A Synchronization Scheme for Position Control of Multiple Rope-Climbing Robots

Abstract— **The dynamic model of rope-climbing robot consisting both the robot dynamics and the actuator dynamics is a high-order system with couplings between each other, which opens up challenges in the development of robot control schemes. This paper presents an adaptive control scheme for the synchronization task among multiple rope-climbing robots. The stability of closed-loop system is rigorously proved with Lyapunov methods, with the consideration of both the robot dynamics and the actuator dynamics. Experimental results are presented to validate the proposed controller.**

Index Terms— **climbing robots, motion control.**

I. Introduction

The overall rope-climbing robotic system consisting both the robot dynamics and the actuator dynamics, is a high-order system with couplings between each other. Hence, controllers for rope-climbing robots should be able to stabilize both subsystems, and neglecting the coupling dynamics is exposed to stability issues [**?**].

Few results have been reported in the literature for the position control of rope-climbing robot. Existing results are limited to a single robot, and not applicable to the coordination between multiple robots. Some applications require the synchronization control of multiple rope-climbing robots.

In this paper, an adaptive controller is proposed to address the synchronization problem of multiple rope-climbing robots. The stability of closed-loop system is rigorously proved with Lyapunov methods, by taking into account the dynamics of both the robot subsystem and the actuator subsystem. The uncertain dynamic parameters are updated with online adaptation laws. Experimental results are presented to validate the proposed controller.

II. Background

A. Mechanical Structure

– briefly introduce the structure –

B. Dynamic Model

The overall dynamic model of a rope-climbing robot consists of two subsystems: the robot and the actuator. In the following analysis, the rotary elements of the rope-climbing robot are converted to equivalent translational elements, without loss of generality. Considering n rope-climbing robots, the overall dynamic model can be described as [**?**]:

$$\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{D}\dot{\boldsymbol{q}}+\boldsymbol{G}=\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q}), \tag{1}$$

$$\boldsymbol{B}\ddot{\boldsymbol{\theta}}+\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})=\boldsymbol{u}, \tag{2}$$

where $\boldsymbol{q}=[q_1,\cdots,q_n]^T\in\Re^n$ is the vector of robot displacement, q_i $(i=1,\cdots,n)$ is the displacement of the i^{th} robot, $\boldsymbol{\theta}\in\Re^n$ is the vector of motor displacement, θ_i is the displacement of the i^{th} motor, $\boldsymbol{M}=\mathrm{diag}(M_1,\cdots,M_n)\in\Re^{n\times n}$ is the diagonal mass matrix of robot where M_i are positive constants, $\boldsymbol{D}$=$\mathrm{diag}(D_1,\cdots,D_n)\in\Re^{n\times n}$ denotes the diagonal friction matrix where D_i are also positive constants, and $\boldsymbol{G}\in\Re^n$ is the gravity matrix, $\boldsymbol{K}=\mathrm{diag}(K_1,\cdots,K_n)\in\Re^{n\times n}$ denotes the stiffness matrix where K_i is the stiffness of the i^{th} rope, $\boldsymbol{B}$=$\mathrm{diag}(B_1,\cdots,B_n)\in\Re^{n\times n}$ is the diagonal mass matrix of actuator, and $\boldsymbol{u}\in\Re^n$ denotes the input forces exerted on actuators.

Equation (1) describes the well-known rigid-body dynamics, and equation (2) describes the actuator dynamics. The two subsystems are linked to each other with the stretching force of the rope, i.e. $\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})$. In this paper, it is assumed the dynamic models for the robot and actuator are well defined, that is, $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{G}$, $\boldsymbol{B}$, and $\boldsymbol{K}$ are exactly known.

III. Synchronization Scheme For Multiple Rope-Climbing Robots

The mechanical dynamics of robot is slower (1) while the dynamics of robot is faster (2), and hence the overall rope-climbing robotic system exhibits the features of two time scales [1]. In this paper, the singular perturbation theory [2] is introduced to design controllers for two subsystems separately, by treating the faster dynamics as perturbation of the slower dynamics.

