ARTIFICIAL INTELLIGENCE

First-Order Logic

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DISCLAIMER

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See presentation notes for sources (chapter pages are for 4th ed.)

OUTLINE

- FOL in the logics landscape
- Syntax & semantics for FOL

INTRO TO FIRST ORDER LOGIC

A logic among others

MAMA MIA

- **Propositional logic** has limited expressive power.
- For Wumpus World
  - Can’t say “pits cause breezes in adjacent squares”
  - Must have 64 rules each for pits, breeze, etc.
- Propositional logic is **declarative.**
  - Declarative – pieces of syntax corresponds to facts
- **First-order logic (FOL)** expands to objects, relations, and functions!

A WHOLE NEW WORLD

	First-Order Logic	Propositional Logic
Objects	People, houses, numbers, theories, etc.	Propositional symbols (true/false values)
Relations / Functions	Red, round, brother of, owns, father of, logical connectives, etc.	Logical connectives

THIS BAD BOY CAN FIT SO MANY PROPOSITIONS IN IT

Propositional Logic

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$
$$B_{\dots} \Leftrightarrow (P_{\dots} \vee P_{\dots} \vee P_{\dots})$$
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$$\dots$$

First-Order Logic

$$\forall y\ Breezy(y) \Rightarrow$$
$$\exists x\ Pit(x) \wedge Adjacent(x, y)$$

MODELING REALITY: TABLE OF CONTENTS

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	Facts	True/false/unknown
First-order logic	Facts, objects, relations	True/false/unknown
Temporal logic	Facts, objects, relations, times	True/false/unknown
Probability theory	Facts	Degree of belief
Fuzzy logic	Facts + degree of truth	Known interval value

FOL SYNTAX & SEMANTICS

Speaking in the first order

A GRAND FANTASTIC POINT OF VIEW

- FOL expands from and adds to propositional logic syntax!
- **Symbols** are expanded from propositional logic to:
  - **Constants** – Objects
  - **Predicates** – Relations (always maps to T/F)
  - **Functions** – Functions (can return objects, T/F, etc.)

SYMBOLS VS. THE COOLER SYMBOLS

Propositional Logic Symbols
- `A = true`
- `B = false`

FOL Objects
- `John = {Attributes={Person, King, …}, Brother={Richard}}`

FOL Predicates
- `3 > 2 = true`
- `2 > 3 = false`

FOL Functions
- `IsEven(x) = x % 2 == 0`
- `NumOfAttributes(x) = len(x.Attributes)`
- `∀x, y HaveAChild(x, y) ⇔ ∃z Parent(x, z) ∧ Parent(y, z)`

FOL Symbols

WORDS FOR THINGS

- FOL sentences have **terms** as well.
- Terms refer to **objects**.
- Examples:
  - $KingJohn$ – a constant symbol, representing an object
  - $LeftLeg(KingJohn)$ – a function which refers to an object
  - x – a **variable** used by a **quantifier**
    - We’ll get to that later

WORDS FOR CONCLUSIONS ABOUT THINGS

- FOL has atomic and complex sentences.
- An **atomic sentence** is a singular predicate (returns true/false).
  - Note $term1 = term2$ is a binary predicate
- A **complex sentence** is a:
  - Modification to a sentence
    - Wrapping in parentheses
    - Negation
  - Two sentences logically connected with $\land$, $\lor$, $\Rightarrow$, or $\Leftrightarrow$
  - A quantifier, variable, sentence combo.

SENTENCE INCEPTION

Complex sentence

$$\forall y\ \underbrace{Breezy(y) \Rightarrow \exists x\ Pit(x) \land Adjacent(x, y)}_{\text{Complex Sentence}}$$

Quantifier

Variable

$$Breezy(y) \Rightarrow \underbrace{\exists x\ Pit(x) \land Adjacent(x, y)}_{\text{Complex Sentence}}$$

