VE215 RC 6

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Overview

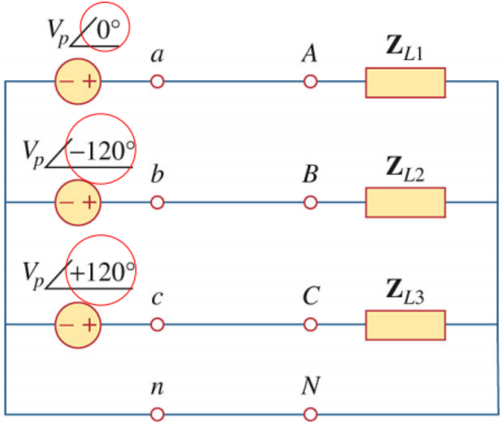
Three-Phase Circuits

Magnetically Coupled Circuits

Frequency Response

Ployphase

Sources operate at the same frequency but different phases. For example, the figure below shows a three-phase four-wire system:



Three-Phase Circuits

Three-Phase Voltage sources

- Type: Y-connected, Δ-connected
- Balanced: $\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$
  $|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$
- Phase sequences: abc/positive, acb/negative.

Types

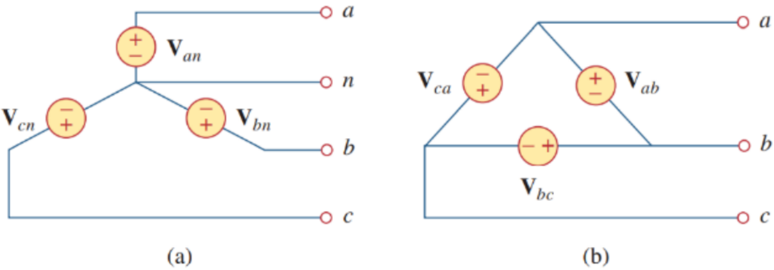


Figure 12.6
Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

Phase sequences

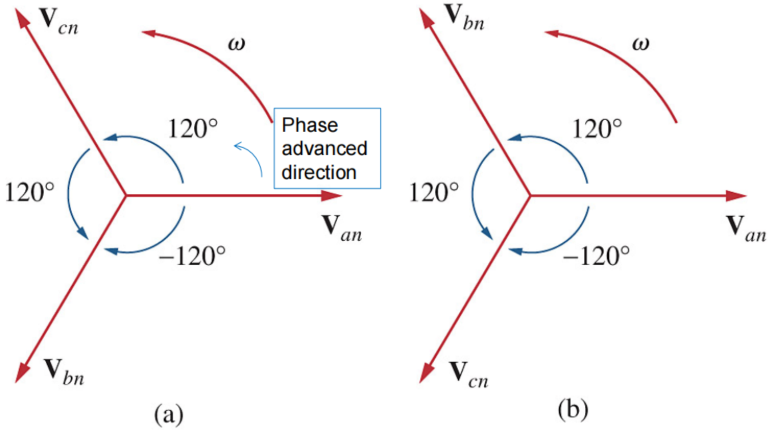


Figure 12.7 Phase sequences: (a) *abc* or positive sequence, (b) *acb* or negative sequence.

abc: a lead b, b leads c (i.e., $\angle a > \angle b > \angle c$)

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Three-Phase Loads

- Type: Y-connected, Δ-connected
- Balanced: $\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$$

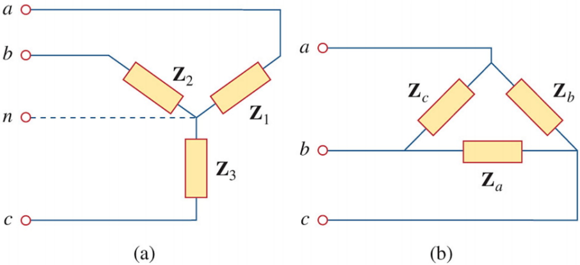


Figure 12.8 Two possible three-phase load configurations: (a) a wye-connected load, (b) a delta-connected load.

