VE215 RC3

Erdao Liang, Chongye Yang

UM-SJTU JI

June 13, 2023

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Nodes, Meshes and Loops

Branch: a single element, such as a voltage source or a resistor
Node: the point of connection between two or more branches
Loop: any closed path in a circuit

- **Mesh:** a loop that does not enclose any other loops, i.e., smallest loop
- **Independent loop:** a loop that contains at least one branch which is not a part of any other independent loop

Fundamental theorem of network topology:

$$b \text{ (branches)} = l(\text{mesh}) + n \text{ (nodes)} - 1$$

Conductance

$$G = \frac{1}{R} = \frac{i}{v},\ 1S = 1\mho = 1A/V$$

where **G** is the conductance, **S** (siemens) is the SI unit of conductance and $\mho$ is the reciprocal ohm.
some useful formula:

$$i = Gv, p = vi = i^2 R = \frac{v^2}{R} = v^2 G = \frac{i^2}{G}$$

Kirchhoff's Law

Kirchhoff's Law	Expression	Based on
KCL	$\sum i_k = 0$ for a node	Conservation of charge
KVL	$\sum v_k = 0$ for a mesh	Conservation of energy

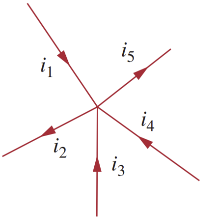


Figure: KCL

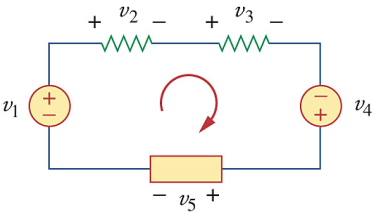


Figure: KVL

KCL

KCL: the algebraic sum of currents entering **a node (or a closed boundary)** is zero.
Steps of applying KCL:

1. Find out all branches connected to the node of interest.
2. Specify **reference** direction for current on each branch.
3. Find all $i_k(k = 1, 2, \cdots, n)$ (Ohm's law $i = \frac{v_a - v_b}{R}$ for linear resistors).
4. List the KCL equation $\sum_k i_k = 0$.

KVL

KVL: the algebraic sum of all voltages around **a closed path (or loop)** is zero.

Steps of applying KVL:

1. Select reference KVL direction (clockwise by convention).
2. Confirm/specify the +/- terminal of each branch.
3. Find $v_k(k = 1, 2, \cdots, n)$ for each branch.
4. List the KVL equation $\sum_k v_k = 0$. Mind that by passive sign convention, the sign in front of a certain term v_k is
   - "+" if the reference KVL direction enters through the positive terminal of the branch.
   - "-" if the reference KVL direction enters through the negative terminal of the branch.

Wye-Delta Transformation

- Motivation: simplify the circuits for easier calculation.
- Two forms of special circuit connections:

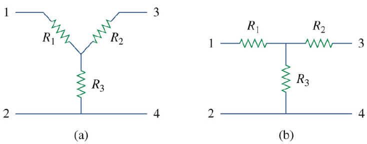


Figure: Wye (Y or T)

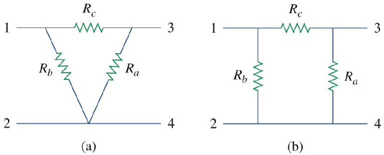


Figure: Delta (Δ or π)

- Goal: transform one type of connection into another.

Wye-Delta Transformation

$$
\begin{cases}
R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c} \\
R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c} \\
R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}
\end{cases}
$$

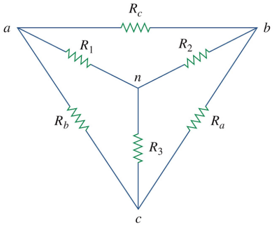


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: $\Delta - Y$

Intuition: parallel $\rightarrow$ series, resistance for each element decreases.

Wye-Delta Transformation

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

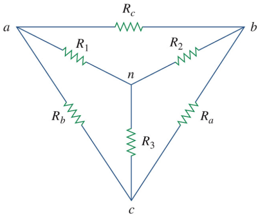


Figure 2.49 Superposition of wye and delta networks as an aid in transforming one to the other.

Figure: Y-Δ

Intuition: series $\rightarrow$ parallel, resistance for each element increases.

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Methods of Analysis

Nodal Analysis:

1. Select a reference node (ground)
2. Apply KCL
3. Solve the equations

Mesh Analysis:

1. Mark the current of all the meshes
2. Apply KVL
3. Solve the equations

Analysis by Inspection

$$\begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ G_{21} & G_{22} & \dots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \dots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

G_{kk} = Sum of the conductances connected to node k

$G_{kj} = G_{jk}$ = Negative of the sum of the conductances directly connecting nodes k and j, $k \neq j$

v_k = Unknown voltage at node k

i_k = Sum of all ~~independent~~ current sources directly connected to node k, with currents entering the node treated as positive

For Nodal Analysis

(only current source in circuit)

$$\begin{bmatrix} R_{11} & R_{12} & \dots & R_{1N} \\ R_{21} & R_{22} & \dots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \dots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

