VE215 RC2

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Overview

Circuit Theorems

Operational Amplifiers

Overview-Chapter4 Circuit Theorems

- Linearity Property
- Superposition
- Source Transformation
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer

Linearity Property

homogeneous: if $x \to y$, then $kx \to ky$
additive: if $x_1 \to y_1$ and $x_2 \to y_2$, then $x_1 + x_2 \to y_1 + y_2$
linear circuit: homogeneous and additive

Exercise

Assume $I_o = 1$ A and use linearity to find the actual value of I_o in the circuit of Fig. 4.4.

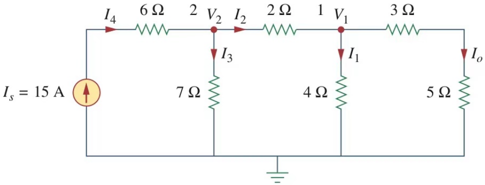
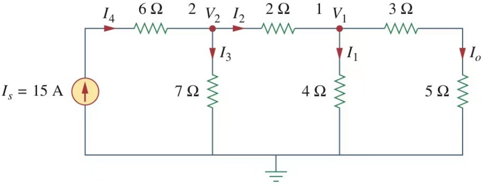


Figure 4.4

Exercise



Answer: $I_0 = 3A$

Superposition

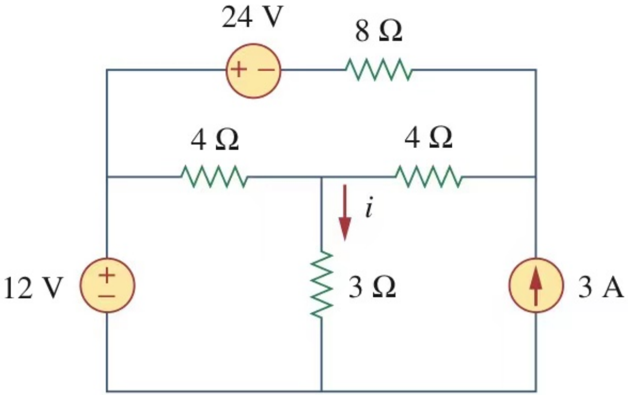
Steps

1. Only consider one **independent** source.
   - voltage source: short circuit
   - current source: open circuit

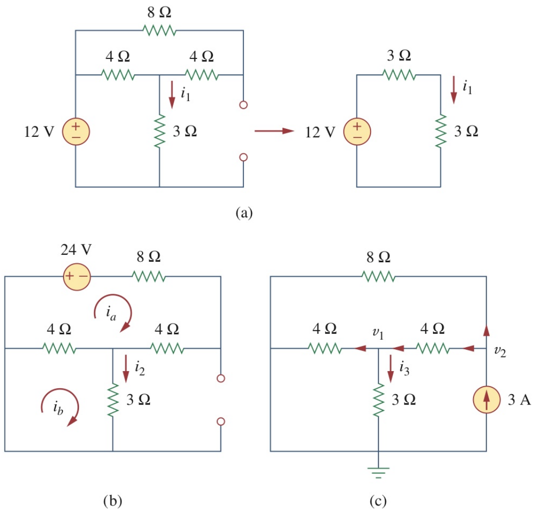
2. Use additivity.

Exercise

Find i in the circuit.