Three-Phase circuits

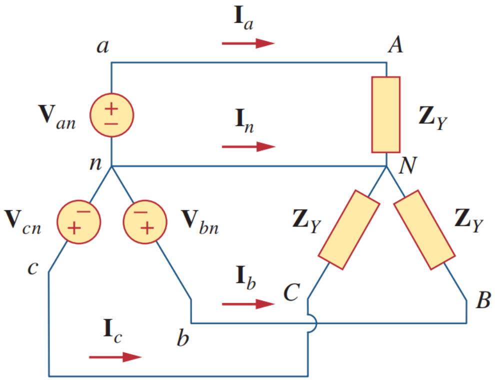
Connect types

- Y-Y
- Y-Δ
- Δ-Y
- Δ-Δ

Some terms

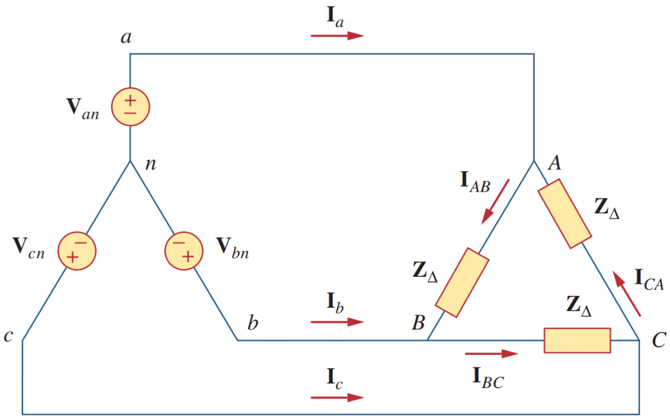
- Line: conductors connecting sources and loads.
- Line voltages: the voltage between any two lines.
- Line currents: the current passing along each line.
- Phase: connected between any pair of line terminals (an element).
- Phase voltages: the voltage across any phase.
- Phase currents: the current passing through any phase.

Y-Y Connection



Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0°$ $\mathbf{V}_{bn} = V_p\angle -120°$ $\mathbf{V}_{cn} = V_p\angle +120°$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30°$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120°$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120°$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120°$ $\mathbf{I}_c = \mathbf{I}_a\angle +120°$

Y-Δ Connection

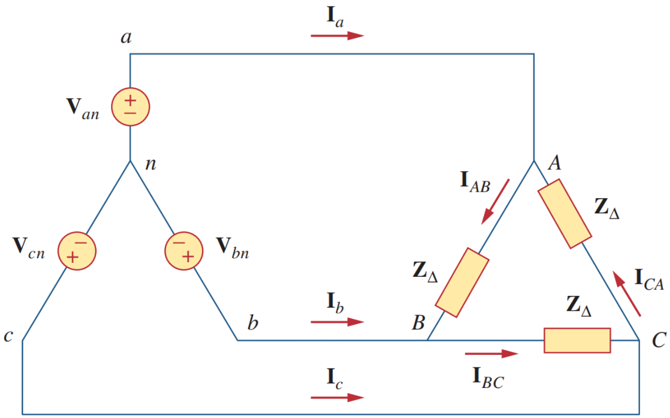


Y-Δ

$$
\begin{aligned}
\mathbf{V}_{an} &= V_p\angle 0^\circ \\
\mathbf{V}_{bn} &= V_p\angle -120^\circ \\
\mathbf{V}_{cn} &= V_p\angle +120^\circ \\
\mathbf{I}_{AB} &= \mathbf{V}_{AB}/\mathbf{Z}_\Delta \\
\mathbf{I}_{BC} &= \mathbf{V}_{BC}/\mathbf{Z}_\Delta \\
\mathbf{I}_{CA} &= \mathbf{V}_{CA}/\mathbf{Z}_\Delta
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{V}_{ab} &= \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ \\
\mathbf{V}_{bc} &= \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ \\
\mathbf{V}_{ca} &= \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ \\
\mathbf{I}_a &= \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ \\
\mathbf{I}_b &= \mathbf{I}_a\angle -120^\circ \\
\mathbf{I}_c &= \mathbf{I}_a\angle +120^\circ
\end{aligned}
$$

Δ-Δ Connection

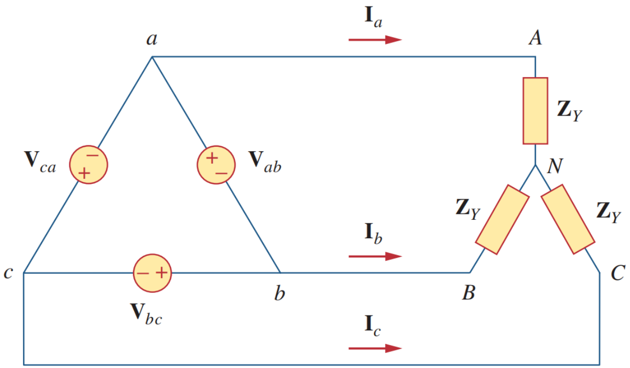


Y-Δ

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{cn} = V_p\angle +120^\circ$$
$$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$$
$$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$$
$$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$$

$$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$$
$$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$$
$$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$$
$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$$
$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$$

