ペアノ算術 (TaPL 会)

るま

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1 5は素数である

$(\exists e_1.5 = 1 + e_1 + 1) \wedge (\forall d_1.(((\exists e_2.1 + e_2 + 1 = d_1) \wedge (\exists e_3.d_1 + e_3 + 1 = 5)) \to (\neg\exists d_2.d_1 \times d_2 = 5)))$

より正式には、

$(\exists e_1.S(S(S(S(S(O))))) = S(O) + e_1 + S(O)) \wedge (\forall d_1.(((\exists e_2.S(O) + e_2 + S(O) = d_1) \wedge (\exists e_3.d_1 + e_3 + S(O) = S(S(S(S(S(O))))))) \to (\neg\exists d_2.d_1 \times d_2 = S(S(S(S(S(O))))))))$

2 素数は無限個ある

$\forall n_1.\exists p_1.((\exists e_1.p_1 = 1 + e_1 + 1) \wedge (\forall d_1.(((\exists e_2.1 + e_2 + 1 = d_1) \wedge (\exists e_3.d_1 + e_3 + 1 = p_1)) \to (\neg\exists d_2.d_1 \times d_2 = p_1))) \wedge (\exists e_4.n_1 + e_4 = p_1))$

3 2の累乗

$$\exists k.\ \ 2^k = \mathbf{m}$$

$\forall q_1.(((\exists e_1.q_1 = 1 + e_1 + 1) \wedge (\forall d_1.(((\exists e_2.1 + e_2 + 1 = d_1) \wedge (\exists e_3.d_1 + e_3 + 1 = q_1)) \to (\neg\exists d_2.d_1 \times d_2 = q_1))) \wedge (\neg(q_1 = 2))) \to (\neg\exists d_3.q_1 \times d_3 = \mathbf{m}))$

4 2のk乗

$$2^{\mathbf{k}} = \mathbf{m}$$

$\exists l_1.\exists p_1.((\exists e_1.p_1 = 1 + e_1 + 1) \wedge (\forall d_1.(((\exists e_2.1 + e_2 + 1 = d_1) \wedge (\exists e_3.d_1 + e_3 + 1 = p_1)) \to (\neg\exists d_2.d_1 \times d_2 = p_1))) \wedge (\forall n_1.((2 \times n_1 = (0 + 1) \times (0 + 1 + 1) + 2 \times 1) \to (\forall r_1.((\exists d_3.l_1 = p_1 \times d_3 + r_1) \to (r_1 = n_1))))) \wedge (\forall pe_1.(((\forall q_1.(((\exists e_4.q_1 = 1 + e_4 + 1) \wedge (\forall d_4.(((\exists e_5.1 + e_5 + 1 = d_4) \wedge (\exists e_6.d_4 + e_6 + 1 = q_1)) \to (\neg\exists d_5.d_4 \times d_5 = q_1))) \wedge (\neg(q_1 = p_1))) \to (\neg\exists d_6.q_1 \times d_6 = pe_1))) \wedge (\exists e_7.pe_1 \times p_1 + e_7 = l_1)) \to ((\forall x_1.((\exists y_1.\forall n_2.((2 \times n_2 = (x_1 + y_1) \times (x_1 + y_1 + 1) + 2 \times y_1) \to (\forall r_3.((\exists d_8.\forall d_7.((\exists r_2.l_1 = pe_1 \times d_7 + r_2) \to (d_7 = p_1 \times d_8 + r_3))) \to (r_3 = n_2))))) \to (\forall x_2.((\exists y_2.\forall n_3.((2 \times n_3 = (x_2 + y_2) \times (x_2 + y_2 + 1) + 2 \times y_2) \to$