R_{kk} = Sum of the resistances in mesh k

$R_{kj} = R_{jk}$ = Negative of the sum of the resistances in common with meshes k and j, $k \neq j$

i_k = Unknown mesh current for mesh k in the clockwise direction

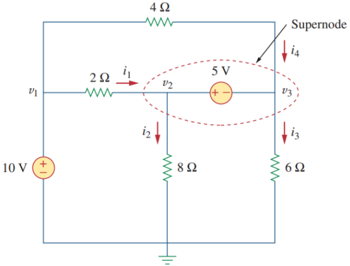
v_k = Sum taken clockwise of all ~~independent~~ voltage sources in mesh k, with voltage rise treated as positive

For Mesh Analysis

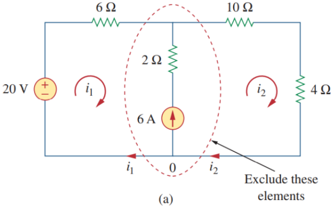
(only voltage source in circuit)

Supernode & Supermesh

- Supernode & Supermesh – simplify the equation

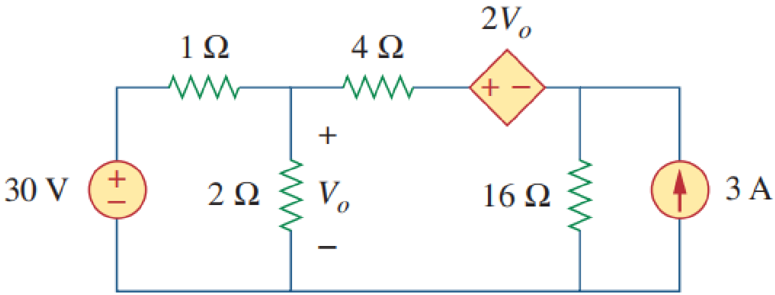


$$i_1 - i_2 - i_3 + i_4 = 0$$

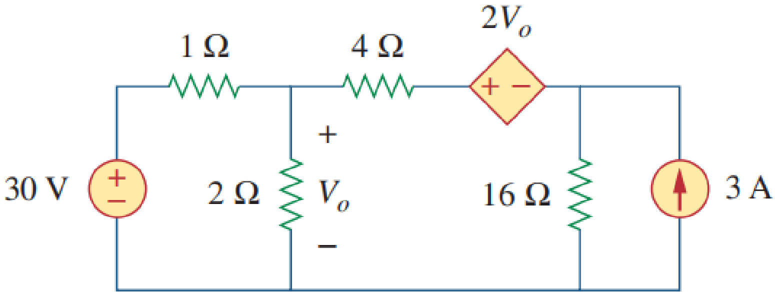


$$20 - 6i_1 - 14i_2 = 0$$

Exercise



Exercise



Answer: $\frac{648}{29}V \approx 22.34V$

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Overview-Chapter4 Circuit Theorems

- Linearity Property
- Superposition
- Source Transformation
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer

Linearity Property

homogeneous: if $x \to y$, then $kx \to ky$
additive: if $x_1 \to y_1$ and $x_2 \to y_2$, then $x_1 + x_2 \to y_1 + y_2$
linear circuit: homogeneous and additive

Exercise

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

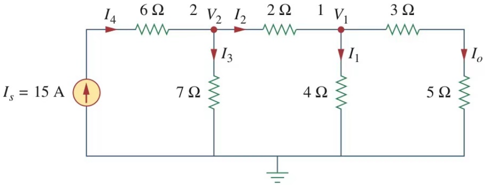
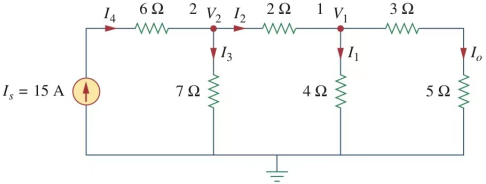


Figure 4.4

Exercise



Answer: $I_0 = 3A$

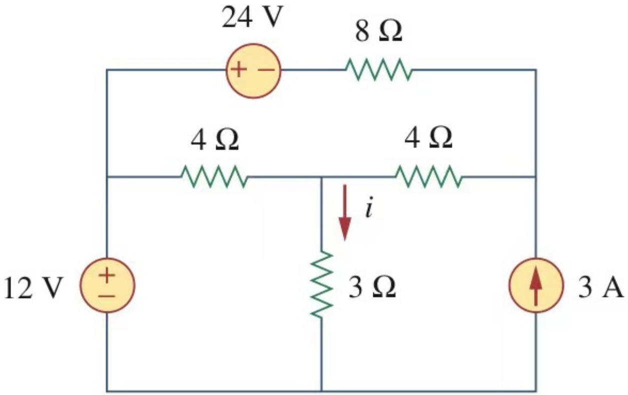
Superposition

Steps

1. Only consider one **independent** source.
   - voltage source: short circuit
   - current source: open circuit
2. Use additivity.

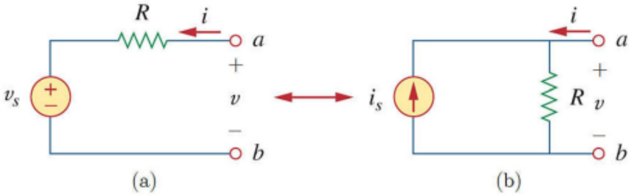
Exercise

Find i in the circuit.



