VE215 RC4

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From DC to AC

Begin our travel in alternating current circuits!

- Ch9: Introduce a new system to represent alternating signals
- Ch10: Analysis tools in AC context (with frequency ω fixed)
- Ch11: Analyze the how power is delivered in AC circuits
- Ch14: Investigate the circuit behavior when frequency ω is changed

Overview

Sinusoids and Phasors

Sinusoidal Steady-State Analysis

Sinosoid

A sinusoid is a signal that has the form of sine or cosine function:

$$v(t) = V_m \sin(\omega t + \phi)$$

where V_m is the amplitude, ω is the frequency, and ϕ is the initial phase.

For $v_1(t) = V_m \sin(\omega t + \phi_1)$ and $v_2(t) = V_m \sin(\omega t + \phi_2)$,

- If $\phi_1 = \phi_2$, v_1 and v_2 are **in phase**
- If $\phi_1 > \phi_2$, v_1 and v_2 are **out of phase**, v_1 **leads** v_2 and v_2 **lags** v_1

Phasors

$e^{ja} = \cos a + j \sin a$

Motivation: want a neat and simple way to represent sinusoidal signals, instead of cos and sin.

Solution: use **phasor** to represent the V_m (amplitude) and ϕ (phase) of a sinusoid.

$\sin(3t)$

$\cos(3t - 90^\circ)$

$+$

-90°

Introducing complex number systems, we have

$$v(t) = V_m \overset{\cos}{\cancel{\sin}}(\omega t + \phi) = Re(V_m e^{j(\omega t+\phi)}) = Re(V_m e^{j\phi} e^{j\omega t})$$

Then we let

$$\tilde{V} = V_m e^{j\phi} = V_m \angle \phi$$

$\phi = -90^\circ$

This is the phasor representation of the sinusoid $v(t)$. Note that it doesn't keep the information of frequency ω. We assume a fixed and known frequency from ch9 to ch13.

$\sin(\omega t + \phi) = \cos(\omega t + \phi - \frac{\pi}{2})$

Phasors

Phasor representation:

- **Polar form**: $z = x + jy$
- **Rectangular form**: $z = |z|\angle\theta$
- Conversion between each other::

$$R = |Z|\cos\theta, X = |Z|\sin\theta$$
$$|Z| = \sqrt{R^2 + X^2}, \theta = \tan(X/R)$$

Phasor calculation:

- Addition/subtraction more convenient in rectangular form:

$$z_1 \pm z_2 = (x_1 \pm x_2) + j(y_1 \pm y_2)$$

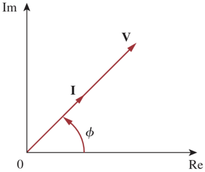
- Multiplication/division more convenient in polar form:

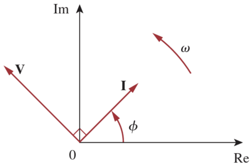
$$z_1 z_2 = |z_1||z_2|\angle(\phi_1 + \phi_2)$$
$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}\angle(\phi_1 - \phi_2)$$

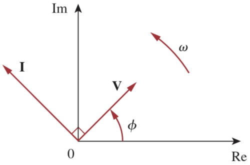
Phasor Relationships for Circuit Elements

We can express the $I - V$ relationship of each type of circuit element in phasor context:

Element	Time domain	Phasor domain	Phase relationship
Resistor	$v = Ri$	$\tilde{V} = R\tilde{I}$	I, V in phase
Inductor	$v = L\frac{di}{dt}$	$\tilde{V} = \boxed{j\omega L}\tilde{I}$	I lags V
Capacitor	$i = C\frac{dv}{dt}$	$\tilde{V} = \boxed{\frac{1}{j\omega C}}\tilde{I}$	I leads V







Impedance and Admittance

Impedance $Z = \tilde{V}/\tilde{I}$: a "generalized" version of resistance.

$$Z = R(\text{resistance}) + jX(\text{reactance}) = |Z|\angle\theta$$

(unit in Ω, same as resistance R)

Elements	Resistor	Inductor	Capacitor
Impedance $Z(\Omega)$	R	$j\omega L$	$\frac{1}{j\omega C}$
Resistance $R(\Omega)$	R	0	0
Reactance $X(\Omega)$	0	ωL	$-\frac{1}{\omega C}$

$Z = X + jY$

Impedance and Admittance

Admittance $Y = 1/Z$: a "generalized" version of conductance.