Atomic Sentence/ Function

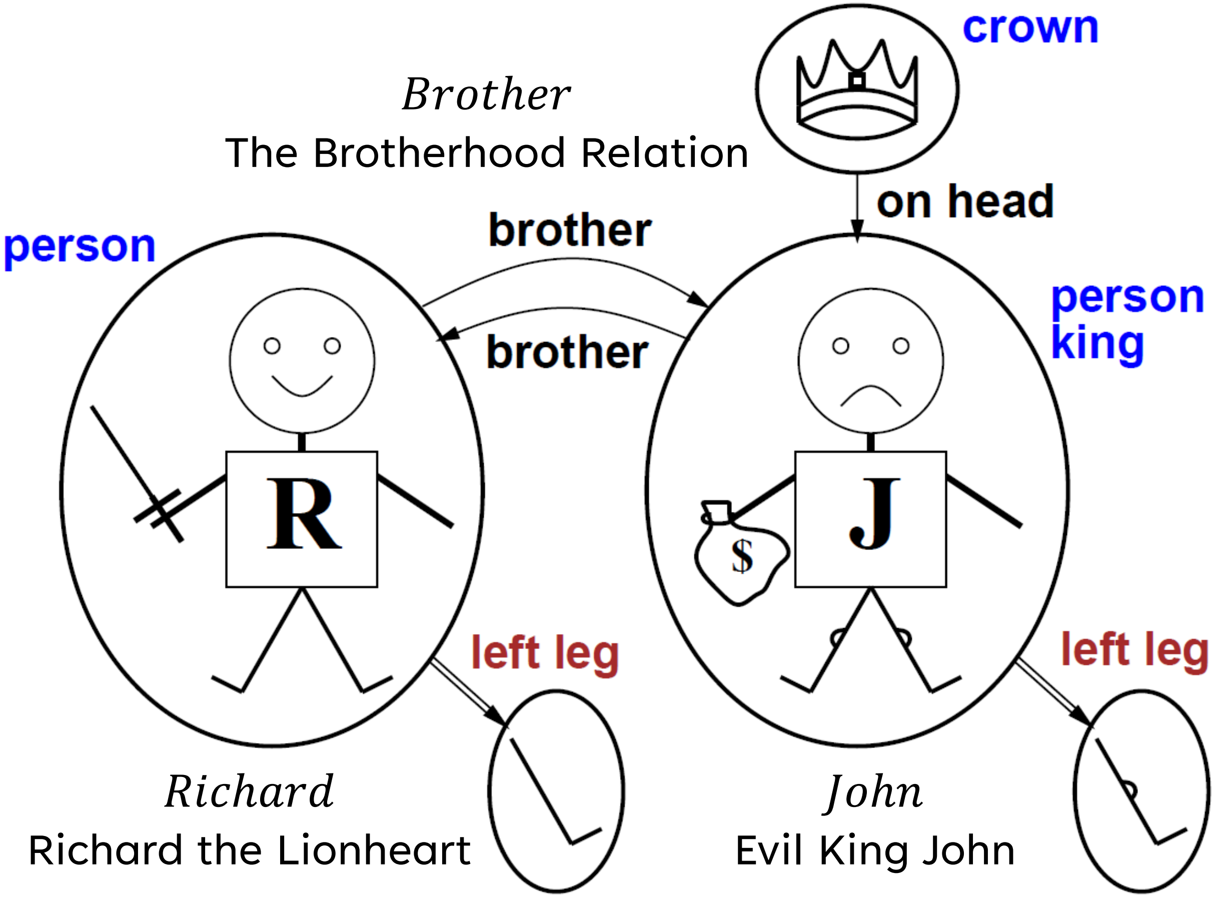
Logical Connective

HANDLING THE TRUTH
(WHICH WE CAN DO)

- Sentences are true with respect to a **model** and an **interpretation**.
- Model contains ≥ 1 objects (**domain elements**) and relations among them.
- An atomic sentence $predicate(term_1, \ldots, term_n)$ is true **iff** the objects referred to by $term_1, \ldots, term_n$ are in the relation referred to by $predicate$.