Source Transformation

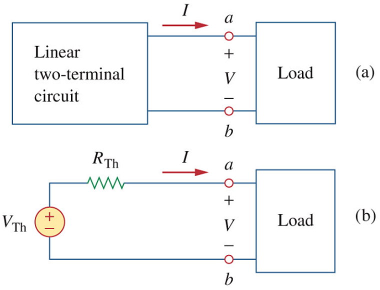
We can replace a voltage source with a resistance with a corresponding current source with the same resistance to simplify the circuit.
In the case shown below, $v_s = i_s \times R$



For dependent sources, the source transformation is also valid.

Thevenin's Theorem

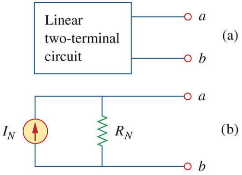
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}.



- V_{Th}: the open-circuit voltage at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

Norton's Theorem

A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_{Th} in parallel with a resistor R_{Th}.



- I_{Th}: the short-circuit current at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

Maximum Power Transfer

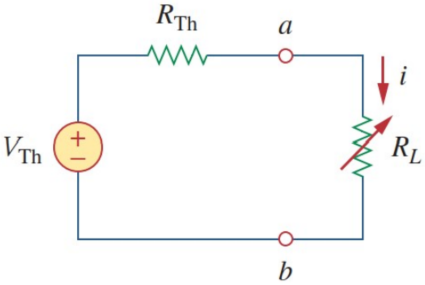
A circuit is usually designed to provide power to a load. For different kinds of circuits, we have different concerns

- **Maximum Power Efficiency:** In power utility systems, the amount of electricity is very large. Therefore, how to increase the efficiency of power transfer becomes an important problem.
- **Maximum Power Transfer:** In communication and instrumental systems, the amount of electricity is small so the problem of efficiency is not so important. Instead, we want to transfer as much of power as possible to the load.

.

Maximum Power Transfer

The Thevenin's equivalent circuit is useful in finding the maximum power delivered to a load. In the circuit below, R_L represents the load.



Maximum Power Theorem

Since

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Let $\frac{dP}{dR_L} = V_{Th}^2 \frac{R_{Th} - R_L}{(R_{Th}+R_L)^3} = 0$, we have $R_L = R_{Th}$.
And when $R_L = R_{Th}$, $\frac{d^2P}{dR_L^2} = V_{Th}^2 \frac{2R_l - 4R_{Th}}{(R_{Th}+R_L)^4} = -\frac{V_{Th}^2}{8R_{Th}^2} < 0$.
Thus p reaches maximum at $R_L = R_{Th}$. $p_{max} = \frac{V_{Th}^2}{4R_{Th}}$

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Ideal Op-amp

Assumption:

- Infinite open-loop gain ($A = \infty$)
- Infinite input resistance ($R_i = \infty$)
- Zero output resistance ($R_0 = 0$)
- (Does not mean that $v_0 = \infty$)

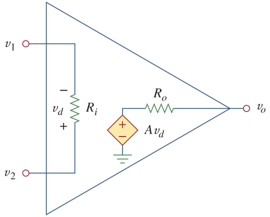


Figure: Op-amp's equivalent circuit

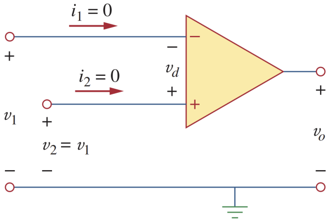


Figure: Symbol of ideal op-amp

Ideal Op-amp

Characteristics of ideal op-amp:

- Open circuit at two input terminals ($i_1 = i_2 = 0$)
- Same voltage at two input terminals ($v_1 = v_2$)
- (**Does not mean that** $i_o = 0$**!**)

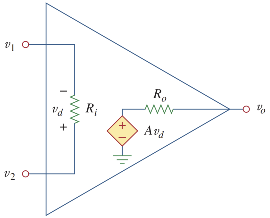


Figure: Op-amp's equivalent circuit

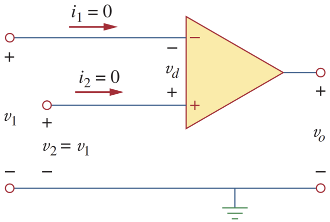


Figure: Symbol of ideal op-amp

Basic Op-amp Circuits: Summary

For basic op-amp circuits:

Op-amp circuits	Input-output relationship
Inverting amplifier	$A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$
Non-inverting amplifier	$A = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$
Voltage follower	$v_o = v_i$
Summing amplifier	$v_o = -(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3)$
Difference amplifier	$v_o = \left[(\frac{R_2}{R_1} + 1)(\frac{R_4/R_3}{1+R_4/R_3})\right] v_2 - \left[\frac{R_2}{R_1}\right] v_1$

For complicated op-amp circuits:

- Identify basic op-amp circuits within it
- Use the formula for cascaded op-amp circuit
- Be proficient in listing nodal analysis equations to obtain v_o/v_i

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

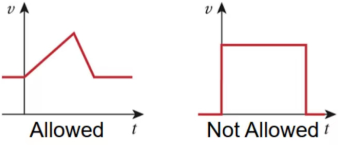
First-Order Circuit

Second-Order Circuit

Duality

Capacitors

1. **Open Circuit Property** When the voltage across a capacitor is not changing with time (**DC steady state**), **the capacitor could be treated as an open circuit**.
2. **Continuity property** The voltage on a capacitor must be continuous.



