The pressure in a gas

— pressure in very simple models

A single particle moving along a line.

- 2 walls seperated by L.
- a collision happens within τ.

For every $\frac{2L}{v}$, there is $2mv$ delivered to a given wall.

$$\langle p \rangle = \langle \frac{\Delta p}{\Delta t} \rangle = \frac{2mv}{\frac{2L}{v}}$$

A group of particles moving along a line.

- speed distribution $f(v)$.
- N: total number of particles.

change in momentum:

$$dp = 2mv \cdot N f(v)\, dv.$$

every $t = \frac{2L}{v}$.

$$\frac{dp}{t} = \frac{mv^2}{L} N \cdot f(v)\, dv.$$

$$\int \frac{dp}{t} = \frac{N}{L} m \int v^2 f(v)\, dv = \frac{N}{L} m \langle v^2 \rangle.$$

Three-dimensional box. mirror-like reflections.

in x-direction: $\langle F_x \rangle = \frac{N}{L_x} m \langle V_x^2 \rangle$

$$P_x = \frac{Nm}{L_x L_y L_z} \langle V_x^2 \rangle$$

n: number of particles per volume.

$$P_x = nm \langle V_x^2 \rangle.$$

$$P_y = nm \langle V_y^2 \rangle. \quad P_z = nm \langle V_z^2 \rangle.$$

In an isotropic condition.

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle.$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

so $$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle.$$

Pressure in isotropic conditions

$$p = \frac{1}{3} n m \langle v^2 \rangle.$$

— Relation between pressure and energy density.

Translational kinetic energy: $K = \frac{1}{2} m v^2$

$$\langle K \rangle = \frac{m}{2} \langle v^2 \rangle.$$

rewrite P : $p = \frac{2}{3} n \langle K \rangle$

$\langle K \rangle$ is the mean kinetic energy of any one particle

$n \langle K \rangle$ is the translational kinetic energy per unit volume.

Energy density: $u = \frac{U}{V}$

so $p = \frac{2}{3} (u - u_{other})$.

u_{other} involves vibration and rotation.

$u_{other} = 0$ for a monatomic gas.

Pressure $\propto$ Energy density.

- Pressure in a more realistic model

  - experimental evidence that mirror-like reflection is wrong.

  - particles don't reflect from the surface; they stick. In steady states, they can also be released.

    desorption (released) adsorption (stick)

  - Dwell time: average time a molecule adsorbed to a surface.

- Flux

  flux of particle per unit area per unit time.

  $$\vec{j} = n\vec{v}$$

  the direction is the flow of particles.

  the magnitude is the number of particles ......

- Pressure from momentum flux.

  - wall perpendicular to x direction
  - v in the range (v_x, v_y, v_z) and $(v_x+dv_x, v_y+dv_y, v_z+dv_z)$

  the number density of such particles:

  $$nF(v_x, v_y, v_z)\, dv_x \cdot dv_y \cdot dv_z$$

  $$dv^3 = dv_x \cdot dv_y \cdot dv_z$$

  the flux of particles in the velocity range through the wall

  $$\vec{j} = n\vec{v}$$

  $$j = nF\, dv^3 \cdot v_x$$

  the momentum received by the wall per unit time and area (only on incidence, not reflection yet).

  $$dP_x = nF\, dv^3 \cdot v_x \cdot m v_x$$

$$P_{x,\ \text{incidence}} = \int_{v_x=0}^{\infty}\int_{v_y=-\infty}^{\infty}\int_{v_z=-\infty}^{\infty} mnF(v_x, v_y, v_z)\, v_x^2\, dv^3$$

Since this same amount of momentum is also given back to the particles, as they leave the wall.

$$P_x = 2P_{x,\ \text{incidence}}.$$

To avoid double-counting, we only include the v_x towards the wall. $v_x: 0 \to \infty$.

To simplify the integral, assume

$$F(v_x, v_y, v_z) = F(-v_x, v_y, v_z).$$

Particles in the entire gas are equally likely to move one away as the opposite.

$$P_x = mn\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(v_x, v_y, v_z)\, v_x^2 \cdot dv^3$$

$$= mn\, \langle v_x^2 \rangle.$$

Consider a wall with unit normal vector $\hat{n}$.

$$P_x = mn \langle (\vec{v}\cdot\hat{n})^2 \rangle.$$