (int i = 1; i <= n; i++)
30     for (int j = 1; j <= n; j++)
31         if ((i + j) % 2 && !g[i][j])
32         {
33             memset(st, 0, sizeof st);
34             if (find(i, j))
35                 res++;
36         }
37     // PROCESS
38 }

```

### 9.11.2 minimum vertex cover

```

1  const int N = 110;
2  int n, m, k, match[N];
3  bool g[N][N], st[N];
4  bool find(int x)
5  {
6      for (int i = 0; i < m; i++)
7          if (!st[i] && g[x][i])
8          {
9              st[i] = true;
10             if (match[i] == -1 || find(match[i]))
11             {
12                 match[i] = x;
13                 return true;
14             }
15         }
16     return false;
17 }
18 int main()
19 {
20     while (cin >> n, n)
21     {
22         cin >> m >> k;
23         memset(g, 0, sizeof g);
24         memset(match, -1, sizeof match);
25         while (k--)
26         {
27             int t, a, b;
28             cin >> t >> a >> b;
29             if (!a || !b) continue;
30             g[a][b] = true;
31         }
32         int res = 0;
33         for (int i = 0; i < n; i++)
34         {

```

```

35             memset(st, 0, sizeof st);
36             if (find(i)) res++;
37         }
38         cout << res << '\n';
39     }
40 }

```

### 9.11.3 maximum independent set

```

1  const int N = 110;
2  int n, m, k;
3  PII match[N][N];
4  bool g[N][N], st[N][N];
5  int dx[8] = {-2, -1, 1, 2, 2, 1, -1, -2};
6  int dy[8] = {1, 2, 2, 1, -1, -2, -2, -1};
7  bool find(int x, int y)
8  {
9      for (int i = 0; i < 8; i++)
10     {
11         int a = x + dx[i], b = y + dy[i];
12         if (a < 1 || a > n || b < 1 || b > m)
13             continue;
14         if (g[a][b]) continue;
15         if (st[a][b]) continue;
16         st[a][b] = true;
17         PII t = match[a][b];
18         if (t.x == 0 || find(t.x, t.y))
19         {
20             match[a][b] = {x, y};
21             return true;
22         }
23     }
24     return false;
25 }
26 int main()
27 {
28     // PROCESS
29     int res = 0;
30     for (int i = 1; i <= n; i++)
31     for (int j = 1; j <= m; j++)
32     {
33         if (g[i][j] || (i + j) % 2)
34             continue;
35         memset(st, 0, sizeof st);
36         if (find(i, j)) res++;
37     }
38     cout << n * m - k - res << '\n';
39 }

```

### 9.11.4 minimum path cover

- Only for DAG.
- If you need to compute the **minimum path cover with repeated nodes**, you need to perform transitive closure as shown in the following code.

```

1  const int N = 210, M = 30010;
2  int n, m, match[N];
3  bool d[N][N], st[N];
4  bool find(int x)

```

```

5 {
6     for (int i = 1; i <= n; i++)
7         if (d[x][i] && !st[i])
8         {
9             st[i] = true;
10            int t = match[i];
11            if (t == 0 || find(t))
12            {
13                match[i] = x;
14                return true;
15            }
16        }
17    return false;
18 }
19 int main()
20 {
21     // 传递闭包
22     for (int k = 1; k <= n; k++)
23         for (int i = 1; i <= n; i++)
24             for (int j = 1; j <= n; j++)
25                 d[i][j] |= d[i][k] & d[k][j];
26     int res = 0;
27     for (int i = 1; i <= n; i++)
28     {
29         memset(st, 0, sizeof st);
30         if (find(i)) res++;
31     }
32     cout << n - res;
33 }

```

```

21     t = i / 2 + 1;
22     if (i & 1) t = -t;
23 }
24 else t = i + 1;
25 int j = e[i];
26 i = ne[i];
27 dfs(j);
28 ans[++cnt] = t;
29 }
30 }
31 int main()
32 {
33     cin >> type >> n >> m;
34     memset(h, -1, sizeof h);
35     for (int i = 0; i < m; i++)
36     {
37         int a, b;
38         cin >> a >> b;
39         add(a, b);
40         if (type == 1) add(b, a);
41         din[b]++, dout[a]++;
42     }
43     if (type == 1)
44     {
45         for (int i = 1; i <= n; i++)
46             if (din[i] + dout[i] & 1)
47             {
48                 cout << "NO\n";
49                 return 0;
50             }
51     }
52     else
53     {
54         for (int i = 1; i <= n; i++)
55             if (din[i] != dout[i])
56             {
57                 cout << "NO\n";
58                 return 0;
59             }
60     }
61     for (int i = 1; i <= n; i++)
62         if (h[i] != -1)
63         {
64             dfs(i);
65             break;
66         }
67 }

```

## 9.12 Eulerian Circuit & Eulerian Path

### 9.12.1 Eulerian Circuit

- **Undirected Graph:** If and only if it is connected and every vertex has even degree.
- **Directed Graph:** If and only if it is strongly connected and each vertex has equal in-degree and out-degree.

```

1 int type, n, m;
2 int h[N], e[M], ne[M], idx;
3 bool used[M];
4 int ans[M], cn, din[N], dout[N];
5 void add(int a, int b)
6 { e[idx] = b, ne[idx] = h[a], h[a] = idx++;
7 }
8 void dfs(int u)
9 {
10     for (int &i = h[u]; ~i;)
11     {
12         if (used[i])
13         {
14             i = ne[i];
15             continue;
16         }
17         used[i] = true;
18         if (type == 1) used[i ^ 1] = true;
19         int t;
20         if (type == 1)
21         {

```

### 9.12.2 Eulerian Path

#### Undirected Graph

If and only if it is connected (ignoring isolated vertices) and has exactly 0 or 2 vertices with odd degree.

```

1 const int N = 510;
2 int n = 500, m, g[N][N];
3 int ans[1100], cnt, d[N];
4 void dfs(int u)
5 {
6     for (int i = 1; i <= n; i++)
7         if (g[u][i])
8         {
9             g[u][i]--, g[i][u]--;
10            dfs(i);

```

```

11     }
12     ans[++cnt] = u;
13 }
14 int main()
15 {
16     cin >> m;
17     while (m--)
18     {
19         int a, b;
20         cin >> a >> b;
21         g[a][b]++, g[b][a]++;
22         d[a]++, d[b]++;
23     }
24     int start = 1;
25     while (!d[start])
26         ++start;
27     for (int i = 1; i <= 500; i++)
28         if (d[i] % 2)
29         {
30             start = i;
31             break;
32         }
33     dfs(start);
34 }

```

## Directed Graph

If and only if it is connected in terms of non-zero degree vertices, and

- At most one vertex has  
(out-degree) — (in-degree) = 1
- At most one vertex has  
(in-degree) — (out-degree) = 1
- All other vertices have  
equal in-degree and out-degree