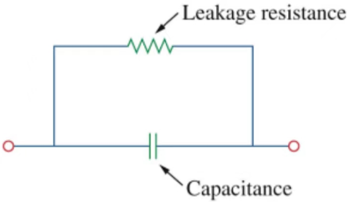
3. **Capacitors IV relationship**

$$i = C\frac{dv}{dt}$$

property 2 can be intuitively shown be property 3. If the voltage across the capacitor is not continuous, say $\frac{dv}{dt} = \infty$, which will cause i to be infinity.

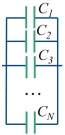
Capacitors

1. **An ideal capacitor will not dissipate energy.** It takes power from the circuit when storing energy in its electric field and returns previously stored energy when delivering power to the circuit.
2. **A real capacitor has a large leakage resistance**



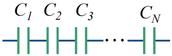
Capacitors in parallel & in series

- **capacitors in parallel**



$$C_{eq} = C_1 + C_2 + C_3 + ... + C_N$$

- **capacitors in series**



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + ... + \frac{1}{C_N}$$

Energy stored in Capacitors

The instantaneous power delivered to the capacitor is

$$p = vi = v(C\frac{dv}{dt})$$

Therefore, **the total energy** stored in the capacitor is

$$w = \frac{1}{2}CV^2$$

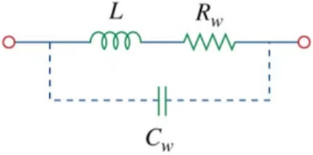
Inductors

1. **Short Circuit Property** When the current through an inductor is not changing with time (**DC steady state**), **the inductor could be treated as a short circuit in the circuit**.
2. **Continuity property** The current through a capacitor must be continuous.
3. **Inductor IV relationship**

$$v = L\frac{di}{dt}$$

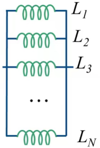
Inductors

1. **An ideal inductor will not dissipate energy.** It takes power from the circuit when storing energy in its magnetic field and returns previously stored energy when delivering power to the circuit.
2. **A real inductor has a significant winding resistance and a small winding capacitance**



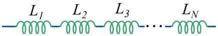
Inductors in parallel & in series

- **inductors in parallel**



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + ... + \frac{1}{L_N}$$

- **inductors in series**



$$L_{eq} = L_1 + L_2 + L_3 + L_4 + ... + L_N$$

Energy stored in Inductors

The instantaneous power delivered to the inductor is

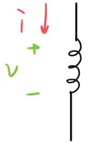
$$p = vi = (L\frac{di}{dt})i$$

Therefore, **the total energy** stored in the inductor is

$$w = \frac{1}{2}Li^2$$

Summary of Capacitors and Inductors

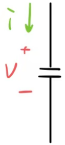
	Capacitor	Inductor
Electric/magnetic	q	ψ
	q=Cv	ψ=Li
i-v (or v-i) relation	i=C×dv/dt	v=L×di/dt
energy	1/2Cv²	1/2Li²



$v = L\frac{di}{dt}$

i increase

→ charging the inductor



$i = C\frac{dv}{dt}$

v increase

→ charging the capacitor

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Introduction

$i = C \cdot \frac{dv}{dt}$ $\qquad$ $v = L\frac{di}{dt}$

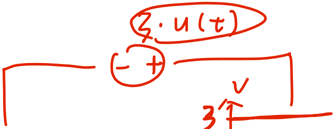
Definition: a first-order circuit is a circuit that contains **only ONE** capacitor/inductor after circuit simplification.

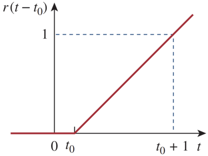
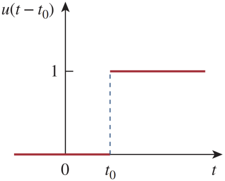
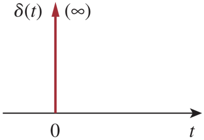
Motivation: we want to investigate how the circuit responses if we

- Store energy to capacitor/inductor
- Let the capacitor/inductor releases energy

	Only one Capacitor	Only one inductor
Store energy	Step input RC	Step input RL
Release energy	Source free RC	Source free RL

Singularity Functions



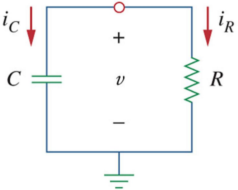
Unit ramp	Unit step	Unit impulse
$r(t) = \begin{cases} 0, t \le 0 \\ t, t > 0 \end{cases}$	$u(t) = \begin{cases} 0, t \le 0 \\ 1, t > 0 \end{cases}$	$\delta(t) = \begin{cases} 0, t \neq 0 \\ \text{Undef.}, t = 0 \end{cases}$
		

Give a nice way to represent "Switch on/off" of the sources/part of circuits.

