

Trabajo T4

Ejercicio 1

Función objetivo original

$$\text{Minimizar } \frac{1}{2}\theta^t\theta + C \sum_{n=1}^N \zeta_n \text{ sujeto a } c_n(\theta^t x_n + \theta_0) \geq 1 - \zeta_n, \zeta_n \geq 0, 1 \leq n \leq N$$

Función primal de Lagrange

$$\Lambda(\theta, \theta_0, \zeta, \alpha, \beta) = \frac{1}{2}\theta^t\theta + C \sum_{n=1}^N \zeta_n - \sum_{n=1}^N \alpha_n (c_n(\theta^t x_n + \theta_0) - 1 + \zeta_n) - \sum_{n=1}^N \beta_n \zeta_n$$

$$\text{Sujeto a: } \alpha_n \geq 0, \beta_n \geq 0, \zeta_n \geq 0, 1 \leq n \leq N$$

$$\text{Resolver } \nabla_{\theta, \theta_0, \zeta} \Lambda(\theta, \theta_0, \zeta, \alpha, \beta) = 0$$

$$\frac{\partial \Lambda(\theta, \theta_0, \zeta, \alpha, \beta)}{\partial \theta} \Rightarrow \theta = \sum_{n=1}^N c_n \alpha_n x_n$$

$$\frac{\partial \Lambda(\theta, \theta_0, \zeta, \alpha, \beta)}{\partial \theta_0} \Rightarrow 0 = \sum_{n=1}^N c_n \alpha_n$$

$$\frac{\partial \Lambda(\theta, \theta_0, \zeta, \alpha, \beta)}{\partial \zeta} \Rightarrow \zeta = C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n$$

Función dual de Lagrange

$$\begin{aligned} \Lambda(\alpha, \beta) = & \frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'} + C \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - \\ & \sum_{n=1}^N \alpha_n \left(c_n \left(\sum_{n=1}^N c_{n'} \alpha_{n'} x_{n'}^t x_n + \theta_0 \right) - 1 + C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n \right) - \\ & \sum_{n=1}^N (\beta_n C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) \end{aligned}$$

$$\begin{aligned} \Lambda(\alpha, \beta) = & -\frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'} - \theta_0 \sum_{n=1}^N \alpha_n c_n + \sum_{n=1}^N \alpha_n + C \sum_{n=1}^N (C - \\ & \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - \sum_{n=1}^N \alpha_n (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - \sum_{n=1}^N \beta_n (C - \\ & \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) \end{aligned}$$

$$\Lambda(\alpha, \beta) = -\frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'} - \theta_0 \sum_{n=1}^N \alpha_n c_n + \sum_{n=1}^N \alpha_n + C \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - \alpha_n \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - \beta_n \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n)$$

$$\Lambda(\alpha, \beta) = -\frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'} - \theta_0 \sum_{n=1}^N \alpha_n c_n + \sum_{n=1}^N \alpha_n + C \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n) - C \sum_{n=1}^N (C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n)$$

$$\Lambda(\alpha, \beta) = -\frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'} - \theta_0 \sum_{n=1}^N \alpha_n c_n + \sum_{n=1}^N \alpha_n$$

$$\Lambda(\alpha, \beta) = \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'}$$

$$\text{Sujeto a: } 0 = \sum_{n=1}^N c_n \alpha_n, C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n, \alpha_n \geq 0, 1 \leq n \leq N$$

Por tanto, hay que maximizar:

$$\Lambda(\alpha) = \sum_{n=1}^N \alpha_n - \frac{1}{2} \sum_{n,n'=1}^N c_n c_{n'} \alpha_n \alpha_{n'} x_n x_{n'}$$

$$\text{Sujeto a: } 0 = \sum_{n=1}^N c_n \alpha_n, C - \sum_{n=1}^N \alpha_n - \sum_{n=1}^N \beta_n, \alpha_n \geq 0, 1 \leq n \leq N$$

Ejercicio 2