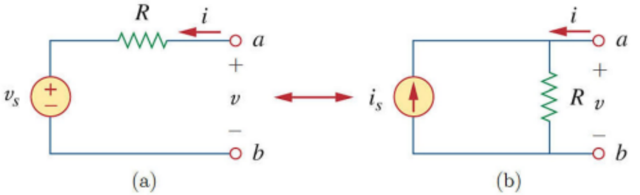
Exercise



Answer: 2A

Source Transformation

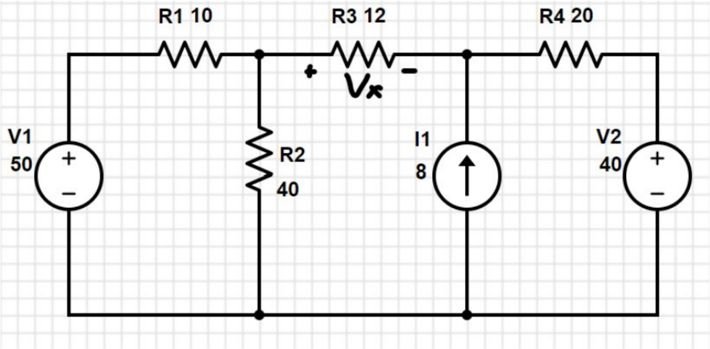
We can replace a voltage source with a resistance with a corresponding current source with the same resistance to simplify the circuit.
In the case shown below, $v_s = i_s \times R$



For dependent sources, the source transformation is also valid.

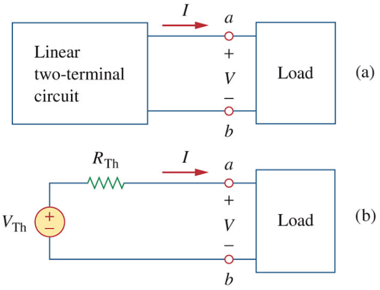
Exercise

Calculate V_x in the circuit below by applying source transformation.



Thevenin's Theorem

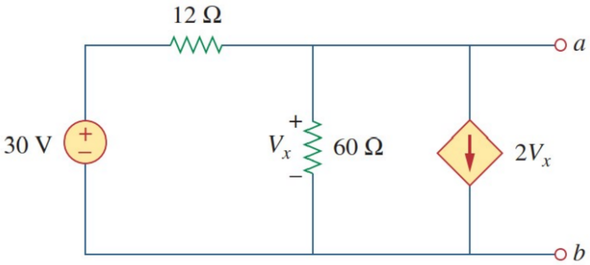
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}.



- V_{Th}: the open-circuit voltage at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

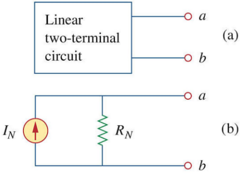
Exercise

Obtain the Thevenin equivalent circuit of this circuit with respect to terminal a and b.



Norton's Theorem

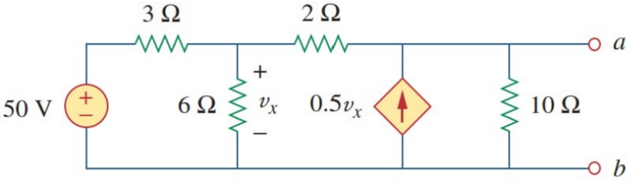
A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_{Th} in parallel with a resistor R_{Th}.



- I_{Th}: the short-circuit current at the terminals.
- R_{Th}: the equivalent resistance at the terminals when all the independent sources are turned off.

Exercise

Obtain the Norton equivalent circuit of this circuit with respect to terminal a and b.



Maximum Power Transfer

A circuit is usually designed to provide power to a load. For different kinds of circuits, we have different concerns

- **Maximum Power Efficiency:** In power utility systems, the amount of electricity is very large. Therefore, how to increase the efficiency of power transfer becomes an important problem.
- **Maximum Power Transfer:** In communication and instrumental systems, the amount of electricity is small so the problem of efficiency is not so important. Instead, we want to transfer as much of power as possible to the load.

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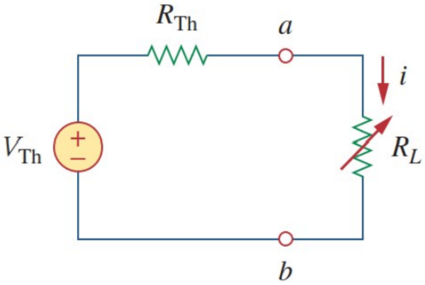
Maximum Power Transfer

The Thevenin's equivalent circuit is useful in finding the maximum power delivered to a load. In the circuit below, R_L represents the load.



