Energy Equation

consider $U(T, V)$

$$dU = \frac{\partial U}{\partial T}\Big|_V \cdot dT + \frac{\partial U}{\partial V}\Big|_T \cdot dV$$

$$= C_V \cdot dT + \frac{\partial U}{\partial V}\Big|_T \cdot dV$$

$$\frac{\partial U}{\partial T}\Big|_p = C_V + \frac{\partial U}{\partial V}\Big|_T \cdot \frac{\partial V}{\partial T}\Big|_p$$

First law: $dU = dQ - p\,dV$

$$\frac{\partial U}{\partial T}\Big|_p = C_p - p\frac{\partial V}{\partial T}\Big|_p$$

Equate these: $C_V + \frac{\partial U}{\partial V}\Big|_T \frac{\partial V}{\partial T}\Big|_p = C_p - p\frac{\partial V}{\partial T}\Big|_p$

$$\left(p + \frac{\partial U}{\partial V}\Big|_T\right) \cdot \frac{\partial V}{\partial T}\Big|_p = C_p - C_V$$

$$\frac{\partial U}{\partial V}\Big|_T = (C_p - C_V)\frac{\partial T}{\partial V}\Big|_p - p$$

Sub it back to $\frac{\partial U}{\partial T}\Big|_p = C_V + \frac{\partial U}{\partial V}\Big|_T \cdot \frac{\partial V}{\partial T}\Big|_p$

$$\frac{\partial U}{\partial T}\Big|_p = C_V + \left[(C_p - C_V)\frac{\partial T}{\partial V}\Big|_p - p\right]\frac{\partial V}{\partial T}\Big|_p$$

$$dU = C_V \cdot dT + \left(\frac{C_p - C_V}{\alpha V} - p\right) dV$$. α expansivity.

For an ideal Gas

$$U = C_V \cdot T = \frac{Nk_BT}{\gamma - 1}$$. C_V independent of T.

$$U = \frac{p \cdot V}{\gamma - 1} \Rightarrow p = (\gamma - 1)\frac{U}{V}.$$

pressure is proportional to energy density.