```

27     dout[a]++, din[b]++;
28     p[find(a)] = find(b);
29 }
30 int start = 0, end = 0;
31 bool success = true;
32 for (int i = 0; i < 26; i++)
33     if (din[i] != dout[i])
34     {
35         if (din[i] == dout[i] + 1) end++;
36         else if (din[i] + 1 == dout[i])
37             start++;
38         else
39         {
40             success = false;
41             break;
42         }
43     }
44 if (success && (!(start && !end || start
45 == 1 && end == 1))
46     success = false;
47 int rep = -1;
48 for (int i = 0; i < 26; i++)
49     if (st[i])
50     {
51         if (rep == -1) rep = find(i);
52         else if (rep != find(i))
53         {
54             success = false;
55             break;
56         }
57     }
58 return 0;
59 }

```

```

1  const int N = 30;
2  int n, p[N], din[N], dout[N];
3  bool st[N];
4  int find(int x)
5  {
6      if (x != p[x]) p[x] = find(p[x]);
7      return p[x];
8  }
9  int main()
10 {
11     char str[1010];
12     int T;
13     cin >> T;
14     while (T--)
15     {
16         cin >> n;
17         memset(din, 0, sizeof din);
18         memset(dout, 0, sizeof dout);
19         memset(st, 0, sizeof st);
20         for (int i = 0; i < 26; i++) p[i] = i;
21         for (int i = 0; i < n; i++)
22         {
23             cin >> str;
24             int a = str[0] - 'a',
25                 b = str[strlen(str) - 1] - 'a';
26             st[a] = st[b] = true;

```

## 10 ★ Advanced Math

### 10.1 Factorial Prime Factorization

```
1  const int N = 1000010;
2  int primes[N], cnt, n;
3  bool st[N];
4  void init(int n)
5  {
6      for (int i = 2; i <= n; i++)
7      {
8          if (!st[i]) primes[cnt++] = i;
9          for (int j = 0; primes[j] * i <= n; j++)
10             {
11                 st[primes[j] * i] = true;
12                 if (i % primes[j] == 0) break;
13             }
14      }
15  }
16
17  int main()
18  {
19      cin >> n;
20      init(n);
21      for (int i = 0; i < cnt; i++)
22      {
23          int p = primes[i];
24          int s = 0;
25          for (int j = n; j; j /= p) s += j / p;
26          cout << p << ' ' << s << '\n';
27      }
28  }
```

### 10.2 Euler's Totient Function

#### 10.2.1 GCD

```
1  const int N = 1e7 + 10;
2  int primes[N], cnt, phi[N];
3  bool st[N];
4  LL s[N];
5  void init(int n)
6  {
7      for (int i = 2; i <= n; i++)
8      {
9          if (!st[i])
10             {
11                 primes[cnt++] = i;
12                 phi[i] = i - 1;
13             }
14             for (int j = 0; primes[j] * i <= n; j++)
15                 {
16                     st[primes[j] * i] = true;
17                     if (i % primes[j] == 0)
18                     {
19                         phi[i * primes[j]] = phi[i] *
20                         primes[j];
21                         break;
22                     }
23                 }
24             }
25  }
```

```
21     }
22     phi[i * primes[j]] = phi[i] * (
23         primes[j] - 1);
24     }
25     for (int i = 1; i <= n; i++)
26         s[i] = s[i - 1] + phi[i];
27 }
28 int main()
29 {
30     int n; cin >> n;
31     init(n);
32     LL res = 0;
33     for (int i = 0; i < cnt; i++)
34     {
35         int p = primes[i];
36         res += s[n / p] * 2 + 1;
37     }
38 }
```

### 10.3 Matrix Multiplication

#### 10.3.1 Simple Matrix Multiplication

- **Conclusion:** For the sequence  $\{a_n\}$ , if

$$a_i = k_1 a_{i-1} + k_2 a_{i-2} + \cdots + k_i a_0,$$

then

$$\begin{bmatrix} k_1 & k_2 & \cdots & k_{i-1} & k_i \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix}^n \begin{bmatrix} a_{i-1} \\ a_{i-2} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} a_{n+i-1} \\ a_{n+i-2} \\ \vdots \\ a_{n+1} \\ a_n \end{bmatrix}$$

```
1  const int N = 3;
2  int n, m;
3  void mul(int c[], int a[], int b[][N])
4  {
5      int temp[N] = {0};
6      for (int i = 0; i < N; i++)
7          for (int j = 0; j < N; j++)
8              temp[i] = (temp[i] + (LL)a[j] * b[
9                  j][i]) % m;
10     memcpy(c, temp, sizeof temp);
11 }
12 void mul(int c[][N], int a[][N], int b[][N])
13 {
14     int temp[N][N] = {0};
15     for (int i = 0; i < N; i++)
16         for (int j = 0; j < N; j++)
17             for (int k = 0; k < N; k++)
18                 temp[i][j] = (temp[i][j] + (LL)
19                     a[i][k] * b[k][j]) % m;
20     memcpy(c, temp, sizeof temp);
21 }
22 int main()
23 {
24     while (n)
25     {
26         if (n & 1) mul(f1, f1, a);
27         mul(a, a, a); n >>= 1;
28     }
29 }
```

```

26     }
27 }

```

### 10.3.2 KMP & Matrix Multiplication

```

1  const int N = 25;
2  int n, m, mod, ne[N], a[N][N];
3  char str[N];
4  void mul(int c[][N], int a[][N], int b[][N])
5  {
6      int tmp[N][N];
7      memset(tmp, 0, sizeof tmp);
8      for (int i = 0; i < m; i++)
9          for (int j = 0; j < m; j++)
10             for (int k = 0; k < m; k++)
11                 tmp[i][j] = (tmp[i][j] + a[i][k] * b
12                     [k][j]) % mod;
13     memcpy(c, tmp, sizeof tmp);
14 }
15 int qmi(int k)
16 {
17     int f0[N][N] = {1};
18     while (k)
19     {
20         if (k & 1) mul(f0, f0, a);
21         mul(a, a, a);
22         k >>= 1;
23     }
24     int res = 0;
25     // 因为简化 f[0][j] 相当于 f[n][j]
26     for (int j = 0; j < m; j++)
27         res = (res + f0[0][j]) % mod;
28     return res;
29 }
30 int main()
31 {
32     cin >> n >> m >> mod;
33     cin >> str + 1;
34     for (int i = 2, j = 0; i <= m; i++)
35     {
36         while (j && str[j + 1] != str[i])
37             j = ne[j];
38         if (str[j + 1] == str[i]) j++;
39         ne[i] = j;
40     }
41     for (int j = 0; j < m; j++)
42         for (int c = '0'; c <= '9'; c++)
43         {
44             int k = j;
45             while (k && str[k + 1] != c)
46                 k = ne[k];
47             if (str[k + 1] == c) k++;
48             if (k < m) a[j][k]++;
49         }
50     cout << qmi(n) << endl;
51     return 0;
52 }

```

## 11 ★ Advanced DP

### 11.1 Advanced Linear DP

#### 11.1.1 Two-pass grid collection problem

In this case we run DP on two different roads at the same time:

```
1  const int N = 15;
2  int n, w[N][N], f[N * 2][N][N];
3  int main()
4  {
5      cin >> n;
6      // INPUT w[N][N]
7      for (int k = 2; k <= n * 2; k++)
8          for (int i1 = 1; i1 <= k; i1++)
9              for (int i2 = 1; i2 <= k; i2++)
10                 {
11                     int j1 = k - i1, j2 = k - i2;
12                     int t = w[i1][j1];
13                     if (i1 != i2) t += w[i2][j2];
14                     int &x = f[k][i1][i2];
15                     x = max(x, f[k - 1][i1 - 1][i2 - 1] +
16                             t);
17                     x = max(x, f[k - 1][i1 - 1][i2] + t);
18                     x = max(x, f[k - 1][i1][i2 - 1] + t);
19                     x = max(x, f[k - 1][i1][i2] + t);
20                 }
21     cout << f[n * 2][n][n] << '\n';
22     return 0;
23 }
```

## 11.2 Advanced LIS

### 11.2.1 Longest Bitonic Subsequence

```
1  const int N = 1010;
2  int n, a[N], f[N], g[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          {
10             f[i] = 1;
11             for (int j = 1; j < i; j++)
12                 if (a[i] > a[j])
13                     f[i] = max(f[i], f[j] + 1);
14             }
15             g[i] = 1;
16             for (int j = n; j > i; j--)
17                 if (a[i] > a[j])
18                     g[i] = max(g[i], g[j] + 1);
19             }
20     int ans = 0;
21     for (int i = 1; i <= n; i++)
22         ans = max(ans, f[i] + g[i] - 1);
23     cout << ans << '\n';
24 }
```

```
26     return 0;
27 }
```

#### 11.2.2 MSIS

MSIS means Maximum Sum Increasing Subsequence

```
1  const int N = 1010;
2  int n, w[N], f[N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 0; i < n; i++) cin >> w[i];
7      int res = 0;
8      for (int i = 0; i < n; i++)
9          {
10             f[i] = w[i];
11             for (int j = 0; j < i; j++)
12                 if (w[i] > w[j])
13                     f[i] = max(f[i], f[j] + w[i]);
14             res = max(res, f[i]);
15         }
16     cout << res;
17 }
```

#### 11.2.3 LCIS

LCIS means Longest Common Increasing Subsequence

```
1  const int N = 3010;
2  int n, a[N], b[N], f[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7          cin >> a[i];
8      for (int i = 1; i <= n; i++)
9          cin >> b[i];
10     for (int i = 1; i <= n; i++)
11         {
12             int maxv = 1;
13             for (int j = 1; j <= n; j++)
14                 {
15                     f[i][j] = f[i - 1][j];
16                     if (a[i] == b[j])
17                         f[i][j] = max(f[i][j], maxv);
18                     if (a[i] > b[j])
19                         maxv = max(maxv, f[i - 1][j] +
20                                     1);
21                 }
22             int res = 0;
23             for (int i = 1; i <= n; i++)
24                 res = max(res, f[n][i]);
25             cout << res;
26         }
```

## 11.3 Knapsack Problem

### 11.3.1 How To Initialize

Initialization for Counting the Number of Solutions:



- **2D Case:**

- When volume is at most  $j$ :  $f[0][i] = 1$  for  $0 \leq i \leq m$ , others are 0
- When volume is exactly  $j$ :  $f[0][0] = 1$ , others are 0
- When volume is at least  $j$ :  $f[0][0] = 1$ , others are 0

- **1D Case:**

- When volume is at most  $j$ :  $f[i] = 1$  for  $0 \leq i \leq m$
- When volume is exactly  $j$ :  $f[0] = 1$ , others are 0
- When volume is at least  $j$ :  $f[0] = 1$ , others are 0

```

13     int hh = 0, tt = -1;
14     for (int j = r; j <= m; j += v)
15     {
16         while (hh <= tt && j - s * v > q[hh])
17             hh++;
18         while (hh <= tt && g[q[tt]] + (j - q[tt]) / v * w <= g[j])
19             tt--;
20         q[++tt] = j;
21         f[j] = g[q[hh]] + (j - q[hh]) / v * w;
22     }
23 }
24 }
25 cout << f[m];
26 }

```

Initialization for Finding Maximum or Minimum Value:

- **2D Case:**

- When volume is at most  $j$ :  $f[i][k] = 0$  for  $0 \leq i \leq n, 0 \leq k \leq m$  (only for maximizing value)
- When volume is exactly  $j$ :
  - \* For minimizing value:  $f[0][0] = 0$ , others are INF
  - \* For maximizing value:  $f[0][0] = 0$ , others are -INF
- When volume is at least  $j$ :  $f[0][0] = 0$ , others are INF (only for minimizing value)

- **1D Case:**

- When volume is at most  $j$ :  $f[i] = 0$  for  $0 \leq i \leq m$  (only for maximizing value)
- When volume is exactly  $j$ :
  - \* For minimizing value:  $f[0] = 0$ , others are INF
  - \* For maximizing value:  $f[0] = 0$ , others are -INF
- When volume is at least  $j$ :  $f[0] = 0$ , others are INF (only for minimizing value)

```

1  const int N = 110;
2  int n, V, M, f[N][N];
3  int main()
4  {
5      cin >> n >> V >> M;
6      for (int i = 0; i < n; i++)
7      {
8          int v, m, w;
9          cin >> v >> m >> w;
10         for (int j = V; j >= v; j--)
11             for (int k = M; k >= m; k--)
12                 f[j][k] = max(f[j][k], f[j - v][k - m] + w);
13     }
14     cout << f[V][M] << '\n';
15 }

```

### 11.3.2 Multiple Knapsack Problem

```

1  const int M = 20010;
2  int n, m, v, w, s;
3  int f[M], g[M], q[M];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8      {
9          cin >> v >> w >> s;
10         memcpy(g, f, sizeof g);
11         for (int r = 0; r < v; r++)
12         {

```

### 11.3.3 Two-Dimensional Cost Knapsack Problem

### 11.3.4 Finding the Actual Solution Set

```

1  const int N = 1010;
2  int n, m;
3  int v[N], w[N], f[N][N];
4  int main()
5  {
6      cin >> n >> m;
7      for (int i = 1; i <= n; i++)
8          cin >> v[i] >> w[i];
9      for (int i = n; i >= 1; i--)
10         for (int j = 0; j <= m; j++)
11         {
12             f[i][j] = f[i + 1][j];
13             if (j >= v[i])
14                 f[i][j] = max(f[i][j], f[i + 1][j - v[i]] + w[i]);
15         }
16     int j = m;
17     for (int i = 1; i <= n; i++)
18         if (j >= v[i] && f[i][j] == f[i + 1][j - v[i]] + w[i])
19             {

```

```

20         cout << i << ' ';
21         j -= v[i];
22     }
23 }

```

### 11.3.5 Maximum Linearly Independent Subset

```

1  const int N = 110, M = 25010;
2  int n, v[N];
3  bool f[M];
4  int main()
5  {
6      int T; cin >> T;
7      while (T--)
8      {
9          cin >> n;
10         for (int i = 1; i <= n; ++i)
11             cin >> v[i];
12         sort(v + 1, v + n + 1);
13         int m = v[n], res = 0;
14         memset(f, 0, sizeof f);
15         f[0] = true;
16         for (int i = 1; i <= n; ++i)
17         {
18             if (f[v[i]]) continue;
19             res++;
20             for (int j = v[i]; j <= m; ++j)
21                 f[j] |= f[j - v[i]];
22         }
23         cout << res << '\n';
24     }
25 }

```

### 11.3.6 Mixed Knapsack Problem