$$(\forall r_5.((\exists d_{10}.\forall d_9.((\exists r_4.l_1 = pe_1 \times p_1 \times d_9 + r_4) \to (d_9 = p_1 \times d_{10} + r_5))) \to (r_5 = n_3))))) \to (x_2 = x_1 + 1))))) \land (\forall y_3.((\exists x_3.\forall n_4.((2 \times n_4 = (x_3 + y_3) \times (x_3 + y_3 + 1) + 2 \times y_3) \to (\forall r_7.((\exists d_{12}.\forall d_{11}.((\exists r_6.l_1 = pe_1 \times d_{11} + r_6) \to (d_{11} = p_1 \times d_{12} + r_7))) \to (r_7 = n_4))))) \to (\forall y_4.((\exists x_4.\forall n_5.((2 \times n_5 = (x_4 + y_4) \times (x_4 + y_4 + 1) + 2 \times y_4) \to (\forall r_9.((\exists d_{14}.\forall d_{13}.((\exists r_8.l_1 = pe_1 \times p_1 \times d_{13} + r_8) \to (d_{13} = p_1 \times d_{14} + r_9))) \to (r_9 = n_5))))) \to (y_4 = y_3 \times 2)))))))) \land (\forall n_6.((2 \times n_6 = (\mathbf{k} + \mathbf{m}) \times (\mathbf{k} + \mathbf{m} + 1) + 2 \times \mathbf{m}) \to (\forall r_{11}.((\exists d_{22}.\forall d_{21}.((\exists r_{10}.\forall v_1.(((\forall q_2.(((\exists e_8.q_2 = 1 + e_8 + 1) \land (\forall d_{15}.(((\exists e_9.1 + e_9 + 1 = d_{15}) \land (\exists e_{10}.d_{15} + e_{10} + 1 = q_2)) \to (\neg\exists d_{16}.d_{15} \times d_{16} = q_2))) \land (\neg(q_2 = p_1))) \to (\neg\exists d_{17}.q_2 \times d_{17} = v_1))) \land (\exists e_{11}.v_1 = l_1 + e_{11} + 1) \land (\forall u_1.(((\forall q_3.(((\exists e_{12}.q_3 = 1 + e_{12} + 1) \land (\forall d_{18}.(((\exists e_{13}.1 + e_{13} + 1 = d_{18}) \land (\exists e_{14}.d_{18} + e_{14} + 1 = q_3)) \to (\neg\exists d_{19}.d_{18} \times d_{19} = q_3))) \land (\neg(q_3 = p_1))) \to (\neg\exists d_{20}.q_3 \times d_{20} = u_1))) \land (\exists e_{15}.u_1 = l_1 + e_{15} + 1)) \to (\exists e_{16}.u_1 = v_1 + e_{16})))) \to (l_1 = v_1 \times d_{21} + r_{10}))) \to (d_{21} = p_1 \times d_{22} + r_{11}))) \to (r_{11} = n_6))))))$$

