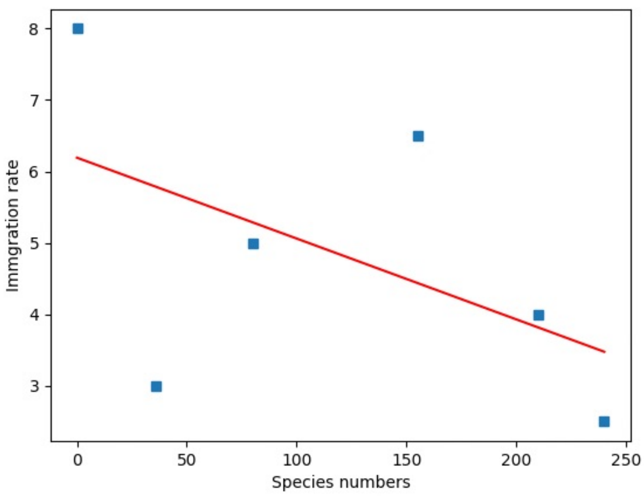
[I]

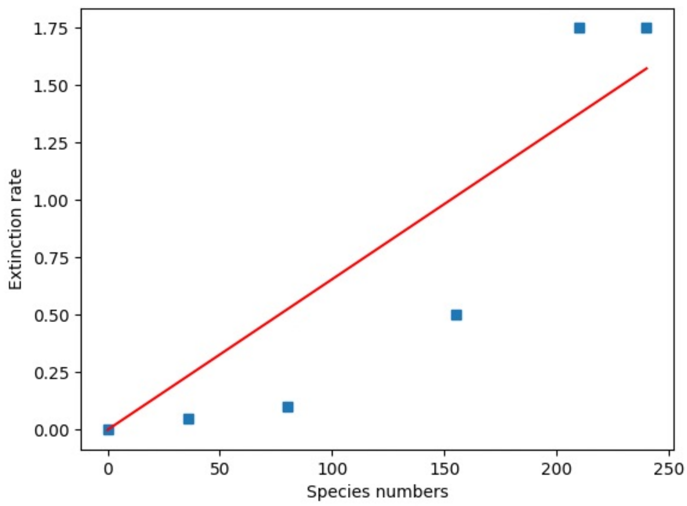
(a)

from the python code

Immigration rate equation = $-0.01132R + 6.193$

Extinction rate equation = $0.00656R$





(b)

$$R_{t+1} = R_t + (\text{Immigration}) - (\text{Extinction})$$

$$R_{t+1} = R_t + (-0.01132R_t + 6.193) - (0.00656R_t)$$

(c)

$$R_{t+1} = R_t + (-0.01132R_t + 6.193) - (0.00656R_t)$$

$$-6.193 = -0.01132R_t - 0.00656R_t$$

$R_t = 346.36$, the equilibrium number $= 346$

(d)

$$0 = I_x - (I_x/P)\hat{R} - (E_x/P)\hat{R}$$

$I_x = 6.19$ $\quad \frac{I_x}{P} = 0.01132$ $\quad P = 546.8$

$P \approx 547$

(e) To treat the island size changed.

I introduced a Costant C

and rewrite the equation

$$R_{t+1} = R_t + (-0.01132R_t + 6.193) - (0.00656R_t) + C \cdot R_t$$

for $R_{t+1} = R_t$, $R_t = 250$

$\downarrow$

$0 = (-0.01132 \times 250 + 6.193) - (0.00656 \times 250) + 250 \cdot C$

$3.363 - 1.64 = -250 \cdot C$

$C = -0.006892$

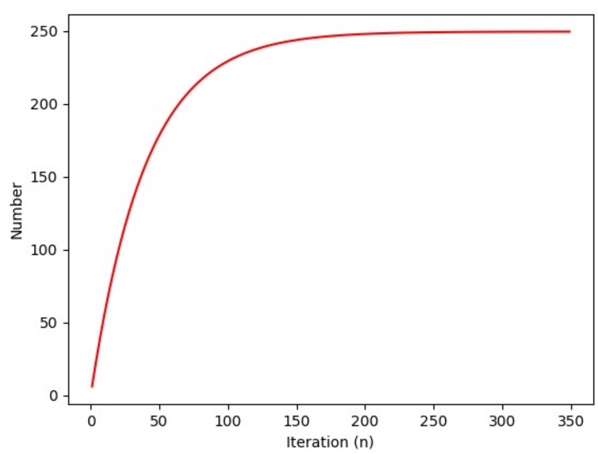
$eq = R_{t+1} = R_t + (-0.01132 R_t + 6.193) - (0.00656 R_t) - 0.006892 R_t$

by the python progam

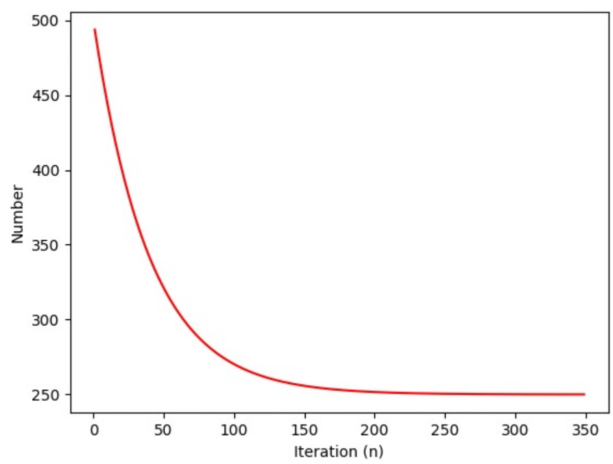
It uses 349 steps from 0 to 250

and also 349 steps from 500 to 250.