$$Y = G(\text{conductance}) + jB(\text{susceptance}) = |Y|\angle\theta$$

(Unit in S, same as conductance G)

Elements	Resistor	Inductor	Capacitor
Impedance $Y(S)$	$1/R$	$-\frac{j}{\omega L}$	$j\omega C$
Resistance $G(S)$	$1/R$	0	0
Reactance $B(S)$	0	$-\frac{1}{\omega L}$	ωC

Impedance Combination

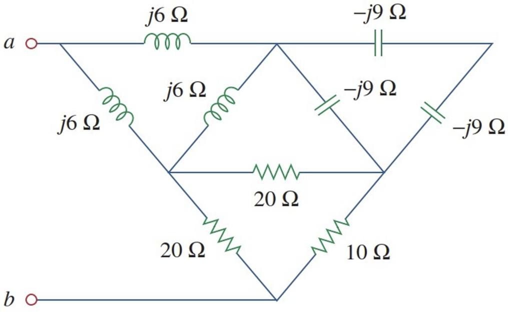
Previous rules still apply, only generalized.

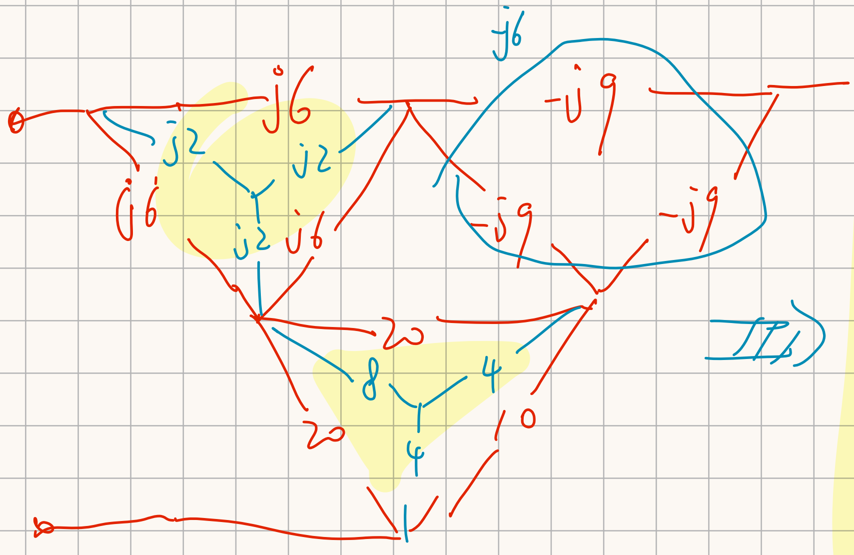
	Series connection	Parallel connection
Impedance $Z(\Omega)$	$Z_{eq} = \sum_{i=1}^{n} Z_i$	$\frac{1}{Y_{eq}} = \sum_{i=1}^{n} \frac{1}{Y_i}$
Admittance $Y(S)$	$\frac{1}{Z_{eq}} = \sum_{i=1}^{n} \frac{1}{Z_i}$	$Y_{eq} = \sum_{i=1}^{n} Y_i$

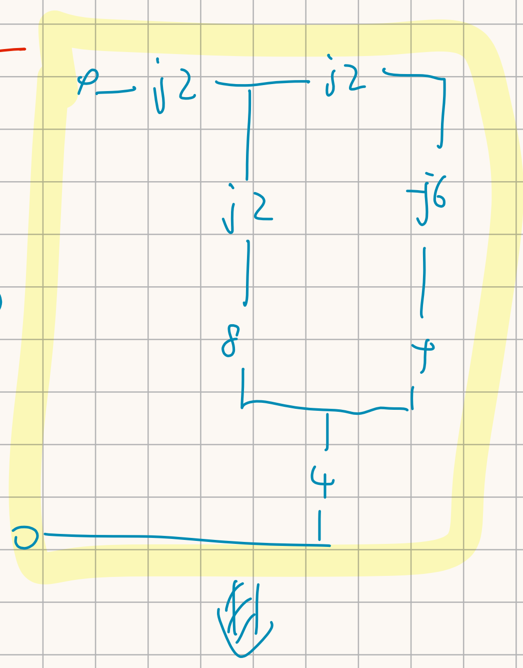
Capacitors and inductors are now treated similarly as resistors!