MODELS FOR FOL

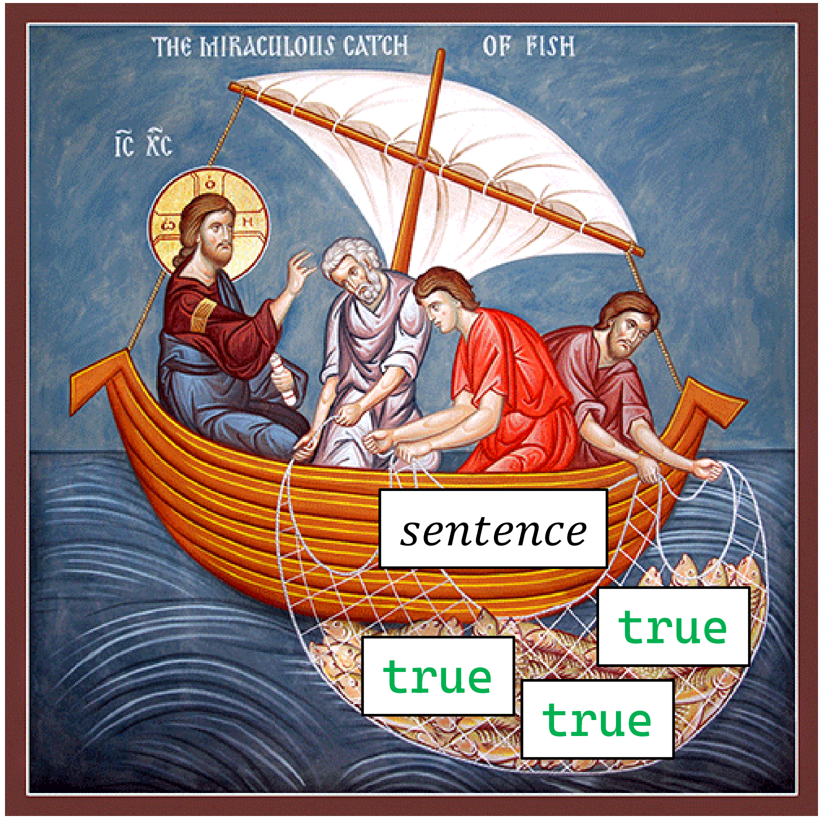
Under this interpretation, $Brother(Richard, John)$ is true in relation to this model.

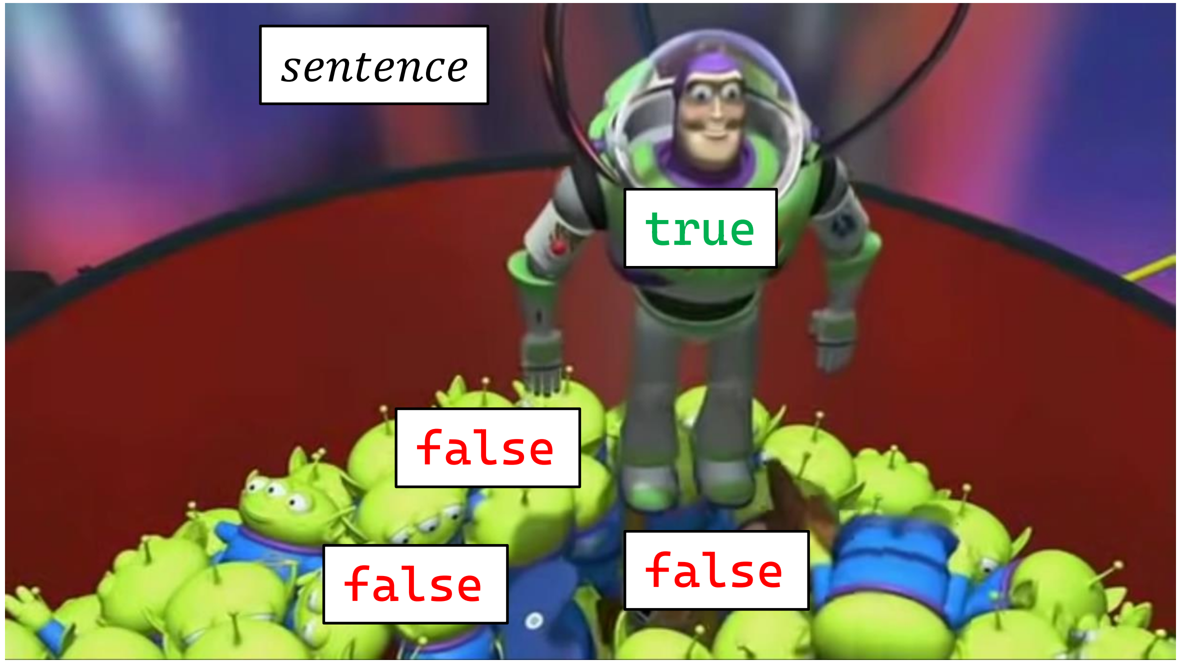


FOL MODELS
(TO INFINITY AND BEYOND)

