Distribution of Each Component.

- let f_x be the PDF for V_x.

  $f_x(V_x)\,dx$ is the probability of V_x

- Assume independent distributions between f_x, f_y and f_z.

$$P(\vec{v}) = \iiint_{\mathcal{V}} f_x(V_x)\cdot f_y(V_y)\cdot f_z(V_z)\, dV_x\, dV_y\, dV_z$$

- Change to Spherical Coordinate

$$dV_x\, dV_y\, dV_z = v^2 \sin\theta\, dv\, d\theta\, d\varphi$$

$$\iiint_{\mathcal{V}} f_x(V_x) f_y(V_y) f_z(V_z)\, v^2 \sin\theta\, dv\, d\theta\, d\varphi = \iiint_{\mathcal{V}} f(v)\, dv \cdot \frac{\sin\theta}{4\pi}\, d\theta\, d\varphi$$

- If the integration is the same for every region $\mathcal{V}$.

  then $f_x(V_x) f_y(V_y) f_z(V_z)\, v^2 \sin\theta = f(v)\,\frac{\sin\theta}{4\pi}$

- Substitute for Maxwell-Boltzmann distribution

$$f_x(V_x) f_y(V_y) f_z(V_z) = \frac{A}{4\pi} e^{-m(V_x^2+V_y^2+V_z^2)/2k_BT}$$

Reminder: $f(v) = Av^2 e^{-\frac{1}{2}\frac{mv^2}{k_BT}}$

- $f_x(V_x) = Ne^{-\frac{1}{2}\frac{mV_x^2}{k_BT}}$

  $f_y(V_y) = Ne^{-\frac{1}{2}\frac{mV_y^2}{k_BT}}$ $\qquad N = \frac{1}{\sqrt{2\pi}\Delta}$, $\Delta = \sqrt{\frac{k_BT}{m}}$

  $f_z(V_z) = Ne^{-\frac{1}{2}\frac{mV_z^2}{k_BT}}$

- In a single dimension, the distribution function is given by the Boltzmann factor alone.

  $f_x(V_x) \propto e^{-\frac{V_x^2}{2\Delta^2}}$

  From this, we can tell Δ is the s.d.