Δ-Y Connection



Δ-Y

$$\mathbf{V}_{ab} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bc} = V_p\angle -120^\circ$$
$$\mathbf{V}_{ca} = V_p\angle +120^\circ$$

Same as phase voltages

Same as line currents

$$\mathbf{I}_a = \frac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$$
$$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$$

Power

- Instantaneous power: $p = 3|\mathbf{V}_p||\mathbf{I}_p|\cos\theta = \sqrt{3}|\mathbf{V}_L||\mathbf{V}_L|\cos\theta$
  - Y-load: $|\mathbf{I}_L| = |\mathbf{I}_p| \qquad |\mathbf{V}_L| = \sqrt{3}|\mathbf{V}_p|$
  - Δ-Load: $|\mathbf{I}_L| = \sqrt{3}|\mathbf{I}_p| \quad |\mathbf{V}_L| = |\mathbf{V}_p|$
- Average power:
  - Total: $P = 3|\mathbf{V}_p||\mathbf{I}_p|\cos\theta = \sqrt{3}|\mathbf{V}_L||\mathbf{I}_L|\cos\theta$
  - Per phase: $P_p = |\mathbf{V}_p||\mathbf{I}_p|\cos\theta$
- Complex power:
  - Total: $S = 3\mathbf{V}_p\mathbf{I}_p^* = 3|\mathbf{I}_p|^2\mathbf{Z}_p$
  - Per phase: $S_p = \mathbf{V}_p\mathbf{I}_p^* = |\mathbf{I}_p|^2\mathbf{Z}_p$

Overview

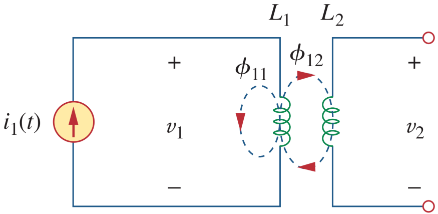
Three-Phase Circuits

Magnetically Coupled Circuits

Frequency Response

Magnetically Coupled Circuits

When two inductors are closed to each other, they affect each other through the magnetic field generated by one of them, here they are said to be **magnetically coupled**.

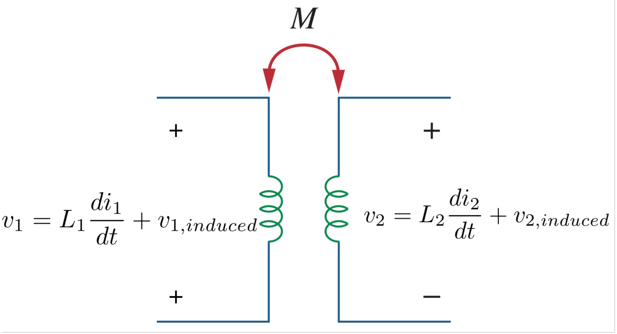


In this case, additional inductance will be generated on both coils, which is called the **mutual inductance** M, with the same unit H as self-inductance L.

(You don't need to be familiar with the derivation (textbook p557-558).)

Mutual Voltage

The existence of mutual inductance will induce voltage in both coils, called **mutual voltage**. This induced voltage must be taken into consideration if we want to analyze the circuit as before.



Mutual Voltage

The magnitude of the mutual voltage on each side are

$$|v_{1,induced}| = |M\frac{di_2}{dt}|$$
$$|v_{2,induced}| = |M\frac{di_1}{dt}|$$

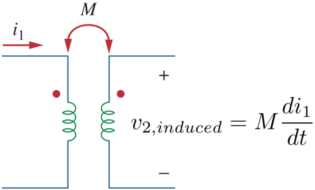
The form of the formula is the same as the i-v relationship of an inductor, but be careful that the induced voltage on one coil is jointly determined by both M and the current in the other coil.

Mutual Voltage: Dot Convention

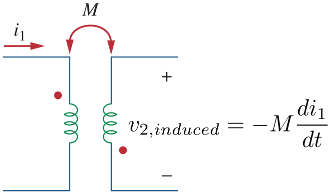
The direction is determined by the **dot convention**.

- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.
- If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.

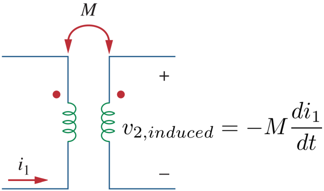
Mutual Voltage: Dot Convention



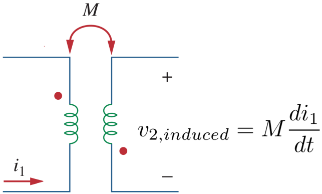
Case 1



Case 1



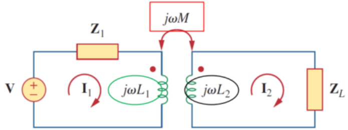
Case 2



Case 2

Applying KCL

KCL for magnetically coupled circuit is basically the same as before. Only to pay attention to the dot convention.
In the following circuit, I_1 enters the node, I_2 leaves the node. So mutual voltage is negative.