A. Controller for Actuator Subsystem

The overall controller for multiple rope-climbing robots can be proposed as:

$$\boldsymbol{u}=\boldsymbol{u}_s+\boldsymbol{u}_f, \tag{3}$$

where $\boldsymbol{u}_s,\boldsymbol{u}_f\in\Re^n$ represent the slow controller for the robot subsystem and the fast controller for the actuator subsystem respectively.

First, the fast controller is specified as:

$$\boldsymbol{u}_f=-\boldsymbol{K}_v(\dot{\boldsymbol{\theta}}-\dot{\boldsymbol{q}}), \tag{4}$$

where $\boldsymbol{K}_v\in\Re^{n\times n}$ is a diagonal and positive-definite matrix. Substituting (3) into (2) yields:

$$\boldsymbol{B}\ddot{\boldsymbol{\theta}}+\boldsymbol{K}_v(\dot{\boldsymbol{\theta}}-\dot{\boldsymbol{q}})+\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})=\boldsymbol{u}_s, \tag{5}$$

which leads to:

$$\boldsymbol{B}(\ddot{\boldsymbol{\theta}}-\ddot{\boldsymbol{q}})+\boldsymbol{K}_v(\dot{\boldsymbol{\theta}}-\dot{\boldsymbol{q}})+\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})=\boldsymbol{u}_s-\boldsymbol{B}\ddot{\boldsymbol{q}}. \tag{6}$$

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Next, a variable $\boldsymbol{y}$ is introduced as the stretching force of the rope as: $\boldsymbol{y}=\boldsymbol{K}(\boldsymbol{\theta}-\boldsymbol{q})$, such that equations (1) and (6) can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{D}\dot{\boldsymbol{q}}+\boldsymbol{G}=\boldsymbol{y}, \tag{7}$$

$$\boldsymbol{B}\ddot{\boldsymbol{y}}+\boldsymbol{K}_v\dot{\boldsymbol{y}}+\boldsymbol{K}\boldsymbol{y}=\boldsymbol{K}(\boldsymbol{u}_s-\boldsymbol{B}\ddot{\boldsymbol{q}}). \tag{8}$$

Expressing $\boldsymbol{K}$ and $\boldsymbol{K}_v$ in terms of ϵ as [1]:

$$\boldsymbol{K}=\boldsymbol{K}_1/\epsilon^2, \tag{9}$$

$$\boldsymbol{K}_v=\boldsymbol{K}_2/\epsilon, \tag{10}$$

where ϵ is a small parameter. Then, equation (8) becomes:

$$\epsilon^2\boldsymbol{B}\ddot{\boldsymbol{y}}+\epsilon\boldsymbol{K}_2\dot{\boldsymbol{y}}+\boldsymbol{K}_1\boldsymbol{y}=\boldsymbol{K}_1(\boldsymbol{u}_s-\boldsymbol{B}\ddot{\boldsymbol{q}}). \tag{11}$$

The system described by (7) and (11) is singularly perturbed. The variables $\boldsymbol{y}$ and $\dot{\boldsymbol{y}}$ can be viewed as fast time-scale variables, while the state variables $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ denote slow time-scale variables.

With $\epsilon=0$, equation (11) becomes:

$$\bar{\boldsymbol{y}}=\boldsymbol{u}_s-\boldsymbol{B}\ddot{\boldsymbol{q}}, \tag{12}$$

in which, the overbar variable denotes the value of variable at ϵ=0. Then, substituting equation (12) into equation (7) at ϵ=0 yields the *quasi-steady-state* model:

$$(\boldsymbol{M}+\boldsymbol{B})\ddot{\boldsymbol{q}}+\boldsymbol{D}\dot{\boldsymbol{q}}+\boldsymbol{G}=\boldsymbol{u}_s. \tag{13}$$

By assuming that $\bar{\boldsymbol{y}}$ is constant on a fast time-scale $\tau=\frac{t}{\epsilon}$, a new variable is introduced as: $\boldsymbol{\eta}=\boldsymbol{y}-\bar{\boldsymbol{y}}$. Substituting the fast time-scale τ and $\boldsymbol{y}=\boldsymbol{\eta}+\bar{\boldsymbol{y}}$ into (11), with $\frac{\mathrm{d}\bar{\boldsymbol{y}}}{\mathrm{d}\tau}=\frac{\mathrm{d}^2\bar{\boldsymbol{y}}}{\mathrm{d}\tau^2}=\boldsymbol{0}$ and $\epsilon=0$, we have:

$$\boldsymbol{B}(\tfrac{\mathrm{d}^2\boldsymbol{\eta}}{\mathrm{d}\tau^2})+\boldsymbol{K}_2(\tfrac{\mathrm{d}\boldsymbol{\eta}}{\mathrm{d}\tau})+\boldsymbol{K}_1\boldsymbol{\eta}=\boldsymbol{0}, \tag{14}$$

which is referred to as *boundary-layer system*.

