XCPC-Template

Created by

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Contents

Part I: Basic Template

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0 ⋆ Preface

0.1 Template

```
#define itn int
#define nit int
#define nti int
#define tin int
#define tni int
#define retrun return
#define reutrn return
#define rutren return
#define fastin                    \
  ios_base::sync_with_stdio(0); \
  cin.tie(0), cout.tie(0);
#include <bits/stdc++.h>
using namespace std;
typedef long long LL;
typedef long double LD;
typedef pair<int, int> PII;
typedef pair<long long, long long> PLL;
typedef pair<double, double> PDD;
typedef vector<int> VI;
#ifndef ONLINE_JUDGE
#define dbg(args...)    \
  do                    \
  {                     \
      cout << "\033[32;1m" << #args << " -> "; \
      err(args);        \
  } while (0)
#else
#define dbg(...)
#endif
void err()
{ cout << "\033[39;0m" << endl; }
template <template <typename...> class T, typename t, typename... Args>
void err(T<t> a, Args... args)
{
  for (auto x : a) cout << x << ' ';
  err(args...);
}
template <typename T, typename... Args>
void err(T a, Args... args)
{ cout << a << ' '; err(args...); }
const int INF = 0x3f3f3f3f;
const int mod = 1e9 + 7;
const double eps = 1e-6;
int main()
{
#ifndef ONLINE_JUDGE
  freopen("test.in", "r", stdin);
  freopen("test.out", "w", stdout);
#endif
  fastin;

  return 0;
}
```

0.2 Operator Precedence

- 括号成员排第一；全体单目排第二；
- 乘除余三加减四；移位五，关系六；
- 等于不等排第七；位与异或和位或；
- 三分天下八九十；逻辑与或十一二；
- 条件赋值十三四；逗号十五最末尾。

0.3 Time Complexity

- In most ACM or coding interview problems, the time limit is usually 1 or 2 seconds. Under such constraints, C++ programs should aim to stay within about $\mathbf{10^7} \sim \mathbf{10^8}$ operations.
- Below is a guide on how to choose algorithms based on different input size ranges:

1. $\mathbf{n \le 30} \to$ Exponential complexity: DFS with pruning, State Compression DP
2. $\mathbf{n \le 100} \to \mathbf{O(n^3)}$: Floyd, DP, Gaussian Elimination
3. $\mathbf{n \le 1000} \to \mathbf{O(n^2)}$, $\mathbf{O(n^2 \log n)}$: DP, Binary Search, Naive Dijkstra, Naive Prim, Bellman-Ford
4. $\mathbf{n \le 10000} \to \mathbf{O(n^{\frac{3}{2}})}$: Block Linked List, Mo's Algorithm
5. $\mathbf{n \le 100000} \to \mathbf{O(n \log n)}$: sort, Segment Tree, Fenwick Tree (BIT), set/map, Heap, Topological Sort, Dijkstra (heap optimized), Prim (heap optimized), Kruskal, SPFA, Convex Hull, Half Plane Intersection, Binary Search, CDQ Divide and Conquer, Overall Binary Search, Suffix Array, Heavy-Light Decomposition, Dynamic Trees
6. $\mathbf{n \le 1000000} \to \mathbf{O(n)}$, or small-constant $\mathbf{O(n \log n)}$: Monotonic Queue, Hashing, Two Pointers, BFS, Union Find, KMP, Aho-Corasick Automaton
7. $\mathbf{n \le 10000000} \to \mathbf{O(n)}$: Two Pointers, KMP, Aho-Corasick Automaton, Linear Sieve for Primes
8. $\mathbf{n \le 10^9} \to \mathbf{O(\sqrt{n})}$: Primality Testing
9. $\mathbf{n \le 10^{18}} \to \mathbf{O(\log n)}$: GCD, Fast Exponentiation, Digit DP
10. $\mathbf{n \le 10^{1000}} \to \mathbf{O((\log n)^2)}$: Big Integer Arithmetic (Add/Subtract/Multiply/Divide)
11. $\mathbf{n \le 10^{100000}} \to \mathbf{O(\log k \cdot \log\log k)}$, where k is the number of digits: Big Integer Add/Subtract, FFT/NTT

0.4 If <bits/stdc++.h> Failed

Replace it with:

```cpp
#include <algorithm>
#include <bitset>
#include <complex>
#include <deque>
#include <exception>
#include <fstream>
#include <functional>
#include <iomanip>
#include <ios>
#include <iosfwd>
#include <iostream>
#include <istream>
#include <iterator>
#include <limits>
#include <list>
#include <locale>
#include <map>
#include <memory>
#include <numeric>
#include <ostream>
#include <queue>
#include <set>
#include <sstream>
#include <stack>
#include <stdexcept>
#include <streambuf>
#include <string>
#include <typeinfo>
#include <utility>
#include <valarray>
#include <vector>
#include <unordered_map>
#include <unordered_set>
```

1 ⋆ Basic Algorithm

1.1 Quick Sort

Sort the given array from index 1 to n.

```
void quick_sort(int l, int r)
{
  if (l >= r) return;
  int x = a[(l + r) >> 1], i = l - 1, j = r + 1;
  while (i < j)
  {
      do i++; while (a[i] < x);
      do j--; while (a[j] > x);
      if (i < j) swap(a[i], a[j]);
  }
  quick_sort(l, j);
  quick_sort(j + 1, r);
  return;
}
```

1.2 Binary Search

```
// 区间 [l, r] 被划分成 [l, mid] 和 [mid + 1, r] 时使用
// 大于等于区间的最小值，check 应为 target <= a[mid]
int bsearch_1(int l, int r)
{
  while (l < r)
  {
      int mid = l + r >> 1;
      if (check(mid)) r = mid;
      else l = mid + 1;
  }
  return l;
}
// 区间 [l, r] 被划分成 [l, mid - 1] 和 [mid, r] 时使用
// 小于等于区间的最大值，check 应为 target >= a[mid]
int bsearch_2(int l, int r)
{
  while (l < r)
  {
      // 为什么要 l + r + 1：因为 l 的更新条件是 mid 本身
      // 当 r == l + 1 时 mid 向下取整必定取 l，有可能在满足 check(mid) 时导致无限循环
      int mid = l + r + 1 >> 1;
      if (check(mid)) l = mid;
      else r = mid - 1;
  }
  return l;
}
// 浮点数二分
double bsearch_3(double l, double r)
{
  // eps 表示精度，取决于题目对精度的要求
  const double eps = 1e-6;
  while (r - l > eps)
  {
      double mid = (l + r) / 2;
      if (check(mid))  r = mid;
      else l = mid;
  }
  return l;
}
```

1.3 Ternary Search

```
// 整数三分
void tsearch_1(int l, int r)
{
  while (l < r)
  {
      int lmid = l + (r - l) / 3, rmid = r - (r - l) / 3;
      lans = cal(lmid), rans = cal(rmid);
      if (lans <= rans) r = rmid - 1;
      else l = lmid + 1;
      if (lans <= rans) l = lmid + 1;
      else r = rmid - 1;
  }
  // 求凹函数的极小值
  cout << min(lans, rans) << endl;
  // 求凸函数的极大值
  cout << max(lans, rans) << endl;
}
// 浮点数三分
void tsearch_2(int l, int r)
{
  const double eps = 1e-6;
  while (r - l < eps)
  {
      double lmid = l + (r - l) / 3;
      double rmid = r - (r - l) / 3;
      lans = cal(lmid), rans = cal(rmid);
      // 求凹函数的极小值
      if (lans <= rans) r = rmid;
      else l = lmid;
      // 求凸函数的极大值
      if (lans <= rans) l = lmid;
      else r = rmid;
  }
}
```

1.4 High Precision

1.4.1 High Precision Add

```
string s1, s2;
vector<int> a, b, c;
void add(vector<int> &a, vector<int> &b)
{
  if (a.size() < b.size())
  { add(b, a); return; }
  int t = 0;
  for (int i = 0; i < a.size(); i++)
  {
      t += a[i];
```

```
        if (i < b.size()) t += b[i];
        c.push_back(t % 10);
        t /= 10;
    }
    while (t)
        c.push_back(t % 10), t /= 10;
}
int main()
{
    cin >> s1 >> s2;
    for (int i = s1.size() - 1; i >= 0; i--)
        a.push_back(s1[i] - '0');
    for (int i = s2.size() - 1; i >= 0; i--)
        b.push_back(s2[i] - '0');
    add(a, b);
    for (int i = c.size() - 1; i >= 0; i--)
        cout << c[i];
    return 0;
}
```

1.4.2 High Precision Subsection

```
vector<int> a, b, c;
string s1, s2;
void sub(vector<int> &a, vector<int> &b)
{
    int t = 0;
    for (int i = 0; i < a.size(); i++)
    {
        t = a[i] - t;
        if (i < b.size()) t -= b[i];
        c.push_back((t + 10) % 10);
        if (t < 0) t = 1;
        else t = 0;
    }
    while (c.size() > 1 && c.back() == 0)
        c.pop_back();
}
int main()
{
    cin >> s1 >> s2;
    for (int i = s1.size() - 1; i >= 0; i--)
        a.push_back(s1[i] - '0');
    for (int i = s2.size() - 1; i >= 0; i--)
        b.push_back(s2[i] - '0');
    if (s1.size() < s2.size())
        cout << '-', sub(b, a);
    else if (s1.size() == s2.size() && s1 < s2
        )
        cout << '-', sub(b, a);
    else sub(a, b);
    for (int i = c.size() - 1; i >= 0; i--)
        cout << c[i];
    return 0;
}
```

1.4.3 High Precision Multiply

```
string s1, s2;
vector<int> a, c;
int b;
void mul(vector<int> &a, int b)
{
    for (int i = 0, t = 0; i < a.size() || t;
        i++)
    {
        if (i < a.size()) t += a[i] * b;
        c.push_back(t % 10);
        t /= 10;
    }
    while (c.size() > 1 && c.back() == 0)
        c.pop_back();
}
int main()
{
    cin >> s1 >> b;
    for (int i = s1.size() - 1; i >= 0; i--)
        a.push_back(s1[i] - '0');
    mul(a, b);
    for (int i = c.size() - 1; i >= 0; i--)
        cout << c[i];
    return 0;
}
```

1.4.4 High Precision Divide

```
string s1, s2;
vector<int> a, c;
int b, r;
void divide(vector<int> &a, int b, int &r)
{
    r = 0;
    for (int i = a.size() - 1; i >= 0; i--)
    {
        r = r * 10 + a[i];
        c.push_back(r / b);
        r %= b;
    }
    reverse(c.begin(), c.end());
    while (c.size() > 1 && c.back() == 0)
        c.pop_back();
}
int main()
{
    cin >> s1 >> b;
    for (int i = s1.size() - 1; i >= 0; i--)
        a.push_back(s1[i] - '0');
    divide(a, b, r);
    for (int i = c.size() - 1; i >= 0; i--)
        cout << c[i];
    cout << '\n' << r;
    return 0;
}
```

1.5 Prefix Sum & Difference Array

1.5.1 1D Prefix Sum

```
S[i] = a[1] + a[2] + ... a[i]
a[l] + ... + a[r] = S[r] - S[l - 1]
```

1.5.2 2D Prefix Sum

```
// S[i, j] = i 行 j 列左上部分所有元素和为:
s[i - 1][j] + s[i][j - 1] - s[i - 1][j - 1] + a[i][j]
// 以 (x1, y1) 为左上角, (x2, y2) 为右下角的子矩阵的和为:
S[x2][y2] - S[x1 - 1][y2] - S[x2][y1 - 1] + S[x1 - 1][y1 - 1]
```

1.5.3 1D Difference Array

```
const int N = 100010;
int n, m;
int a[N], b[N];
void insert(int l, int r, int c)
{ b[l] += c; b[r + 1] -= c; }
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> a[i];
  for (int i = 1; i <= n; i++)
      insert(i, i, a[i]);
  while (m--)
  {
      int l, r, c;
      cin >> l >> r >> c;
      insert(l, r, c);
  }
  for (int i = 1; i <= n; i++)
      b[i] += b[i - 1],
      cout << b[i] << ' ';
  return 0;
}
```

1.5.4 2D Difference Array

```
const int N = 1010;
int n, m, q, a[N][N], b[N][N];
void insert(int x1, int y1, int x2, int y2, int c)
{
  b[x1][y1] += c;
  b[x2 + 1][y2 + 1] += c;
  b[x1][y2 + 1] -= c;
  b[x2 + 1][y1] -= c;
}
int main()
{
  cin >> n >> m >> q;
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
          cin >> a[i][j];
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
          insert(i, j, i, j, a[i][j]);
  while (q--)
  {
      int x1, x2, y1, y2, c;
      cin >> x1 >> y1 >> x2 >> y2 >> c;
      insert(x1, y1, x2, y2, c);
  }
  // 其他过程略
}
```

2 ⋆ Basic Data Structures

2.1 Linked List

2.1.1 Singly Linked List

```
const int N = 100010;
int n, h[N], e[N], ne[N], idx = 1;
void init() { ne[0] = -1; }
void insert(int k, int x) //  第 k 个节点后插入
{ e[idx] = x, ne[idx] = ne[k], ne[k] = idx++; }
void del(int k) // 第 k 个节点后删除
{ ne[k] = ne[ne[k]]; }
```

2.1.2 Bidirectional Linked List

```
const int N = 100010;
int n, r[N], l[N], e[N], idx = 2;
void init() { r[0] = 1; l[1] = 0; }
void insert(int k, int x) // 第 k 个节点后插入
{
  e[idx] = x;
  r[idx] = r[k];
  l[idx] = k;
  l[r[k]] = idx;
  r[k] = idx++;
}
void remove(int k) // 删除 k 本身
{ r[l[k]] = r[k]; l[r[k]] = l[k]; }
```

2.2 Stack & Queue

2.2.1 Monotonic Stack

```
// 常见模型：找出每个数左边离它最近的比它大/小的数
int tt = 0;
for (int i = 1; i <= n; i ++ )
{
  while (tt && check(stk[tt], i)) tt -- ;
  stk[++tt] = i;
}
```

2.2.2 Monotonic Queue

```
// 常见模型：找出滑动窗口中的最大值/最小值
int hh = 0, tt = -1;
for (int i = 0; i < n; i ++ )
{
  while (hh <= tt && check_out(q[hh]))
      hh++; // 判断队头是否滑出窗口
  while (hh <= tt && check(q[tt], i))
      tt-- ;
  q[++tt] = i;
}
```

2.3 KMP

```
const int N = 100010, M = 1000010;
int n, m;
char p[N], s[M];
void getNext(int ne[])
{
  for (int i = 2, j = 0; i <= n; i++)
  {
      while (j && p[j + 1] != p[i])
          j = ne[j];
      if (p[j + 1] == p[i]) j++;
      ne[i] = j;
  }
}
int KMP()
{
  int *ne = new int[n + 1];
  getNext(ne);
  for (int i = 1, j = 0; i <= m; i++)
  {
      while (j && p[j + 1] != s[i])
          j = ne[j];
      if (p[j + 1] == s[i]) j++;
      if (j == n) cout << i - n << ' ';
  }
  return -1;
}
```

