Expansion

Free Expansion : in an isolated system. a gas expands to vacuum.

- in an isolated system : $dQ = 0$
- into vacuum : $p = 0 \Rightarrow dW = -\int p\,dV = 0$.

$dU = dQ + dW = 0$. No change in internal energy.

- From Joule's experiment. within his precision, there was no change in temperature. $\frac{\partial T}{\partial V}\Big|_U = 0$. (this is always true for ideal gas)

- consider reciprocity theorem:

$$\frac{\partial V}{\partial T}\Big|_U \cdot \frac{\partial T}{\partial U}\Big|_V \cdot \frac{\partial U}{\partial V}\Big|_T = -1.$$

so $\frac{\partial T}{\partial U}\Big|_V \cdot \frac{\partial U}{\partial V}\Big|_T = 0$.

- $\frac{\partial T}{\partial U}\Big|_V$ can't be zero as injecting energy increases temperature.

so $\frac{\partial U}{\partial V}\Big|_T = 0$.

Adiabatic Expansion

For an ideal gas.. pV^γ = constant