Attempted on 6/6/23, time taken : 30 mins 5secs.

Score : 92/100 . Take note of $(-x)$ within functions when you do chain rule

① Given $y=\frac{1}{3}x^2$, $\frac{dy}{dx}=\frac{2}{3}x$.

When $x=1$, $y=\frac{1}{3}$ and $\frac{dy}{dx}=\frac{2}{3}$.

∴ tangent line at $x=1$ has eq^n : $y-\frac{1}{3}=\frac{2}{3}(x-1)$ 10

② (a) $\frac{d}{dx}\left(\frac{x}{\sqrt{1-x}}\right) = \frac{(1-x)^{1/2} - -\frac{1}{2}(1-x)^{-1/2}(x)}{1-x}$

$= \frac{1-x+(\frac{1}{2}x)}{(1-x)\sqrt{1-x}}$ 0

$= \frac{1-\cancel{\frac{3}{2}x}\,\frac{1}{2}x}{(1-x)\sqrt{1-x}}$

(b) $\frac{d}{dx}\left[\frac{\cos(2x)}{x}\right] = \frac{(-\sin 2x)(2)(x)-\cos 2x}{x^2}$

$= -\frac{2x\sin 2x+\cos 2x}{x^2}$ 8

(c) $e^{2f(x)}=g(x)$

$\frac{d}{dx}\left[e^{2f(x)}=g(x)\right]$

$2f'(x)e^{2f(x)}=g'(x)$. 5

$2f'(x)g(x)=g'(x)$

$\Rightarrow f'(x)=\frac{g'(x)}{2g(x)}$

(d) $\frac{d}{dx}[\ln(\sin x)] = \frac{\cos x}{\sin x} = \cot x$ 5

③ Given $y^4+xy=4$,

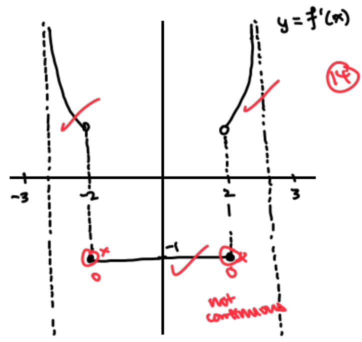
$\frac{d}{dx}(y^4+xy=4)$

$4y^3\frac{dy}{dx}+y+x\frac{dy}{dx}=0$

$\frac{dy}{dx}(4y^3+x)=-y \Rightarrow \frac{dy}{dx}=-\frac{y}{4y^3+x}$

Thus at $(x,y)=(3,1)$, $\frac{dy}{dx}=-\frac{1}{4+3}=-\frac{1}{7}$

④



⑤ $f(x)=\begin{cases} ax+b, & x<1 \\ x^4+x+1, & x\geq 1\end{cases}$

diff ⇒ cont. Since $f(1)=3$, $a+b=3$.

$f'(x)=\begin{cases} a, & x<1 \\ 4x^3+1, & x\geq 1\end{cases}$ 15

diff ⇒ $f'(x)$ is continuous ⇒ $a=4+1=5$ ⇒ $b=-2$

⑥ (a) $\lim_{x\to 0}\frac{(1+2x)^{10}-1}{x} = 2\cdot\lim_{x\to 0}\frac{(1+2x)^{10}-1^{10}}{2x}$

$=2\cdot\lim_{2x\to 0}\frac{(1+2x)^{10}-1^{10}}{2x}$ [∵ $x\to 0 \Rightarrow 2x\to 0$]

$=2\cdot\frac{d}{dx}x^{10}\Big|_{x=1}$

$=2\cdot(10x^9)\Big|_{x=1}=20$ 5

(b) $\lim_{x\to 0}\frac{\sqrt{\cos x}-1}{x} = \lim_{x\to 0}\frac{\sqrt{\cos(0+x)}-\sqrt{\cos(0)}}{x}$

$=\frac{d}{dx}\sqrt{\cos x}\Big|_{x=0}$

$=-\frac{\sin x}{2\sqrt{\cos x}}\Big|_{x=0}$

$=0$ 5

⑦ $\frac{d}{dx}a^x = \lim_{\Delta x\to 0}\frac{a^{x+\Delta x}-a^x}{\Delta x}$

$=\lim_{\Delta x\to 0}\frac{a^x(a^{\Delta x}-1)}{\Delta x}$

$=a^x\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}$

$=m(a)\,a^x$, where $m(a)=\lim_{\Delta x\to 0}\frac{a^{\Delta x}-1}{\Delta x}$ 10