2.4 Trie

```
const int N = 100010;
int trie[N][26], cnt[N], idx = 0;
void insert(string &str)    // 插入到 Trie 数组
{
  int p = 0;
  for (auto c : str)
  {
      int u = c - 'a';
      if (!trie[p][u])
          trie[p][u] = ++idx;
      p = trie[p][u];
  }
  cnt[p]++;
}
int query(string &str)      // 查询字符串出现的次数
{
  int p = 0;
  for (auto c : str)
  {
      int u = c - 'a';
      if (!trie[p][u]) return 0;
      p = trie[p][u];
  }
  return cnt[p];
```

```
}
```

2.5 Disjoint-Set

```
const int N = 100010;
int n, m, p[N], Size[N], D[N];
void init()
{
  for (int i = 1; i <= n; i ++ )
      p[i] = i, Size[i] = 1, D[i] = 0;
}
int find(int x)
{
  if (p[x] != x)
  {
      int u = find(p[x]);
      D[x] += D[p[x]];  // 视具体情况计算
      p[x] = u;
  }
  return p[x];
}
void merge(int a, int b, int distance)
{
  int x = find(a), y = find(b);
  if(x != y)
  {
      p[x] = y;
      D[x] = distance;  // 视具体情况计算
      Size[y] += Size[x];
  }
}
```

2.6 Hash

2.6.1 Simple Hash

```
// (1) 拉链法
int h[N], e[N], ne[N], idx;
void insert(int x)
{
  int k = (x % N + N) % N;
  e[idx] = x, ne[idx] = h[k], h[k] = idx ++ ;
}
bool find(int x)
{
  for (int i = h[(x % N + N) % N]; i != -1; i = ne[i])
      if (e[i] == x) return true;
  return false;
}
// (2) 开放寻址法
int find(int x)
{
  int t = (x % N + N) % N;
  while (h[t] != null && h[t] != x)
  { t ++ ; if (t == N) t = 0; }
  return t;
}
```

2.6.2 String Hash

```
typedef unsigned long long ULL;
ULL h[N], p[N];
void init()
{
  p[0] = 1;
  for (int i = 1; i <= n; i ++ ) { h[i] = h[i - 1] * P + str[i]; p[i] = p[i - 1] * P ; }
}
ULL get(int l, int r) { return h[r] - h[l - 1] * p[r - l + 1]; }
```

2.7 STL

```
// vector
size()      返回元素个数
empty()     返回是否为空
clear()     清空
front()/back()
push_back()/pop_back()
begin()/end()
[]
支持比较运算，按字典序
// pair<int, int>
first       第一个元素
second      第二个元素
支持比较运算，以first为第一关键字，以second为第二关键字（字典序）
// string
size()/length()  返回字符串长度
empty()
clear()
substr(起始下标，(子串长度))  返回子串
c_str()  返回字符串所在字符数组的起始地址
// queue
size()
empty()
push()      向队尾插入一个元素
front()     返回队头元素
back()      返回队尾元素
pop()       弹出队头元素
// priority_queue
size()
empty()
push()      插入一个元素
top()       返回堆顶元素
pop()       弹出堆顶元素
定义成小根堆的方式：priority_queue<int, vector<int>, greater<int>> q;
// stack
size()
empty()
push()      向栈顶插入一个元素
top()       返回栈顶元素
pop()       弹出栈顶元素
// deque
size()
empty()
clear()
front()/back()
push_back()/pop_back()
```

```
push_front()/pop_front()
begin()/end()
[]
// set, map, multiset, multimap: 基于平衡二叉树（红黑树）动态维护有序序列
size()
empty()
clear()
begin()/end()
++, -- 返回前驱和后继，时间复杂度 O(logn)
// set/multiset
  insert()  插入一个数
  find()    查找一个数
  count()   返回某一个数的个数
  erase()
      (1) 输入是一个数x，删除所有x，O(k + logn)
      (2) 输入一个迭代器，删除这个迭代器
  lower_bound()/upper_bound()
      lower_bound(x)  返回大于等于x的最小的数的迭代器
      upper_bound(x)  返回大于x的最小的数的迭代器
// map/multimap
  insert()  插入的数是一个pair
  erase()   输入的参数是pair或者迭代器
  find()
  []        注意multimap不支持此操作。 时间复杂度是 O(logn)
  lower_bound()/upper_bound()
// unordered_set, unordered_map, unordered_multiset, unordered_multimap
增删改查的时间复杂度是 O(1)
不支持 lower_bound()/upper_bound()， 迭代器的++，--
// bitset
bitset<10000> s;
~, &, |, ^
>>, <<
==, !=
[]
count()     返回有多少个1
any()       判断是否至少有一个1
none()      判断是否全为0
set()       把所有位置成1
set(k, v)   将第k位变成v
reset()     把所有位变成0
flip()      等价于~
flip(k)     把第k位取反
```

3 ⋆ Search & Graph Theory

3.1 Representation of Tree & Graph

3.1.1 Adjacency Matrix

```
// g[a][b] = a->b
```

3.1.2 Adjacency List

```
int h[N], e[N], ne[N], idx;
void init() { memeset(h, -1, sizeof h); }
void add(int a, int b) { e[idx] = b, ne[idx] = h[a], h[a] = idx++ ; }
```

3.2 DFS & BFS

3.2.1 DFS

```
int dfs(int u)
{
  st[u] = true; // 表示点 u 已经被遍历过
  for (int i = h[u]; i != -1; i = ne[i])
  { int j = e[i]; if (!st[j]) dfs(j); }
}
```

3.2.2 BFS

```
queue<int> q;
st[1] = true; q.push(1);
while (q.size())
{
  int t = q.front(); q.pop();
  for (int i = h[t]; i != -1; i = ne[i])
     if (!st[e[i]]) { st[e[i]] = true; q.push(e[i]); }
}
```

3.3 Topological Sort

```
const int N = 100010;
int e[2 * N], ne[2 * N], h[N], d[N], idx;
int n, m, q[N];
void init() { memeset(h, -1, sizeof h); }
void add(int a, int b) { e[idx] = b, ne[idx] = h[a], h[a] = idx++, d[b]++; }
bool topSort()
{
  int hh = 0, tt = -1;
  for (int i = 1; i <= n; i++)
    if (!d[i]) q[++tt] = i;
  while (hh <= tt)
    for (int i = h[q[hh++]]; ~i; i = ne[i])
      if (--d[e[i]] == 0) q[++tt] = e[i];
  return tt == n - 1;
}
```

3.4 Shortest Path

3.4.1 Dijkstra

```
const int N = 1010;
int n, dist[N];
int h[N], w[N], e[N], ne[N], idx;
bool st[N];
void add(int a, int b, int c) { e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++; }
int dijkstra()      // 需要初始化 dist 与 h
{
  dist[1] = 0;
  priority_queue<PII, vector<PII>, greater<PII>> heap;
  heap.push({0, 1});
  while (heap.size())
  {
     auto t = heap.top();
     heap.pop();
     int ver = t.second, distance = t.first;
     if (st[ver]) continue;
     st[ver] = true;
     for (int i = h[ver]; i != -1; i = ne[i])
        if (dist[e[i]] > distance + w[i])
        {
           dist[e[i]] = distance + w[i];
           heap.push({dist[e[i]], e[i]});
        }
  }
  if (dist[n] == 0x3f3f3f3f) return -1;
  return dist[n];
}
```

3.4.2 Bellman-Ford

```
const int N = 100010;
int n, m, dist[N], backup[N];
struct Edge
{
  int a, b, w;
}edges[N];
int bellman_ford()
{
  memset(dist, 0x3f, sizeof dist);
  dist[1] = 0;
  for (int i = 0; i < n; i ++ )
  {
     memcpy(backup, dist, sizeof dist);
     for (int j = 0; j < m; j++)
     {
```

```cpp
            int a = edges[j].a, b = edges[j].b, w = edges[j].w;
            dist[b] = min(dist[b], backup[a] + w);
        }
    }
    if (dist[n] > 0x3f3f3f3f / 2) return -1;
    return dist[n];
}
```

3.4.3 SPFA

```cpp
const int N = 100010;
int n, m, dist[N];
int e[2 * N], ne[2 * N], w[2 * N], h[N], idx;
bool vis[N];
void spfa()      // 需要初始化 dist 与 h
{
  queue<int> q;
  q.push(1); vis[1] = true;
  while (q.size())
  {
      int t = q.front();
      q.pop();
      vis[t] = false;
      for (int i = h[t]; ~i; i = ne[i])
          if (dist[e[i]] > dist[t] + w[i])
          {
              dist[e[i]] = dist[t] + w[i];
              if (!vis[e[i]]) vis[e[i]] = true, q.push(j);
          }
  }
  dist[n] > INF / 2 ? cout << "impossible" : cout << dist[n];
}
```

3.4.4 Detecting Negative Circle in SPFA

```cpp
void spfa()      // 只需要初始化 h
{
  queue<int> q;
  // 基于虚拟原点假设，所有点放入队列
  for (int i = 1; i <= n; i++) q.push(i), st[i] = true;
  while (q.size())
  {
      int t = q.front();
      q.pop();
      vis[t] = false;
      for (int i = h[t]; ~i; i = ne[i])
          if (dist[e[i]] > dist[t] + w[i])
          {
              dist[e[i]] = dist[t] + w[i];
              // 新增
              cnt[j] = cnt[t] + 1;
              if (cnt[j] >= n) return true
              if (!st[j]) q.push(j), st[j] = true;
          }
  }
  return false;
}
```

3.4.5 Floyd

```cpp
const int N = 210;
int g[N][N], n, m, k;
int main()
{
  cin >> n >> m >> k;
  memset(g, 0x3f, sizeof g);
  for (int i = 1; i <= n; i++) g[i][i] = 0;
  while (m--)
  {
      int a, b, c;
      cin >> a >> b >> c;
      g[a][b] = min(g[a][b], c);
  }
  for (int k = 1; k <= n; k++)
      for (int i = 1; i <= n; i++)
          for (int j = 1; j <= n; j++)
              g[i][j] = min(g[i][k] + g[k][j], g[i][j]);
  // 后续代码略
  return 0;
}
```

3.5 Minimum Spanning Tree

3.5.1 Prim

```cpp
const int N = 510;
int n, m, g[N][N], dist[N];
bool vis[N];
void prim()
{
  int res = 0;
  for (int i = 0; i < n; i++)
  {
      int t = -1;
      for (int j = 1; j <= n; j++)
          if (!vis[j] && (t == -1 || dist[j] < dist[t])) t = j;
      if (i && dist[t] == INF) { res = INF; break; }
      if (i) res += dist[t];
      vis[t] = true;
      for (int j = 1; j <= n; j++) dist[j] = min(dist[j], g[t][j]);
  }
  res == INF ? cout << "impossible" : cout << res;
}
int main()
{
  memset(g, 0x3f, sizeof g);
  memset(dist, 0x3f, sizeof dist);
  cin >> n >> m;
  while (m--)
```

```
    {
        int a, b, c;
        cin >> a >> b >> c;
        g[a][b] = min(g[a][b], c);
        g[b][a] = min(g[b][a], c);
    }
    prim();
    return 0;
}
```

3.5.2 Kruskal

```
const int N = 100010;
int n, m;
int p[N];
struct Edge
{
  int a, b, w;
  bool operator<(const Edge &e) const { return w < e.w; };
} edge[2 * N];
void init() { for (int i = 1; i <= n; i++) p[i] = i; }
int find(int x)
{
  if (x != p[x]) p[x] = find(p[x]);
  return p[x];
}
void merge(int x, int y) { p[find(x)] = find(y); }
void kruskal()
{
  int res = 0, cnt = 0;
  for (int i = 1; i <= m; i++)
      if (find(edge[i].a) != find(edge[i].b))
      {
          merge(edge[i].a, edge[i].b);
          res += edge[i].w;
          cnt++;
      }
  if (cnt < n - 1) res = INF;
  res == INF ? cout << "impossible" : cout << res;
}
int main()
{
  init();
  cin >> n >> m;
  for (int i = 1; i <= m; i++) cin >> edge[i].a >> edge[i].b >> edge[i].w;
  sort(edge + 1, edge + m + 1);
  kruskal();
  return 0;
}
```

3.6 Bipartite Graph

3.6.1 Coloring Method

To check if a given graph is bipartite.

```
const int N = 100010, M = 200010;
int n, m;
int e[M], ne[M], h[N], color[N], idx;
bool dfs(int u, int c)
{
  color[u] = c;
  for (int i = h[u]; ~i; i = ne[i])
      if (color[e[i]] == -1)
      {
          if (!dfs(e[i], !c)) return false;
      }
      else if (color[e[i]] == c) return false;
  return true;
}
bool check()
{
for (int i = 1; i <= n; i++)
  if (color[i] == -1)
      if (!dfs(i, 0)) return false;
return true;
}
int main()
{
// 注意另外初始化 h 与 color
cin >> n >> m;
while (m--)
{
  int a, b;
  cin >> a >> b;
  add(a, b), add(b, a);
}
// 其余过程略
}
```

3.6.2 Hungarian Algorithm

To find the maximum matching for a given graph.

```cpp
const int N = 510, M = 100010;
int n1, n2, m;
int e[M], ne[M], h[N], match[N], idx;
bool vis[N];
bool find(int x)
{
  for (int i = h[x]; ~i; i = ne[i])
      if (!vis[e[i]])
      {
          vis[e[i]] = true;
          if (match[e[i]] == 0 || find(match
    [e[i]]))
          {
              match[e[i]] = x;
              return true;
          }
      }
  return false;
}
int main()
{
  // 注意初始化 h
  cin >> n1 >> n2 >> m;
  while (m--)
  {
      int a, b;
      cin >> a >> b;
      add(a, b);
  }
  int res = 0;
  for (int i = 1; i <= n1; i++)
  {
      memset(vis, false, sizeof vis);
      if (find(i)) res++;
  }
  cout << res;
  return 0;
}
```

4 ⋆ Basic Math

4.1 Prime Numbers

4.1.1 Judging Prime Numbers

$O(\sqrt{n})$

```
bool is_prime(int x)
{
  if (x < 2) return false;
  for (int i = 2; i <= x / i; i ++ )
      if (x % i == 0) return false;
  return true;
}
```

4.1.2 Prime Factorization

```
void divide(int x)
{
  for (int i = 2; i <= x / i; i ++ )
      if (x % i == 0)
      {   // 此条件成立时 i 一定是质数
          int s = 0;
          while (x % i == 0) x /= i, s ++ ;
          cout << i << ' ' << s << '\n';
      }
  if (x > 1) cout << x << ' ' << 1 << '\n'
}
```

4.1.3 Euler's Sieve

```
int primes[N], cnt;
bool st[N];
void get_primes(int n)
{
  for (int i = 2; i <= n; i ++ )
  {
      if (!st[i]) primes[cnt++] = i;
      for (int j = 0; primes[j] <= n / i; j ++ )
      {
          st[primes[j] * i] = true;
          if (i % primes[j] == 0) break;
      }
  }
}
```

4.2 Divisor

4.2.1 Find All Divisors

```
vector<int> get_divisors(int x)
{
  vector<int> res;
  for (int i = 1; i <= x / i; i ++ )
      if (x % i == 0)
      {
          res.push_back(i);
          if (i != x / i) res.push_back(x / i);
      }
  sort(res.begin(), res.end());
  return res;
}
```

