Tractor Trailer Control

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Dynamics, Systems, and Control Q Exam
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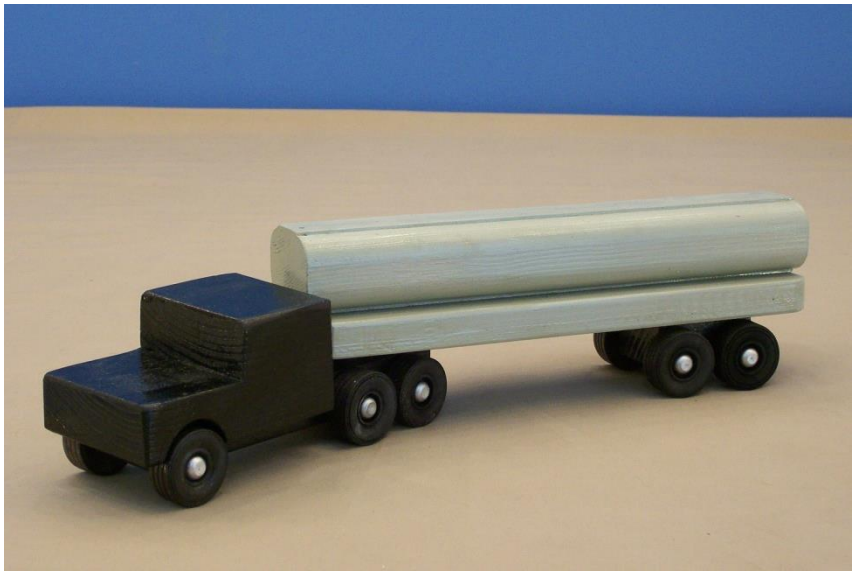
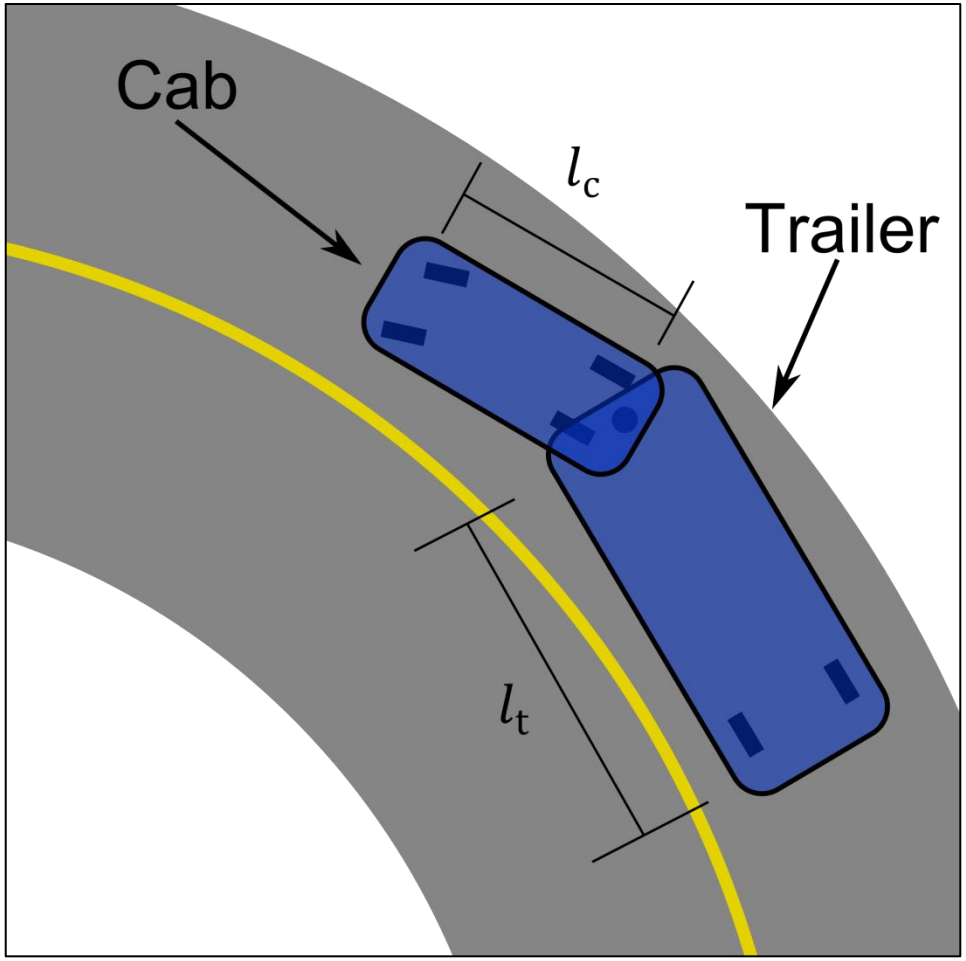
Overview

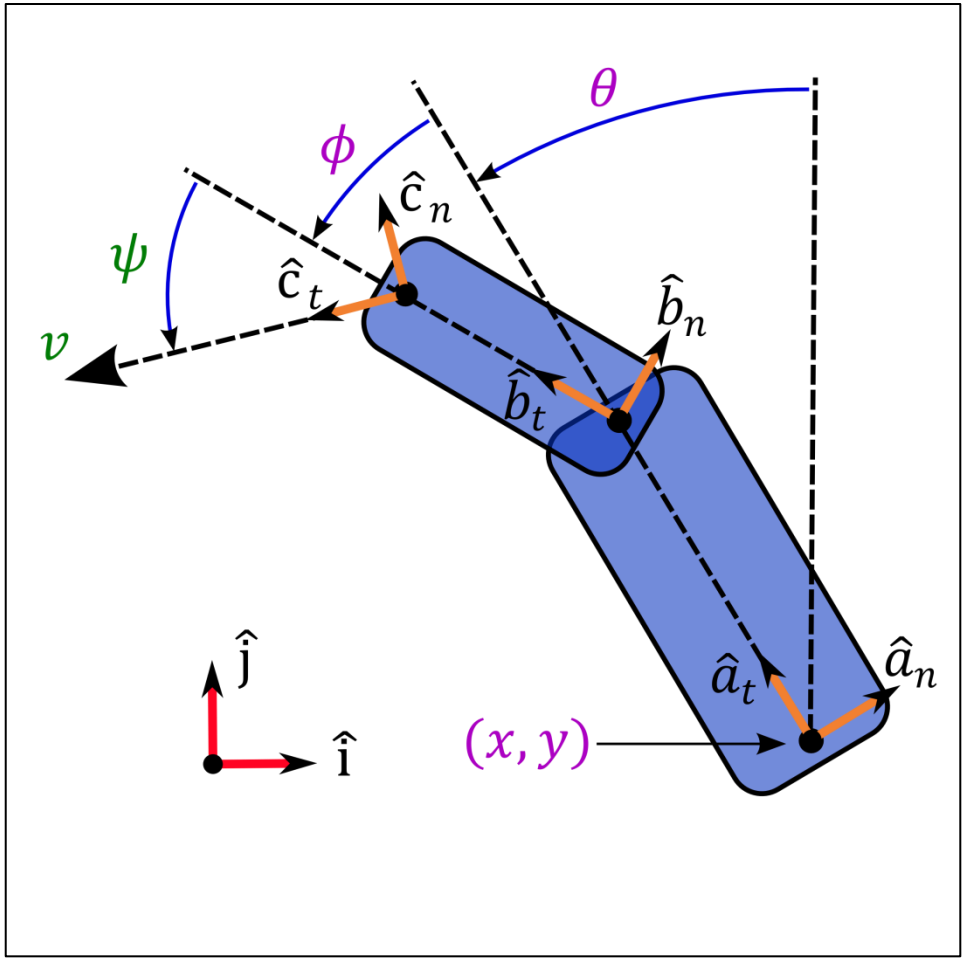
- Problem Statement
- Modeling & Dynamics
- Sensing & Estimation
- Control
- Results



Model - Overview



Model - Details



Dynamics – Constraint Eqns.