According to Tikhonov's theorem [2], the overall system is stable if both the *boundary-layer system* (14) and the *quasi-steady-state system* (13) are exponentially stable. The exponential stability of the *boundary-layer system* can be guaranteed by specifying $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ from (14). Then, the control objective is to design a slow controller $\boldsymbol{u}_s$ which guarantees the exponential stability of the *quasi-steady-state system* (13).

B. Controller for Robot Subsystem

In this section, a slow controller $\boldsymbol{u}_s$ is specified such that the *quasi-steady-state system* is exponentially stable. The slow controller also guarantees the convergence of the position of each robot to the desired position, and achieves the synchronization between each other.

First, a sliding vector is introduced as:

$$\boldsymbol{s}=\dot{\boldsymbol{q}}-\dot{\boldsymbol{q}}_r=\dot{\boldsymbol{q}}-\dot{\boldsymbol{q}}_d+\alpha_q\Delta\boldsymbol{q}+\alpha_s\Delta\boldsymbol{\psi}, \tag{15}$$

where

$$\dot{\boldsymbol{q}}_r=\dot{\boldsymbol{q}}_d-\alpha_q\Delta\boldsymbol{q}-\alpha_s\Delta\boldsymbol{\psi}, \tag{16}$$

is a reference vector, α_q and α_s are positive constants, $\Delta\boldsymbol{q}=\boldsymbol{q}-\boldsymbol{q}_d$ where $\boldsymbol{q}_d=[q_{d1},\cdots,q_{dn}]\in\Re^n$ denotes the vector of desired positions, q_{di} is the desired position for the ith robot, and $\Delta\boldsymbol{\psi}$ denotes the synchronization vector which is specified as: $\Delta\boldsymbol{\psi}=[\psi_1-\psi_n,\psi_2-\psi_1,\cdots,\psi_n-\psi_{n-1}]^T$ where $\psi_1=\Delta q_1-\Delta q_2,\cdots,\psi_n=\Delta q_n-\Delta q_1$. The definition of synchronization errors was introduced in [3], We can now propose the slow controller $\boldsymbol{u}_s$ as:

$$\boldsymbol{u}_s=-\boldsymbol{K}_s\boldsymbol{s}-k_q\Delta\boldsymbol{q}+(\boldsymbol{M}+\boldsymbol{B})\ddot{\boldsymbol{q}}_r+\boldsymbol{D}\dot{\boldsymbol{q}}_r+\boldsymbol{G}, \tag{17}$$

where $\boldsymbol{K}_s\in\Re^{n\times n}$ is positive-definite, and k_q is a positive constant. In (17), the first two terms include the position control, the synchronization control, and the velocity control, and the last three terms represent the dynamic compensation.

The key novelty of the proposed control method is to integrate both the position error $\Delta\boldsymbol{q}$ and the synchronization error $\Delta\boldsymbol{\varepsilon}$ into a single controller, such that multiple robots can move to the desired positions while always maintaining the same height when they are climbing along the ropes. Both the position control task and the synchronization control task are important for the coordination of multiple rope-climbing robots. For example, when a manipulator is installed at the center of multiple rope-climbing robots and the vertical position of the manipulator is varied by controlling the multiple robots as shown in Fig. **??**, the proposed control method allows the manipulator to be transported to a desired vertical position to perform manipulation tasks (e.g. wiping the window), and also guarantees that the base of the manipulator is always parallel to the ground throughout the transportation.

Note that the overall dynamic model described by (1) and (2) is a high-order system, while the complexity of the proposed controller in (3) is the same as that of the dynamics of the robot subsystem. That is, the use of singular perturbation approach helps to decrease the complexity and also the computational const of the controller.