4.2.2 The Number of Divisors

```
const int mod = 1e9 + 7;
int n;
int main()
{
  cin >> n;
  unordered_map<int, int> h;
  while (n--)
  {
      int x;
      cin >> x;
      for (int i = 2; i <= x / i; i++)
          while (x % i == 0) { h[i]++; x = x / i; }
      if (x > 1) h[x]++;
  }
  long long res = 1;
  for (auto iter = h.begin(); iter != h.end(); iter++)
      res = res * (iter->second + 1) % mod;
  cout << res;
  return 0;
}
```

4.2.3 The Sum of Divisors

```
const int mod = 1e9 + 7;
int n;
long long getSum(int x, int c)
{
  long long s = 1;
  while(c--) s = (s * x + 1) % mod;
  return s;
}
int main()
{
  cin >> n;
  unordered_map<int, int> h;
  while (n--)
  {
      int x;
      cin >> x;
      for (int i = 2; i <= x / i; i++)
          while (x % i == 0) { h[i]++; x = x / i; }
      if (x > 1) h[x]++;
  }
  long long res = 1;
  for (auto iter = h.begin(); iter != h.end(); iter++)
      res = res * getSum(iter->first, iter->second) % mod;
```

```
    cout << res;
    return 0;
}
```

4.2.4 Euclidean Algorithm

```
int gcd(int a, int b)
{ return a % b == 0 ? b : gcd(b, a % b); }
```

4.3 Euler Function

4.3.1 Simple Method

```
int phi(int x)
{
    int res = x;
    for (int i = 2; i <= x / i; i ++ )
        if (x % i == 0)
        {
            res = res / i * (i - 1);
            while (x % i == 0) x /= i;
        }
    if (x > 1) res = res / x * (x - 1);
    return res;
}
```

4.3.2 Euler's Sieve Method

```
const int N = 1000010;
int n, primes[N], phi[N], cnt;
bool st[N];
void getEuler()
{
    phi[1] = 1;
    for (int i = 2; i <= n; i++)
    {
        if (!st[i])
        {
            primes[cnt++] = i;
            // i 是质数，它只会被本身整除，所以直接赋值 i - 1
            phi[i] = i - 1;
        }
        for (int j = 0; primes[j] <= n / i; j ++)
        {
            st[i * primes[j]] = true;
            if (i % primes[j] == 0)
            {
                // 如果 i % primes[j] == 0 成立表示 primes[j] 是 i 的最小质因子
                // 也是 primes[j] * i 的最小质因子
                // 1 - 1 / primes[j] 这一项在 phi[i] 中计算过了，只需将基数 N 修正为 primes[j] 倍
                phi[primes[j] * i] = phi[i] * primes[j];
                break;
            }
            // 否则，primes[j] 不是 i 的质因子，只是 primes[j] * i 的最小质因子
            // 不仅需要将基数 N 修正为 primes[j] 倍
            // 还需要补上 1 - 1 / primes[j] 的分子项，因此最终结果为 phi[i] * (primes[j] - 1)
            phi[primes[j] * i] = phi[i] * (primes[j] - 1);
        }
    }
}
```

4.4 Exponentiating by Squaring

```
LL qmi(int m, int k, int p)
{
    LL res = 1 % p, t = m;
    while (k)
    {
        if (k&1) res = res * t % p;
        t = t * t % p;
        k >>= 1;
    }
    return res;
}
```

4.5 Extended Euclidean Algorithm

```
int exgcd(int a, int b, int &x, int &y)
{
    if (!b)
    {
        x = 1;
        y = 0;
        return a;
    }
    int d = exgcd(b, a % b, y, x);
    y -= (a / b) * x;
    return d;
}
```

4.6 Chinese Remainder Theorem

```
LL exgcd(LL a, LL b, LL &x, LL &y)
{
    if (!b) { x = 1, y = 0; return a; }
    LL d = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return d;
}
int main()
{
```

```
    int n;
    cin >> n;
    LL x = 0, m1, a1;
    cin >> m1 >> a1;
    for (int i = 0; i < n - 1; i++)
    {
        LL m2, a2;
        cin >> m2 >> a2;
        LL k1, k2;
        LL d = exgcd(m1, m2, k1, k2);
        if ((a2 - a1) % d) { x = -1; break; }
        k1 *= (a2 - a1) / d;
        k1 = (k1 % (m2 / d) + m2 / d) % (m2 / d);
        x = k1 * m1 + a1;
        LL m = abs(m1 / d * m2);
        a1 = k1 * m1 + a1;
        m1 = m;
    }
    if (x != -1)
        x = (a1 % m1 + m1) % m1;
    cout << x << '\n';
    return 0;
}
```

4.7 Gauss-Jordan Elimination

4.7.1 Linear Equation Group

```
int gauss()
{
    int c, r;
    for (c = 0, r = 0; c < n; c++)
    {
        int t = r;
        for (int i = r; i < n; i++)      // 找绝对值最大的行
            if (fabs(a[i][c]) > fabs(a[t][c]))
                t = i;
        if (fabs(a[t][c]) < eps)         // 此时没必要对该列该行处理
            continue;
        for (int i = c; i <= n; i++)
            swap(a[t][i], a[r][i]);      // 将绝对值最大的行换到最顶端
        for (int i = n; i >= c; i--)
            a[r][i] /= a[r][c];          // 将当前行的首位变成1
        for (int i = r + 1; i < n; i++)  // 用当前行将下面所有的列消成0
            if (fabs(a[i][c]) > eps)
                for (int j = n; j >= c; j--)
                    a[i][j] -= a[r][j] * a[i][c];
        r++;
    }
    if (r < n)
    {
        for (int i = r; i < n; i++)
            if (fabs(a[i][n]) > eps)
                return 2; // 无解
        return 1;          // 有无穷多组解
    }
    for (int i = n - 1; i >= 0; i--)
        for (int j = i + 1; j < n; j++)
            a[i][n] -= a[i][j] * a[j][n];
    return 0;            // 有解
}
```

4.7.2 XOR Linear Equation Group

```
int gauss()
{
    int c, r;
    for (c = 0, r = 0; c < n; c++)
    {
        int t = r;
        for (int i = r; i < n; i++)
            if (a[i][c])
                t = i;
        if (!a[t][c])
            continue;
        for (int i = c; i <= n; i++)
            swap(a[r][i], a[t][i]);
        for (int i = r + 1; i < n; i++)
            if (a[i][c])
                for (int j = n; j >= c; j--)
                    a[i][j] ^= a[r][j];
        r++;
    }
    if (r < n)
    {
        for (int i = r; i < n; i++)
            if (a[i][n])
                return 2;
        return 1;
    }
    for (int i = n - 1; i >= 0; i--)
        for (int j = i + 1; j < n; j++)
            a[i][n] ^= a[i][j] * a[j][n];
    return 0;
}
```

4.8 Combinatorial Counting

4.8.1 Recurrence Relation

```
void init()
{
    for (int i = 0; i < N; i++)
        for (int j = 0; j <= i; j++)
            if (!j) c[i][j] = 1;
            else c[i][j] = (c[i - 1][j] + c[i - 1][j - 1]) % mod;
}
```

4.8.2 Preprocessing & Inverse Element

```
const int N = 100010, mod = 1e9 + 7;
int n, fact[N], infact[N];
int qmi(int a, int b, int p)
{
```

```
    int res = 1;
    while (b)
    {
        if (b & 1)
            res = (LL)res * a % p;
        a = (LL)a * a % p;
        b >>= 1;
    }
    return res;
}
int main()
{
    fact[0] = infact[0] = 1;
    for (int i = 1; i < N; i++)
    {
        fact[i] = (LL)fact[i - 1] * i % mod;
        infact[i] = (LL)infact[i - 1] * qmi(i, mod - 2, mod) % mod;
    }
    // 此后 C(a, b) = (LL)fact[a] * infact[b] % mod * infact[a - b] % mod
}
```

4.8.3 Lucas Theorem

```
int qmi(int a, int k, int p)
{
    int res = 1 % p;
    while (k)
    {
        if (k & 1)
            res = (LL)res * a % p;
        a = (LL)a * a % p;
        k >>= 1;
    }
    return res;
}
int C(int a, int b, int p)
{
    if (a < b) return 0;
    LL x = 1, y = 1;
    // x = a * (a - 1) * (a - 2) * ... * (a - b + 1) = a! / (a - b)! (mod p)
    // y = 1 * 2 * ... * b = b! (mod p)
    for (int i = a, j = 1; j <= b; i--, j++)
    { x = (LL)x * i % p; y = (LL)y * j % p; }
    return x * (LL)qmi(y, p - 2, p) % p;
}
int lucas(LL a, LL b, int p)
{
    if (a < p && b < p)
        return C(a, b, p);
    return (LL)C(a % p, b % p, p) * lucas(a / p, b / p, p) % p;
}
```

4.8.4 Factorization Method

```
const int N = 5010;
int n, primes[N], sum[N], cnt;
bool st[N];
void getPrimes(int n) { // 略 }
// 求 n! 中 p 的幂次
int get(int n, int p)
{
    int res = 0;
    while (n) { res += n / p; n /= p; }
    return res;
}
void mul(vector<int> &a, int b) { // 高精度乘，略 }
int main()
{
    int a, b;
    cin >> a >> b;
    getPrimes(a);
    for (int i = 0; i < cnt; i++)
    {
        int p = primes[i];
        sum[i] = get(a, p) - get(b, p) - get(a - b, p);
    }
    vector<int> res;
    res.push_back(1);
    for (int i = 0; i < cnt; i++)
        for (int j = 0; j < sum[i]; j++)
            mul(res, primes[i]);
    for (int i = res.size() - 1; i >= 0; i--)
        cout << res[i];
}
```

4.8.5 Catalan Number

```
const int N = 100010, mod = 1e9 + 7;
int qmi(int a, int k, int p) { // 略 }
int main()
{
    int n;
    cin >> n;
    int a = n * 2, b = n, res = 1;
    for (int i = a; i > a - b; i--)
        res = (LL)res * i % mod;
    for (int i = 1; i <= b; i++)
        res = (LL)res * qmi(i, mod - 2, mod) % mod;
    res = (LL)res * qmi(n + 1, mod - 2, mod) % mod;
}
```

4.9 Inclusion-Exclusion Principle

```
const int N = 20;
int n, m, res = 0, p[N];
int main()
{
    cin >> n >> m;
    for (int i = 0; i < m; i++)
        cin >> p[i];
    // 使用二进制数字表示数字选取情况
    for (int i = 1; i < 1 << m; i++)
    {
```

```cpp
            int t = 1, cnt = 0;
            // 遍历每个被选取的质数
            for (int j = 0; j < m; j++)
                if (i >> j & 1)
                {
                    cnt++;
                    // 一个质数能被选取的条件应该是其累乘积不超过目标数字
                    if ((LL)t * p[j] > n)
                    { t = -1; break; }
                    t *= p[j];
                }
            if (t != -1)
                // 容斥原理公式中奇数个并集系数为 1, 反之为 -1
                if (cnt % 2) res += n / t;
                else res -= n / t;
    }
    cout << res;
}
```

4.10 Game Theory

4.10.1 NIM Game

```cpp
const int N = 110, M = 100010;
int k, n,  s[N], f[M];
int sg(int x)
{
    if (f[x] != -1) return f[x];
    // 到达节点得 SG 函数集合
    unordered_set<int> S;
    // 能取走石子就说明能到达，并且递归向下求解
    for (int i = 0; i < k; i++)
    {
        int sum = s[i];
        if (x >= sum) S.insert(sg(x - sum));
    }
    // SG 从小到达遍历并返回，找到最小的、不包含在 SG 函数集合中的自然数
    for (int i = 0;; i++)
        if (!S.count(i))
            return f[x] = i;
}

int main()
{
    cin >> k;
    for (int i = 0; i < k; i++) cin >> s[i];
    cin >> n;
    memset(f, -1, sizeof f);
    int res = 0;
    // 每一堆石子都是一个入度为 0 的起始点
    for (int i = 0; i < n; i++)
    {
        int x;
        cin >> x;
        res ^= sg(x);
    }
    res ? cout << "Yes" : cout << "No";
    return 0;
}
```

5 ⋆ Basic DP

5.1 Knapsack Problem

5.1.1 01 Knapsack

```
const int N = 1010;
int n, m, v[N], w[N], f[N];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> v[i] >> w[i];
  for (int i = 1; i <= n; i++)
      for (int j = m; j >= v[i]; j++)
          f[j] = max(f[j], f[j - v[i]] + w[i]);
  cout << f[m];
}
```

5.1.2 Complete Knapsack

```
const int N = 1010;
int n, m, v[N], w[N], f[N];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> v[i] >> w[i];
  for (int i = 1; i <= n; i++)
      for (int j = v[i]; j <= m; j++)
          f[j] = max(f[j], f[j - v[i]] + w[i]);
  cout << f[m];
}
```

5.1.3 Mutiple Knapsack

```
const int N = 25010;
int n, m, f[N];
int main()
{
  cin >> n >> m;
  for (int i = 0; i < n; i++)
  {
    int v, w, s;
    cin >> v >> w >> s;
    for (int k = 1; k <= s; k *= 2)
    {
      for (int j = m; j >= k * v; j--)
        f[j] = max(f[j], f[j - k * v] + k * w);
      s -= k;
    }
    if (s)
      for (int j = m; j >= s * v; j--)
        f[j] = max(f[j], f[j - s * v] + s * w);
  }
  cout << f[m];
}
```

5.1.4 Grouped Knapsack

```
const int N = 120;
int n, m, s[N], v[N][N], w[N][N], f[N];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
  {
      cin >> s[i];
      for (int j = 1; j <= s[i]; j++)
          cin >> v[i][j] >> w[i][j];
  }
  for (int i = 1; i <= n; i++)
      for (int j = m; j >= 0; j--)
          for (int k = 1; k <= s[i]; k++)
              if (v[i][k] <= j)
                  f[j] = max(f[j], f[j - v[i][k]] + w[i][k]);
  cout << f[m];
}
```

5.2 Linear DP

5.2.1 LIS

Here is an $O(n^2)$ solution:

```
const int N = 1010;
int n, a[N], f[N];
int main()
{
  cin >> n;
  for (int i = 1; i <= n; i++)
      cin >> a[i];
  for (int i = 1; i <= n; i++)
  {
      f[i] = 1;
      for (int j = 1; j < i; j++)
          if (a[j] < a[i])
              f[i] = max(f[i], f[j] + 1);
  }
  int res = 0;
  for (int i = 1; i <= n; i++)
      res = max(res, f[i]);
  cout << res;
}
```

Another is an $O(nlogn)$ solution:

```
const int N = 100010;
int n, a[N], q[N];
int main()
{
  cin >> n;
  for (int i = 1; i <= n; i++) cin >> a[i];
  int len = 0;
  q[len] = -INF;
  for (int i = 1; i <= n; i++)
```

```cpp
    {
        int l = 0, r = len;
        while (l < r)
        {
            int mid = l + r + 1 >> 1;
            if (q[mid] < a[i]) l = mid;
            else r = mid - 1;
        }
        len = max(r + 1, len);
        q[r + 1] = a[i];
    }
    cout << len;
}
```

5.2.2 LCS

```cpp
const int N = 1010;
int n, m, f[N][N];
char a[N], b[N];
int main()
{
    cin >> n >> m >> (a + 1) >> (b + 1);
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= m; j++)
        {
            f[i][j] = max(f[i - 1][j], f[i][j - 1]);
            if (a[i] == b[j])
                f[i][j] = max(f[i][j], f[i - 1][j - 1] + 1);
        }
    cout << f[n][m];
}
```