```

1  const int N = 1010;
2  int n, m, f[N];
3  int main()
4  {
5      cin >> n >> m;
6      for (int i = 0; i < n; i++)
7      {
8          int v, w, s;
9          cin >> v >> w >> s;
10         if (!s)
11         {
12             for (int j = v; j <= m; j++)
13                 f[j] = max(f[j], f[j - v] + w);
14         }
15         else
16         {
17             if (s == -1) s = 1;
18             for (int k = 1; k <= s; k *= 2)
19             {
20                 for (int j = m; j >= k * v; j
21                     --)
22                     f[j] = max(f[j], f[j - k *
23                         v] + k * w);
24                 s -= k;
25             }
26         }
27     }
28 }

```

```

23     }
24     if (s)
25     {
26         for (int j = m; j >= s * v; j
27             --)
28             f[j] = max(f[j], f[j - s *
29                 v] + s * w);
30     }
31     cout << f[m] << '\n';
32 }

```

### 11.3.7 Dependent Knapsack Problem

```

1  const int N = 110;
2  int n, m, root;
3  int h[N], e[N], ne[N], idx;
4  int v[N], w[N], [N][N];
5  void add(int a, int b)
6  {
7      e[idx] = b, ne[idx] = h[a], h[a] = idx++;
8  }
9  void dfs(int u)
10 {
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         int son = e[i];
14         dfs(son);
15         for (int j = m - v[u]; j >= 0; --j)
16             for (int k = 0; k <= j; ++k)
17                 f[u][j] = max(f[u][j], f[u][j
18                     - k] + f[son][k]);
19     }
20     for (int j = m; j >= v[u]; --j)
21         f[u][j] = f[u][j - v[u]] + w[u];
22     for (int j = 0; j < v[u]; ++j)
23         f[u][j] = 0;
24 }
25 int main()
26 {
27     memset(h, -1, sizeof h);
28     cin >> n >> m;
29     for (int i = 1; i <= n; ++i)
30     {
31         int p;
32         cin >> v[i] >> w[i] >> p;
33         if (p == -1) root = i;
34         else add(p, i);
35     }
36     dfs(root);
37     cout << f[root][m] << '\n';
38 }

```

### 11.3.8 Number of Solutions

```

1  const int N = 1010, mod = 1e9 + 7;
2  int n, m;
3  int w[N], v[N], f[N], g[N];
4  int main()
5  {
6      cin >> n >> m;

```

```

7   for (int i = 1; i <= n; ++i)
8       cin >> v[i] >> w[i];
9   g[0] = 1;
10  for (int i = 1; i <= n; ++i)
11  {
12      for (int j = m; j >= v[i]; --j)
13      {
14          int temp = max(f[j], f[j - v[i]] +
15                          w[i]), c = 0;
16          if (temp == f[j])
17              c = (c + g[j]) % mod;
18          if (temp == f[j - v[i]] + w[i])
19              c = (c + g[j - v[i]]) % mod;
20          f[j] = temp, g[j] = c;
21      }
22      int res = 0;
23      for (int j = 0; j <= m; ++j)
24          if (f[j] == f[m])
25              res = (res + g[j]) % mod;
26      cout << res << '\n';
27  }

```

```

13     ne[i] = j;
14 }
15 f[0][0] = 1;
16 for (int i = 0; i < n; i++)
17     for (int j = 0; j < m; j++)
18         for (char ch = 'a'; ch <= 'z'; ch++)
19             {
20                 int ptr = j;
21                 while (ptr && s[ptr + 1] != ch)
22                     ptr = ne[ptr];
23                 if (s[ptr + 1] == ch) ptr++;
24                 f[i + 1][ptr] = (f[i + 1][ptr] + f[i
25                     ][j]) % mod;
26             }
27 int res = 0;
28 for (int j = 0; j < m; j++)
29     res = (res + f[n][j]) % mod;
30 cout << res;
31 }

```

### 11.4.3 Linear DP + AC Automaton

## 11.4 FSM

### 11.4.1 Common FSM

```

1   const int N = 100010;
2   int n, w[N], f[N][2];
3   int main()
4   {
5       int T; cin >> T;
6       while (T--)
7       {
8           cin >> n;
9           for (int i = 1; i <= n; i++)
10              cin >> w[i];
11          for (int i = 1; i <= n; i++)
12          {
13              // YOUR_FSM_RULES
14              // f[i][0] =
15              // f[i][1] =
16          }
17          cout << max(f[n][0], f[n][1]) << '\n';
18      }
19  }

```

### 11.4.2 Linear DP + KMP

```

1   const int N = 55, mod = 1e9 + 7
2   int n, m, f[N][N], ne[N];
3   char s[N];
4   int main()
5   {
6       cin >> n >> s + 1;
7       m = strlen(s + 1);
8       for (int i = 2, j = 0; i <= m; i++)
9       {
10          while (j && s[i] != s[j + 1])
11              j = ne[j];
12          if (s[j + 1] == s[i]) j++;

```

```

1   const int N = 1010, INF = 0x3f3f3f3f;
2   int n, m, T = 1, f[N][N];
3   int tr[N][4], dar[N], idx;
4   int q[N], ne[N];
5   char str[N];
6   int get(char c)
7   {
8       if (c == 'A') return 0;
9       if (c == 'T') return 1;
10      if (c == 'G') return 2;
11      return 3;
12  }
13 void insert()
14 {
15     int p = 0;
16     for (int i = 0; str[i]; i++)
17     {
18         int t = get(str[i]);
19         if (tr[p][t] == 0)
20             tr[p][t] = ++idx;
21         p = tr[p][t];
22     }
23     dar[p] = 1;
24 }
25 void build()
26 {
27     int hh = 0, tt = -1;
28     for (int i = 0; i < 4; i++)
29         if (tr[0][i])
30             q[++tt] = tr[0][i];
31     while (hh <= tt)
32     {
33         int t = q[hh++];
34         for (int i = 0; i < 4; i++)
35         {
36             int p = tr[t][i];
37             if (!p)
38                 tr[t][i] = tr[ne[t]][i];
39             else
40             {
41                 ne[p] = tr[ne[t]][i];
42                 q[++tt] = p;

```