$$\delta(t) \xrightarrow{\int} u(t) \xrightarrow{\int} r(t)$$

Source-Free Circuits (I) Response

Source-free RC

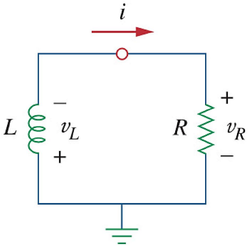


Voltage: $v = v_0 e^{-t/RC}$

Time constant: $\tau = RC$

Current: $i_R = \frac{v}{R} = \frac{v_0}{R} e^{-t/\tau}$
Power: $p = v i_R = \frac{v_0^2}{R} e^{-2t/\tau}$
Energy: $w_R = \int_0^t p\,dt = \frac{1}{2} C v_0^2$

Source-free RL



Current: $i = i_0 e^{-t/(L/R)}$

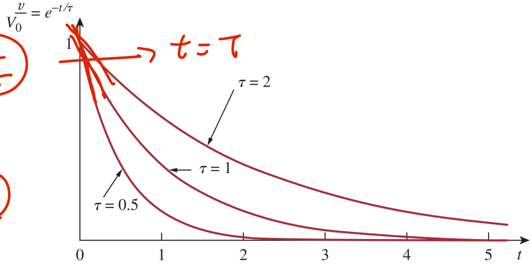
Time constant: $\tau = L/R$

Voltage: $v_R = iR = \frac{i_0}{R} e^{-t/\tau}$
Power: $p = v_R i = i_0^2 R e^{-2t/\tau}$
Energy: $w_R = \int_0^t p\,dt = \frac{1}{2} L i_0^2$

Source-Free Circuits (II) Time Constant

$-\frac{t}{\tau}$

$\to t = \tau$



$v = V_0 \cdot e$

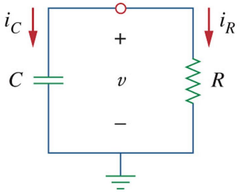
$v(\tau) = \frac{V_0}{e}$

	Source-free RC	Source-free RL
Time constant	$\tau = RC$	$\tau = L/R$
Relation to initial decay rate	$\frac{d}{dt}(\frac{v}{v_0}) = -1/\tau$	$\frac{d}{dt}(\frac{i}{i_0}) = -1/\tau$

- Time required for the response to decay to a factor of $1/e$ or 36.8% of its initial value
- Indicates the initial decaying rate
- Assume complete decay after 5τ

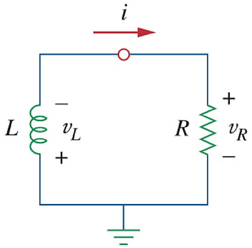
Source-Free Circuits (III) General Steps

Source-free RC



(1) Find initial voltage v_0
(2) Find time constant $\tau = RC$
(3) Obtain v_c, then i_c, v_R, i_R

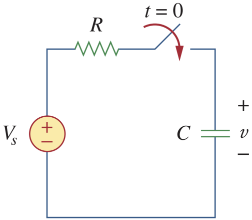
Source-free RL



(1) Find initial voltage i_0
(2) Find time constant $\tau = L/R$
(3) Obtain i_L, then v_L, v_R, i_R

Circuits with Step Input (I) Response

Step-input RC

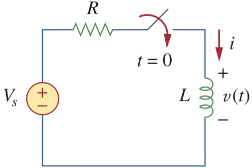


Initial condition:
$$v(0^+) = v(0^-) = V_0$$

Equation:
$$\text{(KVL)}\ (C\frac{dv}{dt}R + v = V_s)$$

Response:
$$v(t) = V_s + (V_0 - V_s)e^{-t/\tau}$$

Step-input RL

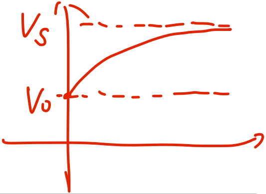


Initial condition:
$$i(0^+) = i(0^-) = I_0$$

Equation:
$$\text{(KCL)}\ iR + L\frac{di}{dt} = V_s$$

Response:
$$i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-t/\tau}$$

Circuits with Step Input (II) Interpretation of Response

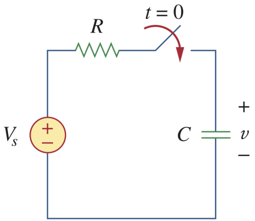


There are three ways to look at the result.
Take $v(t) = \boxed{V_s + (V_0 - V_s)e^{-t/\tau}}$ as example,

Interpretation	First component	Second component
$v(t) = v_n(t) + v_f(t)$	$v_n(t) = (V_o - V_s)e^{-t/\tau}$	$v_f(t) = V_s$
	Natural response	Forced response
$v(t) = v_t(t) + v_{ss}(t)$	$v_t(t) = (V_o - V_s)e^{-t/\tau}$	$v_{ss}(t) = V_s$
	Temporary response	Steady-state response
$v(t) = v_{zp}(t) + v_{zs}(t)$	$v_{zp}(t) = V_0 e^{t/\tau}$	$v_{zs}(t) = (1 - e^{-t/\tau})V_s$
	Zero-input response	Zero-state response