5.3 Interval DP

In this case we focus on an interval, whose sum of its elements can represent the answer we want to find:

```cpp
const int N = 310;
int n, s[N], f[N][N];
int main()
{
    cin >> n;
    for (int i = 1; i <= n; i++)
        cin >> s[i], s[i] += s[i - 1];
    for (int len = 2; len <= n; len++)
        for (int i = 1; i + len - 1 <= n; i++)
        {
            int l = i, r = i + len - 1;
            f[l][r] = INF;
            for (int k = l; k < r; k++)
                f[l][r] = min(f[l][r], f[l][k] + f[k + 1][r] + s[r] - s[l - 1]);
        }
    cout << f[1][n];
}
```

5.4 Counting DP

```cpp
const int N = 1010, M = 1e9 + 7;
int n, f[N][N];
int main()
{
    cin >> n;
    f[0][0] = 1;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= i; j++)
            f[i][j] = (f[i - 1][j - 1] + f[i - j][j]) % M;
    int ans = 0;
    for (int i = 1; i <= n; i++)
        ans = (ans + f[n][i]) % M;
    cout << ans;
}
```

5.5 Digit DP

```cpp
// 求数 n 的位数
int get(int n)
{
    int res = 0;
    while (n) n /= 10, res++;
    return res;
}
int count(int n, int i)
{
    int res = 0, dgt = get(n);
    for (int j = 1; j <= dgt; j++)
    {
        // p 为当前遍历位次(第 j 位)的数大小 <10^(右边的数的位数)>, Ps: 从左往右(从高位到低位)
        // l 为第 j 位的左边的数, r 为右边的数, dj 为第 j 位上的数
        int p = pow(10, dgt - j), l = n / p / 10, r = n % p, dj = n / p % 10;
        // 求要选的数在 i 的左边的数小于 l 的情况:
        //      1)、当 i 不为 0 时 xxx : 0...0 ~ l - 1, 即 l * (右边的数的位数) == l * p 种选法
        //      2)、当 i 为 0 时 由于不能有前导零 故 xxx: 0....1 ~ l - 1, 即 (l - 1) * (右边的数的位数) == (l - 1) * p 种选法
        if (i) res += l * p;
        else res += (l - 1) * p;
        // 求要选的数在 i 的左边的数等于 l 的情况: (即视频中的xxx == l 时)
        //      1)、i > dj 时 0 种选法
        //      2)、i == dj 时 yyy : 0...0 ~ r 即 r + 1 种选法
        //      3)、i < dj 时 yyy : 0...0 ~ 9...9 即 10^(右边的数的位数) == p 种选法 */
        if (i == dj) res += r + 1;
        if (i < dj) res += p;
    }
    return res;
}
int main()
{
    int a, b;
```

```cpp
  while (cin >> a >> b, a)
  {
    if (a > b) swap(a, b);
    for (int i = 0; i <= 9; ++i)
      cout << count(b, i) - count(a - 1, i) << ' ';
    // 利用前缀和思想：[l, r] 的和 = s[r] - s[l - 1]
    cout << '\n';
  }
}
```

5.6 State Compression DP

```cpp
const int N = 12, M = 1 << 12;
int n, m;
LL f[N][M];
bool st[M];
int main()
{
  while (cin >> n >> m, n || m)
  {
    memset(f, 0, sizeof f);
    for (int i = 0; i < 1 << n; i++)
    {
      st[i] = true;
      // 统计连续 0 的个数，若连续 0 为奇数个就不能正好放得下竖放的方格
      int cnt = 0;
      for (int j = 0; j < n && st[i]; j++)
        if (i >> j & 1)
        {
          // 当前格子被使用
          // 如果连续 0 的数量为奇数个，当前格子被使用的后果就是导致格子重合，所以不可取
          if (cnt & 1)
            st[i] = false;
          // 刷新状态
          cnt = 0;
        }
        else cnt++;
      // 最后再判断一次，防止漏判
      if (cnt & 1)
        st[i] = false;
    }
    // 没有摆放任何棋子的状态默认只有 1 种取法
    f[0][0] = 1;
    // 遍历每一列
    for (int i = 1; i <= m; i++)
      // 遍历当前列的每一种用二进制数字表示的摆放状态：1 指横向摆放，0 指空位
      for (int j = 0; j < 1 << n; j++)
        // 遍历上一列的每一种用二进制数字表示的摆放状态：1 指横向摆放，0 指空位
        for (int k = 0; k < 1 << n; k++)
          // 满足两个条件：两列的摆放互不冲突；两列摆放状态的结合状态是一个可取的状态则累加情况数
          if (!(j & k) && st[j | k])
            f[i][j] += f[i - 1][k];
    // 输出摆放好第 m 列且第 (m + 1) 列没有任何方格的状态数
    cout << f[m][0] << '\n';
  }
}
```

5.7 Tree DP

```cpp
// Don't use I/O functions from stdio.h!!!
#define itn int
#define nit int
#define nti int
#define tin int
#define tni int
#define retrun return
#define reutrn return
#define rutren return
#define INF 0x3f3f3f3f
#include <bits/stdc++.h>
using namespace std;
typedef pair<int, int> PII;
typedef long long LL;

const int N = 6010;

int n;
int e[N], ne[N], happy[N], h[N], idx;
int f[N][2];
bool has_father[N];
void add(int a, int b)
{ e[idx] = b, ne[idx] = h[a], h[a] = idx++; }
void dfs(int u)
{
  f[u][1] = happy[u];
  for (int i = h[u]; ~i; i = ne[i])

    dfs(e[i]);
    f[u][0] += max(f[e[i]][0], f[e[i]][1]);
    f[u][1] += f[e[i]][0];
  }
}
int main()
{
  memset(h, -1, sizeof h);
  cin >> n;
  for (int i = 1; i <= n; i++) cin >> happy[i];
  for (int i = 0; i < n - 1; i++)
  {
    int a, b;
    cin >> a >> b;
    has_father[a] = true;
    add(b, a);
  }
  int root = 1;
  while (has_father[root]) root++;
  dfs(root);
  cout << max(f[root][0], f[root][1]);
}
```

5.8 Memoized Search

```cpp
const int N = 310;
int n, m,
h[N][N], f[N][N],
dx[4] = {0, 1, 0, -1}, dy[4] = {1, 0, -1,
    0};
int dp(int x, int y)
{
  int &v = f[x][y];
  if (v != -1) return v;
  v = 1;
  for (int i = 0; i < 4; i++)
  {
      int a = x + dx[i], b = y + dy[i];
      if (a >= 1 && a <= n && b >= 1 && b <=
     m && h[a][b] < h[x][y])
          v = max(v, dp(a, b) + 1);
  }
  return v;
}
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
          cin >> h[i][j];
  memset(f, -1, sizeof f);
  int res = 0;
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
          res = max(res, dp(i, j));
  cout << res;
}
```

Part II: Advanced Template

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6 ⋆ Advanced Basic

6.1 Slow Multiplication

```
LL mul(LL a, LL b, LL p)
{
  LL ans = 0;
  while (b)
  {
      if (b & 1) ans = (ans + a) % p;
      a = a * 2 % p; b >>= 1;
  }
  return ans;
}
```

6.2 Sum of Geometric Series

```
const int mod = 9901;
int a, b;
int qmi(int a, int k)
{
  int res = 1;
  a %= mod;
  while (k)
  {
      if (k & 1)
          res = res * a % mod;
      a = a * a % mod;
      k >>= 1;
  }
  return res;
}
int sum(int p, int k)
{
  if (k == 1) return 1;
  if (k % 2 == 0)
      return (1 + qmi(p, k / 2)) * sum(p, k / 2) % mod;
  return (sum(p, k - 1) + qmi(p, k - 1)) % mod;
}
int main()
{
  // 以 a^b 约数之和为例求等比数列和
  cin >> a >> b;
  int res = 1;
  for (int i = 2; i <= a / i; i++)
      if (a % i == 0)
      {
          int s = 0;
          while (a % i == 0) a /= i, s++;
          res = res * sum(i, b * s + 1) % mod;
      }
  if (a > 1) res = res * sum(a, b + 1) % mod;
}
```

6.3 Sort

6.3.1 Card Balancing Problem

```
cin >> n;
for (int i = 1; i <= n; i++)
  cin >> a[i], avg += a[i];
avg /= n;
for (int i = 1; i <= n; i++)
  if (a[i] != avg)
      a[i + 1] += a[i] - avg, ans++;
cout << ans;
```

6.3.2 2D Card Balancing Problem

```
const int N = 100010;
int row[N], col[N], c[N], s[N];
LL work(int n, int a[])
{
  for (int i = 1; i <= n; i++)
      s[i] = s[i - 1] + a[i];
  if (s[n] % n) return -1;
  int avg = s[n] / n;
  c[1] = 0;
  for (int i = 2; i <= n; i++)
      c[i] = s[i - 1] - (i - 1) * avg;
  sort(c + 1, c + n + 1);
  LL res = 0;
  for (int i = 1; i <= n; i++)
      res += abs(c[i] - c[(n + 1) / 2]);
  return res;
}
int main()
{
  int n, m, cnt;
  cin >> n >> m >> cnt;
  while (cnt--)
  {
      int x, y;
      cin >> x >> y;
      row[x]++; col[y]++;
  }
  LL r = work(n, row);
  LL c = work(m, col);
  if (r != -1 && c != -1)
      cout << "both " << r + c;
  else if (r != -1)
      cout << "row " << r;
  else if (c != -1)
      cout << "column " << c;
  else cout << "impossible";
}
```

6.3.3 Dual Heaps

```
if (down.empty() || x <= down.top())
          down.push(x);
else up.push(x);
if (down.size() > up.size() + 1)
  up.push(down.top()), down.pop();
```

```
if (up.size() > down.size())
  down.push(up.top()), up.pop();
if (i % 2)
{
  cout << down.top() << ' ';
  if (++cnt % 10 == 0) cout << '\n';
}
```

6.4 RMQ

```
const int N = 200010, M = 18;
int n, m, w[N], f[N][M];
void init()
{
  for (int j = 0; j < M; j++)
      for (int i = 1; i + (1 << j) - 1 <= n; i++)
          if (!j) f[i][j] = w[i];
          else    // 也可以是最小值
              f[i][j] = max(f[i][j - 1], f[i + (1 << j - 1)][j - 1]);
}
int query(int l, int r)
{
  int len = r - l + 1;
  int k = log(len) / log(2);
  return max(f[l][k], f[r - (1 << k) + 1][k]);
}
```

7 ⋆ Advanced Data Structures

7.1 Binary Indexed Tree

```
// 支持区间修改、区间查询
// 利用变差分求二阶区间和
const int N = 100010;
int n, m, a[N];
LL tr1[N], tr2[N];
int lowbit(int x) { return x & -x; }
void add(LL tr[], LL x, LL c)
{
  for (int i = x; i <= n; i += lowbit(i))
      tr[i] += c;
}
LL sum(LL tr[], LL x)
{
  LL res = 0;
  for (int i = x; i; i -= lowbit(i))
      res += tr[i];
  return res;
}
LL prefix_sum(LL x)
{ return sum(tr1, x) * (x + 1) - sum(tr2, x); }
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> a[i];
  for (int i = 1; i <= n; i++)
  {
      int b = a[i] - a[i - 1];
      add(tr1, i, b);
      add(tr2, i, (LL)i * b);
  }
  while (m--)
  {
      char op[2];
      int l, r, d;
      cin >> op >> l >> r;
      if (*op == 'Q')
          cout << prefix_sum(r) - prefix_sum(l - 1) << '\n';
      else
      {
          cin >> d;
          add(tr1, l, d), add(tr2, l, (LL)l * d),
          add(tr1, r + 1, -d),
          add(tr2, r + 1, (LL)-(r + 1) * d);
      }
  }
}
```

7.2 Segment Tree

7.2.1 Maintain the Maximum

```
struct Node
{ int l, r, v; } tr[N * 4];
void pushup(int u)
{
  tr[u].v = max(tr[u << 1].v, tr[u << 1 | 1].v);
}
void build(int u, int l, int r)
{
  tr[u] = {l, r};
  if (l == r) return;
  int mid = l + r >> 1;
  build(u << 1, l, mid),
  build(u << 1 | 1, mid + 1, r);
}
int query(int u, int l, int r)
{
  if (tr[u].l >= l && tr[u].r <= r)
      return tr[u].v;
  int mid = tr[u].l + tr[u].r >> 1;
  int v = 0;
  if (l <= mid)
      v = query(u << 1, l, r);
  if (r > mid)
      v = max(v, query(u << 1 | 1, l, r));
  return v;
}
void modify(int u, int x, int v)
{
  if (tr[u].l == x && tr[u].r == x)
      tr[u].v = v;
  else
  {
      int mid = tr[u].l + tr[u].r >> 1;
      if (x <= mid)
          modify(u << 1, x, v);
      else
          modify(u << 1 | 1, x, v);
      pushup(u);
  }
}
```

7.2.2 Maintain the Maximum Subarray Sum

```
struct Node
{ int l, r, sum, lmax, rmax, tmax; } tr[N * 4];
void pushup(Node &u, Node &l, Node &r)
{
  u.sum = l.sum + r.sum;
  u.lmax = max(l.lmax, l.sum + r.lmax);
  u.rmax = max(r.rmax, r.sum + l.rmax);
  u.tmax = max(max(l.tmax, r.tmax), l.rmax + r.lmax);
}
void pushup(int u)
{ pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
void build(int u, int l, int r)
{
  if (l == r)
      tr[u] = {l, r, w[r], w[r], w[r], w[r]};
  else
  {
```

```
      tr[u] = {l, r};
      int mid = l + r >> 1;
      build(u << 1, l, mid),
      build(u << 1 | 1, mid + 1, r);
      pushup(u);
  }
}
void modify(int u, int x, int v)
{
  if (tr[u].l == x && tr[u].r == x)
      tr[u] = {x, x, v, v, v, v};
  else
  {
      int mid = tr[u].l + tr[u].r >> 1;
      if (x <= mid)
          modify(u << 1, x, v);
      else
          modify(u << 1 | 1, x, v);
      pushup(u);
  }
}
Node query(int u, int l, int r)
{
  if (tr[u].l >= l && tr[u].r <= r)
      return tr[u];
  else
  {
      int mid = tr[u].l + tr[u].r >> 1;
      if (r <= mid)
          return query(u << 1, l, r);
      else if (l > mid)
          return query(u << 1 | 1, l, r);
      else
      {
          auto left = query(u << 1, l, r);
          auto right = query(u << 1 | 1, l, r);
          Node res;
          pushup(res, left, right);
          return res;
      }
  }
}
```

