Function of State ⟺ Proper Differential.

1. integrate a proper differential around a loop

$$\oint df = f(A) - f(A) = 0.$$

2. value of a state function only depends on current conditions. There is no cummulative effect.

An ideal gas

Boyle's law : pV is constant at fixed temperature.

Joule's law : The internal energy is independent of pressure at fixed temperature.

Boyle's law ⟹ $pV = f(T)$

Joule's law ⟹ $U = U(T)$.

Real gases tend to be ideal at low pressure.

Derive $pV = Nk_B \cdot T$.

$$dU = \delta Q - p dV.$$

at constant volume. $dU = dU$. $C_v = \frac{dU}{dT}$.

$$\delta Q = C_v \cdot dT + p dV.$$

$$dS = \frac{C_v}{T} dT + \frac{p}{T} dV .$$ ⟵ entropy : state function

so dS is exact. $\frac{\partial}{\partial V}\left(\frac{C_v}{T}\right)\Big|_T = \frac{\partial}{\partial T}\left(\frac{p}{T}\right)\Big|_V = 0$.

Hence. $\frac{p}{T} = g(V)$. ⟹ $p(T, V) = T g(V)$.

$p(T, V) = T g(V)$ ⟹ $T g(V) \cdot V = f(T)$. ⟹ $g(V) V = \frac{f(T)}{T}$.

Left only depends on V; right only depends on T. They are constant.

$$f(T) \propto T. \qquad \frac{pV}{T} = constant.$$