- Rear Trailer Wheels (1 Eqn.)

$$\frac{d}{dt}(x\hat{\imath} + y\hat{\jmath}) \cdot \hat{a}_n = 0$$

- Rear Cab Wheels (1 Eqn.)

$$\frac{d}{dt}(x\hat{\imath} + y\hat{\jmath} + l_t\hat{a}_t) \cdot \hat{b}_n = 0$$

- Front Cab Wheel Velocity (2 Eqns.)

$$\frac{d}{dt}\left(x\hat{\imath} + y\hat{\jmath} + l_t\hat{a}_t + l_c\hat{b}_t\right) = v\hat{c}_t$$

Dynamics - Solution

$$\dot{x} = -vSin(\theta)Cos(\phi)Cos(\psi)$$

$$\dot{y} = vCos(\theta)Cos(\phi)Cos(\psi)$$

$$\dot{\theta} = \left(\frac{v}{l_t}\right)Sin(\phi)Cos(\psi)$$

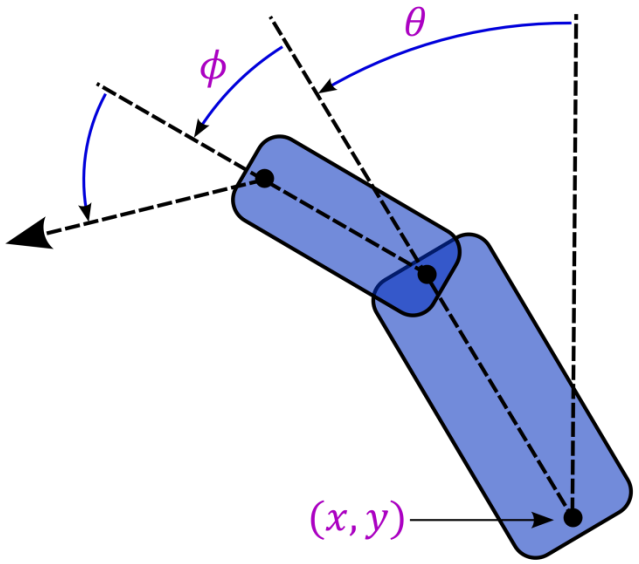
$$\dot{\phi} = \left(\frac{v}{l_c}\right)Sin(\psi) - \left(\frac{v}{l_t}\right)Sin(\phi)Cos(\psi)$$

Estimation

- Measurement :

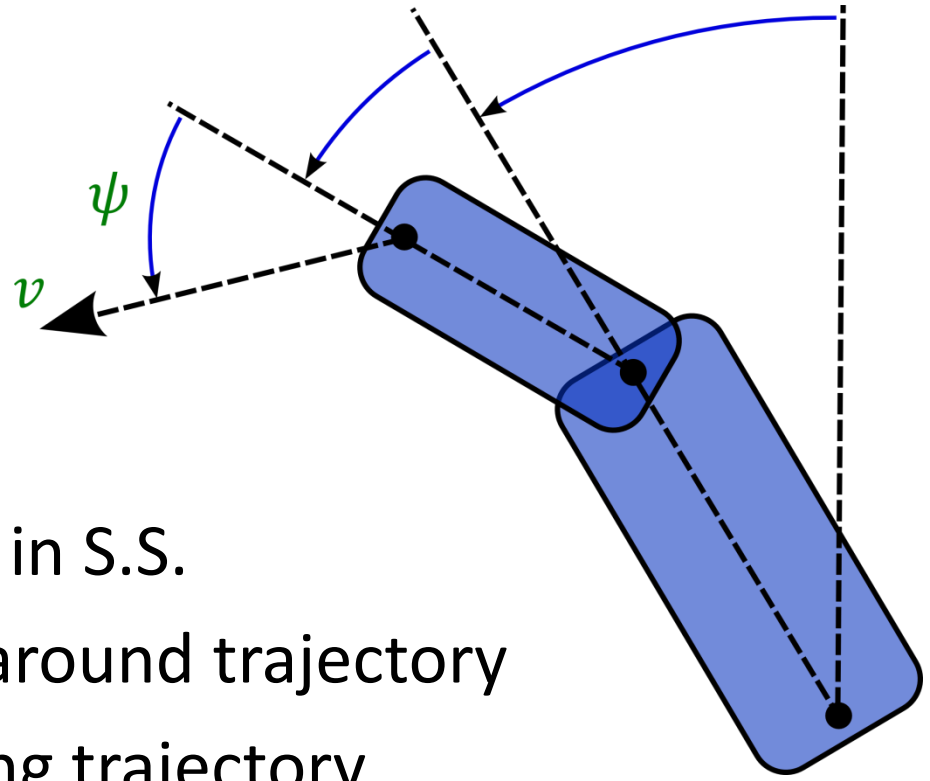
$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \theta \\ \phi \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

- Sensors:
  - GPS (x, y)
  - Encoder (ϕ)
- Extended Kalman Filter:
  - Estimate orientation (θ)
  - Filter out sensor noise