7.2.3 Maintain the GCD

```
struct Node
{ int l, r; LL sum, d; } tr[N * 4];
LL gcd(LL a, LL b)
{ return b ? gcd(b, a % b) : a; }
void pushup(Node &u, Node &l, Node &r)
{
  u.sum = l.sum + r.sum;
  u.d = gcd(l.d, r.d);
}
void pushup(int u)
{ pushup(tr[u], tr[u << 1], tr[u << 1 | 1]); }
void build(int u, int l, int r)
{
  if (l == r)
  {
      LL b = w[r] - w[r - 1];
      tr[u] = {l, r, b, b};
  }
  else
  {
      tr[u].l = l, tr[u].r = r;
      int mid = l + r >> 1;
      build(u << 1, l, mid),
      build(u << 1 | 1, mid + 1, r);
      pushup(u);
  }
}
void modify(int u, int x, LL v)
{
  if (tr[u].l == x && tr[u].r == x)
  {
      LL b = tr[u].sum + v;
      tr[u] = {x, x, b, b};
  }
  else
  {
      int mid = tr[u].l + tr[u].r >> 1;
      if (x <= mid)
          modify(u << 1, x, v);
      else
          modify(u << 1 | 1, x, v);
      pushup(u);
  }
}
Node query(int u, int l, int r)
{
  if (tr[u].l >= l && tr[u].r <= r)
      return tr[u];
  else
  {
      int mid = tr[u].l + tr[u].r >> 1;
      if (r <= mid)
          return query(u << 1, l, r);
      else if (l > mid)
          return query(u << 1 | 1, l, r);
      else
      {
          auto left = query(u << 1, l, r);
          auto right = query(u << 1 | 1, l, r);
          Node res;
          pushup(res, left, right);
          return res;
      }
  }
}
```

7.2.4 Optimize Range Updates

Use this when you need to get summary of a specific range of an array but you also need to modify a specific range of an array:

```
struct Node
{ int l, r; LL sum, add; } tr[N * 4];
void pushup(int u)
{ tr[u].sum = tr[u << 1].sum + tr[u << 1 | 1].sum; }
void pushdown(int u)
{
  auto &root = tr[u],
       &left = tr[u << 1],
```

```
        &right = tr[u << 1 | 1];
    if (root.add)
    {
        left.add += root.add,
        left.sum += (LL)(left.r - left.l + 1) * root.add;
        right.add += root.add,
        right.sum += (LL)(right.r - right.l + 1) * root.add;
        root.add = 0;
    }
}
void build(int u, int l, int r)
{
    if (l == r) tr[u] = {l, r, w[r], 0};
    else
    {
        tr[u] = {l, r};
        int mid = l + r >> 1;
        build(u << 1, l, mid);
        build(u << 1 | 1, mid + 1, r);
        pushup(u);
    }
}
void modify(int u, int l, int r, int d)
{
    if (tr[u].l >= l && tr[u].r <= r)
    {
        tr[u].sum += (LL)(tr[u].r - tr[u].l + 1) * d;
        tr[u].add += d;
    }
    else
    {
        pushdown(u);
        int mid = tr[u].l + tr[u].r >> 1;
        if (l <= mid)
            modify(u << 1, l, r, d);
        if (r > mid)
            modify(u << 1 | 1, l, r, d);
        pushup(u);
    }
}
LL query(int u, int l, int r)
{
    if (tr[u].l >= l && tr[u].r <= r)
        return tr[u].sum;
    pushdown(u);
    int mid = tr[u].l + tr[u].r >> 1;
    LL sum = 0;
    if (l <= mid)
        sum += query(u << 1, l, r);
    if (r > mid)
        sum += query(u << 1 | 1, l, r);
    return sum;
}
```

7.3 Persistent Data Structure

7.3.1 Persistent Trie

```
const int N = 600010, M = N * 25;
int n, m, s[N], root[N], idx;
int trie[M][2], max_id[M];
void insert(int i, int k, int p, int q)
{
    if (k < 0)
    {
        max_id[q] = i;
        return;
    }
    int v = s[i] >> k & 1;
    if (p)
        trie[q][v ^ 1] = trie[p][v ^ 1];
    trie[q][v] = ++idx;
    insert(i, k - 1, trie[p][v], trie[q][v]);
    max_id[q] = max(max_id[trie[q][0]], max_id[trie[q][1]]);
}
int query(int root, int C, int L)
{
    int p = root;
    for (int i = 23; i >= 0; i--)
    {
        int v = C >> i & 1;
        if (max_id[trie[p][v ^ 1]] >= L)
            p = trie[p][v ^ 1];
        else
            p = trie[p][v];
    }
    return C ^ s[max_id[p]];
}
// insert(i, 23, root[i - 1], root[i]);
// query(root[r - 1], l - 1, x ^ s[n]);
```

7.3.2 Persistent Segment Tree

```
const int N = 100010, M = 10010;
int n, m, a[N], root[N], idx;
vector<int> nums;
struct Node
{
    int l, r;
    int cnt;
} tr[N * 4 + N * 17];
int find(int x)
{
    return lower_bound(nums.begin(), nums.end(), x) - nums.begin();
}
int build(int l, int r)
{
    int p = ++idx;
    if (l == r)
        return p;
    int mid = l + r >> 1;
    tr[p].l = build(l, mid), tr[p].r = build(mid + 1, r);
    return p;
}
int insert(int p, int l, int r, int x)
{
    int q = ++idx;
    tr[q] = tr[p];
    if (l == r)
    {
        tr[q].cnt++;
```

```
        return q;
    }
    int mid = l + r >> 1;
    if (x <= mid)
        tr[q].l = insert(tr[p].l, l, mid, x);
    else
        tr[q].r = insert(tr[p].r, mid + 1, r, x);
    tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].cnt;
    return q;
}
int query(int q, int p, int l, int r, int k)
{
    if (l == r)
        return r;
    int cnt = tr[tr[q].l].cnt - tr[tr[p].l].cnt;
    int mid = l + r >> 1;
    if (k <= cnt)
        return query(tr[q].l, tr[p].l, l, mid, k);
    else
        return query(tr[q].r, tr[p].r, mid + 1, r, k - cnt);
}
```

7.4 Treap

```
const int N = 100010, INF = 1e8;
int n, root, idx;
struct Node
{ int l, r, key, val, cnt, size; } tr[N];
void pushup(int p)
{
    tr[p].size = tr[tr[p].l].size +
                 tr[tr[p].r].size + tr[p].cnt;
}
int get_node(int key)
{
    tr[++idx].key = key;
    tr[idx].val = rand();
    tr[idx].cnt = tr[idx].size = 1;
    return idx;
}
void zig(int &p)
{
    int q = tr[p].l;
    tr[p].l = tr[q].r, tr[q].r = p, p = q;
    pushup(tr[p].r), pushup(p);
}
void zag(int &p)
{
    int q = tr[p].r;
    tr[p].r = tr[q].l, tr[q].l = p, p = q;
    pushup(tr[p].l), pushup(p);
}
void build()
{
    get_node(-INF), get_node(INF);
    root = 1, tr[1].r = 2;
    pushup(root);
    if (tr[1].val < tr[2].val) zag(root);
}
void insert(int &p, int key)
{
    if (!p) p = get_node(key);
    else if (tr[p].key == key) tr[p].cnt++;
    else if (tr[p].key > key)
    {
        insert(tr[p].l, key);
        if (tr[tr[p].l].val > tr[p].val)
            zig(p);
    }
    else
    {
        insert(tr[p].r, key);
        if (tr[tr[p].r].val > tr[p].val)
            zag(p);
    }
    pushup(p);
}
void remove(int &p, int key)
{
    if (!p) return;
    if (tr[p].key == key)
    {
        if (tr[p].cnt > 1) tr[p].cnt--;
        else if (tr[p].l || tr[p].r)
        {
            if (!tr[p].r || tr[tr[p].l].val > tr[tr[p].r].val)
            {
                zig(p);
                remove(tr[p].r, key);
            }
            else
            {
                zag(p);
                remove(tr[p].l, key);
            }
        }
        else p = 0;
    }
    else if (tr[p].key > key)
        remove(tr[p].l, key);
    else remove(tr[p].r, key);
    pushup(p);
}
int get_rank_by_key(int p, int key)
{
    if (!p) return 0;
    if (tr[p].key == key)
        return tr[tr[p].l].size + 1;
    if (tr[p].key > key)
        return get_rank_by_key(tr[p].l, key);
    return tr[tr[p].l].size + tr[p].cnt + get_rank_by_key(tr[p].r, key);
}
int get_key_by_rank(int p, int rank)
{
    if (!p) reutrn INF;
    if (tr[tr[p].l].size >= rank)
        reutrn get_key_by_rank(tr[p].l, rank);
    if (tr[tr[p].l].size + tr[p].cnt >= rank)
        reutrn tr[p].key;
    return get_key_by_rank(tr[p].r, rank - tr[tr[p].l].size - tr[p].cnt);
}
```

```
int get_prev(int p, int key)
{
  if (!p) return -INF;
  if (tr[p].key >= key)
      reutrn get_prev(tr[p].l, key);
  return max(tr[p].key, get_prev(tr[p].r,
    key));
}
int get_next(int p, int key)
{
  if (!p) reutrn INF;
  if (tr[p].key <= key)
      return get_next(tr[p].r, key);
  return min(tr[p].key, get_next(tr[p].l,
    key));
}
```

7.5 AC Automaton

```
const int N = 10010, M = 1000010, S = 55;
int n, tr[N * S][26], cnt[N * S], idx;
int q[N * S], ne[N * S];
char str[M];
void insert()
{
  int p = 0;
  for (int i = 0; str[i]; i++)
  {
      int t = str[i] - 'a';
      if (!tr[p][t]) tr[p][t] = ++idx;
      p = tr[p][t];
  }
  cnt[p]++;
}
void build()
{
  int hh = 0, tt = -1;
  for (int i = 0; i < 26; i++)
      if (tr[0][i]) q[++tt] = tr[0][i];
  while (hh <= tt)
  {
      int t = q[hh++];
      for (int i = 0; i < 26; i++)
      {
          int p = tr[t][i];
          if (!p) tr[t][i] = tr[ne[t]][i];
          else
          {
              ne[p] = tr[ne[t]][i];
              q[++tt] = p;
          }
      }
  }
}
```

8 ⋆ Advanced Search

8.1 Flood-Fill

```
const int N = 1010, M = N * N;
int n, m;
char g[N][N];
PII q[M];
bool st[N][N];
void bfs(int sx, int sy)
{
  int hh = 0, tt = 0;
  q[0] = {sx, sy}; st[sx][sy] = true;
  while (hh <= tt)
  {
      PII t = q[hh++];
      for (int i = t.first - 1; i <= t.first + 1; i++)
          for (int j = t.second - 1; j <= t.second + 1; j++)
          {
              if (i == t.first && j == t.second)
                  continue;
              if (i < 0 || i >= n || j < 0 || j >= m)
                  continue;
              if (g[i][j] == '.' || st[i][j])
                  continue;
              q[++tt] = {i, j};
              st[i][j] = true;
          }
  }
}
int main()
{
  int cnt = 0;
  for (int i = 0; i < n; i++)
      for (int j = 0; j < m; j++)
          if (g[i][j] == 'W' && !st[i][j])
          { bfs(i, j); cnt++; }
}
```

8.2 Multi-source BFS

```
const int N = 1010, M = N * N;
int n, m, dist[N][N];
char g[N][N];
PII q[M];
int dx[4] = {-1, 0, 1, 0},
  dy[4] = {0, 1, 0, -1};
void bfs()
{
  memset(dist, -1, sizeof dist);
  int hh = 0, tt = -1;
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
          if (g[i][j] == '1')
          {
              dist[i][j] = 0;
              q[++tt] = {i, j};
          }
  while (hh <= tt)
  {
      auto t = q[hh++];
      for (int i = 0; i < 4; i++)
      {
          int a = t.x + dx[i], b = t.y + dy[i];
          if (a < 1 || a > n | b < 1 || b > m) continue;
          if (dist[a][b] != -1) continue;
          dist[a][b] = dist[t.x][t.y] + 1;
          q[++tt] = {a, b};
      }
  }
}
```

8.3 BFS with Deque

```
const int N = 510, M = N * N;
int n, m, dist[N][N];
char g[N][N];
bool st[N][N];
int dx[4] = {-1, -1, 1, 1},
  dy[4] = {-1, 1, 1, -1},
  ix[4] = {-1, -1, 0, 0},
  iy[4] = {-1, 0, 0, -1};
int bfs()
{
  memset(dist, 0x3f, sizeof dist);
  memset(st, 0, sizeof st);
  dist[0][0] = 0;
  deque<PII> q;
  q.push_back({0, 0});
  char cs[] = "\\/\\/";
  while (q.size())
  {
      PII t = q.front();
      q.pop_front();
      if (st[t.x][t.y]) continue;
      st[t.x][t.y] = true;
      for (int i = 0; i < 4; i++)
      {
          int a = t.x + dx[i], b = t.y + dy[i];
          if (a < 0 || a > n || b < 0 || b > m) continue;
          int ca = t.x + ix[i], cb = t.y + iy[i];
          int d = dist[t.x][t.y] +
          (g[ca][cb] != cs[i]);
          if (d < dist[a][b])
          {
              dist[a][b] = d;
              if (g[ca][cb] != cs[i])
                  q.push_back({a, b});
              else
                  q.push_front({a, b});
          }
      }
  }
  return dist[n][m];
```

```
}
```

8.4 Bidirectional BFS

```
int bfs()
{
  if (A == B) return 0;
  queue<string> qa, qb;
  unordered_map<string, int> da, db;
  qa.push(A), qb.push(B);
  da[A] = db[B] = 0;
  int step = 0;
  while (qa.size() && qb.size())
  {
      int t;
      if (qa.size() < qb.size())
          // PROCESS
      else
          // PROCESS
      if (t <= 10) return t;
      if (++step == 10) return -1;
  }
  return -1;
}
```

8.5 A*

```
const int N = 1010, M = 200010;
int n, m, S, T, K;
int h[N], rh[N], e[M], w[M], ne[M], idx;
int dist[N], cnt[N];
bool st[N];
void dijkstra()
{
  priority_queue<PII, vector<PII>, greater<PII>> heap;
  heap.push({0, T});
  memset(dist, 0x3f, sizeof dist);
  dist[T] = 0;
  while (heap.size())
  {
      auto t = heap.top();
      heap.pop();
      int ver = t.y;
      if (st[ver]) continue;
      st[ver] = true;
      for (int i = rh[ver]; ~i; i = ne[i])
      {
          int j = e[i];
          if (dist[j] > dist[ver] + w[i])
          {
              dist[j] = dist[ver] + w[i];
              heap.push({dist[j], j});
          }
      }
  }
}

int astar()
{
  priority_queue<PIII, vector<PIII>, greater<PIII>> heap;
  heap.push({dist[S], {0, S}});
  while (heap.size())
  {
      auto t = heap.top();
      heap.pop();
      int ver = t.y.y, distance = t.y.x;
      cnt[ver]++;
      if (cnt[T] == K) return distance;
      for (int i = h[ver]; ~i; i = ne[i])
      {
          int j = e[i];
          if (cnt[j] < K)
              heap.push({distance + w[i] + dist[j], {distance + w[i], j}});
      }
  }
  return -1;
}
int main()
{
  // PROCESS
  dijkstra(); cout << astar();
  // PROCESS
}
```

8.6 DFS Connectivity Model

```
char g[N][N];
int xa, ya, xb, yb;
int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0, -1};
bool st[N][N];
bool dfs(int x, int y)
{
  if (g[x][y] == '#') return false;
  if (x == xb && y == yb) return true;
  st[x][y] = true;
  for (int i = 0; i < 4; i++)
  {
      int a = x + dx[i], b = y + dy[i];
      if (a < 0 || a >= n || b < 0 || b >= n) continue;
      if (st[a][b]) continue;
      if (dfs(a, b)) return true;
  }
  return false;
}
```