By using the sliding vector (15), the dynamics of the *quasi-steady-state system* can be rewritten as:

$$(\boldsymbol{M}+\boldsymbol{B})\dot{\boldsymbol{s}}+\boldsymbol{D}\boldsymbol{s}+(\boldsymbol{M}+\boldsymbol{B})\ddot{\boldsymbol{q}}_r+\boldsymbol{D}\dot{\boldsymbol{q}}_r+\boldsymbol{G}=\boldsymbol{u}_s. \tag{18}$$

Substituting (17) into (18) yields the following dynamic equation:

$$(\boldsymbol{M}+\boldsymbol{B})\dot{\boldsymbol{s}}+(\boldsymbol{D}+\boldsymbol{K}_s)\boldsymbol{s}+k_q\Delta\boldsymbol{q}=\boldsymbol{0}. \tag{19}$$

Now we are in the position to state the following theorem. **Theorem**: *The proposed controller described by (3), (4), and (17) for multiple rope-climbing robots guarantees the stability of the closed-loop system and the convergence of tracking errors and synchronization errors to zero, if the control parameters are chosen such that*

$$\lambda_{\min}[\boldsymbol{K}_s]\geq\lambda_{\max}[\tfrac{1}{2}(\boldsymbol{M}+\boldsymbol{B})-\boldsymbol{D}], \tag{20}$$

$$\alpha_q\geq\tfrac{1}{2}, \tag{21}$$

where $\lambda_{\min}[\cdot]$ and $\lambda_{\max}[\cdot]$ represent the minimum and the maximum eigenvalues respectively.

Proof: A Lyapunov-like candidate is proposed as:

$$V=\tfrac{1}{2}\boldsymbol{s}^T(\boldsymbol{M}+\boldsymbol{B})\boldsymbol{s}+\tfrac{k_q}{2}\Delta\boldsymbol{q}^T\Delta\boldsymbol{q}. \tag{22}$$

Differentiating (22) with respect to time yields:

$$\dot{V} = \boldsymbol{s}^T(\boldsymbol{M}+\boldsymbol{B})\dot{\boldsymbol{s}}+k_q\Delta\boldsymbol{q}^T\Delta\dot{\boldsymbol{q}}. \tag{23}$$

Substituting (15) and (19) into (23), we have:

$$\begin{aligned}\dot{V} &= -\boldsymbol{s}^T(\boldsymbol{D}+\boldsymbol{K}_s)\boldsymbol{s} - \boldsymbol{s}^T\boldsymbol{K}_q\Delta\boldsymbol{q}+k_q\Delta\boldsymbol{q}^T\Delta\dot{\boldsymbol{q}} \\ &= -\boldsymbol{s}^T(\boldsymbol{D}+\boldsymbol{K}_s)\boldsymbol{s}-\alpha_q k_q\Delta\boldsymbol{q}^T\Delta\boldsymbol{q}-\alpha_s k_q\Delta\boldsymbol{\psi}^T\Delta\boldsymbol{q}.\end{aligned} \tag{24}$$

Note that

$$\begin{aligned}\Delta\boldsymbol{\psi}^T\Delta\boldsymbol{q} &= \sum_{i=1}^{n}[\Delta q_i(\psi_i - \psi_{i-1})] \\ &= \Delta q_1\psi_1 - \Delta q_1\psi_n + \Delta q_2\psi_2 - \Delta q_2\psi_1 + \cdots \\ &\quad +\Delta q_n\psi_n - \Delta q_n\psi_{n-1} \\ &= (\Delta q_1 - \Delta q_2)\psi_1 + (\Delta q_2 - \Delta q_3)\psi_2 + \cdots \\ &\quad +(\Delta q_n - \Delta q_1)\psi_n \\ &= \psi_1^2 + \psi_2^2 + \cdots + \psi_n^2 = \boldsymbol{\psi}^T\boldsymbol{\psi},\end{aligned} \tag{25}$$

where $\boldsymbol{\psi} = [\psi_1, \cdots, \psi_n]^T \in \Re^n$.