```

43     dar[p] |= dar[ne[p]];
44 }
45 }
46 }
47 }
48 int main()
49 {
50     int T = 1;
51     while (cin >> n, n)
52     {
53         memset(tr, 0, sizeof tr);
54         memset(dar, 0, sizeof dar);
55         memset(ne, 0, sizeof ne);
56         idx = 0;
57         for (int i = 0; i < n; i++)
58         {
59             cin >> str;
60             insert();
61         }
62         build();
63         cin >> (str + 1);
64         m = strlen(str + 1);
65         memset(f, 0x3f, sizeof f);
66         f[0][0] = 0;
67         for (int i = 0; i < m; i++)
68             for (int j = 0; j <= idx; j++)
69                 for (int k = 0; k < 4; k++)
70                 {
71                     int t = get(str[i + 1]) != k;
72                     int p = tr[j][k];
73                     if (!dar[p])
74                         f[i + 1][p] = min(f[i + 1][p], f[i][j] + t);
75                 }
76         int res = INF;
77         for (int i = 0; i <= idx; i++)
78             res = min(res, f[m][i]);
79         if (res == INF) res = -1;
80         cout << "Case " << T++
81              << ": " << res << '\n';
82     }
83 }

```

## 11.5 State Compression DP

### 11.5.1 Bitmask DP & Rolling Array & Target-State Optimization

```

1  const int N = 12, M = 1 << 10, K = 110;
2  int n, m, cnt[M];
3  vector<int> state, head[M];
4  LL f[N][K][M];
5  bool check(int state)
6  {
7      for (int i = 0; i < n; i++)
8          if ((state >> i & 1) && (state >> i + 1 & 1))
9              return false;
10     return true;
11 }
12 int count(int state)
13 {
14     int res = 0;

```

```

15     for (int i = 0; i < n; i++)
16         res += state >> i & 1;
17     return res;
18 }
19 int main()
20 {
21     cin >> n >> m;
22     for (int i = 0; i < 1 << n; i++)
23         if (check(i))
24         {
25             state.push_back(i);
26             cnt[i] = count(i);
27         }
28     for (int i = 0; i < state.size(); i++)
29         for (int j = 0; j < state.size(); j++)
30         {
31             int a = state[i], b = state[j];
32             if ((a & b) == 0 && check(a | b))
33                 head[i].push_back(j);
34         }
35     f[0][0][0] = 1;
36     for (int i = 1; i <= n + 1; i++)
37         for (int j = 0; j <= m; j++)
38             for (int a = 0; a < state.size(); a++)
39                 for (int b : head[a])
40                 {
41                     int c = cnt[state[a]];
42                     if (j >= c)
43                         f[i][j][a] += f[i - 1][j - c][b];
44                 }
45     cout << f[n + 1][m][0];
46 }

```

### 11.5.2 Bitmask DP & Transfer Optimization

```

1  const int N = 10, M = 1 << 10;
2  int n, m, g[1010], cnt[M];
3  int f[2][M][M];
4  vector<int> state;
5  bool check(int state)
6  {
7      for (int i = 0; i < m; i++)
8          if ((state >> i & 1) && ((state >> i + 1 & 1) || (state >> i + 2 & 1)))
9              return false;
10     return true;
11 }
12 int count(int state)
13 {
14     int res = 0;
15     for (int i = 0; i < m; i++)
16         if (state >> i & 1) res++;
17     return res;
18 }
19 int main()
20 {
21     cin >> n >> m;
22     for (int i = 1; i <= n; i++)
23         for (int j = 0; j < m; j++)
24         {
25             char c;
26             cin >> c;

```

```

27     g[i] += (c == 'H') << j;
28 }
29 for (int i = 0; i < 1 << m; i++)
30     if (check(i))
31     {
32         state.push_back(i);
33         cnt[i] = count(i);
34     }
35 for (int i = 1; i <= n; i++)
36     for (int j = 0; j < state.size(); j++)
37         for (int k = 0; k < state.size(); k++)
38             for (int u = 0; u < state.size(); u
39                 ++){
40                 int a = state[j], b = state[k], c
41                 = state[u];
42                 if (a & b | a & c | b & c)
43                     continue;
44                 if (g[i] & b | g[i - 1] & a)
45                     continue;
46                 f[i & 1][j][k] = max(f[i & 1][j][k]
47                     , f[i - 1 & 1][u][j] + cnt[b]);
48             }
49 int res = 0;
50 for (int i = 0; i < state.size(); i++)
51     for (int j = 0; j < state.size(); j++)
52         res = max(res, f[n & 1][i][j]);
53 cout << res;
54 }

```

```

31     }
32 }
33 }
34 }
35 void solve()
36 {
37     cin >> n >> m;
38     for (int i = 0; i < n; i++)
39         cin >> ver[i].x >> ver[i].y;
40     init_path();
41     memset(f, 0x3f, sizeof f);
42     f[0] = 0;
43     for (int cur_st = 0; cur_st + 1 < 1 << n;
44         cur_st++){
45         {
46             int t = -1;
47             for (int i = 0; i < n; i++)
48                 if (!(cur_st >> i & 1)) t = i;
49             for (int i = 0; i < n; i++)
50             {
51                 int ne_st = path[t][i] | cur_st;
52                 f[ne_st] = min(f[ne_st], f[cur_st] +
53                     1);
54             }
55         }
56         cout << f[(1 << n) - 1] << '\n';
57     }
58 }
59 int main()
60 {
61     cin >> T;
62     while (T--) solve();
63 }

```

### 11.5.3 Bitmask DP & Repeated Coverage Problem

```

1  const double eps = 1e-8;
2  const int N = 20, M = 1 << N;
3  int n, m, T, path[N][N], f[M]
4  PDD ver[N];
5  int cmp_lf(double a, double b)
6  {
7      if (fabs(a - b) < eps) return 0;
8      if (a > b) return 1;
9      return -1;
10 }
11 void init_path()
12 {
13     memset(path, 0, sizeof path);
14     for (int i = 0; i < n; i++)
15     {
16         path[i][i] = 1 << i;
17         for (int j = 0; j < n; j++)
18         {
19             double x1 = ver[i].x, y1 = ver[i].y;
20             double x2 = ver[j].x, y2 = ver[j].y;
21             if (!cmp_lf(x1, x2)) continue;
22             double a = (y1 / x1 - y2 / x2) / (x1 -
23                 x2);
24             double b = (y1 / x1) - a * x1;
25             if (cmp_lf(a, 0.0) >= 0)
26                 continue;
27             for (int k = 0; k < n; k++)
28             {
29                 double x = ver[k].x, y = ver[k].y;
30                 if (!cmp_lf(y, a * x * x + b * x))
31                     path[i][j] += 1 << k;
32             }
33         }
34     }
35 }

```

### 11.5.4 Bitmask DP & MST

```

1  const int N = 12, M = 1 << N;
2  int n, m, d[N][N], f[M][N], g[M];
3  int main()
4  {
5      cin >> n >> m;
6      memset(d, 0x3f, sizeof d);
7      for (int i = 0; i < n; i++)
8          d[i][i] = 0;
9      while (m--)
10     {
11         int a, b, c;
12         cin >> a >> b >> c;
13         a--, b--;
14         d[a][b] = d[b][a] = min(d[a][b], c);
15     }
16     for (int i = 1; i < 1 << n; i++)
17         for (int j = 0; j < n; j++)
18             if (i >> j & 1)
19                 for (int k = 0; k < n; k++)
20                     if (d[j][k] != INF)
21                         g[i] |= 1 << k;
22     memset(f, 0x3f, sizeof f);
23     for (int i = 0; i < n; i++)
24         f[1 << i][0] = 0;
25     for (int i = 1; i < 1 << n; i++)
26         for (int j = (i - 1); j; j = (j - 1) & i)
27             if ((g[j] & i) == i)
28             {
29                 for (int k = 0; k < n; k++)
30                     if (d[j][k] != INF)
31                         f[i] = min(f[i], f[j] + d[j][k]);
32             }
33     int ans = INF;
34     for (int i = 0; i < n; i++)
35         ans = min(ans, f[1 << i][i]);
36     cout << ans;
37 }

```