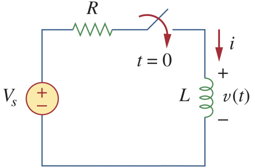
Circuits with Step Input (III) General Steps

Step-input RC



(1) Find initial voltage $v(0^+)$
(2) Find final voltage $v(\infty)$
(3) Find time constant

Step-input RL



(1) Find initial current $i(0^+)$
(2) Find find current $i(\infty)$
(3) Find time constant

General Formula for First-Order Circuits

General formula for RC:

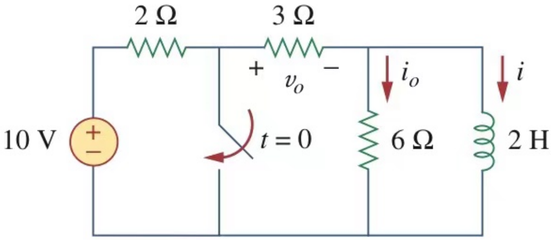
$$v(t) = v(\infty) + \left[v(0^+) - v(\infty)\right] e^{-t/\tau}$$

General formula for RL:

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right] e^{-t/\tau}$$

Exercise

Find i_0, v_o and i for all time, assuming that the switch was open for a long time.



Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

Duality

Introduction

Definition: a second-order circuit is a circuit that consists of resistors and **TWO** capacitors/inductors after circuit simplification.

Workflow to solve 2nd-order circuits:

Find initial and final values and its derivative

$\downarrow$

List differential equation and find its solution

$\downarrow$

Use initial and final values to determine the coefficients in the solution

Initial and Final Values

- $v(0)$,$i(0)$,$v(\infty)$,$i(\infty)$: same method as the first-order circuit.
- $dv(0)/dt = I_C/C$ (Trick here is to use the property that the current across an inductor cannot change abruptly.)
- $di(0)/dt = V_L/L$ (Trick here is to use the property that the voltage of a capacitor cannot change abruptly.)

Caution

Please do care about the polarity when calculating the initial derivatives! You should always remember that current flows from high voltage to low voltage.

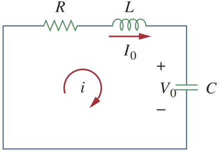
Basic RLC Circuits

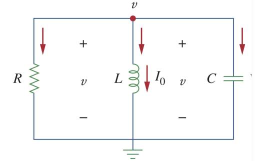
$iR + V_L + V_c = 0$

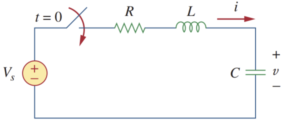
$\Rightarrow iR + L\frac{di}{dt} + v_c = 0$ $i = C\frac{dv_c}{dt}$

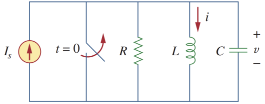
$\Rightarrow R\frac{di}{dt} + L\frac{d^2i}{dt^2} + C i = 0$

	Series connection	Parallel connection
Source-free	(By KVL) $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$	(By KCL) $\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$
Step input	(By KVL) $\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{1}{LC}v = V_s$	(By KCL) $\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = I_s$









Solving 2nd-Order Differential Equations

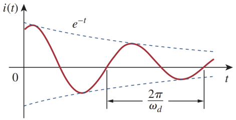
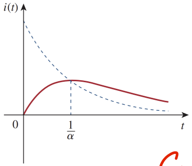
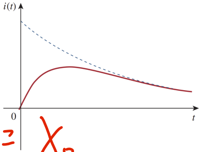
$as^2 + bs + c = 0$

Through analysis we will obtain the differential equation for x,

- For source-free circuits, $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$ (homogeneous, 2nd-order, const coefficient)
- For step input circuits, $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = X_0$ (non-homogeneous, 2nd-order, const coefficient)

$s.$

$x = A$

Underdamped	Critically damped	Overdamped
$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$
No root	One root s	Two roots s_1, s_2
		

$c \cdot A = X_0$

$A = \frac{X_0}{c}$

Solving 2nd-Order Differential Equations

For $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$ (homogeneous),

- No root:

$$r = -\frac{b}{2a}, \omega = \frac{\sqrt{4ac - b^2}}{2a}$$

$$x(t) = e^{rt}(C_1 \sin \omega t + C_2 \cos \omega t)$$

- One root s:

$$x(t) = (C_1 + C_2 t)e^{st}$$

- ~~To~~ Two roots s_1, s_2:

$$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

For $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = X_0$ (non-homogeneous), adopt

$$x(t) = \boxed{x_{homogeneous}(t)} + \boxed{x_{particular}(t)}$$

General Steps for Second-Order Circuits

What we want: $v(t)$ or $i(t)$ of some part of the circuit

- For source-free circuit:
  1. Draw the circuit for $t < 0$, obtain $i(0^+)$ and $v(0^+)$
  2. Draw the circuit for $t > 0$, list equations to obtain $\frac{di(0^+)}{dt}$ or $\frac{dv(0^+)}{dt}$
  3. Get the differential equation for the variable we want to study
     $\longrightarrow$ Judge how many roots in its characteristic equation
     $\longrightarrow$ Select the form of solution (with two coefficients unsolved)
  4. Use the initial conditions to solve the two coefficients C_1, C_2
- For step-input circuit:
  1. Follow the same steps to obtain the differential equation
  2. **Solve its corresponding homogeneous differential equation** $y_{homogeneous}$ (with two coefficients unsolved)
  3. Find a **constant** particular solution, i.e. $y_{particular} = C$ such that $a \times 0 + b \times 0 + c \times C = X_0 \Rightarrow C = X_0/c$
  4. General solution $x = x_{homogeneous} + x_{particular}$
  5. Use the initial conditions to solve the two coefficients C_1, C_2

