① (a) $\frac{d}{dx} \frac{\sqrt{x}}{1+2x} \Big|_{x=1} = \frac{\frac{1}{2}(x^{-1/2})(1+2x) - 2(x^{1/2})}{(1+2x)^2}$

$= \frac{1}{2(1+2x)\sqrt{x}} - \frac{2\sqrt{x}}{(1+2x)^2}$ //

Value at $x=1$?

$\frac{1}{6} - \frac{2}{9} = -\frac{1}{18}.$

(b) $\frac{d}{du}(u \ln 2u) = \ln(2u) + u\left(\frac{2}{2u}\right)$

$= \ln(2u) + 1$ //

② (a) $\frac{d}{dt}\sqrt{1-k\cos^2 t} = \frac{1}{2}(1-k\cos^2 t)^{-1/2} \cdot (-k)(2\cos t)(-\sin t)$

$= \frac{k \sin 2t}{2\sqrt{1-k\cos^2 t}}$ //

(b) When $k=1$, $\frac{d}{dt}\sqrt{1-k\cos^2 t} = \frac{d}{dt}\sqrt{1-\cos^2 t}$

$= \frac{d}{dt}\sqrt{\sin^2 t}$

$= \frac{d}{dt}\sin t = \cos t.$

See that $\frac{\sin 2t}{2\sqrt{1-\cos^2 t}} = \frac{2\sin t\cos t}{2\sin t} = \cos t$ //

③ $\frac{d}{dx}\left(\frac{1}{x^2}\right) = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)^2} - \frac{1}{x^2}}{\Delta x}$

$= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left[\frac{x^2-(x+\Delta x)^2}{[x(x+\Delta x)]^2}\right]$

$= \lim_{\Delta x\to 0} \frac{1}{\Delta x}\left[\frac{-2x\Delta x - (\Delta x)^2}{[x(x+\Delta x)]^2}\right]$

$= \lim_{\Delta x\to 0}\left(\frac{-2x-\Delta x}{[x(x+\Delta x)]^2}\right)$

$= -\frac{2x}{(x^2)^2} = -\frac{2}{x^3}$ //

④ $y = \sin^{-1}x \iff x = \sin y$ ∧ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\frac{d}{dx}(x = \sin y)$

$1 = (\cos y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$ $\Rightarrow \cos y \ge 0.$

$= \frac{1}{\sqrt{1-\sin^2 y}}$

$= \frac{1}{\sqrt{1-x^2}}$ //

⑤ $f(x) = \begin{cases} ax+b, & x>1 \\ x^2-3x+2, & x\le 1 \end{cases}$

"Diff $\Rightarrow$ cont.". See $f(1) = 1-3+2=0 \Rightarrow a+b=0 \Rightarrow a=-b.$

Then $f'(x) = \begin{cases} a, & x>1 \\ 2x-3, & x\le 1. \end{cases}$

$f'(1) = -1 \Rightarrow a = -1$, for $f'(x)$ to be continuous $\Rightarrow b=1$ //

⑥ (a) $\lim_{u\to 0}\frac{\tan 2u}{u} = \lim_{u\to 0}\frac{\sin 2u}{u}\cdot\frac{1}{\cos 2u}$

$= \lim_{u\to 0}\frac{\sin u}{u}\cdot\frac{2\cos u}{\cos 2u}$

$= \left(\lim_{u\to 0}\frac{\sin u}{u}\right)\left(\lim_{u\to 0}\frac{2\cos u}{\cos 2u}\right)$

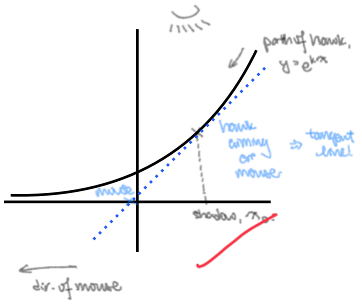
$= (1)(2) = 2$ //

(b) $\lim_{h\to 0}\frac{e^h-1}{h} = \lim_{h\to 0}\frac{e^{0+h}-e^0}{h}$

$= \frac{d}{dx}e^x\Big|_{x=0}$

$= 1$ //

⑦



We aim to find the x-intercept of the tangent line of $y=e^{kx}$ at $x=x_0$, $k>0$.

First, see that $\frac{dy}{dx} = ke^{kx}$.

The tangent line at (x_0, e^{kx_0}) would have the eq^n:

$y - e^{kx_0} = ke^{kx_0}(x-x_0).$

Set $y=0$: $x = x_0 - \frac{e^{kx_0}}{ke^{kx_0}}$

$= x_0 - \frac{1}{k}$

This means the mouse is at $(x_0 - \frac{1}{k}, 0)$ //