Exercise

Calculate Z_{ab} in the figure below.



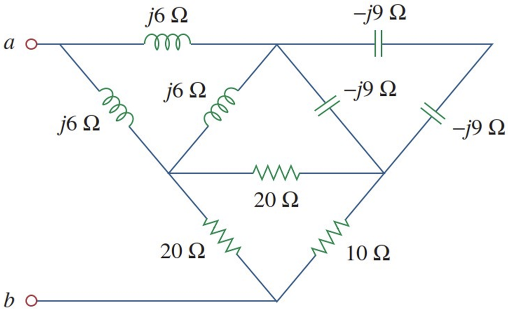




$$Z_{ab} = (2j+4) + (2j+8) \parallel (4-4j) = 7.57 + j0.59$$
$$= 7.59 \angle 4.49^\circ$$

Exercise

Calculate Z_{ab} in the figure below.

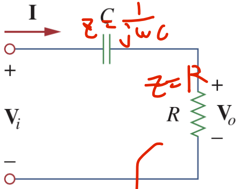


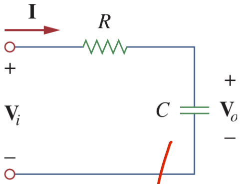
Answer: $Z_{ab} = 7.57 + j0.59 = 7.59\angle 4.49^\circ (\Omega)$

Application: Phase Shifters

Goal: change the phase of the original signal, lagging or leading
Solution: adopt an RC (or RL) circuit

Leading shifter	Lagging shifter
V_o acorss resistor	V_o across capacitor



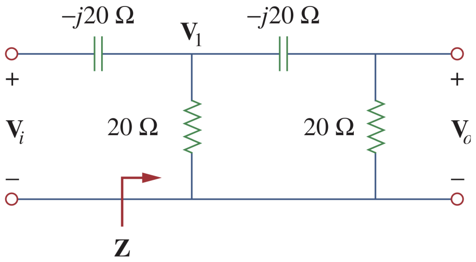


$$V_o = \frac{R}{R + \frac{1}{j\omega C}} V_i$$

$$V_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_i$$

Application: Phase Shifters

In a single shifter, $0 < \Delta\theta < 90^\circ$. Cascade shifters to achieve a $\geq 90^\circ$ phase shift.



TODO

Overview

Sinusoids and Phasors

Sinusoidal Steady-State Analysis

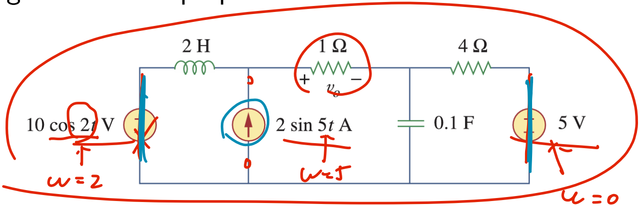
Sinusoidal Steady-State Analysis

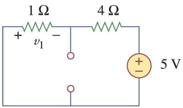
Basically, all the laws and methods we learned in DC circuit can still be applied in AC circuit.

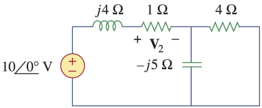
- Ohm's Law
- KCL & KVL
- Nodal & Mesh Analysis
- $Y - \Delta$ Transformation
- Superposition Theorem
- Source Transformation
- Thevenin & Norton Theorem
- Op-amp Circuits

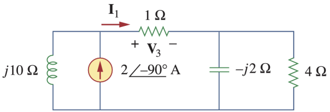
Importance of Superposition Theorem

In a AC circuit, there might be sources operating at different frequencies. Analysis should be separate in each frequency, and added together with superposition theorem.









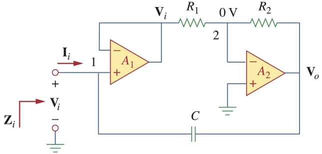
$v_o = v_1(\omega = 0, \mathrm{DC}) + v_2(\omega = 2) + v_3(\omega = 5)$ in time domain

Applications

Capacitance Multiplier

Small capacitance + op-amp to produce large capacitance

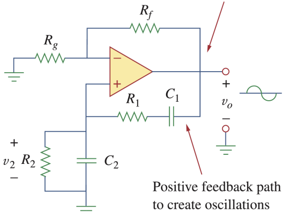
$$I_i/V_i = j\omega(1 + \frac{R_2}{R_1})C$$
$$Z_i = V_i/I_i = \frac{1}{j\omega(1+R_2/R_1)C}$$