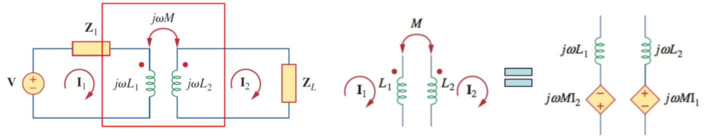


Loop 1 $\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - \boxed{j\omega M \mathbf{I}_2}$

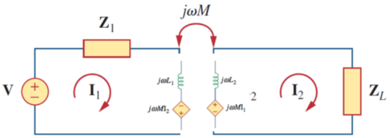
Loop 2 $0 = \boxed{-j\omega M \mathbf{I}_1} + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$

Apply KCL: Physically Not Connected

To make the analysis easier, we assign a dependent voltage based on the sign of the mutual voltage.



Two circuits physically not connected



Loop 1

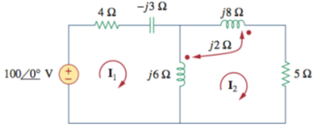
$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2$$

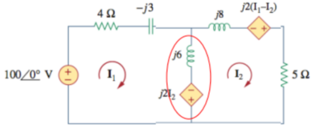
Loop 2

$$0 = -j\omega M \mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

Apply KCL: Physically Connected

For circuit physically connected, pay attention to the values and polarity.





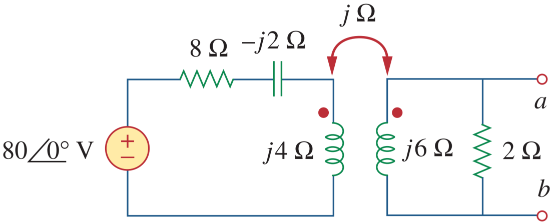
Loop 1: $-100 + \mathbf{I}_1(4 - j3 + j6) - j6\mathbf{I}_2 - j2\mathbf{I}_2 = 0$

Two circuits physically connected.

Loop 2: $0 = -2j\mathbf{I}_1 - j6\mathbf{I}_1 + (j6 + j8 + j2 \times 2 + 5)\mathbf{I}_2$

Exercise

Obtain the Norton equivalent at terminals $a - b$ of the circuit in the following circuit.



Energy in a Coupled Circuit

The energy stored in a coupled circuit is

$$w = \frac{1}{2}L_1 i_1^2(t) + \frac{1}{2}L_2 i_2^2(t) \pm i_1(t) i_2(t)$$

Minus sign occurs when the mutual voltage is negative, i.e., current enters one dotted terminal and leaves the other dotted terminal. ($i_1(t)$, $i_2(t)$ should be sinusoidal. Plug in value of t in calculation.)

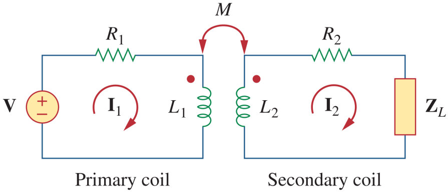
The coupling coefficient

$$k = \frac{M}{L_1 L_2}$$

describes the extent to which M approaches $\sqrt{L_1 L_2}$.
$k > 0.5 \rightarrow$ loosely coupled, $k > 0.5 \rightarrow$ tightly coupled.

Transformers

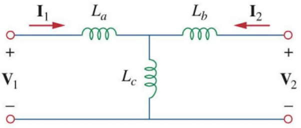
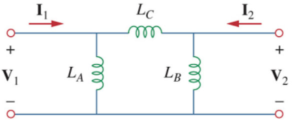
A transformer is a four-terminal circuit element comprising two magnetically coupled coils.



Transformers

It is convenient to replace a transformer by an equivalent circuit with no magnetic coupling.

- When two dots are on the same side:

Equivalent T circuit	Equivalent Π circuit
	
$L_a = L_1 - M$ $L_b = L_2 - M$ $L_c = M$	$L_A = \frac{L_1L_2 - M^2}{L_2 - M}$ $L_B = \frac{L_1L_2 - M^2}{L_1 - M}$ $L_C = \frac{L_1L_2 - M^2}{M}$

- When two dots are on opposite sides, replace all M by $-M$.

