① (a) $\frac{d}{dt}\left(\frac{3t}{\ln t}\right)\Big|_{e^2} = \frac{3\ln t - 3}{(\ln t)^2}\Big|_{e^2}$

$= \frac{3}{4}$

(b) $\lim\limits_{u\to 0} \frac{3u}{\tan 2u} = \frac{3}{2}\lim\limits_{u\to 0}\frac{2u}{\tan 2u}$

$= \frac{3}{2}\lim\limits_{u\to 0}\frac{\cos 2u}{\sin 2u/2u}$

$= \frac{3}{2}\left(\frac{1}{1}\right) = \frac{3}{2}$

(c) $\frac{d^3}{dx^3}\sin kx = \frac{d^2}{dx^2}(k\cos kx)$

$= \frac{d}{dx}(-k^2\sin kx)$

$= -k^3\cos kx$

(d) $\frac{d}{d\theta}\sqrt[3]{a+k\sin^2\theta} = \frac{d}{d\theta}(a+k\sin^2\theta)^{1/3}$

$= \frac{1}{3}(a+k\sin^2\theta)^{-2/3}(2k\sin\theta\cos\theta)$

$= \frac{k}{3}\sin 2\theta \cdot (a+k\sin^2\theta)^{-2/3}$

② $\frac{d}{dx}x^3\Big|_{x=x_0} = \lim\limits_{\Delta x\to 0}\frac{(x_0+\Delta x)^3 - x_0^3}{\Delta x}$

$= \lim\limits_{\Delta x\to 0}\frac{x_0^3 + 3x_0^2\Delta x + 3x_0(\Delta x)^2 + (\Delta x)^3 - x_0^3}{\Delta x}$

$= \lim\limits_{\Delta x\to 0}(3x_0^2 + 3x_0\Delta x + (\Delta x)^2)$

$= 3x_0^2$

③ If we let $f(x) = \sqrt[3]{x}$,

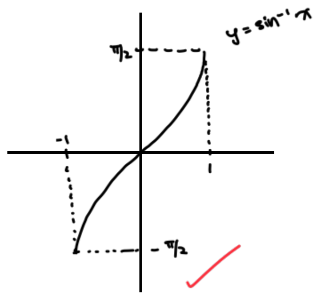
$\lim\limits_{h\to 0}\frac{1-\sqrt[3]{1+h}}{h} = -\lim\limits_{h\to 0}\frac{\sqrt[3]{1+h}-\sqrt[3]{1}}{h}$

$= -\lim\limits_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$= -\frac{d}{dx}f(x)\Big|_{x=1}$

$= \left[-\frac{1}{3}x^{-2/3}\right]\Big|_{x=1} = -\frac{1}{3}$

④



letting $y = \sin^{-1}x$, $x = \sin y$

$\frac{d}{dx}(x = \sin y)$

$\Rightarrow 1 = (\cos y)\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

⑤ $f(x) = \begin{cases} ax+b, & x>0 \\ 1-x+x^2, & x\le 0 \end{cases}$

For $f(x)$ to be continuous, $\lim\limits_{x\to 0^+}f(x) = \lim\limits_{x\to 0^-}f(x)$.

$\Rightarrow b=1,\ a\in\mathbb{R}$

For $f(x)$ to be differentiable, $\lim\limits_{x\to 0^+}f'(x) = \lim\limits_{x\to 0^-}f'(x)$,

and by the "diff $\Rightarrow$ cont" theorem, $f(x)$ must be continuous as well $\Rightarrow b=1$.

$f'(x) = \begin{cases} a, & x>0 \\ -1+2x, & x\le 0. \end{cases}$ Since $f'(0) = -1$, $a=-1$ too

⑥ Given $x^2y + y^3 + x^2 = 8$, diff. both sides wrt x,

$2xy + x^2y' + 3y^2y' + 2x = 0$

$y'(x^2+3y^2) = -2x(y+1)$

$\Rightarrow y' = -\frac{2x(y+1)}{x^2+3y^2}$

Horizontal tangent line $\Rightarrow y'=0 \Rightarrow x=0$ or $y=-1$

When $x=0$, $y=2$; when $y=-1$, $x\to\pm\infty$

So the only point w/ a horizontal tangent line is $(0,2)$

⑦ Tangent line of $y=f(x)$ at (x_0, y_0) has eqn $y-y_0 = f'(x_0)\cdot(x-x_0)$

"intersect w/ x-axis" $\Rightarrow y=0$.

Subst. $y=0$: $-y_0 = [f'(x_0)](x-x_0) \Rightarrow x = x_0 - \frac{y_0}{f'(x_0)}$, ie. it is at point $\left(x_0 - \frac{y_0}{f'(x_0)}, 0\right)$

⑧ $V = \frac{4}{3}\pi r^3$, for volume V and radius r.

Diff. both sides wrt to time t,

$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\frac{dr}{dt}\right) = 4\pi r^2\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$.

$\therefore \frac{dr}{dt}\Big|_{r=20} = \frac{1}{4\pi(20)^2}(-10) = -\frac{1}{10\pi}$ cm s^{-1}

Rate of decrease is $\frac{1}{10\pi}$ cm s^{-1} $-\frac{1}{160\pi}$

⑨ (a) $\sec x = \frac{1}{\cos x}$

$\cos x = 0 \iff x = \frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$.

$\therefore \sec x$ is discontinuous when $x = \frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

(b) $\frac{1+x^2}{1-x^2} = \frac{1+x^2}{(1-x)(1+x)} \Rightarrow$ discontinuous when $x=\pm 1$.

(c) $|x| = \begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases}$

$\Rightarrow \frac{d}{dx}|x| = \begin{cases} 1, & x\ge 0 \\ -1, & x<0 \end{cases} \Rightarrow$ discontinuous at $x=0$

⑩ (a) Given $A = \frac{1}{4}A_0$,

$e^{-rt} = \frac{1}{4} \Rightarrow rt = \ln 4 \Rightarrow t = \frac{2}{r}\ln 2$

(b) $\frac{dA}{dt} = A_0e^{-rt}(-r) = -A_0re^{-rt}$ $= A(-r) = \frac{1}{4}(-r) = -\frac{r}{4}$

$\Rightarrow \frac{dA}{dt}\Big|_{t=\frac{2}{r}\ln 2} = -A_0r(4) = -4A_0r$ g s^{-1}

can be simplified