Oscillator (Lab 7)

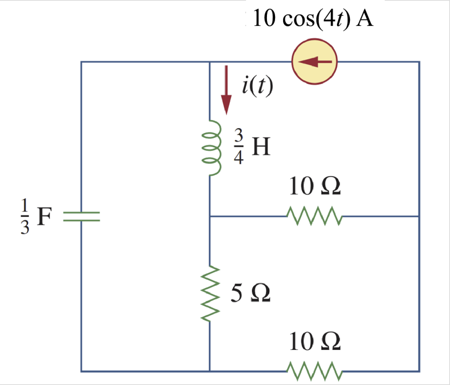
Produces an AC waveform as output when powered by a DC input

$$Z_p = R_2 \| \frac{1}{j\omega C_2} \quad Z_s = R_1 + \frac{1}{j\omega C_1}$$
$$\text{gain} = \frac{V_2}{V_o} = \frac{Z_p}{Z_p+Z_s} = \frac{R_g}{R_f+R_g}$$

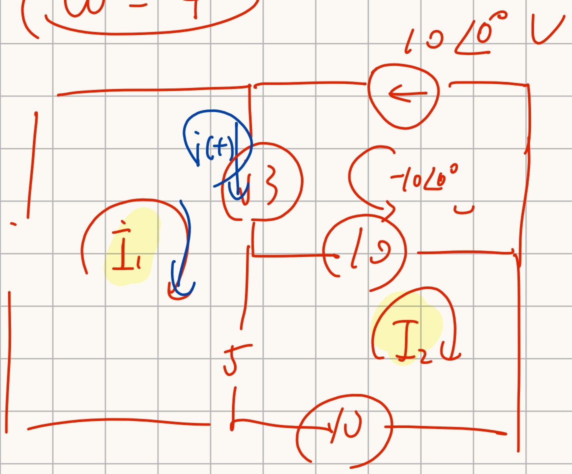


Exercise

Find current $i(t)$ in the circuit below.



$\omega = 4$



$$\begin{cases} \frac{1}{3}F \to \frac{1}{j\omega C} = \frac{1}{j4\cdot\frac{1}{3}} = -j\frac{3}{4} \\ \frac{3}{4}H \to j\omega L = j\cdot\frac{3}{4}\cdot 4 = j3 \end{cases}$$

Mesh analysis

I_1

$$\begin{cases} -\frac{3}{4}j I_1 + j3(I_1 + 10) + 5(I_1 - I_2) = 0 \\ 5(I_2 - I_1) + 10(I_2 + 10) + 10 I_2 = 0 \end{cases}$$

$$\to \begin{cases} I_1 = -7.0 - j3.56 \\ I_2 = -5.4 - j0.71 \end{cases}$$

solve $I = I_1 + 10\angle 0^\circ$

$\downarrow \quad = 3.0 - j3.56$

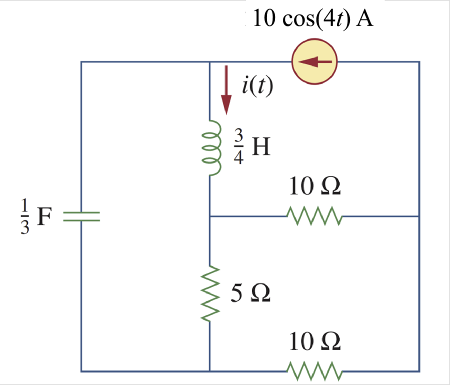
to time domain

$$i(t) = \sqrt{3.0^2 + 3.56^2}\cos\left(\omega t - \tan^{-1}\left(\frac{3.56}{3.0}\right)\right)$$

$$= 4.66\cos(4t - 50.0^\circ)$$

Exercise

Find current $i(t)$ in the circuit below.



Answer: $\mathbf{I} = \cancel{3 - j4} \rightarrow i(t) = \cancel{5\cos(4t - 53.13°)}$

$= 3{,}0 - j3{,}56 \longrightarrow i(t) = 4{,}66 \cos(4t - 53{,}0°)$

References

1. 2023 Summer VE215 slides, Rui Yang
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2022 Fall RC5, Yuxuan Peng
4. 2022 Fall RC6, Zhiyu Zhou

Thank you!