8.7 IDDFS

```
const int N = 110;
int n, path[N];
bool dfs(int u, int k)
{
  if (u == k)
      return path[u - 1] == n;
  bool st[N] = {0};
  for (int i = u - 1; i >= 0; i--)
```

```
        for (int j = i; j >= 0; j--)
        {
            int s = path[i] + path[j];
            if (s > n || s <= path[u - 1] || st[s]) continue;
            st[s] = true;
            path[u] = s;
            if (dfs(u + 1, k)) return true;
        }

    return false;
}
```

```
{
    if (f() > maxn - depth) return false;
    if (depth == maxn) return true;
    for (int i = 0; i <= n; i++)
    {
        // OPERATION
        if (IDAstar(depth + 1, maxn))
            return true;
        // OPERATION
    }
    return false;
}
```

8.8 Bidirectional DFS

```
const int N = 1 << 24;
int n, m, k, cnt = 0, ans;
int g[50], weights[N];
void dfs(int u, int s)
{
  if (u == k)
  {
      weights[cnt++] = s;
      return;
  }
  if ((LL)s + g[u] <= m)
      dfs(u + 1, s + g[u]);
  dfs(u + 1, s);
}
void dfs2(int u, int s)
{
  if (u == n)
  {
      int l = 0, r = cnt - 1;
      while (l < r)
      {
          int mid = l + r + 1 >> 1;
          if (weights[mid] + (LL)s <= m)
              l = mid;
          else r = mid - 1;
      }
      if (weights[l] + (LL)s <= m)
          ans = max(ans, weights[l] + s);
      return;
  }
  if ((LL)s + g[u] <= m)
      dfs2(u + 1, s + g[u]);
  dfs2(u + 1, s);
}
```

8.9 IDA*

```
const int N = 1e2;
int n, a[N];
string t;
int f()
{
  // YOUR_F_FUNCTION
}
bool IDAstar(int depth, int maxn)
```

9 ⋆ Advanced Graph Theory

9.1 Detecting Negative Cycles

```
int n, m1, m2;
int h[N], e[M], w[M], ne[M], idx;
int dist[N], q[N], cnt[N];
bool st[N];
bool spfa()
{
  memset(dist, 0, sizeof dist);
  memset(cnt, 0, sizeof cnt);
  memset(st, 0, sizeof st);
  int hh = 0, tt = 0;
  for (int i = 1; i <= n; i++)
  {
  q[tt++] = i;
  st[i] = true;
  }
  while (hh != tt)
  {
  int t = q[hh++];
  if (hh == N) hh = 0;
  st[t] = false;
  for (int i = h[t]; ~i; i = ne[i])
  {
    int j = e[i];
    if (dist[j] > dist[t] + w[i])
    {
      dist[j] = dist[t] + w[i];
      cnt[j] = cnt[t] + 1;
      if (cnt[j] >= n) return true;
      if (!st[j])
      {
        q[tt++] = j;
        if (tt == N) tt = 0;
        st[j] = true;
      }
    }
  }
  }
  return false;
}
```

9.2 SPFA-SLF

Using deque to solve SPFA question.

```
void spfa()
{
  memset(dist, 0x3f, sizeof dist);
  memset(st, 0, sizeof st);
  deque<int> q;
  q.push_back(s);
  st[s] = 1, dist[s] = 0;
  while (q.size())
  {
  int t = q.front();
  q.pop_front();
  st[t] = 0;
  for (int i = h[t]; ~i; i = ne[i])
  {
    int j = e[i];
    if (dist[j] > dist[t] + w[i])
    {
      dist[j] = dist[t] + w[i];
      if (!st[j])
      {
        st[j] = true;
        if (q.size() && dist[j] < dist[q.front()])
          q.push_front(j);
        else
          q.push_back(j);
      }
    }
  }
  }
}
```

9.3 SPFA-Stack

```
bool spfa()
{
  int hh = 0, tt = 1;
  memset(dist, -0x3f, sizeof dist);
  dist[0] = 0;
  q[0] = 0;
  while (hh != tt)
  {
  int t = q[--tt];
  st[t] = false;
  for (int i = h[t]; ~i; i = ne[i])
  {
    int j = e[i];
    if (dist[j] < dist[t] + w[i])
    {
      dist[j] = dist[t] + w[i];
      cnt[j] = cnt[t] + 1;
      if (cnt[j] >= n + 1) return true;
      if (!st[j])
      {
        st[j] = true;
        q[tt++] = j;
      }
    }
  }
  }
  return false;
}
```

9.4 SPFA & MIN & MAX

Using SPFA to maintain the minimum and maximum. In this case we need **Original Graph** and **Reverse Graph**, in which we can use **type == 0** or **type == 1** to describe.

```
void spfa(int h[], int dist[], int type)
{
  int hh = 0, tt = 1;
  if (type == 0)
  {
```

```cpp
    memset(dist, 0x3f, sizeof dmin);
    dist[1] = w[1];
    q[0] = 1;
    }
    else
    {
    memset(dist, -0x3f, sizeof dmax);
    dist[n] = w[n];
    q[0] = n;
    }
    while (hh != tt)
    {
    int t = q[hh++];
    if (hh == N) hh = 0;
    st[t] = false;
    for (int i = h[t]; ~i; i = ne[i])
    {
      int j = e[i];
      if (type == 0 && dist[j] > min(dist[t], w[j]) || type == 1 && dist[j] < max(dist[t], w[j]))
      {
        if (type == 0)
          dist[j] = min(dist[t], w[j]);
        else
          dist[j] = max(dist[t], w[j]);
        if (!st[j])
        {
          q[tt++] = j;
          if (tt == N) tt = 0;
          st[j] = true;
        }
      }
    }
    }
}
```

9.5 Second Shortest Path

```cpp
const int N = 1010, M = 20010;
struct Ver
{
  int id, type, dist;
  bool operator>(const Ver &W) const
  {
  return dist > W.dist;
  }
};
int n, m, S, T, dist[N][2], cnt[N][2];
int h[N], e[M], w[M], ne[M], idx;
bool st[N][2];
void add(int a, int b, int c)
{
  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++;
}
int dijkstra()
{
  memset(st, 0, sizeof st);
  memset(dist, 0x3f, sizeof dist);
  memset(cnt, 0, sizeof cnt);
  dist[S][0] = 0, cnt[S][0] = 1;
  priority_queue<Ver, vector<Ver>, greater<Ver>> heap;
  heap.push({S, 0, 0});
  while (heap.size())
  {
  Ver t = heap.top();
  heap.pop();
  int ver = t.id, type = t.type, distance = t.dist, count = cnt[ver][type];
  if (st[ver][type])
    continue;
  st[ver][type] = true;
  for (int i = h[ver]; ~i; i = ne[i])
  {
    int j = e[i];
    if (dist[j][0] > distance + w[i])
    {
      dist[j][1] = dist[j][0], cnt[j][1] = cnt[j][0];
      heap.push({j, 1, dist[j][1]});
      dist[j][0] = distance + w[i], cnt[j][0] = count;
      heap.push({j, 0, dist[j][0]});
    }
    else if (dist[j][0] == distance + w[i])
      cnt[j][0] += count;
    else if (dist[j][1] > distance + w[i])
    {
      dist[j][1] = distance + w[i], cnt[j][1] = count;
      heap.push({j, 1, dist[j][1]});
    }
    else if (dist[j][1] == distance + w[i])
      cnt[j][1] += count;
  }
  }
  int res = cnt[T][0];
  if (dist[T][0] + 1 == dist[T][1])
  res += cnt[T][1];
  return res;
}
```

9.6 Second Minimum Spanning Tree

9.6.1 brute-force

```cpp
const int N = 510, M = 10010;
int n, m, p[N], dist1[N][N], dist2[N][N];
int h[N], e[N * 2], w[N * 2], ne[N * 2], idx;
struct Edge
{
  int a, b, w;
  bool f;
  bool operator<(const Edge &e) const
  { return w < e.w; }
} edge[M];
void add(int a, int b, int c)
{
  e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++;
}
int find(int x)
```

```cpp
{
  if (p[x] != x) p[x] = find(p[x]);
  return p[x];
}
void dfs(int u, int fa, int maxd1, int maxd2, int d1[], int d2[])
{
  d1[u] = maxd1, d2[u] = maxd2;
  for (int i = h[u]; ~i; i = ne[i])
  {
  int j = e[i];
  if (j != fa)
  {
    int td1 = maxd1, td2 = maxd2;
    if (w[i] > td1)
      td2 = td1, td1 = w[i];
    else if (w[i] < td1 && w[i] > td2)
      td2 = w[i];
    dfs(j, u, td1, td2, d1, d2);
  }
  }
}
int main()
{
  cin >> n >> m;
  memset(h, -1, sizeof h);
  for (int i = 0; i < m; i++)
  cin >> edge[i].a >> edge[i].b >> edge[i].w;
  sort(edge, edge + m);
  for (int i = 1; i <= n; i++) p[i] = i;
  LL sum = 0;
  for (int i = 0; i < m; i++)
  {
  int a = edge[i].a, b = edge[i].b, w = edge[i].w;
  int pa = find(a), pb = find(b);
  if (pa != pb)
  {
    p[pa] = pb;
    sum += w;
    add(a, b, w), add(b, a, w);
    edge[i].f = true;
  }
  }
  for (int i = 1; i <= n; i++)
  dfs(i, -1, -1e9, -1e9, dist1[i], dist2[i]);
  LL res = 1e18;
  for (int i = 0; i < m; i++)
  if (!edge[i].f)
  {
    int a = edge[i].a, b = edge[i].b, w = edge[i].w;
    LL t;
    if (w > dist1[a][b])
      t = sum + w - dist1[a][b];
    else if (w > dist2[a][b])
      t = sum + w - dist2[a][b];
    res = min(res, t);
  }
}
```

9.6.2 LCA

```cpp
const int N = 100010, M = 300010;
int n, m, p[N], q[N];
int h[N], e[M], w[M], ne[M], idx;
int depth[N], fa[N][17], d1[N][17], d2[N][17];
struct Edge
{
  int a, b, w;
  bool used;
  bool operator<(const Edge &t) const
  { return w < t.w; }
} edge[M];
void add(int a, int b, int c)
{ e[idx] = b, w[idx] = c, ne[idx] = h[a], h[a] = idx++; }
int find(int x)
{
  if (p[x] != x) p[x] = find(p[x]);
  return p[x];
}
LL kruskal()
{
  for (int i = 1; i <= n; i++) p[i] = i;
  sort(edge, edge + m);
  LL res = 0;
  for (int i = 0; i < m; i++)
  {
  int a = find(edge[i].a), b = find(edge[i].b), w = edge[i].w;
  if (a != b)
  {
    p[a] = b; res += w;
    edge[i].used = true;
  }
  }
  return res;
}
void build()
{
  memset(h, -1, sizeof h);
  for (int i = 0; i < m; i++)
  if (edge[i].used)
  {
    int a = edge[i].a, b = edge[i].b, w = edge[i].w;
    add(a, b, w), add(b, a, w);
  }
}
void bfs()
{
  memset(depth, 0x3f, sizeof depth);
  depth[0] = 0, depth[1] = 1, q[0] = 1;
  int hh = 0, tt = 0;
  while (hh <= tt)
  {
  int t = q[hh++];
  for (int i = h[t]; ~i; i = ne[i])
  {
    int j = e[i];
    if (depth[j] > depth[t] + 1)
    {
      depth[j] = depth[t] + 1;
      q[++tt] = j;
      fa[j][0] = t;
      d1[j][0] = w[i], d2[j][0] = -INF;
```

```cpp
      for (int k = 1; k <= 16; k++)
      {
        int anc = fa[j][k - 1];
        fa[j][k] = fa[anc][k - 1];
        int distance[4] = {d1[j][k - 1],
                           d2[j][k - 1],
                           d1[anc][k - 1],
                           d2[anc][k - 1]};
        d1[j][k] = d2[j][k] = -INF;
        for (int u = 0; u < 4; u++)
        {
          int d = distance[u];
          if (d > d1[j][k])
            d2[j][k] = d1[j][k], d1[j][k] = d;
          else if (d != d1[j][k] && d > d2[j][k])
            d2[j][k] = d;
        }
      }
    }
  }
  }
}
int lca(int a, int b, int w)
{
  static int distance[N * 2];
  int cnt = 0;
  if (depth[a] < depth[b])
  swap(a, b);
  for (int k = 16; k >= 0; k--)
  if (depth[fa[a][k]] >= depth[b])
  {
    distance[cnt++] = d1[a][k];
    distance[cnt++] = d2[a][k];
    a = fa[a][k];
  }
  if (a != b)
  {
  for (int k = 16; k >= 0; k--)
    if (fa[a][k] != fa[b][k])
    {
      distance[cnt++] = d1[a][k];
      distance[cnt++] = d2[a][k];
      distance[cnt++] = d1[b][k];
      distance[cnt++] = d2[b][k];
      a = fa[a][k], b = fa[b][k];
    }
  distance[cnt++] = d1[a][0];
  distance[cnt++] = d1[b][0];
  }
  int dist1 = -INF, dist2 = -INF;
  for (int i = 0; i < cnt; i++)
  {
  int d = distance[i];
  if (d > dist1)
    dist2 = dist1, dist1 = d;
  else if (d != dist1 && d > dist2)
    dist2 = d;
  }
  if (w > dist1) return w - dist1;
  if (w > dist2) return w - dist2;
  return INF;
}
int main()
{
  cin >> n >> m;
  for (int i = 0; i < m; i++)
  {
  int a, b, c;
  cin >> a >> b >> c;
  edge[i] = {a, b, c};
  }
  LL sum = kruskal();
  build();
  bfs();
  LL res = 1e18;
  for (int i = 0; i < m; i++)
  if (!edge[i].used)
  {
    int a = edge[i].a, b = edge[i].b, w = edge[i].w;
    res = min(res, sum + lca(a, b, w));
  }
  cout << res;
}
```

9.7 Difference Constraints

- **size == N**: Feasible Solution
- **size == 1**: Maximum/Minimum
- **Maximum**: Shortest Path
- **Minimum**: Longest Path

9.7.1 Maximum-Shortest Path

```cpp
bool spfa(int size)
{
  int hh = 0, tt = 0;
  memset(dist, 0x3f, sizeof dist);
  memset(st, 0, sizeof st);
  memset(cnt, 0, sizeof cnt);
  for (int i = 1; i <= size; i++)
  {
  q[tt++] = i;
  dist[i] = 0;
  st[i] = true;
  }
  while (hh != tt)
  {
  int t = q[hh++];
  if (hh == N) hh = 0;
  st[t] = false;
  for (int i = h[t]; ~i; i = ne[i])
  {
    int j = e[i];
    if (dist[j] > dist[t] + w[i])
    {
      dist[j] = dist[t] + w[i];
      cnt[j] = cnt[t] + 1;
      if (cnt[j] >= n) return true;
      if (!st[j])
      {
        st[j] = true;
        q[tt++] = j;
        if (tt == N) tt = 0;
      }
```

```
      }
    }
    }
    return false;
  }
  int main()
  {
    // add(a, b, k) means x_b <= x_a + k
    // PROCESS
  }
```