5　完全数は無限個ある

$$\forall n_{10}.\exists m_1.((\forall d_{21}.((\exists l_1.\exists p_1.((\forall n_1.((2 \times n_1 = (0 + m_1) \times (0 + m_1 + 1) + 2 \times m_1) \to (\forall r_1.((\exists d_1.l_1 = p_1 \times d_1 + r_1) \to (r_1 = n_1))))) \land (\forall pe_1.(((\forall q_1.(((\exists e_1.q_1 = 1 + e_1 + 1) \land (\forall d_2.(((\exists e_2.1 + e_2 + 1 = d_2) \land (\exists e_3.d_2 + e_3 + 1 = q_1)) \to (\neg\exists d_3.d_2 \times d_3 = q_1))) \land (\neg(q_1 = p_1))) \to (\neg\exists d_4.q_1 \times d_4 = pe_1))) \land (\exists e_4.pe_1 \times p_1 + e_4 = l_1)) \to ((\forall x_1.((\exists y_1.\forall n_2.((2 \times n_2 = (x_1 + y_1) \times (x_1 + y_1 + 1) + 2 \times y_1) \to (\forall r_3.((\exists d_6.\forall d_5.((\exists r_2.l_1 = pe_1 \times d_5 + r_2) \to (d_5 = p_1 \times d_6 + r_3))) \to (r_3 = n_2))))) \to (\forall x_2.((\exists y_2.\forall n_3.((2 \times n_3 = (x_2 + y_2) \times (x_2 + y_2 + 1) + 2 \times y_2) \to (\forall r_5.((\exists d_8.\forall d_7.((\exists r_4.l_1 = pe_1 \times p_1 \times d_7 + r_4) \to (d_7 = p_1 \times d_8 + r_5))) \to (r_5 = n_3))))) \to (x_2 = x_1 + 1))))) \land (\forall r_{10}.((((\exists d_{11}.\forall x_3.((\exists y_3.\forall n_4.((2 \times n_4 = (x_3 + y_3) \times (x_3 + y_3 + 1) + 2 \times y_3) \to (\forall r_7.((\exists d_{10}.\forall d_9.((\exists r_6.l_1 = pe_1 \times p_1 \times d_9 + r_6) \to (d_9 = p_1 \times d_{10} + r_7))) \to (r_7 = n_4))))) \to (x_3 \times d_{11} = m_1))) \land (\forall x_4.((\exists y_4.\forall n_5.((2 \times n_5 = (x_4 + y_4) \times (x_4 + y_4 + 1) + 2 \times y_4) \to (\forall r_9.((\exists d_{13}.\forall d_{12}.((\exists r_8.l_1 = pe_1 \times p_1 \times d_{12} + r_8) \to (d_{12} = p_1 \times d_{13} + r_9))) \to (r_9 = n_5))))) \to (r_{10} = x_4)))) \lor ((\neg\exists d_{16}.\forall x_5.((\exists y_5.\forall n_6.((2 \times n_6 = (x_5 + y_5) \times (x_5 + y_5 + 1) + 2 \times y_5) \to (\forall r_{12}.