Maximum Power Theorem

Since

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Let $\frac{dP}{dR_L} = V_{Th}^2 \frac{R_{Th} - R_L}{(R_{Th}+R_L)^3} = 0$, we have $R_L = R_{Th}$.
And when $R_L = R_{Th}$, $\frac{d^2 P}{dR_L^2} = V_{Th}^2 \frac{2R_l - 4R_{Th}}{(R_{Th}+R_L)^4} = -\frac{V_{Th}^2}{8R_{Th}^2} < 0$.
Thus p reaches maximum at $R_L = R_{Th}$. $p_{max} = \frac{V_{Th}^2}{4R_{Th}}$

Overview

Circuit Theorems

Operational Amplifiers

Operational Amplifiers

Definition: an circuit element that can

- amplify an input electrical signal
- perform mathematical operations (e.g. $+$, $-$, $\times$, $\div$) on this signal when combined with feedback circuits

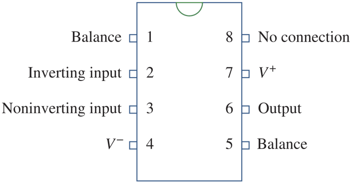


Figure: Structure of op-amp

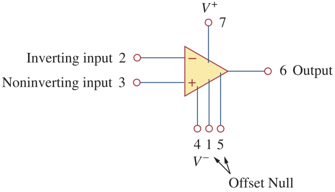


Figure: Symbol of op-amp

Operational Amplifiers

Limitation: the magnitude of the output voltage cannot be as large as we want

- It cannot exceed the supply power V^+
- Otherwise: saturation
- **Ignored in ideal op-amps**

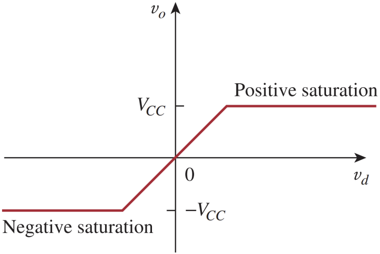


Figure: Saturation

Ideal Op-amp

Assumption:

- Infinite open-loop gain ($A = \infty$)
- Infinite input resistance ($R_i = \infty$)
- Zero output resistance ($R_0 = 0$)
- (Does not mean that $v_0 = \infty$)

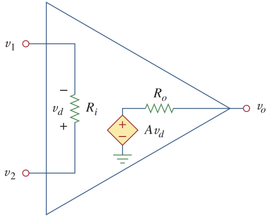


Figure: Op-amp's equivalent circuit

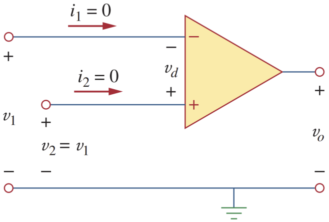


Figure: Symbol of ideal op-amp

Ideal Op-amp

Characteristics of ideal op-amp:

- Open circuit at two input terminals ($i_1 = i_2 = 0$)
- Same voltage at two input terminals ($v_1 = v_2$)
- (**Does not mean that** $i_o = 0$**!**)

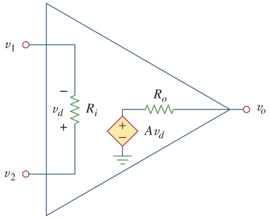


Figure: Op-amp's equivalent circuit

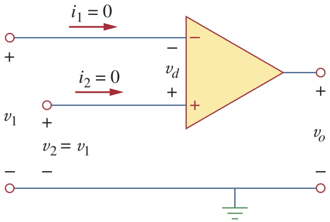


Figure: Symbol of ideal op-amp

Basic Op-amp Circuits: Inverting Op-amp

Open-loop gain:

$$A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$$

Deduction:

$$\begin{cases} \dfrac{v_i - v_1}{R_1} = \dfrac{v_1 - v_o}{R_f} \\ v_1 = v_2 = 0 \end{cases}$$

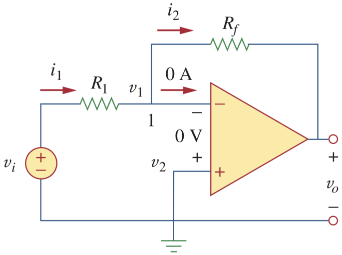


Figure: Inverting op-amp

An interesting variant:

Basic Op-amp Circuits: Non-inverting Op-amp

Open-loop gain:

$$A = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$$

Deduction:

$$\begin{cases} \dfrac{0 - v_1}{R_1} = \dfrac{v_1 - v_o}{R_f} \\ v_1 = v_2 = v_i \end{cases}$$

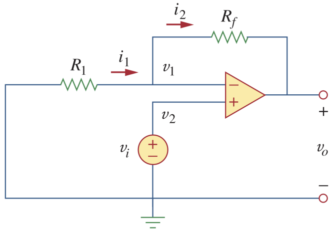


Figure: Non-inverting op-amp

A variant: voltage follower

- $v_o = v_i$
- Let $R_f = 0$ or $R_1 = \infty$
- Isolate two parts of circuits
- Decrease inter-stage loads

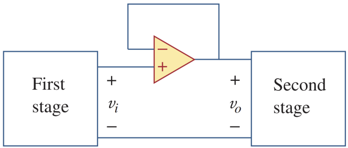


Figure: Voltage follower

Basic Op-amp Circuits: Summing Op-amp

Input-output relationship:

$$v_o = -(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3)$$

Notes:

- Be aware of the minus sign, summing and also inverting
- Treat it as a advanced version of inverting op-amp

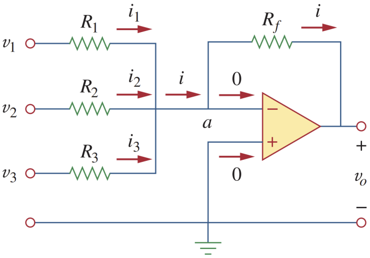


Figure: Summing op-amp

Special cases:

- $R_1 = R_2 = R_3 = R \Rightarrow v_o = -\frac{R_f}{R}(v_1 + v_2 + v_3)$
- $R_1 = R_2 = R_3 = R_f = R \Rightarrow v_o = -(v_1 + v_2 + v_3)$

Basic Op-amp Circuits: Difference Op-amp

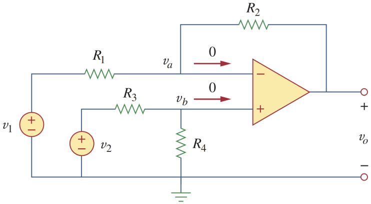


Figure: Difference op-amp

Input-output relationship:

$$v_o = \left[\left(\frac{R_2}{R_1} + 1\right)\left(\frac{R_4/R_3}{1 + R_4/R_3}\right) \right] v_2 - \left[\frac{R_2}{R_1} \right] v_1$$

Special cases:

- $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow v_o = \frac{R_2}{R_1}(v_2 - v_1)$
- $R_1 = R_2, R_3 = R_4 \Rightarrow v_o = v_2 - v_1$

Cascaded Op-amp Circuits

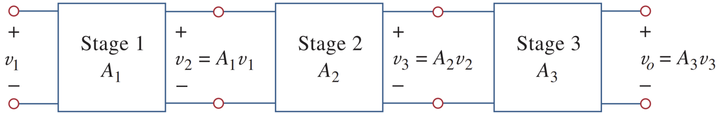


Figure: Cascaded op-amp circuits

$$A_{total} = \frac{v_2}{v_1}\frac{v_3}{v_2}\frac{v_o}{v_1} = A_1 A_2 A_3$$

Basic Op-amp Circuits: Summary

For basic op-amp circuits:

Op-amp circuits	Input-output relationship
Inverting amplifier	$A = \frac{v_0}{v_i} = -\frac{R_f}{R_1}$
Non-inverting amplifier	$A = \frac{v_0}{v_i} = 1 + \frac{R_f}{R_1}$
Voltage follower	$v_o = v_i$
Summing amplifier	$v_o = -(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3)$
Difference amplifier	$v_o = \left[(\frac{R_2}{R_1} + 1)(\frac{R_4/R_3}{1+R_4/R_3})\right] v_2 - \left[\frac{R_2}{R_1}\right] v_1$