9.7.2 Minimum-Longest Path

```
  bool spfa(int size)
  {
    int hh = 0, tt = 0;
    memset(dist, -0x3f, sizeof dist);
    memset(st, 0, sizeof st);
    memset(cnt, 0, sizeof cnt);
    for (int i = 1; i <= size; i++)
    {
    q[tt++] = i;
    dist[i] = 0;
    st[i] = true;
    }
    while (hh != tt)
    {
    int t = q[hh++];
    if (hh == N) hh = 0;
    st[t] = false;
    for (int i = h[t]; ~i; i = ne[i])
    {
      int j = e[i];
      if (dist[j] < dist[t] + w[i])
      {
        dist[j] = dist[t] + w[i];
        cnt[j] = cnt[t] + 1;
        if (cnt[j] >= n) return false;
        if (!st[j])
        {
          st[j] = true;
          q[tt++] = j;
          if (tt == N) tt = 0;
        }
      }
    }
    }
    return ture;
  }
  int main()
  {
    // add(a, b, k) means x_a + k <= x_b
    // PROCESS
  }
```

9.8 LCA

```
  int n, m, h[N], e[M], ne[M], idx;
  int depth[N], fa[N][16], q[N];
  void bfs(int root)
  {
    memset(depth, 0x3f, sizeof depth);
    depth[0] = 0;
    depth[root] = 1;
    int hh = 0, tt = 0;
    q[0] = root;
    while (hh <= tt)
    {
    int t = q[hh++];
    for (int i = h[t]; ~i; i = ne[i])
    {
      int j = e[i];
      if (depth[j] > depth[t] + 1)
      {
        depth[j] = depth[t] + 1;
        q[++tt] = j;
        fa[j][0] = t;
        for (int k = 1; k <= 15; k++)
          fa[j][k] = fa[fa[j][k - 1]][k - 1];
      }
    }
    }
  }
  int lca(int a, int b)
  {
    if (depth[a] < depth[b]) swap(a, b);
    for (int k = 15; k >= 0; k--)
    if (depth[fa[a][k]] >= depth[b])
      a = fa[a][k];
    if (a == b) return a;
    for (int k = 15; k >= 0; k--)
    if (fa[a][k] != fa[b][k])
    {
      a = fa[a][k];
      b = fa[b][k];
    }
    return fa[a][0];
  }
```

9.9 SCC

```
  void tarjan(int u)
  {
    dfn[u] = low[u] = ++timestap;
    stack[++top] = u, in_stk[u] = true;
    for (int i = h[u]; ~i; i = ne[i])
    {
    int j = e[i];
    if (!dfn[j])
    {
      tarjan(j);
      low[u] = min(low[u], low[j]);
    }
    else if (in_stk[j])
      low[u] = min(low[u], dfn[j]);
    }
    if (dfn[u] == low[u])
    {
    int y;
    ++scc_cnt;
    do
    {
      y = stk[top--];
      in_stk[y] = false;
```

```
      id[y] = scc_cnt;
    } while (y != u);
    }
}
```

9.10 DCC

9.10.1 e-DCC

```
const int N = 5010, M = 20010;
int n, m, h[N], e[M], ne[M], idx;
int dfn[N], low[N], timestamp;
int stk[N], top, id[N], dcc_cnt, d[N];
bool is_bridge[M];
void tarjan(int u, int from)
{
  dfn[u] = low[u] = ++timestamp;
  stk[++top] = u;
  for (int i = h[u]; ~i; i = ne[i])
  {
  int j = e[i];
  if (!dfn[j])
  {
    tarjan(j, i);
    low[u] = min(low[u], low[j]);
    if (dfn[u] < low[j])
      is_bridge[i] = is_bridge[i ^ 1] = true
      ;
  }
  else if (i != (from ^ 1))
    low[u] = min(low[u], dfn[j]);
  }
  if (dfn[u] == low[u])
  {
  ++dcc_cnt;
  int y;
  do
  {
    y = stk[top--];
    id[y] = dcc_cnt;
  } while (y != u);
  }
}
```

9.10.2 v-DCC

```
const int N = 1010, M = 1010;
int n, m, h[N], e[M], ne[M], idx;
int dfn[N], low[N], timestamp;
int stk[N], top, dcc_cnt, root;
vector<int> dcc[N];
bool cut[N];
void init()
{
  for (int i = 1; i <= dcc_cnt; i++)
  dcc[i].clear();
  idx = n = timestamp = top = dcc_cnt = 0;
  memset(h, -1, sizeof h);
  memset(dfn, 0, sizeof dfn);
  memset(cut, 0, sizeof cut);
}
void tarjan(int u)
{
  dfn[u] = low[u] = ++timestamp;
  stk[++top] = u;
  if (u == root && h[u] == -1)
  {
  dcc_cnt++;
  dcc[dcc_cnt].push_back(u);
  return;
  }
  int cnt = 0;
  for (int i = h[u]; ~i; i = ne[i])
  {
  int j = e[i];
  if (!dfn[j])
  {
    tarjan(j);
    low[u] = min(low[u], low[j]);
    if (dfn[u] <= low[j])
    {
      cnt++;
      if (u != root || cnt > 1)
        cut[u] = true;
      ++dcc_cnt;
      int y;
      do
      {
        y = stk[top--];
        dcc[dcc_cnt].push_back(y);
      } while (y != j);
      dcc[dcc_cnt].push_back(u);
    }
  }
  else
    low[u] = min(low[u], dfn[j]);
  }
}
```

9.11 Bipartite Graph

The maximum matching
(by the Hungarian algorithm) =
the minimum vertex cover =
total number of vertices -
maximum independent set =
total number of vertices -
minimum path cover.

9.11.1 maximum matching

```
const int N = 110;
int n, m;
int dx[4] = {-1, 0, 1, 0}, dy[4] = {0, 1, 0,
    -1};
PII match[N][N];
bool g[N][N], st[N][N];
bool find(int x, int y)
{
  for (int i = 0; i < 4; i++)
  {
  int a = x + dx[i], b = y + dy[i];
  if (a && a <= n && b && b <= n && !g[a][b]
     && !st[a][b])
```

```
  {
    st[a][b] = true;
    PII t = match[a][b];
    if (t.x == -1 || find(t.x, t.y))
    {
      match[a][b] = {x, y};
      return true;
    }
  }
  }
  return false;
}
int main()
{
  // PROCESS
  memset(match, -1, sizeof match);
  int res = 0;
  for (int i = 1; i <= n; i++)
  for (int j = 1; j <= n; j++)
    if ((i + j) % 2 && !g[i][j])
    {
      memset(st, 0, sizeof st);
      if (find(i, j))
        res++;
    }
  // PROCESS
}
```

9.11.2 minimum vertex cover

```
const int N = 110;
int n, m, k, match[N];
bool g[N][N], st[N];
bool find(int x)
{
  for (int i = 0; i < m; i++)
  if (!st[i] && g[x][i])
  {
    st[i] = true;
    if (match[i] == -1 || find(match[i]))
    {
      match[i] = x;
      return true;
    }
  }
  return false;
}
int main()
{
  while (cin >> n, n)
  {
  cin >> m >> k;
  memset(g, 0, sizeof g);
  memset(match, -1, sizeof match);
  while (k--)
  {
    int t, a, b;
    cin >> t >> a >> b;
    if (!a || !b) continue;
    g[a][b] = true;
  }
  int res = 0;
  for (int i = 0; i < n; i++)
  {
    memset(st, 0, sizeof st);
    if (find(i)) res++;
  }
  cout << res << '\n';
  }
}
```

9.11.3 maximum independent set

```
const int N = 110;
int n, m, k;
PII match[N][N];
bool g[N][N], st[N][N];
int dx[8] = {-2, -1, 1, 2, 2, 1, -1, -2};
int dy[8] = {1, 2, 2, 1, -1, -2, -2, -1};
bool find(int x, int y)
{
  for (int i = 0; i < 8; i++)
  {
      int a = x + dx[i], b = y + dy[i];
      if (a < 1 || a > n || b < 1 || b > m)
          continue;
      if (g[a][b]) continue;
      if (st[a][b]) continue;
      st[a][b] = true;
      PII t = match[a][b];
      if (t.x == 0 || find(t.x, t.y))
      {
          match[a][b] = {x, y};
          return true;
      }
  }
  return false;
}
int main()
{
  // PROCESS
  int res = 0;
  for (int i = 1; i <= n; i++)
      for (int j = 1; j <= m; j++)
      {
          if (g[i][j] || (i + j) % 2)
              continue;
          memset(st, 0, sizeof st);
          if (find(i, j)) res++;
      }
  cout << n * m - k - res << '\n';
}
```

9.11.4 minimum path cover

- **Only for DAG.**
- If you need to compute the **minimum path cover with repeated nodes**, you need to perform transitive closure as shown in the following code.

```
const int N = 210, M = 30010;
int n, m, match[N];
bool d[N][N], st[N];
bool find(int x)
```

```cpp
{
  for (int i = 1; i <= n; i++)
  if (d[x][i] && !st[i])
  {
    st[i] = true;
    int t = match[i];
    if (t == 0 || find(t))
    {
      match[i] = x;
      return true;
    }
  }
  return false;
}
int main()
{
  // 传递闭包
  for (int k = 1; k <= n; k++)
  for (int i = 1; i <= n; i++)
    for (int j = 1; j <= n; j++)
      d[i][j] |= d[i][k] & d[k][j];
  int res = 0;
  for (int i = 1; i <= n; i++)
  {
  memset(st, 0, sizeof st);
  if (find(i)) res++;
  }
  cout << n - res;
}
```

9.12 Eulerian Circuit & Eulerian Path

9.12.1 Eulerian Circuit

- **Undirected Graph**: If and only if it is connected and every vertex has even degree.
- **Directed Graph**: If and only if it is strongly connected and each vertex has equal in-degree and out-degree.

```cpp
int type, n, m;
int h[N], e[M], ne[M], idx;
bool used[M];
int ans[M], cn, din[N], dout[N];
void add(int a, int b)
{ e[idx] = b, ne[idx] = h[a], h[a] = idx++; }
void dfs(int u)
{
  for (int &i = h[u]; ~i;)
  {
  if (used[i])
  {
    i = ne[i];
    continue;
  }
  used[i] = true;
  if (type == 1) used[i ^ 1] = true;
  int t;
  if (type == 1)
  {
    t = i / 2 + 1;
    if (i & 1) t = -t;
  }
  else t = i + 1;
  int j = e[i];
  i = ne[i];
  dfs(j);
  ans[++cnt] = t;
  }
}
int main()
{
  cin >> type >> n >> m;
  memset(h, -1, sizeof h);
  for (int i = 0; i < m; i++)
  {
  int a, b;
  cin >> a >> b;
  add(a, b);
  if (type == 1) add(b, a);
  din[b]++, dout[a]++;
  }
  if (type == 1)
  {
  for (int i = 1; i <= n; i++)
    if (din[i] + dout[i] & 1)
    {
      cout << "NO\n";
      return 0;
    }
  }
  else
  {
  for (int i = 1; i <= n; i++)
    if (din[i] != dout[i])
    {
      cout << "NO\n";
      return 0;
    }
  }
  for (int i = 1; i <= n; i++)
  if (h[i] != -1)
  {
    dfs(i);
    break;
  }
}
```

9.12.2 Eulerian Path

Undirected Graph

If and only if it is connected (ignoring isolated vertices) and has exactly 0 or 2 vertices with odd degree.

```cpp
const int N = 510;
int n = 500, m, g[N][N];
int ans[1100], cnt, d[N];
void dfs(int u)
{
  for (int i = 1; i <= n; i++)
  if (g[u][i])
  {
    g[u][i]--, g[i][u]--;
    dfs(i);
```

```
    }
    ans[++cnt] = u;
}
int main()
{
    cin >> m;
    while (m--)
    {
    int a, b;
    cin >> a >> b;
    g[a][b]++, g[b][a]++;
    d[a]++, d[b]++;
    }
    int start = 1;
    while (!d[start])
    ++start;
    for (int i = 1; i <= 500; i++)
    if (d[i] % 2)
    {
        start = i;
        break;
    }
    dfs(start);
}
```

Directed Graph

If and only if it is connected in terms of non-zero degree vertices, and

- At most one vertex has **(out-degree) − (in-degree) = 1**
- At most one vertex has **(in-degree) − (out-degree) = 1**
- All other vertices have **equal in-degree and out-degree**

```
const int N = 30;
int n, p[N], din[N], dout[N];
bool st[N];
int find(int x)
{
    if (x != p[x]) p[x] = find(p[x]);
    return p[x];
}
int main()
{
    char str[1010];
    int T;
    cin >> T;
    while (T--)
    {
    cin >> n;
    memset(din, 0, sizeof din);
    memset(dout, 0, sizeof dout);
    memset(st, 0, sizeof st);
    for (int i = 0; i < 26; i++) p[i] = i;
    for (int i = 0; i < n; i++)
    {
        cin >> str;
        int a = str[0] - 'a',
            b = str[strlen(str) - 1] - 'a';
        st[a] = st[b] = true;
        dout[a]++, din[b]++;
        p[find(a)] = find(b);
    }
    int start = 0, end = 0;
    bool success = true;
    for (int i = 0; i < 26; i++)
        if (din[i] != dout[i])
        {
            if (din[i] == dout[i] + 1) end++;
            else if (din[i] + 1 == dout[i])
                start++;
            else
            {
                success = false;
                break;
            }
        }
    if (success && !(!start && !end || start
        == 1 && end == 1))
        success = false;
    int rep = -1;
    for (int i = 0; i < 26; i++)
        if (st[i])
        {
            if (rep == -1) rep = find(i);
            else if (rep != find(i))
            {
                success = false;
                break;
            }
        }
    }
    return 0;
}
```

10 ⋆ Advanced Math

10.1 Euler's Totient Function

10.1.1 GCD

```
const int N = 1e7 + 10;
int primes[N], cnt, phi[N];
bool st[N];
LL s[N];
void init(int n)
{
  for (int i = 2; i <= n; i++)
  {
      if (!st[i])
      {
          primes[cnt++] = i;
          phi[i] = i - 1;
      }
      for (int j = 0; primes[j] * i <= n; j++)
      {
          st[primes[j] * i] = true;
          if (i % primes[j] == 0)
          {
              phi[i * primes[j]] = phi[i] * primes[j];
              break;
          }
          phi[i * primes[j]] = phi[i] * (primes[j] - 1);
      }
  }
  for (int i = 1; i <= n; i++)
      s[i] = s[i - 1] + phi[i];
}
int main()
{
  int n; cin >> n;
  init(n);
  LL res = 0;
  for (int i = 0; i < cnt; i++)
  {
      int p = primes[i];
      res += s[n / p] * 2 + 1;
  }
}
```

10.2 Matrix Multiplication

```
const int N = 3;
int n, m;
void mul(int c[], int a[], int b[][N])
{
  int temp[N] = {0};
  for (int i = 0; i < N; i++)
      for (int j = 0; j < N; j++)
          temp[i] = (temp[i] + (LL)a[j] * b[j][i]) % m;
  memcpy(c, temp, sizeof temp);
}
void mul(int c[][N], int a[][N], int b[][N])
{
  int temp[N][N] = {0};
  for (int i = 0; i < N; i++)
      for (int j = 0; j < N; j++)
          for (int k = 0; k < N; k++)
              temp[i][j] = (temp[i][j] + (LL)a[i][k] * b[k][j]) % m;
  memcpy(c, temp, sizeof temp);
}
int main()
{
  while (n)
  {
      if (n & 1) mul(f1, f1, a);
      mul(a, a, a); n >>= 1;
  }
}
```