((\exists d_{15}.\forall d_{14}.((\exists r_{11}.l_1 = pe_1 \times p_1 \times d_{14} + r_{11}) \to (d_{14} = p_1 \times d_{15} + r_{12}))) \to (r_{12} = n_6))))) \to (x_5 \times d_{16} = m_1))) \land (r_{10} = 0))) \to (\forall y_6.((\exists x_6.\forall n_7.((2 \times n_7 = (x_6 + y_6) \times (x_6 + y_6 + 1) + 2 \times y_6) \to (\forall r_{14}.((\exists d_{18}.\forall d_{17}.((\exists r_{13}.l_1 = pe_1 \times d_{17} + r_{13}) \to (d_{17} = p_1 \times d_{18} + r_{14}))) \to (r_{14} = n_7))))) \to (\forall y_7.((\exists x_7.\forall n_8.((2 \times n_8 = (x_7 + y_7) \times (x_7 + y_7 + 1) + 2 \times y_7) \to (\forall r_{16}.((\exists d_{20}.\forall d_{19}.((\exists r_{15}.l_1 = pe_1 \times p_1 \times d_{19} + r_{15}) \to (d_{19} = p_1 \times d_{20} + r_{16}))) \to (r_{16} = n_8))))) \to (y_7 = y_6 + r_{10})))))))))) \land (\forall n_9.((2 \times n_9 = (m_1 + d_{21}) \times (m_1 + d_{21} + 1) + 2 \times d_{21}) \to (\forall r_{18}.((\exists d_{29}.\forall d_{28}.((\exists r_{17}.\forall v_1.(((\forall q_2.(((\exists e_5.q_2 = 1 + e_5 + 1) \land (\forall d_{22}.(((\exists e_6.1 + e_6 + 1 = d_{22}) \land (\exists e_7.d_{22} + e_7 + 1 = q_2)) \to (\neg\exists d_{23}.d_{22} \times d_{23} = q_2))) \land (\neg(q_2 = p_1))) \to (\neg\exists d_{24}.q_2 \times d_{24} = v_1))) \land (\exists e_8.v_1 = l_1 + e_8 + 1) \land (\forall u_1.(((\forall q_3.(((\exists e_9.q_3 = 1 + e_9 + 1) \land (\forall d_{25}.(((\exists e_{10}.1 + e_{10} + 1 = d_{25}) \land (\exists e_{11}.d_{25} + e_{11} + 1 = q_3)) \to (\neg\exists d_{26}.d_{25} \times d_{26} = q_3))) \land (\neg(q_3 = p_1))) \to (\neg\exists d_{27}.q_3 \times d_{27} = u_1))) \land (\exists e_{12}.u_1 = l_1 + e_{12} + 1)) \to (\exists e_{13}.u_1 = v_1 + e_{13})))) \to (l_1 = v_1 \times d_{28} + r_{17}))) \to (d_{28} = p_1 \times d_{29} + r_{18}))) \to (r_{18} = n_9))))))) \to (2 \times d_{21} = m_1))) \land (\exists e_{14}.n_{10} + e_{14} = m_1))$$