Ideal Transformers

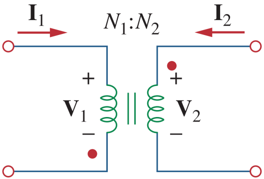
An **ideal transformer** is one with perfect coupling ($k = 1$). With this idealization, we have a quite simple relationship for the currents and voltages at both sides:

$$|\frac{\tilde{V}_2}{\tilde{V}_1}| = |\frac{\tilde{I}_1}{\tilde{I}_2}| = \frac{N_2}{N_1} = n$$

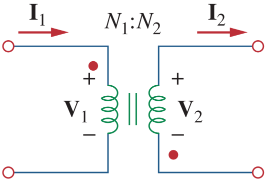
The rules for determining the sign:

- For $\tilde{V}$: If $\tilde{V}_1$ and $\tilde{V}_2$ are both positive or both negative at the dotted terminal, $\tilde{V}_2/\tilde{V}_1 = +n$, otherwise $\tilde{V}_2/\tilde{V}_1 = -n$.
- for $\tilde{I}$: If $\tilde{I}_1$ and $\tilde{I}_2$ both enter or both leave the dotted terminals, $\tilde{I}_1/\tilde{I}_2 = -n$, otherwise $\tilde{I}_1/\tilde{I}_2 = +n$,

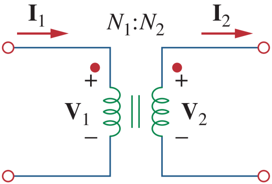
Ideal Transformers



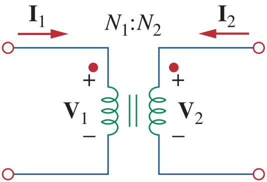
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = -\frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$



$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2}$$



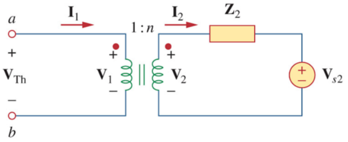
$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} \qquad \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{N_1}{N_2}$$

Ideal Transformers: Equivalent Circuit

It is more convenient to replace the transformer with an equivalent circuit.

Thevenin equivalent circuit

(i) V_{Th}



Terminals a-b are open, therefore, $\mathbf{I}_1 = 0 = \mathbf{I}_2 \rightarrow \mathbf{V}_2 = \mathbf{V}_{s2}$

Using the turns ratio, $\mathbf{V}_2 = n\mathbf{V}_1$

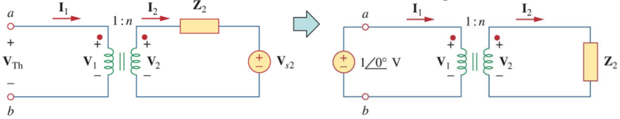
Thevenin voltage is $\mathbf{V}_{\text{Th}} = \mathbf{V}_1 = \dfrac{\mathbf{V}_2}{n} = \dfrac{\mathbf{V}_{s2}}{n}$

Ideal Transformers: Equivalent Circuit

It is more convenient to replace the transformer with a normal circuit.

(ii) Z_{Th}

Remove independent source
Connect test voltage



$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$ Using the turns ratio, $\mathbf{V}_1 = \frac{\mathbf{V}_2}{n}$ $\mathbf{I}_1 = n\mathbf{I}_2$

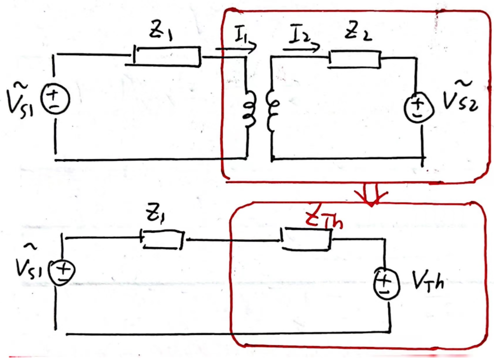
We get Thevenin impedance Z_{Th}

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{\mathbf{V}_2/n}{n\mathbf{I}_2} = \frac{\mathbf{Z}_2}{n^2}, \qquad \mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}_2$$

Ideal Transformers: Equivalent Circuit

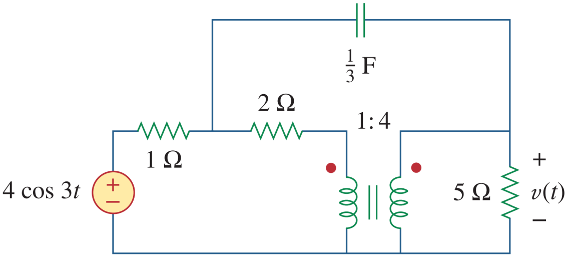
Overall,

$$\mathbf{V}_{th} = \frac{\mathbf{V}_{s2}}{n} \qquad \mathrm{Z}_{th} = \frac{\mathbf{Z}_2}{n^2}$$



Exercise

Find $v(t)$ in the following circuit.