```

29     int remain = i ^ j;
30     int cost = 0;
31     for (int k = 0; k < n; k++)
32         if (remain >> k & 1)
33         {
34             int t = INF;
35             for (int u = 0; u < n; u++)
36                 if (j >> u & 1)
37                     t = min(t, d[k][u]);
38             cost += t;
39         }
40     for (int k = 1; k < n; k++)
41         f[i][k] = min(f[i][k], f[j][k - 1]
42             + cost * k);
43     int res = INF;
44     for (int i = 0; i < n; i++)
45         res = min(res, f[(1 << n) - 1][i]);
46     cout << res;
47 }

```

## 11.6 Interval DP

### 11.6.1 Interval DP & Circular Interval

```

1  const int N = 410;
2  int n, w[N], s[N], f[N][N], g[N][N];
3  int main()
4  {
5      cin >> n;
6      for (int i = 1; i <= n; i++)
7      {
8          cin >> w[i];
9          w[i + n] = w[i];
10     }
11     for (int i = 1; i <= n * 2; i++)
12         s[i] = s[i - 1] + w[i];
13     memset(f, 0x3f, sizeof f);
14     memset(g, -0x3f, sizeof g);
15     for (int len = 1; len <= n; len++)
16         for (int l = 1, r; r = l + len - 1, r <=
17             n << 1; l++)
18             if (l == r)
19                 f[l][r] = g[l][r] = 0;
20             else
21                 for (int k = l; k < r; k++)
22                     {
23                         f[l][r] = min(f[l][r], f[l][k] + f
24                             [k + 1][r] + s[r] - s[l - 1]);
25                         g[l][r] = max(g[l][r], g[l][k] + g
26                             [k + 1][r] + s[r] - s[l - 1]);
27                     }
28     int minv = INF, maxv = -INF;
29     for (int i = 1; i <= n; i++)
30     {
31         minv = min(minv, f[i][i + n - 1]);
32         maxv = max(maxv, g[i][i + n - 1]);
33     }
34     cout << minv << '\n' << maxv;
35 }

```

### 11.6.2 Interval DP & High Precision

```

1  const int N = 55, M = 35;
2  int n, w[N];
3  LL f[N][N][N];
4  void add(LL a[], LL b[])
5  {
6      static LL c[M];
7      memset(c, 0, sizeof c);
8      for (int i = 0, t = 0; i < M; i++)
9      {
10         t += a[i] + b[i];
11         c[i] = t % 10;
12         t /= 10;
13     }
14     memcpy(a, c, sizeof c);
15 }
16 void mul(LL a[], LL b)
17 {
18     static LL c[M];
19     memset(c, 0, sizeof c);
20     LL t = 0;
21     for (int i = 0; i < M; i++)
22     {
23         t += a[i] * b;
24         c[i] = t % 10;
25         t /= 10;
26     }
27     memcpy(a, c, sizeof c);
28 }
29 int cmp(LL a[], LL b[])
30 {
31     for (int i = M - 1; i >= 0; i--)
32         if (a[i] > b[i])
33             return 1;
34         else if (a[i] < b[i])
35             return -1;
36     return 0;
37 }
38 void print(LL a[])
39 {
40     int k = M - 1;
41     while (k && !a[k]) k--;
42     while (k >= 0) cout << a[k--];
43     cout << '\n';
44 }
45 int main()
46 {
47     cin >> n;
48     for (int i = 1; i <= n; i++) cin >> w[i];
49     LL temp[M];
50     for (int len = 3; len <= n; len++)
51         for (int l = 1, r; r = l + len - 1, r <=
52             n; l++)
53         {
54             f[l][r][M - 1] = 1;
55             for (int k = l + 1; k < r; k++)
56             {
57                 memset(temp, 0, sizeof temp);
58                 temp[0] = w[l];
59                 mul(temp, w[k]);
60                 add(temp, f[l][k]);
61                 add(temp, f[k][r]);
62                 if (cmp(f[l][r], temp) > 0)

```

```

63         memcpy(f[l][r], temp, sizeof temp)
64     ;
65 }
66 print(f[1][n]);
67 }

```

```

44     cout << fixed << setprecision(3) << sqrt(
45         dp(1, 1, 1, m, m) / n);

```

## 11.7 Tree DP

### 11.6.3 2D Interval DP

```

1  const int N = 16;
2  int n, m = 8, s[N][N];
3  double f[N][N][N][N][N], X;
4  double get(int x1, int y1, int x2, int y2)
5  {
6      double delta = s[x2][y2] - s[x1 - 1][y2] -
7          s[x2][y1 - 1] + s[x1 - 1][y1 - 1];
8      delta = delta - X;
9      return delta * delta;
10 }
11 double dp(int k, int x1, int y1, int x2, int
12     y2)
13 {
14     if (f[k][x1][y1][x2][y2] >= 0)
15         return f[k][x1][y1][x2][y2];
16     if (k == n)
17         return f[k][x1][y1][x2][y2] = get(x1, y1
18             , x2, y2);
19     double t = 1e9;
20     for (int i = x1; i < x2; i++)
21     {
22         t = min(t, dp(k + 1, x1, y1, i, y2) +
23             get(i + 1, y1,
24                 x2, y2));
25         t = min(t, dp(k + 1, i + 1, y1, x2, y2)
26             + get(x1, y1,
27                 i, y2));
28     }
29     for (int i = y1; i < y2; i++)
30     {
31         t = min(t, dp(k + 1, x1, y1, x2, i) +
32             get(x1, i + 1,
33                 x2, y2));
34         t = min(t, dp(k + 1, x1, i + 1, x2, y2)
35             + get(x1, y1,
36                 x2, i));
37     }
38     return f[k][x1][y1][x2][y2] = t;
39 }
40 int main()
41 {
42     cin >> n;
43     for (int i = 1; i <= m; i++)
44         for (int j = 1; j <= m; j++)
45             cin >> s[i][j];
46     for (int i = 1; i <= m; i++)
47         for (int j = 1; j <= m; j++)
48             s[i][j] += s[i - 1][j] + s[i][j - 1] -
49                 s[i - 1][j - 1];
50     memset(f, -1, sizeof f);
51     X = (double)s[m][m] / n;

```