- If you wanted to enumerate FOL models:
- For each number of domain elements n from 1 to ∞
  - For each k-ary predicate P_k in the vocabulary
    - For each possible k-ary relation on n objects
      - For each constant symbol C in the vocabulary
        - For each choice of referent for C from n objects…
- Computing entailment by enumerating FOL models is not easy…

UNIVERSAL VS. EXISTENTIAL QUANTIFIERS





UNIVERSAL QUANTIFICATION

- Expressing the statement: **Everyone at Berkeley is smart**
- **Universal Quantifier:** $\forall < variables > < sentence >$
  - $\forall \approx$"for all"
  - $\forall x\ P$ is true in a model m **iff** P is true with x being **each** possible object in the model.

Propositional Logic*

$(At(KingJohn, Berkeley) \Rightarrow Smart(KingJohn)) \land$
$(At(Richard, Berkeley) \Rightarrow Smart(Richard)) \land$
$(At(Berkeley, Berkeley) \Rightarrow Smart(Berkeley)) \land$
$\ldots$

FOL

$$\underbrace{\forall}_{\text{Quantifier}} \underbrace{x}_{\text{Variable}} \overbrace{At(x, Berkeley) \Rightarrow Smart(x)}^{\text{Sentence}}$$

- Additional example: $\forall x\ King(x) \Rightarrow Person(x)$

POP QUIZ!

Why $\Rightarrow$ instead of $\wedge$?

EXAM QUESTION: $\forall x\ King(x)(\land/\Rightarrow)Person(x)$

x value	Relationship	$\forall x\ King(x) \land Person(x)$ Result	$\forall x\ King(x) \Rightarrow Person(x)$ Result
Richard the Lionheart	Richard the Lionheart is a king (∧/⇒) Richard the Lionheart is a person.	false ∧ true = false	false ⇒ true = true
King John	King John is a king (∧/⇒) King John is a person.	true ∧ true = true	true ⇒ true = true
Richard's Left Leg	Richard's Left Leg is a king (∧/⇒) Richard's Left Leg is a person.	false ∧ false = false	false ⇒ false = true
John's Left Leg	John's Left Leg is a king (∧/⇒) John's Left Leg is a person.	false ∧ false = false	false ⇒ false = true
The Crown	The Crown is a king (∧/⇒) The Crown is a person.	false ∧ false = false	false ⇒ false = true

EXISTENTIAL QUANTIFICATION

- Expressing the statement: **Someone at Stanford is smart.**
- **Existential Quantifier:** $\exists < variables > < sentence >$
  - $\exists \approx$"there exists a" or "for some"
  - $\exists x\ P$ is true in a model m **iff** P is true with x being **some** possible object in the model.

Propositional Logic*

$\left(At(KingJohn, Stanford) \wedge Smart(KingJohn)\right) \vee$
$\left(At(Richard, Stanford) \wedge Smart(Richard)\right) \vee$
$\left(At(Berkeley, Stanford) \wedge Smart(Berkeley)\right) \vee$
…

$\longleftrightarrow$

FOL

$$\underbrace{\exists}_{\text{Quantifier}}\underbrace{x}_{\text{Variable}}\ \overbrace{At(x, Stanford) \wedge Smart(x)}^{\text{Sentence}}$$

- Additional example: $\exists x\ Crown(x) \wedge OnHead(x, John)$

POP QUIZ!