Overview

Three-Phase Circuits

Magnetically Coupled Circuits

Frequency Response

Frequency Response

Motivation: previously we analyzed all the AC circuits with frequency ω fixed. Now we study how a circuit's behavior varies with change in frequency. This variation is so-called the **frequency response**.

Specifically, we want to study the ratio of two phasor signals, as a function of frequency ω, i.e. the **transfer function**

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

Transfer Function

The transfer function $\mathbf{H}(\omega)$ can always be expressed in the ratio of two polynomials. Also, actually every ω in the function takes with a j (no need to worry about why in VE215). So often we let the variable be $j\omega$ instead of ω, let $s = j\omega$, we have

$$\mathbf{H}(\omega) = \mathbf{H}(j\omega) = \mathbf{H}(s) = \frac{\mathbf{Y}(s)}{\mathbf{X}(s)}$$

Note the equivalency of these notations.

- Zeros: values of $s = j\omega$ that make $\mathbf{H}(s) = 0$ ($\mathbf{Y}(s) = 0$)
- Poles: values of $s = j\omega$ that make $\mathbf{H}(s) = \infty$ ($\mathbf{X}(s) = 0$)

Exercise: Transfer Function

For the RC circuit in Fig.14.2(a), obtain the transfer function V_o/V_s.

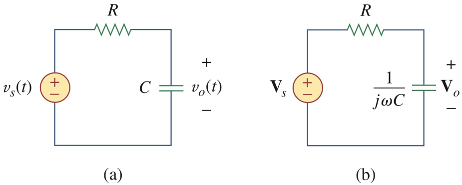


Figure 14.2
For Example 14.1: (a) time-domain *RC* circuit, (b) frequency-domain *RC* circuit.

Exercise: Transfer Function

First convert the time-domain circuit to **frequency-domain** circuit.

Using voltage division, we obtain transfer function:

$$\mathbf{H}(\omega) = \frac{V_o}{V_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

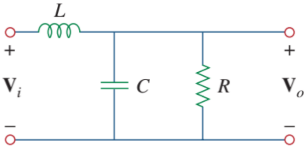
Then we obtain the magnitude and phase

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \quad \phi = -tan^{-1}\frac{\omega}{\omega_0}$$

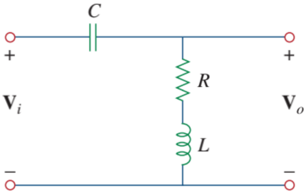
where $\omega_0 = 1/RC$

Exercise: Transfer Function

Find the transfer function $\mathbf{H}(\omega) = V_o/V_i$ of the circuits below.



(a)



(b)

Exercise: Transfer Function

A.

$R||\frac{1}{j\omega C} = \frac{R}{1+j\omega RC}$

$$\mathbf{H}(\omega) = \frac{V_o}{V_i} = \frac{R/(1+j\omega RC)}{j\omega L + R/(1+j\omega RC)} = \frac{R}{-\omega^2 RLC + R + j\omega L}$$

B.

$$\mathbf{H}(\omega) = \frac{V_o}{V_i} = \frac{R + j\omega L}{R + j\omega L + 1/j\omega C} = \frac{-\omega^2 LC + j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

Bode Plots

$\mathbf{H}(\omega)$ is definitely a complex number $\mathbf{H}(\omega) = H(\omega)\angle\phi(\omega)$, and can be written as $\ln\mathbf{H} = \ln H + j\phi$. The real part is a function of the magnitude while the imaginary part is the phase. So we can analyze its magnitude and phase separately, i.e.

- $H(\omega)$: magnitude frequency response
- $\phi(\omega)$: phase frequency response

by drawing the approximated diagrams of $H - \omega$ and $\phi - \omega$, respectively. This paired plots are so-called **Bode plots**.

For convenience, the Bode magnitude plot $H - \omega$ uses decibels for the vertical axis,

$$H_{dB} = 20\log_{10} H$$

Bode Plots

To draw a bode plot:

Step 1 Write the targeted transfer function as the standard form

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)(1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2)\cdots}{(1 + j\omega/p_1)(1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2)\cdots}$$