General Steps for Second-Order Circuits

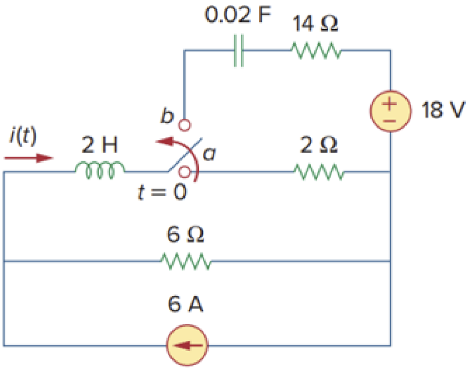
$i = C\frac{dv}{dt}$ $\quad$ $v = L\frac{di}{dt}$

Tips for finding the differential equation:

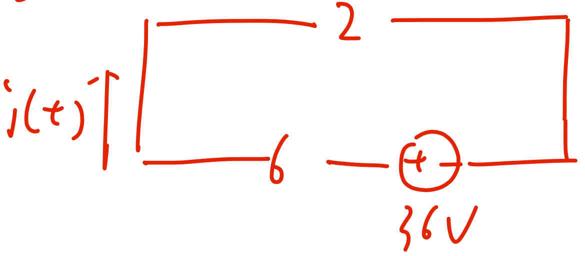
- Start from KCL or KVL?
  Better to try KVL in a scenario similar to series connection, and to try KCL in a scenario similar to parallel connection
- Take advantage of the property of capacitor & inductor:
  - There might be both v and i in the original equation, but only one of them is desired.
  - So consider using $i = C\frac{dv}{dt}$ and $v = L\frac{di}{dt}$ to “kill” one of them

Exercise

The switch in the circuit has been in position for a long time. At $t = 0$ the switch move instantaneously to position b. Find $i(t)$.



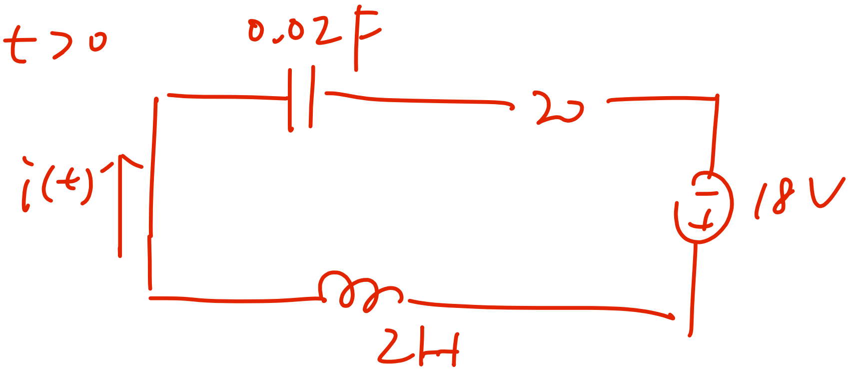
$t<0$



$$i(0^-) = i(0^+) = \frac{36}{8} A$$

$$v(0^-) = v(0^+) = 0 V$$

$t>0$



KVL: $-18 + V_L + V_c + 20i = 0$

$$\Rightarrow -18 + L\frac{di}{dt} + v + 20i = 0$$

$$\Rightarrow \frac{di(0^+)}{dt} = \frac{18 - 20 \cdot i(0^+) - v(0^+)}{2} = -36 A/s$$

List KVL to find the differential eq.

$$-18 + V_L + V_c + 20i = 0$$

$$-18 + L\frac{di}{dt} + \frac{1}{c}\int i \, dt + 20i = 0$$

$$2\frac{d^2 i}{dt^2} + \frac{1}{c} i + 20\frac{di}{dt} = 18.$$

$$\boxed{i''(t) + 10 i'(t) + 25 i(t) = 9}$$

Characteristic equation

$$s^2+10s+25=0 \rightarrow s=-5$$

$$i_{homogeneous}(t)=(C_1+C_2 t)e^{-5t}.$$

Also, $i_{particular}(t)=\frac{9}{25}$.

