Parametric uncertainties for decision making on automation car

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https://github.com/slowy07/clifter_highway

Abstract

The problem of behaviour prediction for linear parameter-varying systems is considered in the interval framework. It is assumed that the system is subject to uncertain inputs and the vector of scheduling parameters is unmeasurable, but all uncertainties take values in a given admissible set. Then an interval predictor is designed and its stability is guaranteed by applying Lyapunov function with a novel structure. The conditions of stability are formulated in the form of linear matrix inequalities. Efficiency of the theoretical results is demonstrated in the application to safe motion planning for autonomous vehicles.

Introduction

There are plenty of emerging application domains nowadays, where the decision algorithms have to operate in the conditions of a severe uncertainty. Therefore, the decision procedures need more information, then the estimation, identification and prediction algorithms come to the attention. In most of these applications, even the nominal simplified models are nonlinear, and in order to solve the problem of estimation and control in nonlinear and uncertain systems, a popular approach is based on the Linear Parameter-Varying (LPV) representation of their dynamics [1], [2], [3], [4], since it allows to reduce the problem to the linear context at the price of augmented parametric variation.

In the presence of uncertainty (unknown parameters or/and external disturbances) the design of a conventional estimator or predictor, approaching the ideal value of the state, can be realized under restrictive assumptions only. However, an interval estimation/prediction remains frequently feasible: using input-output information an algorithm evaluates the set of admissible values (interval) for the state at each instant of time [5], [6]. The interval length must be minimized via a parametric tuning of the system, and it is typically proportional to the size of the model uncertainty [7].

preliminaries

We denote the real numbers by R, the integers by ***Z, R+ = {τ ∈ R : τ ≥ 0}, Z+ = Z∩R+*** and the sequence of integers 1, ..., k as 1, k. Euclidean norm for a vector x ∈ R n will be denoted as |x|, and for a measurable and locally essentially bounded input *u : R+ → R* we denote its L∞ norm by ||u||[t0,t1] = esssuper∈[t0,t1) |u(t)| . If t1 = +∞ then we will simply write ||u||.

We will denote as L∞ the set of all inputs u with the property ||u|| < ∞. The symbols In, En×m and Ep denote the identity matrix with dimension n × n, and the matrices with all elements equal 1 with dimensions n × m and p × 1, respectively.

For a matrix A ∈ R n×n the vector of its eigenvalues is denoted as *λ(A)*, *||A||max = maxi=1,n,j=1,n |Ai,j |* (the element wise maximum norm, it is not submultiplicative) and *||A||2 = q maxi=1,n λi(A>A)* (the induced L2 matrix norm), the relation *||A||max ≤ ||A||2 ≤ n||A||max* is satisfied between these norms.

A. interval arithmetic

the relations x1 ≤ x2 and A1 ≤ A2 are understood elementwise. The relation P ≺ 0 (P 0) means that a symmetric matrix P ∈ R n×n is negative (positive) definite. Given a matrix A ∈ R m×n, define A+ = max{0, A}, A− = A+ − A (similarly for vectors) and denote the matrix of absolute values of all elements by |A| = A+ + A−.

Lemma 1

Let *x ∈ R n* be a vector variable, *x ≤ x ≤ x* for some *x, x ∈ R n*.
(1) If A ∈ R m×n is a constant matrix, then *A +x − A −x ≤ Ax ≤ A +x − A −x*.
(2) *If A ∈ R m×n* is a matrix variable and *A ≤ A ≤ A* for some *A, A ∈ R m×n, then A +x + − A + x − − A −x + + A − x − ≤ Ax* **(2)** *≤ A + x + − A +x − − A − x + + A −x −*. Furthermore, if *−A = A ≤ 0 ≤ A*, then the inequality **(2)** can be simplified: ***−A(xr + + xr −x) ≤ Ax ≤ A(xr + + x −x)***. B. Nonnegative systems A matrix A ∈ R n×n is called Hurwitz if all its eigenvalues have negative real parts, it is called Metzler if all its elements outside the main diagonal are nonnegative. Any solution of the linear system *x˙(t) = Ax(t) + Bω(t), t ≥ 0*,
(3) *y(t) = Cx(t) + Dω(t), with x(t) ∈ R n, y(t) ∈ R p* and *a Metzler matrix A ∈ R n×n*, is elementwise nonnegative for all t ≥ 0 provided that *x(0) ≥ 0, ω : R+ → R q + and B ∈ R n×q +*. The output solution y(t) is nonnegative if C ∈ R p×n + and D ∈ R p×q + . Such dynamical systems are called cooperative (monotone) or nonnegative if only initial conditions in R n + are considered.
This result allows us to represent the system in its nonnegative form via a similarity transformation of coordinates.

Lemma 2

Given the matrices $A \in R^{n\times n}$, $Y \in R^{n\times n}$ and $C \in R^{p\times n}$. If there is a matrix $L \in R^{n\times p}$ such that the matrices A - LC and Y have the same eigenvalues, then there is a matrix $S \in R^{n\times n}$ such that $Y = S(A - LC)S^{-1}$ provided that the pairs $(A - LC, \chi_1)$ and (Y, χ_2) are observable for some $\chi_1 \in R^{1\times n}$, $\chi_2 \in R^{1\times n}$

Lemma 3

Let $D \in \Xi \subset R^{n\times n}$ be a matrix variable satisfying the interval constraints $\Xi = \{D \in R^{n\times n} : D_a - \Delta \leq D \leq D_a + \Delta\}$ for some DT $a = D_a \in R^{n\times n}$ and $\Delta \in R^{n\times n}_+$. If for some constant $\mu \in R_+$ and a diagonal matrix $Y \in R^{n\times n}$ the Metzler matrix $Y = \mu E_{n\times n} - Y$ has the same eigenvalues as the matrix D_a, then there is an orthogonal matrix $S \in R^{n\times n}$ such that the matrices $S^T D S$ are Metzler for all $D \in \Xi$ provided that $\mu > n||\Delta||_{max}$.