6 フェルマーの最終定理

$$\forall a, b, c \geq 1. \quad \forall n \geq 3. \;\; a^n + b^n \neq c^n$$

$\forall n_1.((\exists e_1.n_1 = 2 + e_1 + 1) \to (\forall a_1.\forall b_2.\forall c_1.(\neg\forall b_1.((\exists l_1.\exists p_1.((\exists e_2.p_1 = 1 + e_2+1)\wedge(\forall d_1.(((\exists e_3.1+e_3+1 = d_1)\wedge(\exists e_4.d_1+e_4+1 = p_1)) \to (\neg\exists d_2.d_1\times d_2 = p_1)))\wedge(\forall n_2.((2\times n_2 = (0+1)\times(0+1+1)+2\times 1) \to (\forall r_1.((\exists d_3.l_1 = p_1\times d_3+r_1) \to (r_1 = n_2)))))\wedge(\forall pe_1.(((\forall q_1.(((\exists e_5.q_1 = 1+e_5+1)\wedge(\forall d_4.(((\exists e_6.1+e_6+1 = d_4)\wedge(\exists e_7.d_4+e_7+1 = q_1)) \to (\neg\exists d_5.d_4\times d_5 = q_1)))\wedge(\neg(q_1 = p_1))) \to (\neg\exists d_6.q_1\times d_6 = pe_1)))\wedge(\exists e_8.pe_1\times p_1+e_8 = l_1)) \to ((\forall x_1.((\exists y_1.\forall n_3.((2\times n_3 = (x_1+y_1)\times(x_1+y_1+1)+2\times y_1) \to (\forall r_3.((\exists d_8.\forall d_7.((\exists r_2.l_1 = pe_1\times d_7+r_2) \to (d_7 = p_1\times d_8+r_3))) \to (r_3 = n_3))))) \to (\forall x_2.((\exists y_2.\forall n_4.((2\times n_4 = (x_2+y_2)\times(x_2+y_2+1)+2\times y_2) \to (\forall r_5.((\exists d_{10}.\forall d_9.((\exists r_4.l_1 = pe_1\times p_1\times d_9+r_4) \to (d_9 = p_1\times d_{10}+r_5))) \to (r_5 = n_4))))) \to (x_2 = x_1+1)))))\wedge(\forall y_3.((\exists x_3.\forall n_5.((2\times n_5 = (x_3+y_3)\times(x_3+y_3+1)+2\times y_3) \to (\forall r_7.((\exists d_{12}.\forall d_{11}.((\exists r_6.l_1 = pe_1\times d_{11}+r_6) \to (d_{11} = p_1\times d_{12}+r_7))) \to (r_7 = n_5))))) \to (\forall y_4.((\exists x_4.\forall n_6.((2\times n_6 = (x_4+y_4)\times(x_4+y_4+1)+2\times y_4) \to (\forall r_9.((\exists d_{14}.\forall d_{13}.((\exists r_8.l_1 = pe_1\times p_1\times d_{13}+r_8) \to (d_{13} = p_1\times d_{14}+r_9))) \to (r_9 = n_6))))) \to (y_4 = y_3\times c_1))))))))\wedge(\forall n_7.((2\times n_7 = (n_1+b_1)\times(n_1+b_1+1)+2\times b_1) \to (\forall r_{11}.((\exists d_{22}.\forall d_{21}.((\exists r_{10}.\forall v_1.(((\forall q_2.(((\exists e_9.q_2 = 1+e_9+1)\wedge(\forall d_{15}.(((\exists e_{10}.1+e_{10}+1 = d_{15})\wedge(\exists e_{11}.d_{15}+e_{11}+1 = q_2)) \to (\neg\exists d_{16}.d_{15}\times d_{16} = q_2)))\wedge(\neg(q_2 = p_1))) \to (\neg\exists d_{17}.q_2\times d_{17} = v_1)))\wedge(\exists e_{12}.v_1 = l_1+e_{12}+1)\wedge(\forall u_1.(((\forall q_3.(((\exists e_{13}.q_3 = 1+e_{13}+1)\wedge(\forall d_{18}.(((\exists e_{14}.1+e_{14}+1 = d_{18})\wedge(\exists e_{15}.d_{18}+e_{15}+1 = q_3)) \to (\neg\exists d_{19}.d_{18}\times d_{19} = q_3)))\wedge(\neg(q_3 = p_1))) \to (\neg\exists d_{20}.q_3\times d_{20} = u_1)))\wedge(\exists e_{16}.u_1 = l_1+e_{16}+1)) \to (\exists e_{17}.u_1 = v_1+e_{17})))) \to (l_1 = v_1\times d_{21}+r_{10}))) \to (d_{21} = p_1\times d_{22}+r_{11}))) \to (r_{11} = n_7))))))) \to (\forall b_3.