```

1  const int N = 10010, M = N << 1;
2  int n, ans;
3  int h[N], e[M], ne[M], w[M], idx;
4  void add(int a, int b, int c)
5  {
6      e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
7          a] = idx++;
8  }
9  int dfs(int u, int father)
10 {
11     int dist = 0, d1 = 0, d2 = 0;
12     for (int i = h[u]; ~i; i = ne[i])
13     {
14         int j = e[i];
15         if (j == father) continue;
16         int d = dfs(j, u) + w[i];
17         dist = max(dist, d);
18         if (d >= d1) d2 = d1, d1 = d;
19         else if (d > d2) d2 = d;
20     }
21     ans = max(ans, d1 + d2);
22     return dist;
23 }
24 int main()
25 {
26     cin >> n;
27     memset(h, -1, sizeof h);
28     for (int i = 0; i < n - 1; i++)
29     {
30         int a, b, c;
31         cin >> a >> b >> c;
32         add(a, b, c), add(b, a, c);
33     }
34     dfs(1, -1);
35     cout << ans;

```

### 11.7.2 Tree Center(Rerooting DP)

```

1  const int N = 10010, M = N << 1;
2  int n;
3  int h[N], e[M], ne[M], w[M], idx;
4  int d1[N], d2[N], p1[N], up[N];
5  bool is_leaf[N];
6  void add(int a, int b, int c)
7  {
8      e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
9          a] = idx++;
10 }
11 int dfs_d(int u, int father)
12 {
13     d1[u] = d2[u] = -INF;
14     for (int i = h[u]; ~i; i = ne[i])

```

```

14 {
15     int j = e[i];
16     if (j == father) continue;
17     int d = dfs_d(j, u) + w[i];
18     if (d >= d1[u])
19         d2[u] = d1[u], d1[u] = d, p1[u] = j;
20     else if (d > d2[u])
21         d2[u] = d;
22 }
23 if (d1[u] == -INF)
24     d1[u] = d2[u] = 0, is_leaf[u] = true;
25 return d1[u];
26 }
27 void dfs_u(int u, int father)
28 {
29     for (int i = h[u]; ~i; i = ne[i])
30     {
31         int j = e[i];
32         if (j == father) continue;
33         if (p1[u] == j)
34             up[j] = max(up[u], d2[u]) + w[i];
35         else
36             up[j] = max(up[u], d1[u]) + w[i];
37         dfs_u(j, u);
38     }
39 }
40 int main()
41 {
42     cin >> n;
43     memset(h, -1, sizeof h);
44     for (int i = 0; i < n - 1; i++)
45     {
46         int a, b, c;
47         cin >> a >> b >> c;
48         add(a, b, c), add(b, a, c);
49     }
50     dfs_d(1, -1); dfs_u(1, -1);
51     int res = d1[1];
52     for (int i = 2; i <= n; i++)
53         if (is_leaf[i])
54             res = min(res, up[i]);
55     else
56         res = min(res, max(d1[i], up[i]));
57     cout << res;
58 }

```

### 11.7.3 Tree DP & Knapsack

```

1 const int N = 110, M = N << 1;
2 int n, m, f[N][N];
3 int h[N], e[M], w[M], ne[M], idx;
4 void add(int a, int b, int c)
5 {
6     e[idx] = b, w[idx] = c, ne[idx] = h[a], h[
7     a] = idx++;
8 }
9 void dfs(int u, int father)
10 {
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         if (e[i] == father) continue;
14         dfs(e[i], u);
15         for (int j = m; j; j--)
16             for (int k = 0; k + 1 <= j; k++)

```

```

16         f[u][j] = max(f[u][j], f[u][j - k -
17         1] + f[e[i]][k] + w[i]);
18     }
19 }
20 int main()
21 {
22     cin >> n >> m;
23     memset(h, -1, sizeof h);
24     for (int i = 0; i < n - 1; i++)
25     {
26         int a, b, c;
27         cin >> a >> b >> c;
28         add(a, b, c), add(b, a, c);
29     }
30     dfs(1, -1);
31     cout << f[1][m];
32 }

```

### 11.7.4 Tree DP & FSM

```

1 const int N = 1510, M = N << 1;
2 int n, f[N][2];
3 int h[N], e[M], ne[M], idx;
4 void add(int a, int b)
5 {
6     e[idx] = b, ne[idx] = h[a], h[a] = idx++;
7 }
8 void dfs(int u, int fa)
9 {
10     f[u][0] = 0, f[u][1] = 1;
11     for (int i = h[u]; ~i; i = ne[i])
12     {
13         int j = e[i];
14         if (j == fa) continue;
15         dfs(j, u);
16         f[u][1] += min(f[j][0], f[j][1]);
17         f[u][0] += f[j][1];
18     }
19 }
20 int main()
21 {
22     while (cin >> n)
23     {
24         memset(h, -1, sizeof(h));
25         idx = 0;
26         for (int i = 1; i <= n; i++)
27         {
28             int x, k;
29             char ch;
30             cin >> x >> ch >> ch >> k >> ch;
31             x++;
32             while (k--)
33             {
34                 int y;
35                 cin >> y;
36                 add(x, ++y), add(y, x);
37             }
38         }
39         dfs(1, 0);
40         cout << min(f[1][0], f[1][1]) << '\n';
41     }
42 }

```



## 11.8 Digit DP

### 11.8.1 Common Digit DP

```
1  const int N = 35;
2  int l, r, k, b, a[N], al, f[N][N];
3  int dp(int pos, int st, int op)
4  {
5      if (!pos) return st == k;
6      if (!op && ~f[pos][st]) return f[pos][st];
7      int res = 0, maxx = op ? min(a[pos], 1) : 1;
8      for (int i = 0; i <= maxx; i++)
9      {
10         if (st + i > k) continue;
11         res += dp(pos - 1, st + i, op && i == a[pos]);
12     }
13     return op ? res : f[pos][st] = res;
14 }
15 int calc(int x)
16 {
17     al = 0, memset(f, -1, sizeof f);
18     while (x) a[++al] = x % b, x /= b;
19     return dp(al, 0, 1);
20 }
21 int main()
22 {
23     cin >> l >> r >> k >> b;
24     cout << calc(r) - calc(l - 1) << '\n';
25 }
```

### 11.8.2 Digit DP & Divide and Conquer & Merging

```
1  const int N = 20, MOD = 1e9 + 7;
2  LL T, l, r;
3  int a[N], al, Pow10[N];
4  struct F
5  {
6      LL s0, s1, s2;
7      F() : s0(0), s1(0), s2(0) {}
8      F(LL _0, LL _1, LL _2) : s0(_0), s1(_1), s2(_2) {}
9      void operator+=(const F &t)
10     {
11         s2 = (s2 + t.s2) % MOD;
12         s1 = (s1 + t.s1) % MOD;
13         s0 = (s0 + t.s0) % MOD;
14     }
15 } f[N][N][N];
16 F dp(int pos, int val, int sum, int op)
17 {
18     if (!pos)
19     {
20         if (val && sum) return {1, 0, 0};
21         else return {0, 0, 0};
22     }
23     if (!op && ~f[pos][val][sum].s0)
24         return f[pos][val][sum];
25     F res(0, 0, 0);
26     int maxx = op ? a[pos] : 9;
```