$0 \to 250$



$500 \to 250$



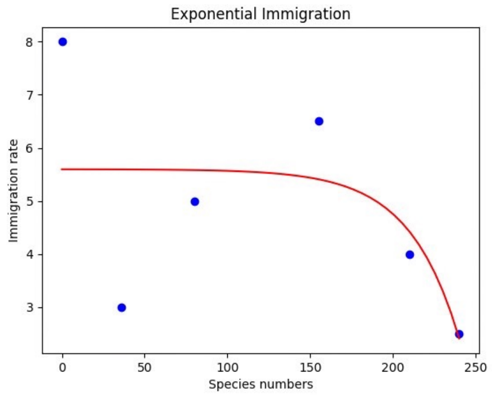
[II]

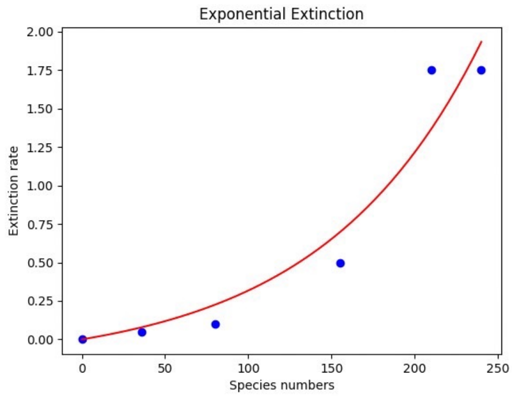
(a)

exponential

$$\text{Immigration} = 0.00109 \times (-e^{R\cdot 0.033258}) + 5.5962$$

$$\text{Extinction} = (-0.1733 \times (-e^{R \times 0.01041}) + 1)$$

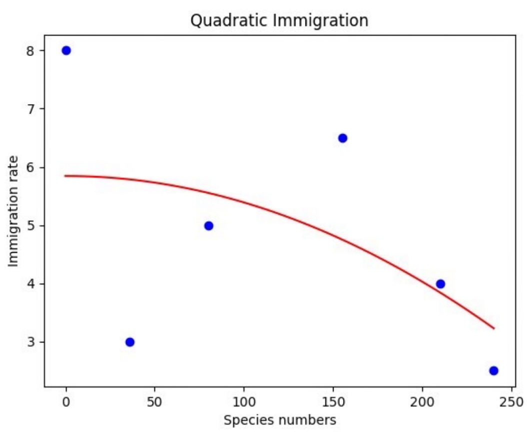


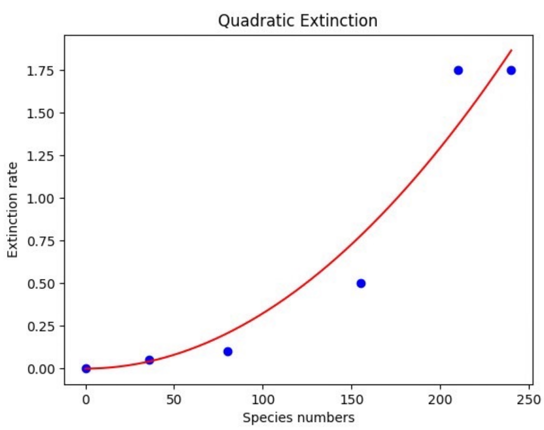


quadratic

$$\text{Immigration} = -4.5417 \times 10^{-5} \cdot R^2 + 1.1774 \times 10^{-11} \cdot R + 5.843$$