((\exists l_2.\exists p_2.((\exists e_{18}.p_2 = 1+e_{18}+1)\wedge(\forall d_{23}.(((\exists e_{19}.1+e_{19}+1 = d_{23})\wedge(\exists e_{20}.d_{23}+e_{20}+1 = p_2)) \to (\neg\exists d_{24}.d_{23}\times d_{24} = p_2)))\wedge(\forall n_8.((2\times n_8 = (0+1)\times(0+1+1)+2\times 1) \to (\forall r_{12}.((\exists d_{25}.l_2 = p_2\times d_{25}+r_{12}) \to (r_{12} = n_8)))))\wedge(\forall pe_2.(((\forall q_4.(((\exists e_{21}.q_4 = 1+e_{21}+1)\wedge(\forall d_{26}.(((\exists e_{22}.1+e_{22}+1 = d_{26})\wedge(\exists e_{23}.d_{26}+e_{23}+1 = q_4)) \to (\neg\exists d_{27}.d_{26}\times d_{27} = q_4)))\wedge(\neg(q_4 = p_2))) \to (\neg\exists d_{28}.q_4\times d_{28} = pe_2)))\wedge(\exists e_{24}.pe_2\times p_2+e_{24} = l_2)) \to ((\forall x_5.((\exists y_5.\forall n_9.((2\times n_9 = (x_5+y_5)\times(x_5+y_5+1)+2\times y_5) \to (\forall r_{14}.((\exists d_{30}.\forall d_{29}.((\exists r_{13}.l_2 = pe_2\times d_{29}+r_{13}) \to (d_{29} = p_2\times d_{30}+r_{14}))) \to (r_{14} = n_9))))) \to (\forall x_6.((\exists y_6.\forall n_{10}.((2\times n_{10} = (x_6+y_6)\times(x_6+y_6+1)+2\times y_6) \to (\forall r_{16}.((\exists d_{32}.\forall d_{31}.((\exists r_{15}.l_2 = pe_2\times p_2\times d_{31}+r_{15}) \to (d_{31} = p_2\times d_{32}+r_{16}))) \to (r_{16} = n_{10}))))) \to (x_6 = x_5+1)))))\wedge(\forall y_7.((\exists x_7.\forall n_{11}.((2\times n_{11} = (x_7+y_7)\times(x_7+y_7+1)+2\times y_7) \to (\forall r_{18}.((\exists d_{34}.\forall d_{33}.((\exists r_{17}.l_2 = pe_2\times d_{33}+r_{17}) \to (d_{33} = p_2\times d_{34}+r_{18}))) \to (r_{18} = n_{11}))))) \to (\forall y_8.((\exists x_8.\forall n_{12}.((2\times n_{12} = (x_8+y_8)\times(x_8+y_8+1)+2\times y_8) \to (\forall r_{20}.((\exists d_{36}.\forall d_{35}.((\exists r_{19}.l_2 = pe_2\times p_2\times d_{35}+r_{19}) \to (d_{35} = p_2\times d_{36}+r_{20}))) \to (r_{20} = n_{12}))))) \to (y_8 = y_7\times b_2))))))))\wedge(\forall n_{13}.((2\times n_{13} = (n_1+b_3)\times(n_1+b_3+1)+2\times b_3) \to (\forall r_{22}.((\exists d_{44}.\forall d_{43}.((\exists r_{21}.\forall v_2.(((\forall q_5.(((\exists e_{25}.q_5 = 1+e_{25}+1)\wedge(\forall d_{37}.(((\exists e_{26}.1+e_{26}+1 = d_{37})\wedge(\exists e_{27}.d_{37}+e_{27}+1 = q_5)) \to (\neg\exists d_{38}.d_{37}\times d_{38} = q_5)))\wedge(\neg(q_5 = p_2))) \to (\neg\exists d_{39}.q_5\times d_{39} = v_2)))\wedge(\exists e_{28}.v_2 = l_2+e_{28}+1)\wedge(\forall u_2.(((\forall q_6.(((\exists e_{29}.q_6 = 1+e_{29}+1)\wedge(\forall d_{40}.(((\exists e_{30}.1+e_{30}+1 = d_{40})\wedge(\exists e_{31}.d_{40}+e_{31}+1 = q_6)) \to (\neg\exists d_{41}.d_{40}\times d_{41} = q_6)))\wedge(\neg(q_6 = p_2))) \to (\neg\exists d_{42}.q_6\times d_{42} = u_2)))\wedge(\exists e_{32}.u_2 = l_2+e_{32}+1)) \to (\exists e_{33}.u_2 = v_2+e_{33})))) \to (l_2 = v_2\times d_{43}+r_{21}))) \to (d_{43} = p_2\times d_{44}+r_{22}))) \to (r_{22} =$