Then $i(t)=i_{homogeneous}(t)+i_{particular}(t)$

$$=(C_1+C_2 t)e^{-5t}+\frac{9}{25}$$

$$\begin{cases} i(0^+)=\frac{36}{8} \Rightarrow C_1+\frac{9}{25}=\frac{36}{8} \\ \frac{di(0^+)}{dt}=-36 \Rightarrow C_2-5C_1=-36 \end{cases} \longrightarrow \begin{cases} C_1=\frac{207}{50} \\ C_2=-\frac{153}{10} \end{cases}$$

$$\boxed{i(t)=\left(\frac{207}{50}-\frac{153}{10}t\right)e^{-5t}+\frac{9}{25}}$$

Overview

Basic Laws

Methods of Analysis

Circuit Theorems

Operational Amplifiers

Capacitors and Inductors

First-Order Circuit

Second-Order Circuit

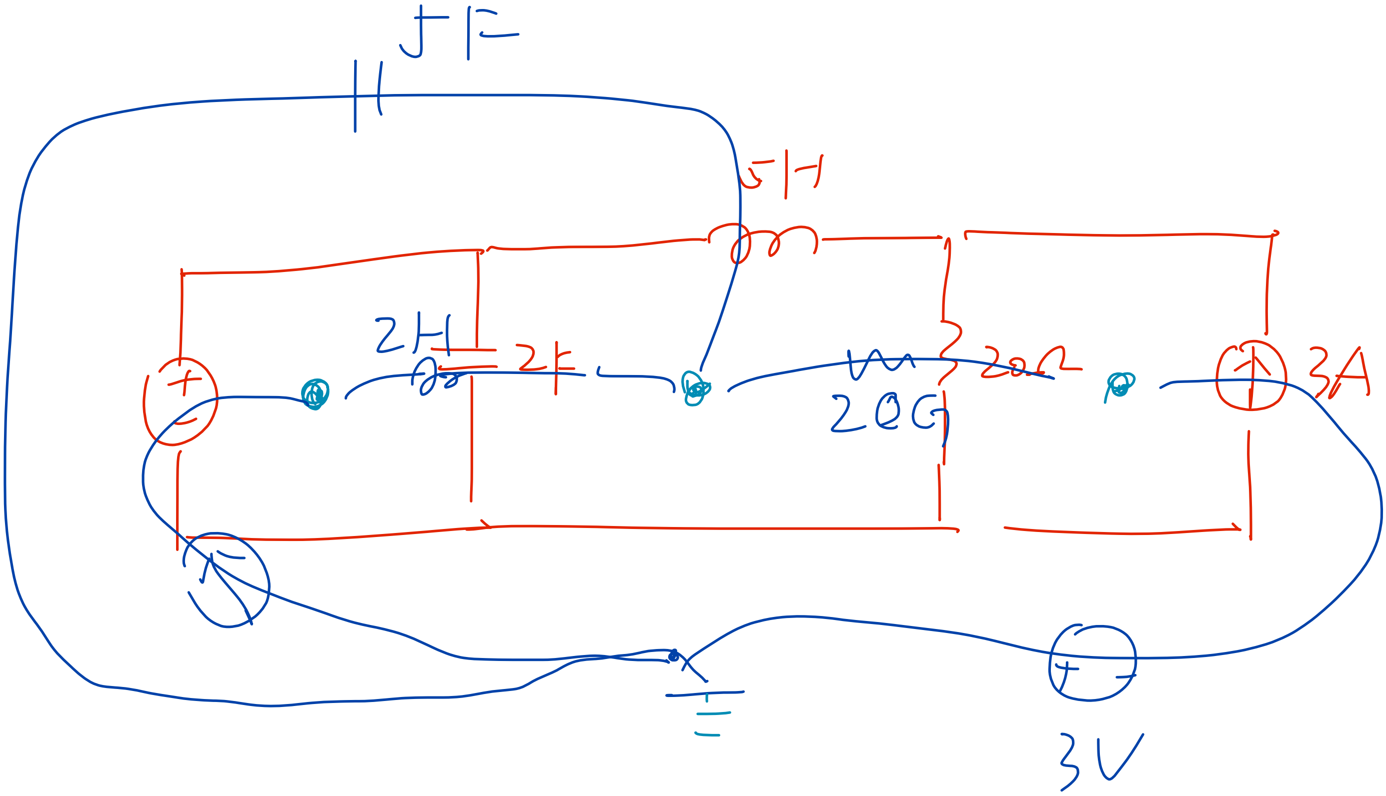
Duality

Dual Pairs

Resistance R	$\longleftrightarrow$	Conductance G
Inductance L	$\longleftrightarrow$	Capacitance C
Voltage v	$\longleftrightarrow$	Current i
Node	$\longleftrightarrow$	Mesh
Serie path	$\longleftrightarrow$	Parallel path
Open circuit	$\longleftrightarrow$	Short circuit
KVL	$\longleftrightarrow$	KCL
Thevenin	$\longleftrightarrow$	Norton

Steps to Draw Dual Circuits

- Place a node at the center of each mesh of the given circuit. Place the reference node (the ground) of the dual circuit outside the given circuit.
- Draw lines between the nodes such that each line crosses an element. Replace that element by its dual.
- To determine the polarity of voltage sources and direction of current sources, follow this rule: A voltage source that produces a positive (clockwise) mesh current has as its dual a current source whose reference direction is from the ground to the non-reference node.



clockwise $\rightleftharpoons$ from ground to non-reference.

References

1. 2023 Summer VE215 slides, Rui Yang
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2022 Fall RC4, Zhiyu Zhou
4. 2022 Fall Mid RC, Zhiyu Zhou, Yifei Cai, Yuxuan Peng

Thank you!