$$\text{Extinction} = 3.2417 \times 10^{-5} \cdot R^2 + 5.9761 \times 10^{-12}$$





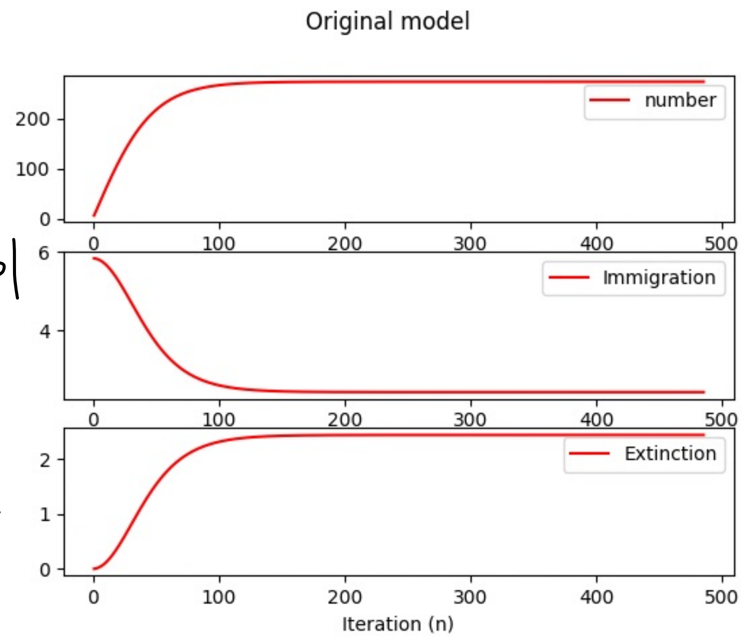
Contrast

equilibrium

$= |R_{t+1} - R_t| < 0.001$

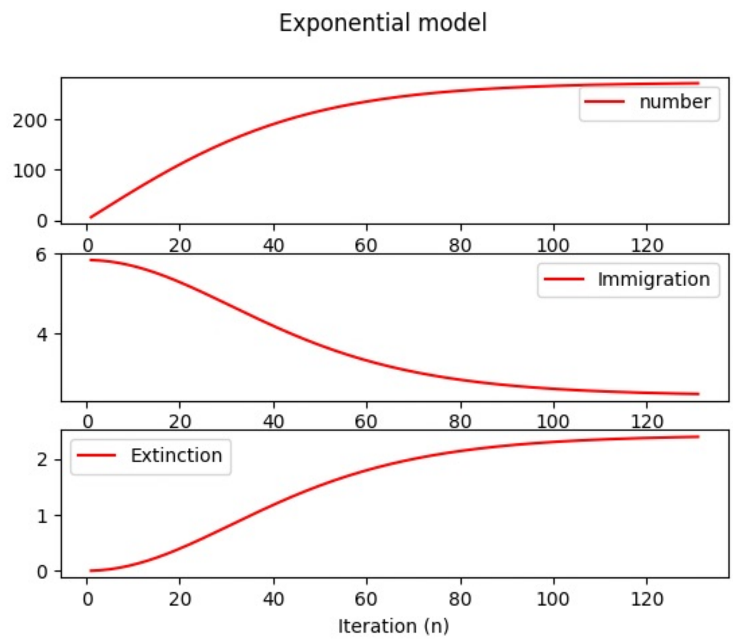
$R = 345$

Steps $= 485$



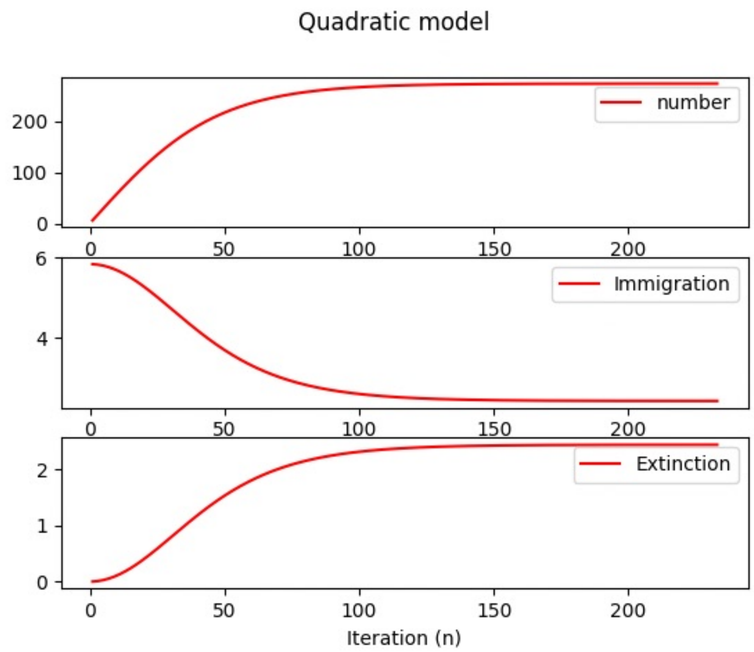
$R = 234$

Steps $= 131$



$R = 273$

Steps $= 233$



(b)

When equilibrium

$$R_{\cancel{t}+1} = R_{\cancel{t}} + (\text{Immigration}) - (\text{Extinction}),\ R_{t+1} = R_t$$

$I = E$ from the Problem 2-b

$$\frac{I_x}{E_x} = R^2 e^{\alpha R} \qquad I_x e^{-\alpha R} = E_x R^2$$

$$R_{t+1} = R_t + (I_x e^{-\alpha R_t}) - (E_x R_t^2) \ \#$$

(c)

若原始模型假設為線性的狀態，其移入率與移出率為線性變化，而實際看起來移入率會隨著目前的物種的飽和程度呈現趨緩，而當島上越多物種時，會有物種間會因為資源而產生敬爭，所以滅絕率與會有上升的趨勢，而這兩種狀態非單純線性模型可解釋，應採用曲線的模型更為適合。