$n_{13}))))))) \to (\forall b_4.((\exists l_3.\exists p_3.((\exists e_{34}.p_3 = 1 + e_{34} + 1) \wedge (\forall d_{45}.(((\exists e_{35}.1 + e_{35} + 1 =$
$d_{45}) \wedge (\exists e_{36}.d_{45} + e_{36} + 1 = p_3)) \to (\neg\exists d_{46}.d_{45} \times d_{46} = p_3))) \wedge (\forall n_{14}.((2 \times n_{14} =$
$(0 + 1) \times (0 + 1 + 1) + 2 \times 1) \to (\forall r_{23}.((\exists d_{47}.l_3 = p_3 \times d_{47} + r_{23}) \to (r_{23} =$
$n_{14})))) \wedge (\forall pe_3.(((\forall q_7.(((\exists e_{37}.q_7 = 1 + e_{37} + 1) \wedge (\forall d_{48}.(((\exists e_{38}.1 + e_{38} + 1 =$
$d_{48}) \wedge (\exists e_{39}.d_{48} + e_{39} + 1 = q_7)) \to (\neg\exists d_{49}.d_{48} \times d_{49} = q_7))) \wedge (\neg(q_7 = p_3))) \to$
$(\neg\exists d_{50}.q_7 \times d_{50} = pe_3))) \wedge (\exists e_{40}.pe_3 \times p_3 + e_{40} = l_3)) \to ((\forall x_9.((\exists y_9.\forall n_{15}.((2 \times$
$n_{15} = (x_9 + y_9) \times (x_9 + y_9 + 1) + 2 \times y_9) \to (\forall r_{25}.((\exists d_{52}.\forall d_{51}.((\exists r_{24}.l_3 = pe_3 \times$
$d_{51} + r_{24}) \to (d_{51} = p_3 \times d_{52} + r_{25}))) \to (r_{25} = n_{15}))))) \to (\forall x_{10}.((\exists y_{10}.\forall n_{16}.((2 \times$
$n_{16} = (x_{10} + y_{10}) \times (x_{10} + y_{10} + 1) + 2 \times y_{10}) \to (\forall r_{27}.((\exists d_{54}.\forall d_{53}.((\exists r_{26}.l_3 =$
$pe_3 \times p_3 \times d_{53} + r_{26}) \to (d_{53} = p_3 \times d_{54} + r_{27}))) \to (r_{27} = n_{16}))))) \to (x_{10} =$
$x_9 + 1))))) \wedge (\forall y_{11}.((\exists x_{11}.\forall n_{17}.((2 \times n_{17} = (x_{11} + y_{11}) \times (x_{11} + y_{11} + 1) + 2 \times y_{11}) \to$
$(\forall r_{29}.((\exists d_{56}.\forall d_{55}.((\exists r_{28}.l_3 = pe_3 \times d_{55} + r_{28}) \to (d_{55} = p_3 \times d_{56} + r_{29}))) \to$
$(r_{29} = n_{17}))))) \to (\forall y_{12}.((\exists x_{12}.\forall n_{18}.((2 \times n_{18} = (x_{12} + y_{12}) \times (x_{12} + y_{12} +$
$1) + 2 \times y_{12}) \to (\forall r_{31}.((\exists d_{58}.\forall d_{57}.((\exists r_{30}.l_3 = pe_3 \times p_3 \times d_{57} + r_{30}) \to (d_{57} =$
$p_3 \times d_{58} + r_{31}))) \to (r_{31} = n_{18}))))) \to (y_{12} = y_{11} \times a_1)))))))) \wedge (\forall n_{19}.((2 \times n_{19} =$
$(n_1 + b_4) \times (n_1 + b_4 + 1) + 2 \times b_4) \to (\forall r_{33}.((\exists d_{66}.\forall d_{65}.((\exists r_{32}.\forall v_3.(((\forall q_8.(((\exists e_{41}.q_8 =$
$1 + e_{41} + 1) \wedge (\forall d_{59}.(((\exists e_{42}.1 + e_{42} + 1 = d_{59}) \wedge (\exists e_{43}.d_{59} + e_{43} + 1 = q_8)) \to$
$(\neg\exists d_{60}.d_{59} \times d_{60} = q_8))) \wedge (\neg(q_8 = p_3))) \to (\neg\exists d_{61}.q_8 \times d_{61} = v_3))) \wedge (\exists e_{44}.v_3 =$
$l_3 + e_{44} + 1) \wedge (\forall u_3.(((\forall q_9.(((\exists e_{45}.q_9 = 1 + e_{45} + 1) \wedge (\forall d_{62}.(((\exists e_{46}.1 + e_{46} + 1 =$
$d_{62}) \wedge (\exists e_{47}.d_{62} + e_{47} + 1 = q_9)) \to (\neg\exists d_{63}.d_{62} \times d_{63} = q_9))) \wedge (\neg(q_9 = p_3))) \to$
$(\neg\exists d_{64}.q_9 \times d_{64} = u_3))) \wedge (\exists e_{48}.u_3 = l_3 + e_{48} + 1)) \to (\exists e_{49}.u_3 = v_3 + e_{49})))) \to$
$(l_3 = v_3 \times d_{65} + r_{32}))) \to (d_{65} = p_3 \times d_{66} + r_{33}))) \to (r_{33} = n_{19}))))))) \to (b_4 + b_3 =$
$b_1)))))))))$