In the last lemma, the existence of similarity transformation is proven for an interval of matrices, e.g. LPV dynamics

iii. Problem statement

Consider an LPV system: $\dot{x}(t) = A(\theta(t))x(t) + Bd(t)$, $t \geq 0$, where $x(t) \in R^n$ is the state, $\theta(t) \in \Pi \subset R^r$ is the vector of scheduling parameters with a known set of admissible values Π, $\theta \in L^r_\infty$; the signal $d : R_+ \to R^m$ is the external input.

The values of the scheduling vector $\theta(t)$ are not available for measurements, and only the set of admissible values Π is known. The matrix $B \in R^{n\times m}$ is known, the matrix function $A : \Pi \to R^{n\times n}$ is locally bounded (continuous) and known. The following assumptions will be used in this work.

Assumption 1 . *In the system, $x \in L^n_\infty$. In addition, $x(0) \in [\underline{x}_0, \overline{x}_0]$ for some known $\underline{x}_0, \overline{x}_0 \in R^n$.*

Assumption 2 *There exists known signals $\underline{d}, \overline{d} \in L^n_\infty$ such that $\underline{d}(t) \leq d(t) \leq \overline{d}(t)$ for all $t \geq 0$.*

Assumption 1 means that the system generates stable trajectories with a bounded state x for the applied class of inputs d, and the initial conditions x(0) are constrained to belong to a given interval $[\underline{x}_0, \overline{x}_0]$. In Assumption 2, it is supposed that the input d(t) belongs to a known bounded interval $[\underline{d}(t), \overline{d}(t)]$ for all $t \in R_+$, which is the standard hypothesis for the interval estimation [5],

[6]. Note that since the function A and the set Π are known, and θ ∈ Π, then there exist matrices A, A ∈ R n×n, which can be easily computed, such that

$$A \le A(\theta) \le A, \ \forall \theta \in$$

A. The goal

The objective of this work is to design an interval predictor for the system (4), which takes the information on the initial conditions [x0 , x0], the admissible bounds on the values of the exogenous input [d(t), d(t)], the information about A and Π (e.g. the matrices A, A, but not the instant value of θ(t)) and generates bounded interval estimates x(t), x(t) ∈ R n such that

$$x(t) \le x(t) \le x(t), \ \forall t \ge 0$$

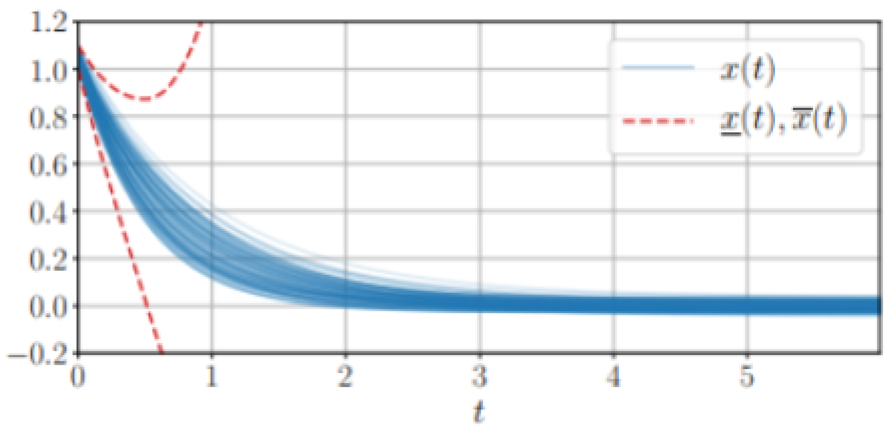
B. Motivation Example

Following the result of Lemma 1, there is a straightforward solution to the problem used in :

$$\dot{x}(t) = A^{+}x^{+}(t) - \overline{A}^{+}x^{-}(t)\ \overline{A}^{-}x^{+}(t) + A^{-}x^{-}(t) + B^{+}d(t) - B^{-}d(t),$$

$$\dot{x}(t) = A^{+}x^{+}(t) - A^{+}x^{-}(t) - A^{-}x^{+}(t) + A^{-}x^{-}(t) + B^{+}d(t) - B^{-}d(t),$$
$$x(0) = x0 , x(0) = x0,$$

where x(t) ∈ R with x(0) ∈ [x0 , x0] = [1.0, 1.1], θ(t) ∈ Π = [θ , θ] = [0.5, 1.5] and d(t) ∈ [d, d] = [-0.1, 0.1] for all t ≥ 0. Obviously, assumptions 1 and 2 are satisfied, and this uncertain dynamics produces bounded trajectories



this consider a Lyapunov function V (x) = x^2. Then the interval predictor (6) takes the form:

$\dot{x}(t) = -\theta x^{+}(t) + \theta x^{-}(t) + d,$

$\dot{x}(t) = -\theta x^{+}(t) + \theta x^{-}(t) + d,$

The results of simulation are shown in Pic. 1. As we can conclude, additional consideration and design are needed to properly solve the posed problem.

iv. interval predictor design

Various interval observers for LPV systems have been proposed, but in those works the cooperativity and stability of the estimation error dynamics are ensured by a proper selection of observer gains and/or by design of control algorithms, which can be dependent on x, x and guarantee the observer robust stability. For interval predictors there is no such a freedom, then a careful selection of hypotheses has to be made in order to provide a desired solution.