11 ⋆ Advanced DP

11.1 Advanced Linear DP

11.1.1 Two-pass grid collection problem

In this case we run DP on two different roads at the same time:

```
const int N = 15;
int n, w[N][N], f[N * 2][N][N];
int main()
{
  cin >> n;
  // INPUT w[N][N]
  for (int k = 2; k <= n * 2; k++)
  for (int i1 = 1; i1 <= k; i1++)
    for (int i2 = 1; i2 <= k; i2++)
    {
      int j1 = k - i1, j2 = k - i2;
      int t = w[i1][j1];
      if (i1 != i2) t += w[i2][j2];
      int &x = f[k][i1][i2];
      x = max(x, f[k - 1][i1 - 1][i2 - 1] + t);
      x = max(x, f[k - 1][i1 - 1][i2] + t);
      x = max(x, f[k - 1][i1][i2 - 1] + t);
      x = max(x, f[k - 1][i1][i2] + t);
    }
  cout << f[n * 2][n][n] << '\n';
  return 0;
}
```

11.2 Advanced LIS

11.2.1 Longest Bitonic Subsequence

```
const int N = 1010;
int n, a[N], f[N], g[N];
int main()
{
  cin >> n;
  for (int i = 1; i <= n; i++)
  cin >> a[i];
  for (int i = 1; i <= n; i++)
  {
  f[i] = 1;
  for (int j = 1; j < i; j++)
    if (a[i] > a[j])
      f[i] = max(f[i], f[j] + 1);
  }
  for (int i = n; i >= 1; i--)
  {
  g[i] = 1;
  for (int j = n; j > i; j--)
    if (a[i] > a[j])
      g[i] = max(g[i], g[j] + 1);
  }
  int ans = 0;
  for (int i = 1; i <= n; i++)
  ans = max(ans, g[i] + f[i] - 1);
  cout << ans << '\n';
  return 0;
}
```

11.2.2 MSIS

MSIS means Maximum Sum Increasing Subsequence

```
const int N = 1010;
int n, w[N], f[N];
int main()
{
  cin >> n;
  for (int i = 0; i < n; i++) cin >> w[i];
  int res = 0;
  for (int i = 0; i < n; i++)
  {
      f[i] = w[i];
      for (int j = 0; j < i; j++)
          if (w[i] > w[j])
              f[i] = max(f[i], f[j] + w[i]);
      res = max(res, f[i]);
  }
  cout << res;
}
```

11.2.3 LCIS

LCIS means Longest Common Increasing Subsequence

```
const int N = 3010;
int n, a[N], b[N], f[N][N];
int main()
{
  cin >> n;
  for (int i = 1; i <= n; i++)
      cin >> a[i];
  for (int i = 1; i <= n; i++)
      cin >> b[i];
  for (int i = 1; i <= n; i++)
  {
      int maxv = 1;
      for (int j = 1; j <= n; j++)
      {
          f[i][j] = f[i - 1][j];
          if (a[i] == b[j])
              f[i][j] = max(f[i][j], maxv);
          if (a[i] > b[j])
              maxv = max(maxv, f[i - 1][j] + 1);
      }
  }
  int res = 0;
  for (int i = 1; i <= n; i++)
      res = max(res, f[n][i]);
  cout << res;
}
```

11.3 Knapsack Problem

11.3.1 How To Initialize

Initialization for Counting the Number of Solutions:

- **2D Case**:
  - When volume is at most j: `f[0][i] = 1` for $0 \le i \le m$, others are 0
  - When volume is exactly j: `f[0][0] = 1`, others are 0
  - When volume is at least j: `f[0][0] = 1`, others are 0
- **1D Case**:
  - When volume is at most j: `f[i] = 1` for $0 \le i \le m$
  - When volume is exactly j: `f[0] = 1`, others are 0
  - When volume is at least j: `f[0] = 1`, others are 0

Initialization for Finding Maximum or Minimum Value:

- **2D Case**:
  - When volume is at most j: `f[i][k] = 0` for $0 \le i \le n$, $0 \le k \le m$ (only for maximizing value)
  - When volume is exactly j:
    - For minimizing value: `f[0][0] = 0`, others are `INF`
    - For maximizing value: `f[0][0] = 0`, others are `-INF`
  - When volume is at least j: `f[0][0] = 0`, others are `INF` (only for minimizing value)
- **1D Case**:
  - When volume is at most j: `f[i] = 0` for $0 \le i \le m$ (only for maximizing value)
  - When volume is exactly j:
    - For minimizing value: `f[0] = 0`, others are `INF`
    - For maximizing value: `f[0] = 0`, others are `-INF`
  - When volume is at least j: `f[0] = 0`, others are `INF` (only for minimizing value)

11.3.2 Multiple Knapsack Problem

```
const int M = 20010;
int n, m, v, w, s;
int f[M], g[M], q[M];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
  {
    cin >> v >> w >> s;
    memcpy(g, f, sizeof g);
    for (int r = 0; r < v; r++)
    {
      int hh = 0, tt = -1;
      for (int j = r; j <= m; j += v)
      {
        while (hh <= tt && j - s * v > q[hh])
          hh++;
        while (hh <= tt && g[q[tt]] + (j - q[tt]) / v * w <= g[j])
          tt--;
        q[++tt] = j;
        f[j] = g[q[hh]] + (j - q[hh]) / v * w;
      }
    }
  }
  cout << f[m];
}
```

11.3.3 Two-Dimensional Cost Knapsack Problem

```
const int N = 110;
int n, V, M, f[N][N];
int main()
{
  cin >> n >> V >> M;
  for (int i = 0; i < n; i++)
  {
      int v, m, w;
      cin >> v >> m >> w;
      for (int j = V; j >= v; j--)
          for (int k = M; k >= m; k--)
              f[j][k] = max(f[j][k], f[j - v][k - m] + w);
  }
  cout << f[V][M] << '\n';
}
```

11.3.4 Finding the Actual Solution Set

```
const int N = 1010;
int n, m;
int v[N], w[N], f[N][N];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> v[i] >> w[i];
  for (int i = n; i >= 1; i--)
      for (int j = 0; j <= m; j++)
      {
          f[i][j] = f[i + 1][j];
          if (j >= v[i])
              f[i][j] = max(f[i][j], f[i + 1][j - v[i]] + w[i]);
      }
  int j = m;
  for (int i = 1; i <= n; i++)
      if (j >= v[i] && f[i][j] == f[i + 1][j - v[i]] + w[i])
      {
```

```
            cout << i << ' ';
            j -= v[i];
        }
}
```

11.3.5 Maximum Linearly Independent Subset

```
const int N = 110, M = 25010;
int n, v[N];
bool f[M];
int main()
{
  int T; cin >> T;
  while (T--)
  {
      cin >> n;
      for (int i = 1; i <= n; ++i)
          cin >> v[i];
      sort(v + 1, v + n + 1);
      int m = v[n], res = 0;
      memset(f, 0, sizeof f);
      f[0] = true;
      for (int i = 1; i <= n; ++i)
      {
          if (f[v[i]]) continue;
          res++;
          for (int j = v[i]; j <= m; ++j)
              f[j] |= f[j - v[i]];
      }
      cout << res << '\n';
  }
}
```

11.3.6 Mixed Knapsack Problem

```
const int N = 1010;
int n, m, f[N];
int main()
{
  cin >> n >> m;
  for (int i = 0; i < n; i++)
  {
      int v, w, s;
      cin >> v >> w >> s;
      if (!s)
      {
          for (int j = v; j <= m; j++)
              f[j] = max(f[j], f[j - v] + w);
      }
      else
      {
          if (s == -1) s = 1;
          for (int k = 1; k <= s; k *= 2)
          {
              for (int j = m; j >= k * v; j--)
                  f[j] = max(f[j], f[j - k * v] + k * w);
              s -= k;
          }
          if (s)
          {
              for (int j = m; j >= s * v; j--)
                  f[j] = max(f[j], f[j - s * v] + s * w);
          }
      }
  }
  cout << f[m] << '\n';
}
```

11.3.7 Dependent Knapsack Problem

```
const int N = 110;
int n, m, root;
int h[N], e[N], ne[N], idx;
int v[N], w[N], [N][N];
void add(int a, int b)
{
  e[idx] = b, ne[idx] = h[a], h[a] = idx++;
}
void dfs(int u)
{
  for (int i = h[u]; ~i; i = ne[i])
  {
      int son = e[i];
      dfs(son);
      for (int j = m - v[u]; j >= 0; --j)
          for (int k = 0; k <= j; ++k)
              f[u][j] = max(f[u][j], f[u][j - k] + f[son][k]);
  }
  for (int j = m; j >= v[u]; --j)
      f[u][j] = f[u][j - v[u]] + w[u];
  for (int j = 0; j < v[u]; ++j)
      f[u][j] = 0;
}
int main()
{
  memset(h, -1, sizeof h);
  cin >> n >> m;
  for (int i = 1; i <= n; ++i)
  {
      int p;
      cin >> v[i] >> w[i] >> p;
      if (p == -1) root = i;
      else add(p, i);
  }
  dfs(root);
  cout << f[root][m] << '\n';
}
```

11.3.8 Number of Solutions

```
const int N = 1010, mod = 1e9 + 7;
int n, m;
int w[N], v[N], f[N], g[N];
int main()
{
  cin >> n >> m;
```

```
    for (int i = 1; i <= n; ++i)
        cin >> v[i] >> w[i];
    g[0] = 1;
    for (int i = 1; i <= n; ++i)
    {
        for (int j = m; j >= v[i]; --j)
        {
            int temp = max(f[j], f[j - v[i]] + w[i]), c = 0;
            if (temp == f[j])
                c = (c + g[j]) % mod;
            if (temp == f[j - v[i]] + w[i])
                c = (c + g[j - v[i]]) % mod;
            f[j] = temp, g[j] = c;
        }
    }
    int res = 0;
    for (int j = 0; j <= m; ++j)
        if (f[j] == f[m])
            res = (res + g[j]) % mod;
    cout << res << '\n';
}
```

11.4 FSM

11.4.1 Common FSM

```
const int N = 100010;
int n, w[N], f[N][2];
int main()
{
  int T; cin >> T;
  while (T--)
  {
      cin >> n;
      for (int i = 1; i <= n; i++)
          cin >> w[i];
      for (int i = 1; i <= n; i++)
      {
          // YOUR_FSM_RULES
          // f[i][0] =
          // f[i][1] =
      }
      cout << max(f[n][0], f[n][1]) << '\n';
  }
}
```

11.4.2 Linear DP + KMP

```
const int N = 55, mod = 1e9 + 7
int n, m, f[N][N], ne[N];
char s[N];
int main()
{
  cin >> n >> s + 1;
  m = strlen(s + 1);
  for (int i = 2, j = 0; i <= m; i++)
  {
    while (j && s[i] != s[j + 1])
      j = ne[j];
    if (s[j + 1] == s[i]) j++;
    ne[i] = j;
  }
  f[0][0] = 1;
  for (int i = 0; i < n; i++)
    for (int j = 0; j < m; j++)
      for (char ch = 'a'; ch <= 'z'; ch++)
      {
        int ptr = j;
        while (ptr && s[ptr + 1] != ch)
          ptr = ne[ptr];
        if (s[ptr + 1] == ch) ptr++;
        f[i + 1][ptr] = (f[i + 1][ptr] + f[i][j]) % mod;
      }
  int res = 0;
  for (int j = 0; j < m; j++)
    res = (res + f[n][j]) % mod;
  cout << res;
}
```

11.4.3 Linear DP + AC Automaton

```
const int N = 1010, INF = 0x3f3f3f3f;
int n, m, T = 1, f[N][N];
int tr[N][4], dar[N], idx;
int q[N], ne[N];
char str[N];
int get(char c)
{
  if (c == 'A') return 0;
  if (c == 'T') return 1;
  if (c == 'G') return 2;
  return 3;
}
void insert()
{
  int p = 0;
  for (int i = 0; str[i]; i++)
  {
    int t = get(str[i]);
    if (tr[p][t] == 0)
      tr[p][t] = ++idx;
    p = tr[p][t];
  }
  dar[p] = 1;
}
void build()
{
  int hh = 0, tt = -1;
  for (int i = 0; i < 4; i++)
    if (tr[0][i])
      q[++tt] = tr[0][i];
  while (hh <= tt)
  {
    int t = q[hh++];
    for (int i = 0; i < 4; i++)
    {
      int p = tr[t][i];
      if (!p)
        tr[t][i] = tr[ne[t]][i];
      else
      {
        ne[p] = tr[ne[t]][i];
        q[++tt] = p;
```

```
        dar[p] |= dar[ne[p]];
      }
    }
  }
}
int main()
{
  int T = 1;
  while (cin >> n, n)
  {
    memset(tr, 0, sizeof tr);
    memset(dar, 0, sizeof dar);
    memset(ne, 0, sizeof ne);
    idx = 0;
    for (int i = 0; i < n; i++)
    {
      cin >> str;
      insert();
    }
    build();
    cin >> (str + 1);
    m = strlen(str + 1);
    memset(f, 0x3f, sizeof f);
    f[0][0] = 0;
    for (int i = 0; i < m; i++)
      for (int j = 0; j <= idx; j++)
        for (int k = 0; k < 4; k++)
        {
          int t = get(str[i + 1]) != k;
          int p = tr[j][k];
          if (!dar[p])
            f[i + 1][p] = min(f[i + 1][p], f[i][j] + t);
        }
    int res = INF;
    for (int i = 0; i <= idx; i++)
      res = min(res, f[m][i]);
    if (res == INF) res = -1;
    cout << "Case " << T++
    << ": " << res << '\n';
  }
}
```

11.5 Digit DP

```
const int N = 35;
int l, r, k, b, a[N], al, f[N][N];
int dp(int pos, int st, int op)
{
  if (!pos) return st == k;
  if (!op && ~f[pos][st])
      return f[pos][st];
  int res = 0, maxx = op ? min(a[pos], 1) : 1;
  for (int i = 0; i <= maxx; i++)
  {
      if (st + i > k) continue;
      res += dp(pos - 1, st + i, op && i == a[pos]);
  }
  return op ? res : f[pos][st] = res;
}
int calc(int x)
{
  al = 0;
  memset(f, -1, sizeof f);
  while (x) a[++al] = x % b, x /= b;
  return dp(al, 0, 1);
}
int main()
{
  cin >> l >> r >> k >> b;
  cout << calc(r) - calc(l - 1) << '\n';
}
```

11.6 Queue Optimization for DP

```
int n, m, s[300010], q[300010];
int main()
{
  cin >> n >> m;
  for (int i = 1; i <= n; i++)
      cin >> s[i], s[i] += s[i - 1];
  int res = INT_MIN, hh = 0, tt = 0;
  for (int i = 1; i <= n; i++)
  {
      if (q[hh] < i - m) hh++;
      res = max(res, s[i] - s[q[hh]]);
      while (hh <= tt && s[q[tt]] >= s[i]) tt--;
      q[++tt] = i;
  }
}
```