Substituting (25) into (24) yields:

$$\dot{V} = -\boldsymbol{s}^T(\boldsymbol{D}+\boldsymbol{K}_s)\boldsymbol{s}-\alpha_q k_q\Delta\boldsymbol{q}^T\Delta\boldsymbol{q}-\alpha_s k_q\boldsymbol{\psi}^T\boldsymbol{\psi}. \tag{26}$$

If the control parameters $\boldsymbol{K}_s$ and α_q are chosen such that conditions (20) and (21) are satisfied, it is obtained that

$$\begin{aligned}\dot{V} &\leq -\boldsymbol{s}^T(\boldsymbol{D}+\boldsymbol{K}_s)\boldsymbol{s}-\alpha_q k_q\Delta\boldsymbol{q}^T\Delta\boldsymbol{q} \\ &\leq -\gamma V \leq 0,\end{aligned} \tag{27}$$

where γ is a positive constant. The inequality (27) indicates that $V \leq V(0)\mathrm{e}^{-\gamma t}$ where $V(0)$ denotes the initial value of the Lyapunov-like candidate. Therefore, the exponential stability of the *quasi-steady-state system* is guaranteed. Since the fast controller (4) ensures the exponential stability of the *quasi-steady-state system* (13), from the analysis in the previous section, we can also conclude that the overall controller (3) guarantees the stability of the rope-climbing robotic system described by (1) and (2).

In addition, the inequality (27) implies that $\boldsymbol{s}, \Delta\boldsymbol{q} \to \boldsymbol{0}$ as $t \to \infty$. Then, from (15), it is obtained that:

$$\Delta\dot{\boldsymbol{q}} + \alpha_s\Delta\boldsymbol{\psi} \to \boldsymbol{0}. \tag{28}$$

In the system of multiple rope-climbing robots, it is usually specified that $q_{d1} = q_{d2} = \cdots = q_{dn}$. Hence, the convergence of $\Delta\boldsymbol{q} \to \boldsymbol{0}$ leads to the convergence of $\Delta\boldsymbol{\psi} \to \boldsymbol{0}$. Then, from (28), it is concluded that $\Delta\dot{\boldsymbol{q}} \to \boldsymbol{0}$. That is, $\boldsymbol{q} \to \boldsymbol{q}_d$, $\dot{\boldsymbol{q}} \to \dot{\boldsymbol{q}}_d$, and $q_i \to q_j$ as $t \to \infty$. Therefore, the convergence of tracking errors and synchronization errors to zero is guaranteed. ■

IV. Experiment

An experimental setup of inspection climbing robot has been established in The Chinese University of Hong Kong.

V. Conclusion

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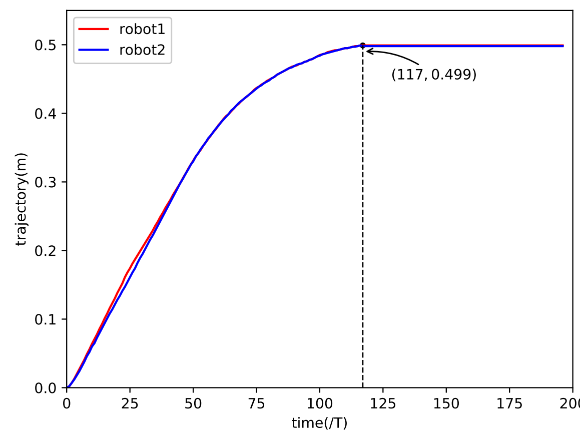

Fig. 1. setpoint control trajectory error of two robots

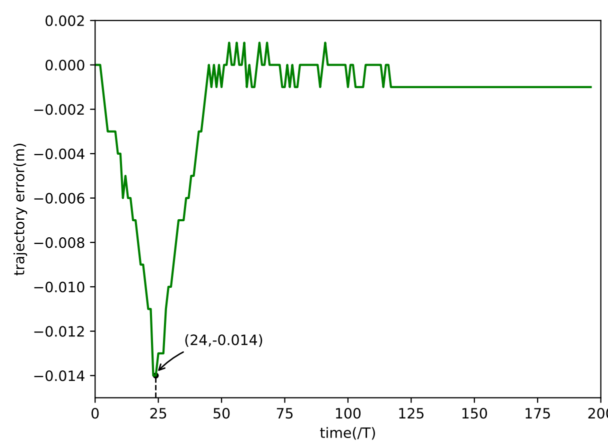

Fig. 2. setpoint control trajectory error of two robots