Speed and Velocity.

- distribution of speed

$$\int_0^\infty f(v)\, dv = 1$$

- distribution of velocity

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(v_x, v_y, v_z)\, dv^3 = 1$$

- Isotropic Velocity Distribution

Isotropic means the same in all directions.

eg. $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3}\langle v^2 \rangle.$

$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0.$

volume element in the spherical coordinate.

$$dv\,(v\,d\theta)(v\sin\theta\, d\varphi)$$

$$= v^2 \sin\theta\, dv\, d\varphi\, d\theta.$$

since isotropic. φ doesn't matter.

$$f_{3p} = \frac{1}{4\pi} f(v) \sin\theta.$$

Normalisation:

$$\frac{1}{4\pi}\int_0^\infty f(v)\, dv \cdot \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi = 1.$$

$$\int_0^\infty f(v)\, dv = 1$$

$f(v)$ is the speed distribution

$$f_{3p}(v,\theta,\varphi)\, dv\, d\theta\, d\varphi = F(v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z$$

For any distribution

$$f_{3p}(v,\theta,\varphi) = v^2 \sin\theta\, F(v_x, v_y, v_z).$$