For complicated op-amp circuits:

- Identify basic op-amp circuits within it
- Use the formula for cascaded op-amp circuit
- Be proficient in listing nodal analysis equations to obtain v_o/v_i

Application: Digital-to-Analog Converter converter

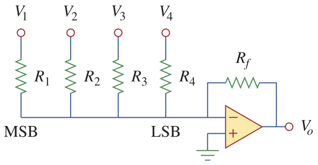


Figure: Circuit of D2A converter

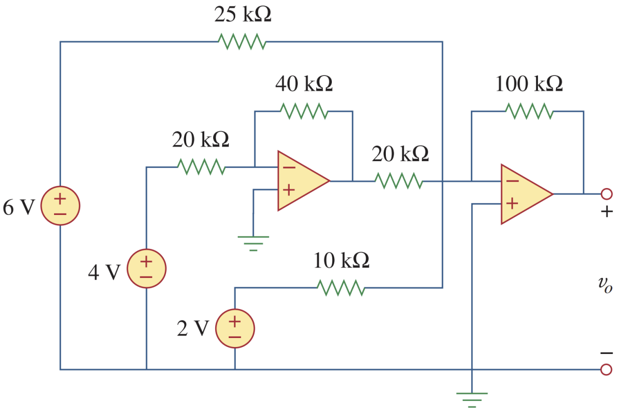
Input V_1, V_2, V_3, V_4 takes only 0 or 1.
Resistors satisfy $R_4 = 2R_3, R_3 = 2R_2, R_2 = 2R_1$.

$$V_o = c(8V_1 + 4V_2 + 2V_3 + V_4)$$

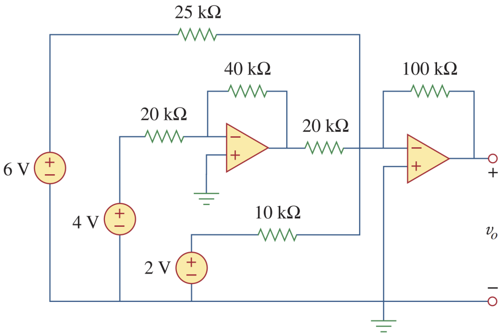
V_o is propositional to the binary representation $[V_1 V_2 V_3 V_4]$.

Exercise

For the circuit below, find v_o.
Hint: cascading of basic op-amp circuits



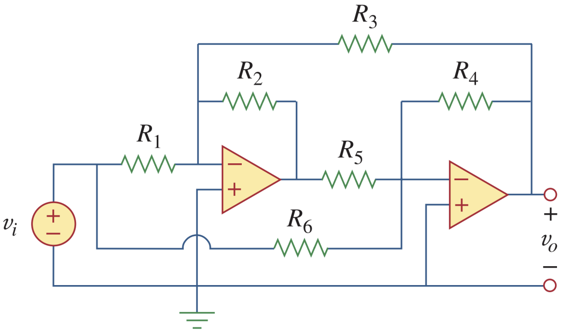
Exercise



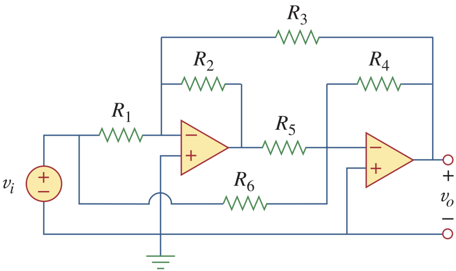
Answer: -4V

Exercise

Determine the gain v_o/v_i of the circuit below.
Hint: list the equations yourself!



Exercise



Answer: $v_o/v_i = \frac{R_2 R_4/R_1 R_5 - R_4 R_6}{1 - R_2 R_4/R_3 R_5}$

(Reminder: don't list KCL on the output node)

References

1. 2023 Summer VE215 slides, Rui Yang
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2022 Fall RC3, Yifei Cai

Thank you!