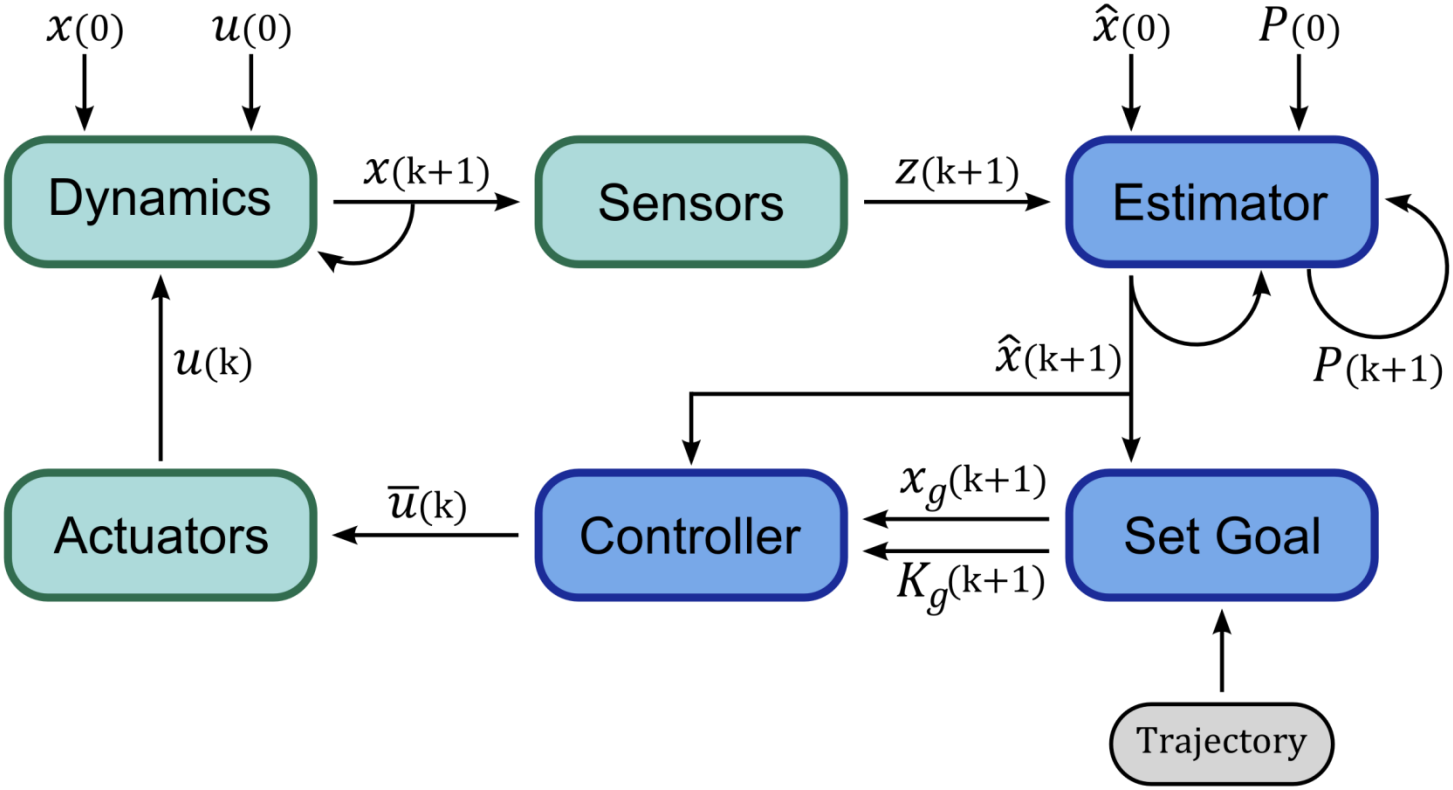


Control

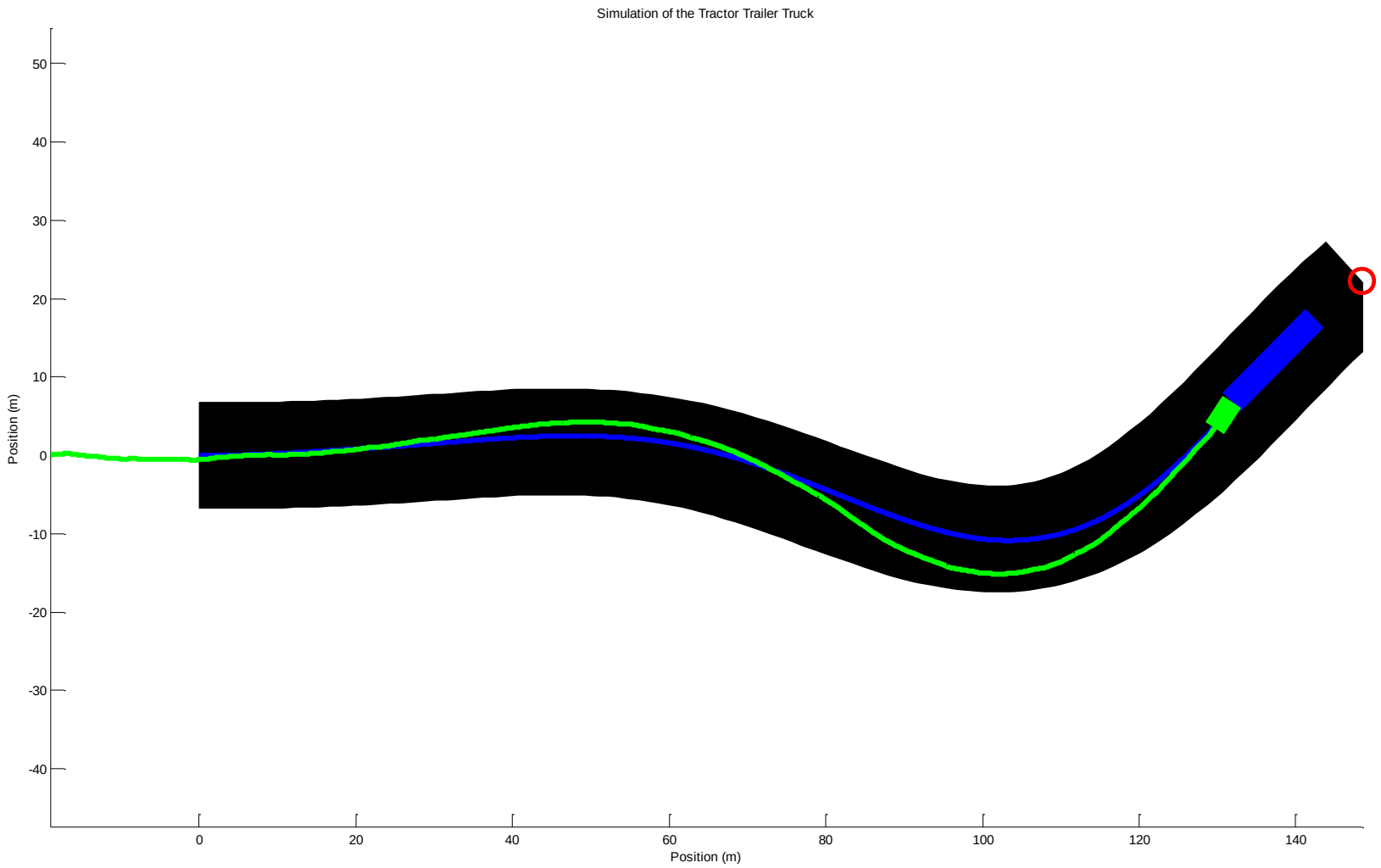
- Actuators:
  - Steering
  - Speed
- LQR Controller
  - Road = Trajectory in S.S.
  - Linearize system around trajectory
  - Set LQR gains along trajectory
  - Select target point using state estimate