```
27     for (int i = 0; i <= maxx; i++)
28     {
29         if (i != 7)
30         {
31             F t = dp(pos - 1, (val * 10 + i) % 7,
32                 (sum + i) % 7, op && i == a[pos]);
33             LL k = (LL)i * Pow10[pos - 1] % MOD;
34             t.s2 = (t.s2 + 211 * k % MOD * t.s1 % MOD) % MOD;
35             t.s2 = (t.s2 + k * k % MOD * t.s0 % MOD) % MOD;
36             t.s1 = (t.s1 + k * t.s0 % MOD) % MOD;
37             res += t;
38         }
39         return op ? res : f[pos][val][sum] = res;
40     }
41     LL calc(LL x)
42     {
43         memset(f, -1, sizeof f), al = 0;
44         for (; x; x /= 10) a[++al] = x % 10;
45         return dp(al, 0, 0, 1).s2;
46     }
47     int main()
48     {
49         Pow10[0] = 1;
50         for (int i = 1; i < 20; i++)
51             Pow10[i] = 1011 * Pow10[i - 1] % MOD;
52         cin >> T;
53         while (T--)
54         {
55             cin >> l >> r;
56             cout << ((calc(r) - calc(l - 1)) % MOD + MOD) % MOD << '\n';
57         }
58     }
```

## 11.9 Queue Optimization for DP

### 11.9.1 Simple Queue Optimization

- $f[i] = \min/\max(f[j] + \text{cost}(j, i))$ ,
- where  $j$  satisfies certain constraints.

```
1  int n, m, s[300010], q[300010];
2  int main()
3  {
4      cin >> n >> m;
5      for (int i = 1; i <= n; i++)
6          cin >> s[i], s[i] += s[i - 1];
7      int res = INT_MIN, hh = 0, tt = 0;
8      for (int i = 1; i <= n; i++)
9      {
10         if (q[hh] < i - m) hh++;
11         res = max(res, s[i] - s[q[hh]]);
12         while (hh <= tt && s[q[tt]] >= s[i])
13             tt--;
14         q[++tt] = i;
15     }
```

## 11.9.2 Queue Optimization & Binary Search

```

1  const int N = 50010;
2  int n, m, w[N], q[N], f[N];
3  bool check(int limit)
4  {
5      int hh = 0, tt = 0;
6      for (int i = 1; i <= n; i++)
7      {
8          while (hh <= tt && i - q[hh] > limit + 1)
9              hh++;
10         f[i] = f[q[hh]] + w[i];
11         while (hh <= tt & f[q[tt]] >= f[i])
12             tt--;
13         q[++tt] = i;
14     }
15     if (n + 1 - q[hh] > limit + 1) hh++;
16     return f[q[hh]] <= m;
17 }
18 int main()
19 {
20     cin >> n >> m;
21     for (int i = 1; i <= n; i++)
22         cin >> w[i];
23     int l = 0, r = n;
24     while (l < r)
25     {
26         int mid = l + r >> 1;
27         if (check(mid)) r = mid;
28         else l = mid + 1;
29     }
30     cout << r;
31 }

```

## 11.9.3 2D Queue Optimization

```

1  const int N = 1010;
2  int n, m, k, res = 1e9;
3  int w[N][N], minv[N][N], maxv[N][N], q[N];
4  int a[N], b[N], c[N], d[N];
5  void get_max(int a[], int f[], int m)
6  {
7      int hh = 0, tt = -1;
8      for (int i = 1; i <= m; i++)
9      {
10         while (hh <= tt && i - q[hh] >= k)
11             hh++;
12         while (hh <= tt && a[i] >= a[q[tt]])
13             tt--;
14         q[++tt] = i;
15         f[i] = a[q[hh]];
16     }
17 }
18 void get_min(int a[], int f[], int m)
19 {
20     int hh = 0, tt = -1;
21     for (int i = 1; i <= m; i++)
22     {
23         while (hh <= tt && i - q[hh] >= k)
24             hh++;
25         while (hh <= tt && a[i] <= a[q[tt]])

```

```

26         tt--;
27         q[++tt] = i;
28         f[i] = a[q[hh]];
29     }
30 }
31 int main()
32 {
33     cin >> n >> m >> k;
34     for (int i = 1; i <= n; i++)
35         for (int j = 1; j <= m; j++)
36             cin >> w[i][j];
37     for (int i = 1; i <= n; i++)
38         get_min(w[i], minv[i], m), get_max(w[i], maxv[i], m);
39     for (int i = k; i <= m; i++)
40     {
41         for (int j = 1; j <= n; j++)
42             a[j] = maxv[j][i], b[j] = minv[j][i];
43         get_max(a, c, n);
44         get_min(b, d, n);
45         for (int j = k; j <= n; j++)
46             res = min(res, c[j] - d[j]);
47     }
48     cout << res;
49 }

```

## 11.10 CHT-Optimized DP

### 11.10.1 Common CHT-Optimized DP

```

1  const int N = 300010;
2  int n, S, q[N];
3  LL st[N], sc[N], f[N];
4  int main()
5  {
6      cin >> n >> S;
7      for (int i = 1; i <= n; i++)
8      {
9          cin >> st[i] >> sc[i];
10         st[i] += st[i - 1], sc[i] += sc[i - 1];
11     }
12     int hh = 0, tt = 0;
13     q[0] = 0;
14     for (int i = 1; i <= n; i++)
15     {
16         while (hh < tt &&
17             (f[q[hh + 1]] - f[q[hh]]) <=
18             (sc[q[hh + 1]] - sc[q[hh]]) * (st[i] + S))
19             hh++;
20         f[i] = f[q[hh]] + S * (sc[n] - sc[q[hh]]) + st[i] * (sc[i] - sc[q[hh]]);
21         while (hh < tt &&
22             (f[q[tt]] - f[q[tt - 1]]) * (sc[i] - sc[q[tt]]) >=
23             (f[i] - f[q[tt]]) * (sc[q[tt]] - sc[q[tt - 1]]))
24             tt--;
25         q[++tt] = i;
26     }
27     cout << f[n];
28 }

```

### 11.10.2 CHT-Optimized DP & Binary Search

```
1  const int N = 300010;
2  int n;
3  LL S, sc[N], st[N], f[N], q[N];
4  int main()
5  {
6      cin >> n >> S;
7      for (int i = 1; i <= n; i++)
8      {
9          cin >> st[i] >> sc[i];
10         st[i] += st[i - 1], sc[i] += sc[i - 1];
11     }
12     int tt = 0;
13     for (int i = 1; i <= n; i++)
14     {
15         int l = 0, r = tt;
16         while (l < r)
```

```
17     {
18         int mid = (l + r) >> 1;
19         if (f[q[mid + 1]] - f[q[mid]] >
20             (S + st[i]) * (sc[q[mid + 1]] - sc
21             [q[mid]]))
22             r = mid;
23         else l = mid + 1;
24     }
25     f[i] = f[q[r]] + S * (sc[n] - sc[q[r]])
26     + st[i] * (sc[i] - sc[q[r]]);
27     while (tt && (double)(f[q[tt]] - f[q[tt
28     - 1]]) * (sc[i] - sc[q[tt]]) >=
29         (double)(f[i] - f[q[tt
30     ]]) * (sc[q[tt]] - sc[q[tt - 1]]))
31     tt--;
32     q[++tt] = i;
33     cout << f[n];
34 }
```