Relationship between Temperature & Mean Translational Kinetic Energy

$$\langle v^2\rangle = \int_0^\infty v^2 f(v)\,dv = \int v^2 A \cdot v^2 e^{-\frac{1}{2}\frac{mv^2}{k_BT}}\,dv.$$

since $\int_0^\infty f(v)\,dv = 1.$

$$\langle v^2\rangle = \frac{A\int_0^\infty v^4 e^{-\frac{1}{2}\frac{mv^2}{k_BT}}\,dv}{A\int_0^\infty v^2 e^{-\frac{1}{2}\frac{mv^2}{k_BT}}\,dv}$$

$$I_n = \int_0^\infty v^n e^{-\frac{1}{2}\frac{mv^2}{k_BT}}\,dv \quad , \quad \text{let } \alpha = \frac{m}{2k_BT}$$

$$= \int_0^\infty v^n e^{-\alpha v^2}\,dv$$

$$= \left[-\frac{1}{2\alpha} v^{n-1} e^{-\alpha v^2}\right]_0^\infty + \frac{n-1}{2\alpha}\int_0^\infty v^{n-2} e^{-\alpha v^2}\,dv$$

$$= \frac{n-1}{2\alpha} I_{n-2}.$$

$$\langle v^2\rangle = \frac{I_4}{I_2} = \frac{4-1}{2\alpha} = \frac{3}{2\alpha} = \frac{3}{2}\cdot 2k_BT/m = \frac{3k_BT}{m}$$

$$\bar{E} = \frac{1}{2}m\bar{v^2} = \frac{3}{2}k_BT$$

Equation of State

$$pV = Nk_BT \Rightarrow k_B\cdot T = \frac{pV}{N}$$

$$\bar{E} = \frac{3}{2}\frac{pV}{N}$$

$$p = \frac{2}{3}\bar{E}\frac{N}{V} = \frac{2}{3}u\cancel{N}. \quad \text{where } u = \frac{\bar{E}N}{V}$$

average Energy density.