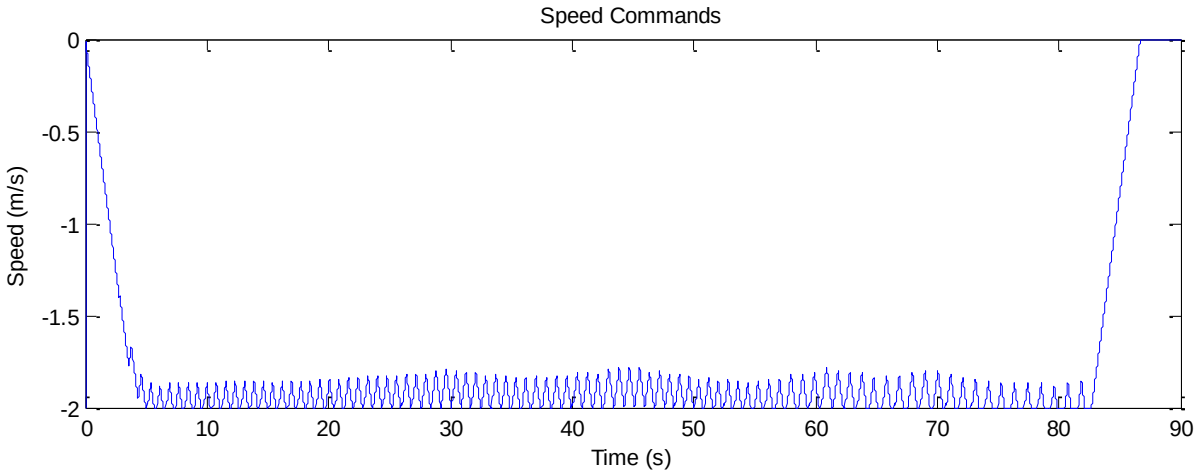
Simulator Block Diagram

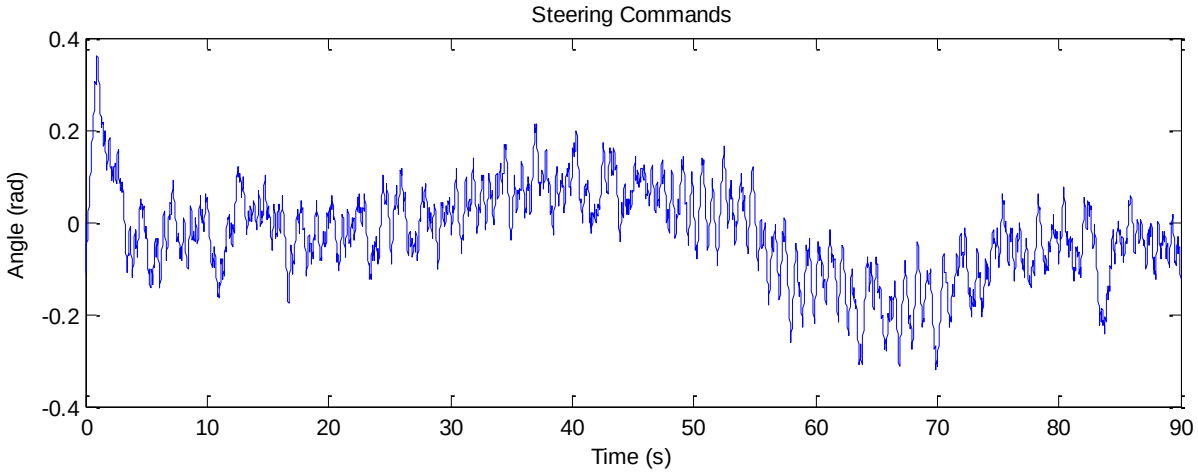


Results

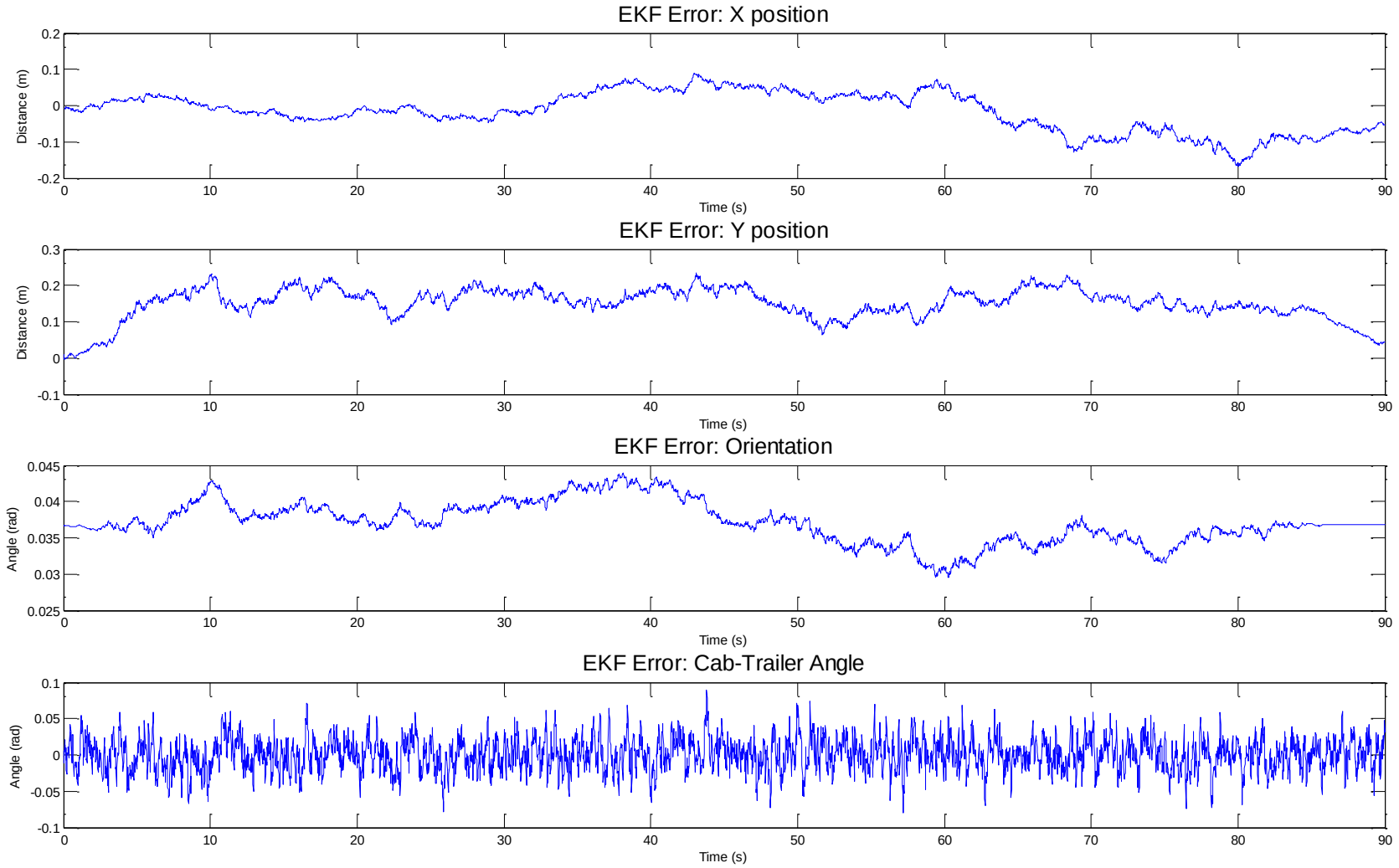

Simulation of the Tractor Trailer Truck
Position (m)
50
40
30
20
10
0
-10
-20
-30
-40
0
20
40
60
80
100
120
140
Position (m)

