Heat Capacities of Ideal Gas

- ideal gas : $pV = NkT$

$U = U(T)$.

Hence: $\left.\frac{\partial U}{\partial T}\right|_V = \left.\frac{\partial U}{\partial T}\right|_p = \frac{dU}{dT}$

$C_V = \frac{dU}{dT}$ $\qquad C_p = \frac{dU}{dT} + p\left.\frac{\partial V}{\partial T}\right|_p$.

Rearrange: $V = \frac{Nk_B}{p}T$. $\qquad \left.\frac{\partial V}{\partial T}\right|_p = \frac{Nk_B}{p}$

$C_p = \frac{dU}{dT} + Nk_B$

$C_p - C_V = Nk_B$

Introduce adiabatic index

$\frac{C_p}{C_V} = \gamma$.

$(\gamma - 1)C_V = Nk_B$

$C_V = \frac{Nk_B}{\gamma - 1}$ $\qquad C_p = \frac{\gamma}{\gamma - 1}Nk_B$