Why $\land$ instead of $\Rightarrow$?

$$\exists x\ Crown(x)(\Rightarrow/\land)OnHead(x, John)$$

x **value**	**Relationship**	$\exists x\ Crown(x) \Rightarrow OnHead(x, John)$ **Result**	$\exists x\ Crown(x) \land OnHead(x, John)$ **Result**
Richard the Lionheart	Richard the Lionheart is a crown (∧/⇒) Richard the Lionheart is on John's head.	false ⇒ false = true	false ∧ false = false
King John	King John is a crown (∧/⇒) King John is on John's head.	false ⇒ false = true	false ∧ false = false
Richard's Left Leg	Richard's Left Leg is a crown (∧/⇒) Richard's Left Leg is on John's head.	false ⇒ false = true	false ∧ false = false
John's Left Leg	John's Left Leg is a crown (∧/⇒) John's Left Leg is on John's head.	false ⇒ false = true	false ∧ false = false
The Crown	The Crown is a crown (∧/⇒) The Crown is on John's head.	true ⇒ true = true	true ∧ true = true

PROPERTIES OF QUANTIFIERS

- Consecutive, same-type quantifiers are **commutative**.
  - $\forall x \forall y = \forall y \forall x = \forall \mathrm{x}, \mathrm{y}$
  - $\exists x \exists y = \exists y \exists x = \exists \mathrm{x}, \mathrm{y}$
- $\exists x \forall y \neq \forall y \exists x$
  - $\forall x \exists y \; Loves(x, y)$ = Everybody loves somebody
  - $\exists y \forall x \; Loves(x, y)$ = There is someone who is loved by everyone

A QUANTIFIER'S GUIDE TO DUALITY & NEGATION **(EXAM QUESTIONS!)**

- **Quantifier duality** – each can be expressed using the other!
  - $\forall x\ Likes(x, Money) \equiv \neg\exists x\ \neg Likes(x, Money)$
  - $\exists x\ Likes(x, Tests) \equiv \neg\forall x\ Likes(x, Tests)$
- De Morgan's rules for quantified sentences:
  - $\neg\exists x\ P \equiv \forall x\ \neg P$
  - $\neg\forall x\ P \equiv \exists x\ \neg P$
  - $\forall x\ P \equiv \neg\exists x\ \neg P$
  - $\exists x\ P \equiv \neg\forall x\ \neg P$

EQUALITY

- $term_1 = term_2$ is true under a given interpretation **iff** $term_1$ and $term_2$ refer to the same object.
  - $1 = 2$ and $\forall x\ \times\left(Sqrt(x), Sqrt(x)\right) = x$ are satisfiable.
  - $2 = 2$ is valid.
  - $Sibling$ in terms of $Parent$:
    $\forall x, y\ Sibling(x, y) \Leftrightarrow$
    $[\neg(x = y) \wedge \exists m, f\ \neg(m = f) \wedge Parent(m, x)$
    $\wedge Parent(f, x) \wedge Parent(m, y) \wedge Parent(f, y)]$
    - For all objects, x and y are siblings if
    - x and y are not the same and
    - There are two objects m and f and
    - m and f are not the same and
    - m and f are both parents to x and y

SUBSTITUTION

- **Substitutions** (also called **binding lists**) allow us to replace variables with values.
- Given a sentence S and a substitution σ, $S\sigma$ means plugging σ into S.
  - $S = Smarter(x, y)$
  - $\sigma = \{x/Hillary, y/Bill\}$
  - $S\sigma = Smarter(Hillary, Bill)$
- $ASK(KB, S)$ returns some/all σ such that $KB \models S\sigma$

SUBSTITUTION REGARDING KBS

- $ASKVARS(KB, Person(x))$
- The KB returns multiple answers in the form of substitutions:
  - $\{x/John\}$
  - $\{x/Richard\}$
- Plugging these substitutions into the sentence $Person(x)$ makes the sentence true.
- So, these are the objects that are applicable to the sentence.

APPLYING FOL

English Representation	FOL Representation
Brothers are siblings.	$\forall x, y\ Brother(x, y) \Rightarrow Sibling(x, y)$
If A is B's sibling, then B is A's sibling. ("Sibling" is symmetric)	$\forall x, y\ Sibling(x, y) \Leftrightarrow Sibling(y, x)$
One's mother is one's female parent.	$\forall x, y\ Mother(x, y) \Leftrightarrow (Female(x) \land Parent(x, y))$
A first cousin is a child of a parent's sibling.	$\forall x, y\ FirstCousin(x, y) \Leftrightarrow$ $\exists p, ps\ Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$

THANK YOU

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