Results – Control Signal

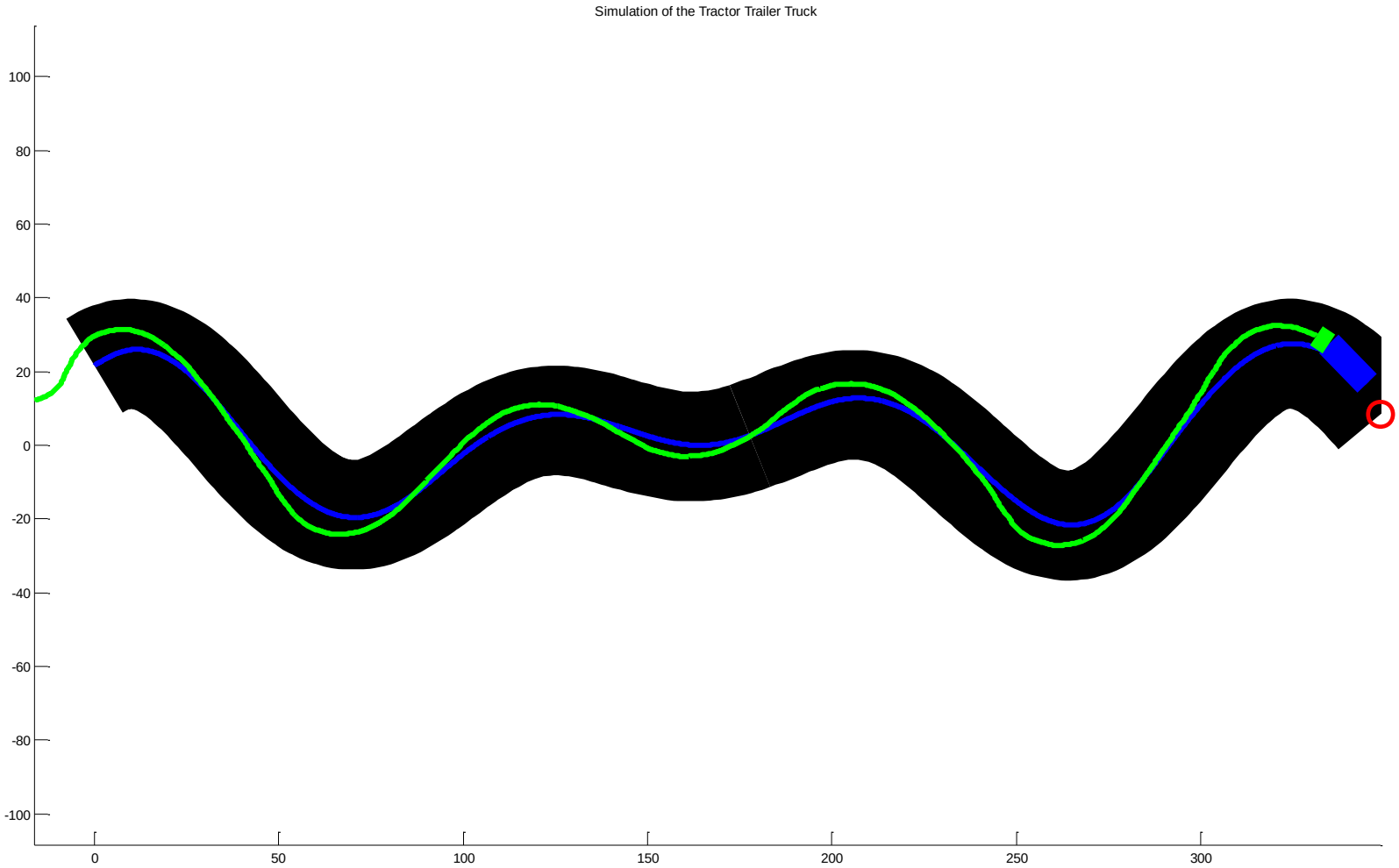




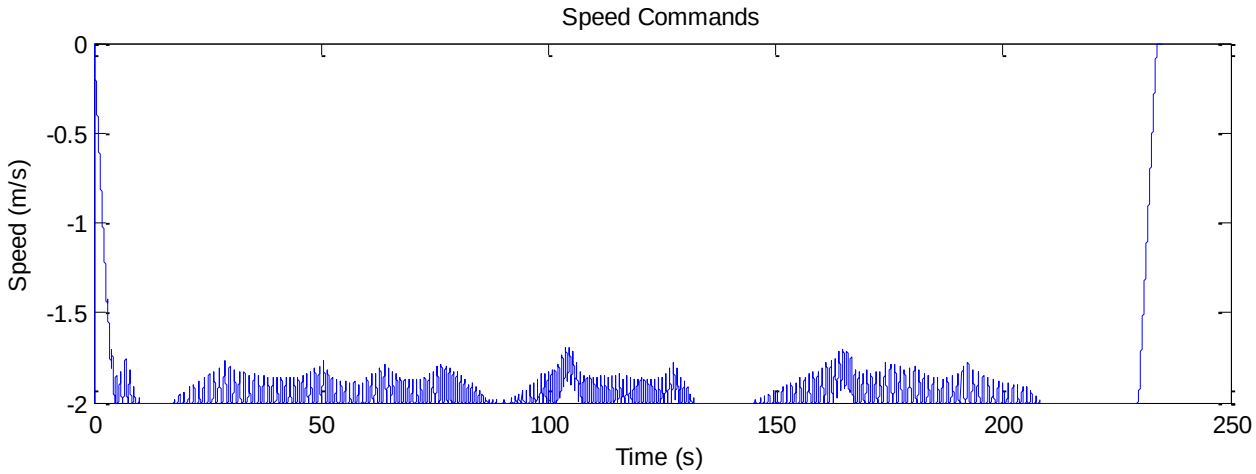
Results – EKF Error

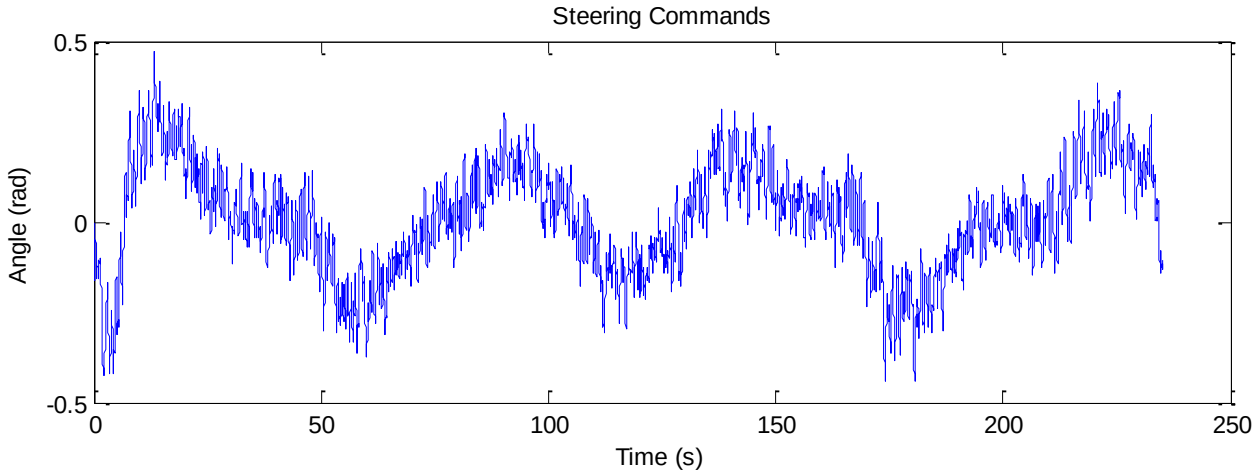


Long Road - Trace

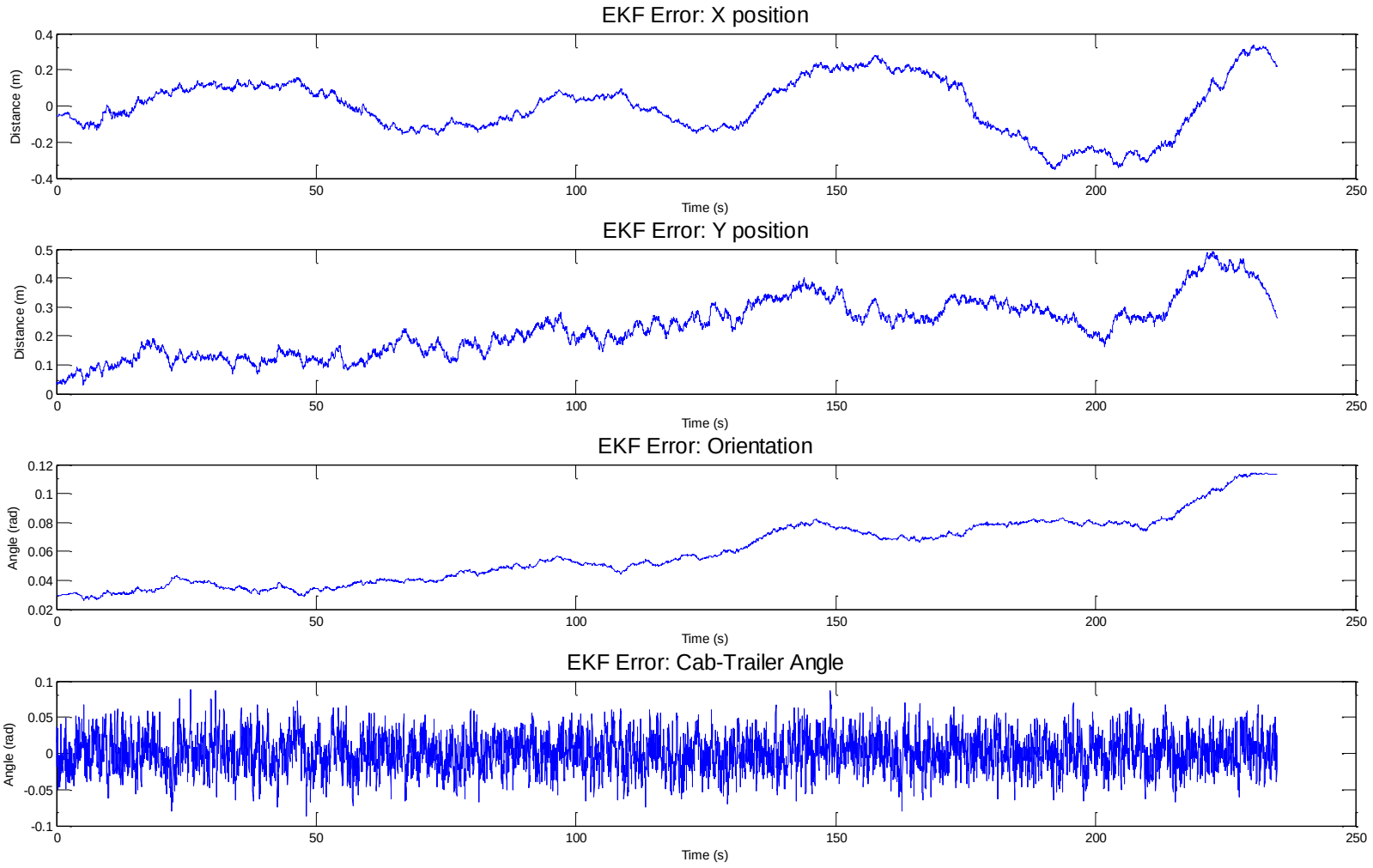


Long Road – Control Signal

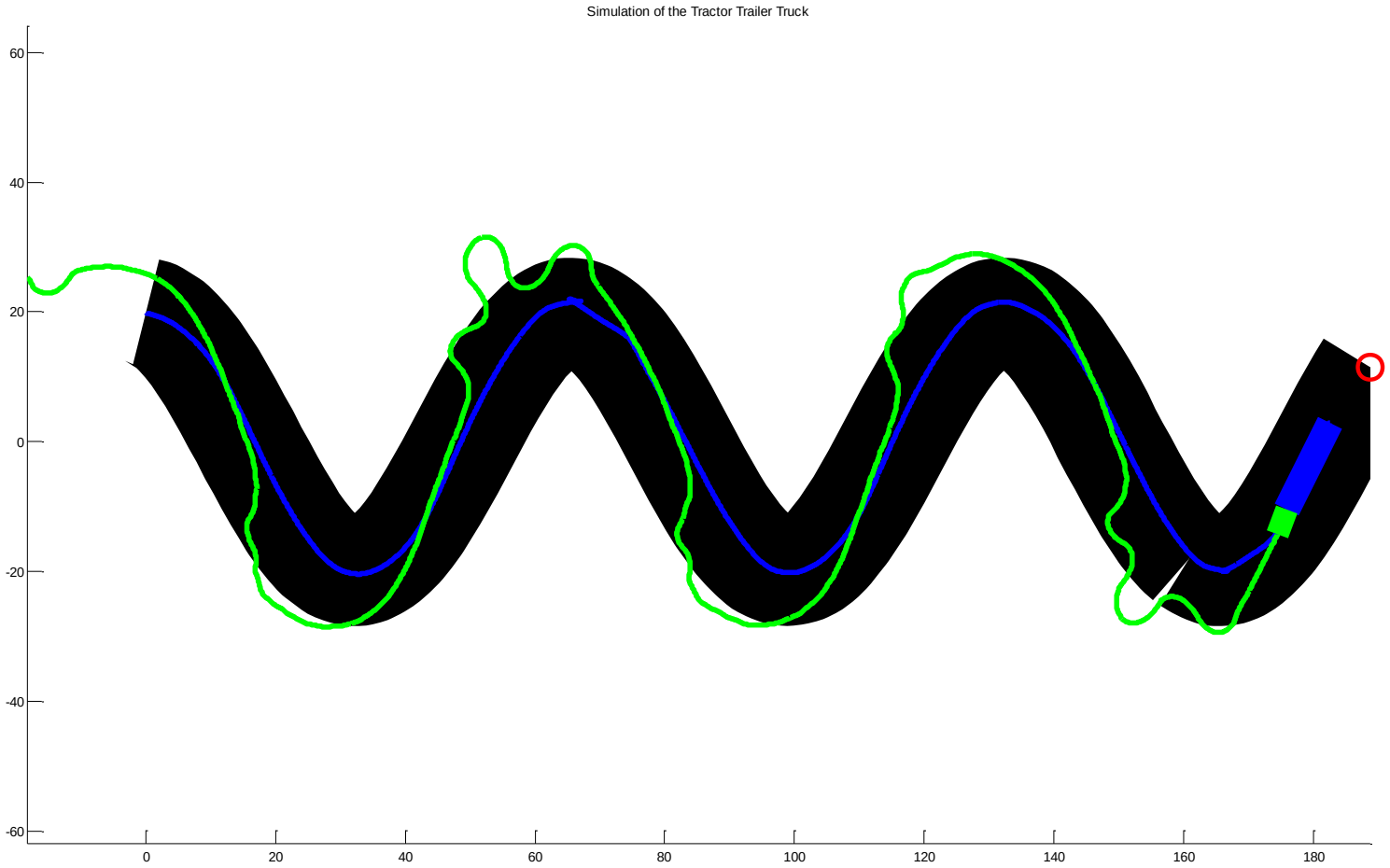




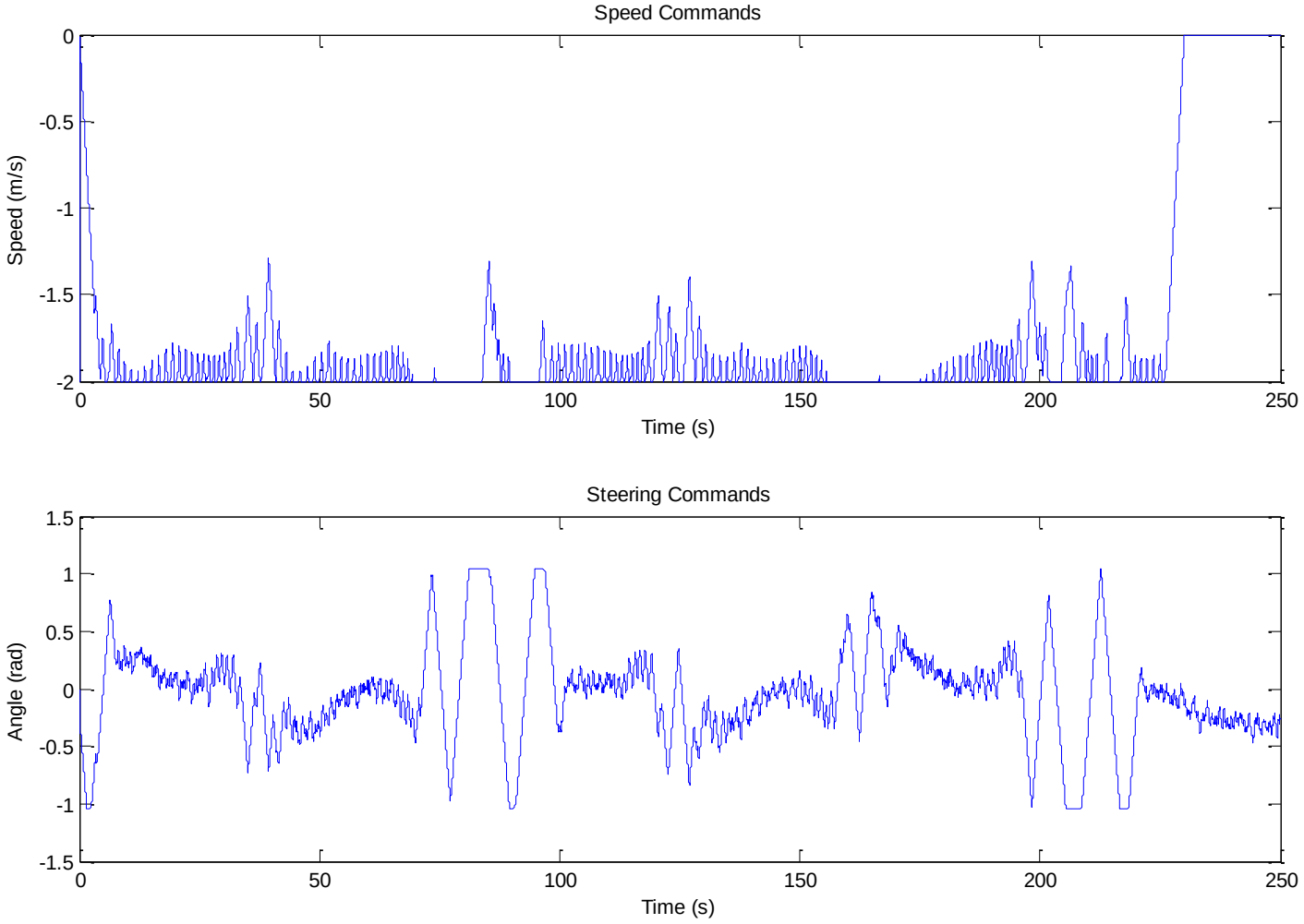
Long Road – EKF Error



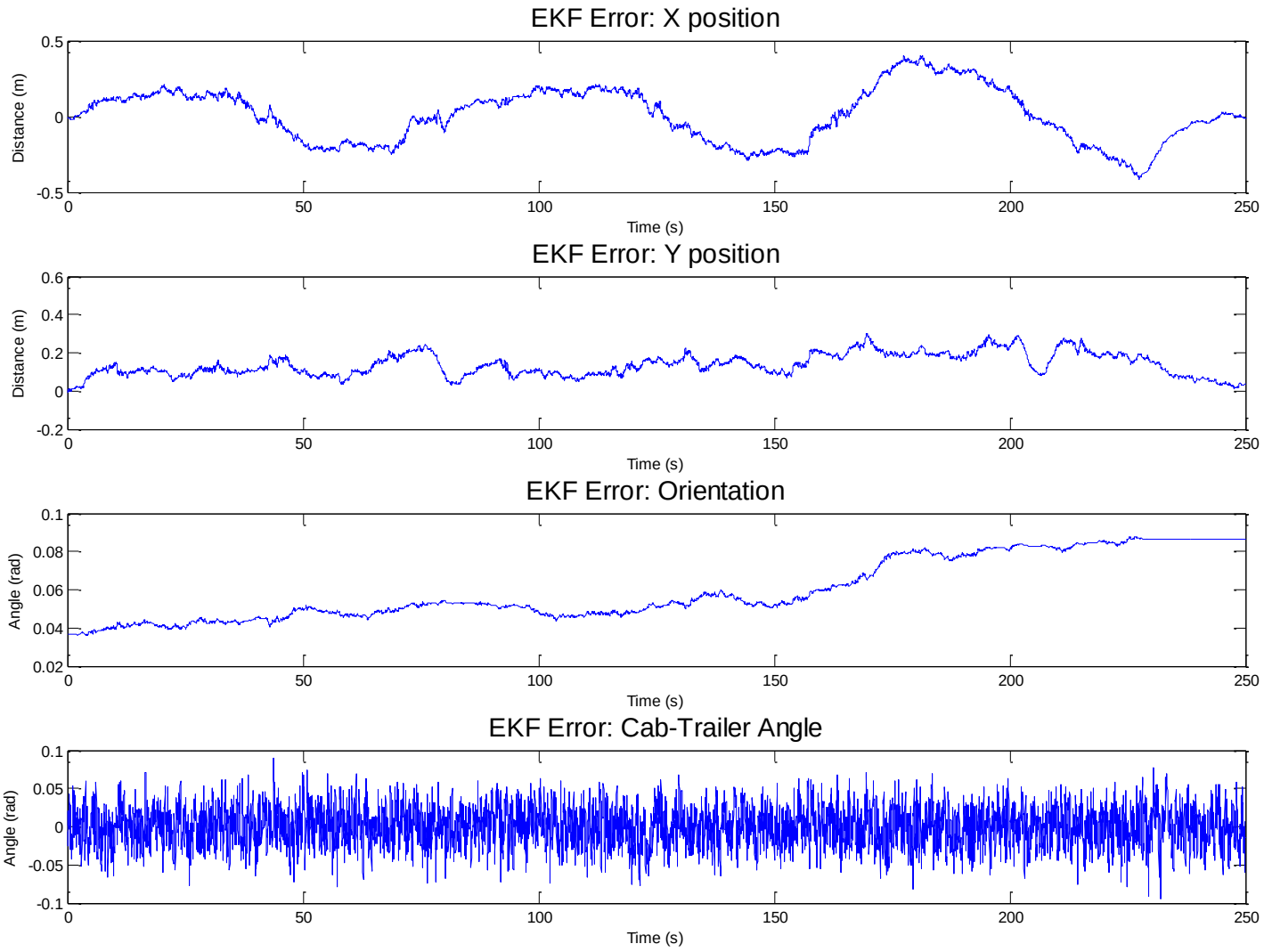
Too Curvy - Trace



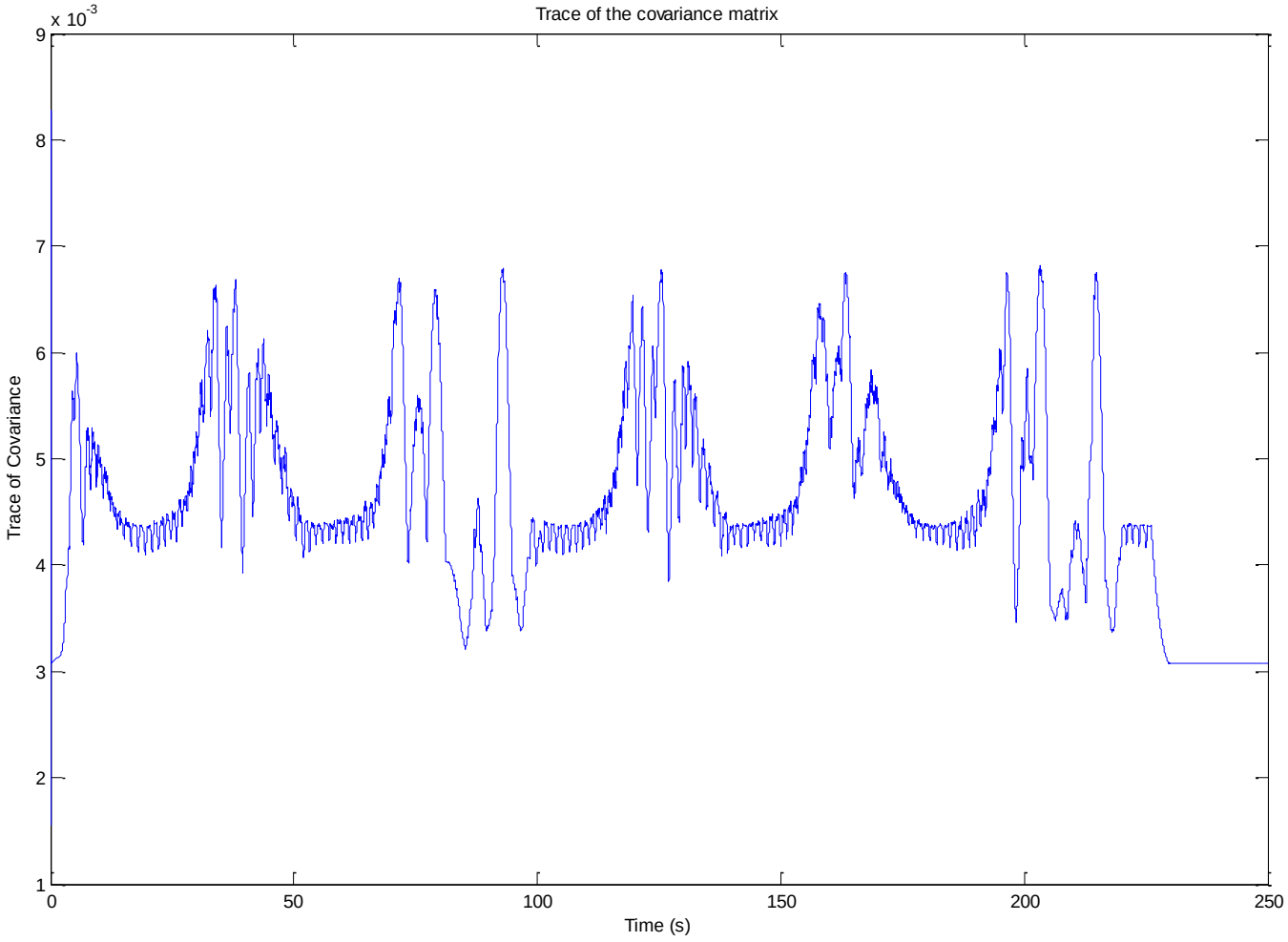
Too Curvy – Control Signal



Too Curvy – EKF Error



Too Curvy – EKF Covariance



Parameters

Simulator:
- -State initialized on first point in trajectory
- -Simulator Frequency: 50 Hertz
- -Integration Method: Euler
- -Control interpolation: none (zero order hold)

Control:
- -LQR Gains based on relative deviation from target state:
  - -Position (x and y) = 1.5 meters
  - -Cab-Trailer Angle (phi) = IGNORE
  - -Orientation (theta) = 30 degrees
  - -Input costs based on saturation limits
- -Input Saturation
  - -Steering Angle = [-60,60] degrees
  - -Speed = [-2,2] meters per second
  - -Steering Rate = [-30,30] degrees per second
  - -Acceleration = [-0.5,0.5] meters per second squared
- -Target Selection: Closest point that is at least:
  - 1 meter from current position estimate
  - [-45,45] degrees from heading estimate
- -Control Frequency: 50 Hertz

Estimator:
- -Sensor Noise
  - -GPS Position standard deviation (x and y) is 0.5 meters
  - -Encoder angle standard deviation (phi) is 4 degrees
- -Process Noise
  - -Position Rate standard deviation is 0.03 meters per second
  - -Orientation Rate standard deviation is 1 degree per second
  - -Cab-Trailer Angle Rate is 2 degrees per second
- -Initial State Estimate Error standard deviation
  - -Position (x and y) = 0.1 meters
  - -Cab-Trailer Angle (phi) = 2 degrees
  - -Orientation (theta) = 4 degrees
- -Initial State Error Covariance is derived from state variance above
- -Estimator Frequency = 50 Hertz