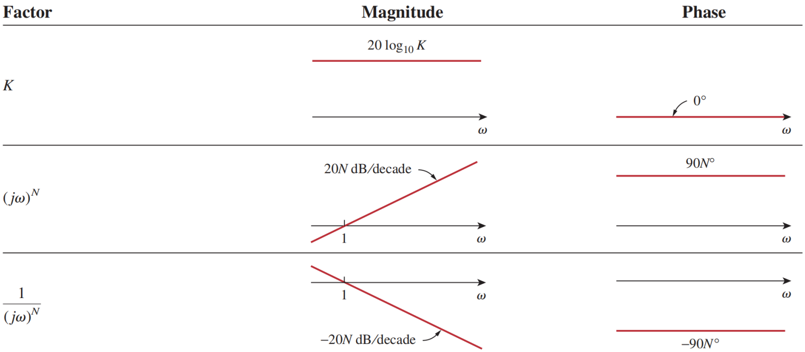
Then we will identify these kinds of factors:

1. gain/constant term: K
2. zero/pole at the origin: $(j\omega)^N$ or $\frac{1}{(j\omega)^N}$
3. simple zero/pole: $(1 + j\omega/z)^N$ or $\frac{1}{(1 + j\omega/p)^N}$
4. quadratic zero/pole: $[1 + \frac{2j\omega\zeta}{\omega_n} + (\frac{j\omega}{\omega_n})^2]^N$ or $\frac{1}{[1 + 2j\omega\zeta/\omega_n + (j\omega/\omega_n)^2]^N}$

Bode Plots

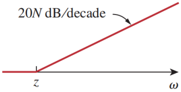
Step 2 Following the summary table, superpose the plots for every factor of $\mathbf{H}(\omega)$, to obtain the magnitude plot

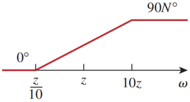
Step 3 Following the summary table, superpose the plots for every factor of $\mathbf{H}(\omega)$, to obtain the phase plot



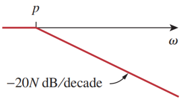
Bode Plots

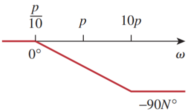
$$\left(1 + \frac{j\omega}{z}\right)^N$$



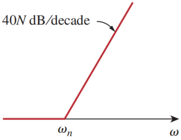


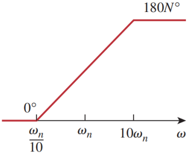
$$\frac{1}{(1 + j\omega/p)^N}$$



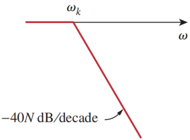


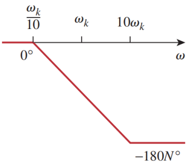
$$\left[1 + \frac{2j\omega\zeta}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right]^N$$





$$\frac{1}{[1 + 2j\omega\zeta/\omega_k + (j\omega/\omega_k)^2]^N}$$





Bode Plots

Tips on Summary Table:

- zeros: upward turn
- poles: downward turn
- for the factor constant K, be careful with the phase angle.

$$\phi = \begin{cases} 0^\circ & \text{K}>0 \\ \pm 180^\circ & \text{K}<0 \end{cases}$$

- Be careful with N. It influences the slope.
- for simple/quadratic poles/zeros: straight-line approximation
- for quadratic poles/zeros: can be treated as a double pole/zero

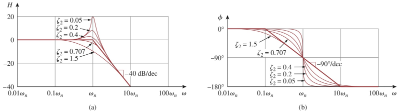


Figure 14.12
Bode plots of quadratic pole $[1 + j2\zeta\omega/\omega_n - \omega^2/\omega_n^2]^{-1}$: (a) magnitude plot, (b) phase plot.

Bode Plots

Steps:

1. Convert the transfer function to the standard form.
2. Get equation of magnitude and phase. (optional)
3. Use the summary table to sketch magnitude and phase plots.
   - dotted lines for each factor
   - solid line for final result

Label:

- x-label
- slope / H_{dB} and ϕ coordinate label

Exercise: Bode Plots

Draw the bode plots of

$$\mathbf{H}(\omega) = \frac{200j\omega}{(j\omega + 2)(j\omega + 10)}$$

Exercise: Bode Plots

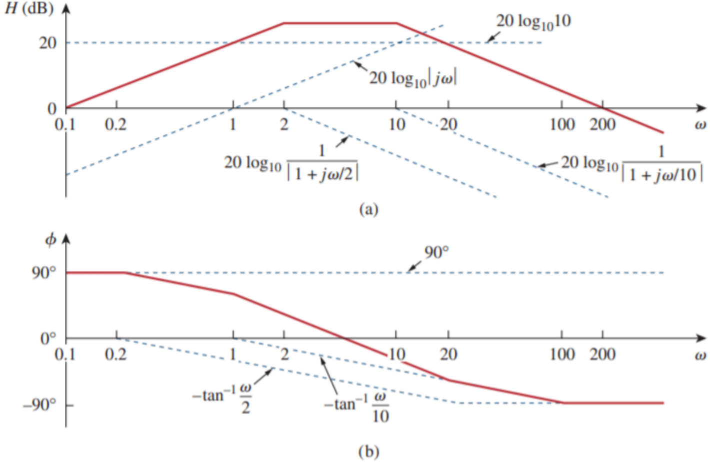
First, write it in the standard form:

$$
\begin{aligned}
\mathrm{H}(\omega) &= \frac{10j\omega}{(1+j\omega/2)(1+j\omega/10)} \\
&= \frac{10|j\omega|}{|1+j\omega/2||1+j\omega/10|}\angle 90^\circ - \tan^{-1}\omega/2 - \tan^{-1}\omega/10
\end{aligned}
$$

Hence, the magnitude and phase are:

$$
\begin{aligned}
H_{dB} &= 20\log_{10}10 + 20\log_{10}|j\omega| - 20\log_{10}|1+\frac{j\omega}{2}| - 20\log_{10}|1+\frac{j\omega}{10}| \\
\phi &= 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}
\end{aligned}
$$

Exercise: Bode Plots



Resonance

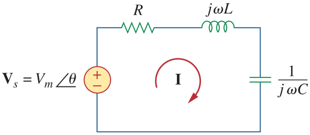
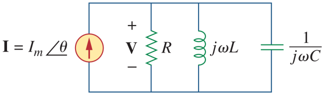
Resonance happens in an RLC circuit (series or parallel), when the capacitive and inductive reactances of the circuit are equal.
When resonance happens,

- The total impedance is purely resistive, $\mathrm{Im}(Z) = 0$.
- Source voltage **V** and current **I** are in phase.
- The magnitude of current, $|\mathbf{I}|$, is maximum.
  (since $|\mathbf{Z}|$ is minimum in $|\mathbf{I}| = \frac{|\mathbf{V}|}{|\mathbf{Z}|}$)
- The power P dissipated on the load is maximum.
  (since $P = |\mathbf{I}|^2 R$)

With a given RLC circuit, our main interest is to find the frequency ω_0 to achieve the resonance condition.

Resonance

- Q: quality factor, a measure of the circuit's energy storage capacity
- ω_1, ω_2: half-power frequencies, the frequencies at which the power dissipated is half of the maximum
- $B = \omega_2 - \omega_1$: bandwidth

	Series RLC	Parallel RLC
Diagram		
ω_0	$1/\sqrt{LC}$	$1/\sqrt{LC}$
Q	$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$	$= \frac{R}{\omega_0 L} = \omega_0 RC$
B	ω_0/Q	ω_0/Q
ω_1, ω_2	$\omega_0\sqrt{1+(\frac{1}{2Q})^2} \pm \omega_0/2Q$	$\omega_0\sqrt{1+(\frac{1}{2Q})^2} \pm \omega_0/2Q$

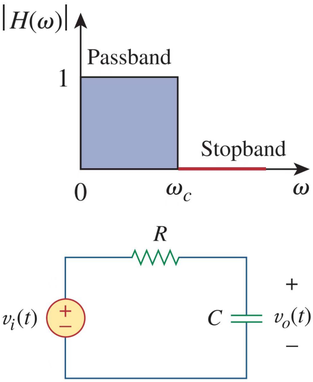
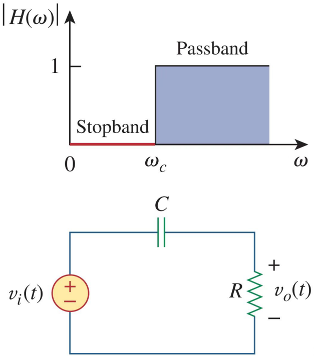
Filters

A **filter** is a circuit that pass signals with desired frequencies and reject others.
(There is no need to know too much about the mechanism of filters in VE215. It is a topic of VE216. In this section, memorizing various types of filters and their formulas is enough.)

Two types of filters

- Passive filters (built with RC or RLC circuit)
- Active filters (built with R + C + op-amp)

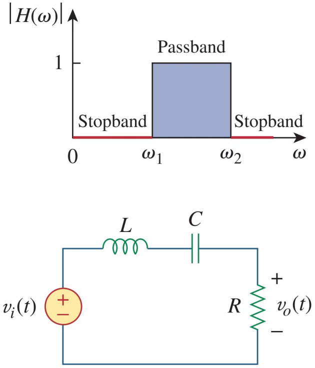
Filters: Passive Filters

Lowpass filter	Highpass filter
	
$\omega_c = 1/RC$	$\omega_c = 1/RC$

Their difference is on where the output is connected across.

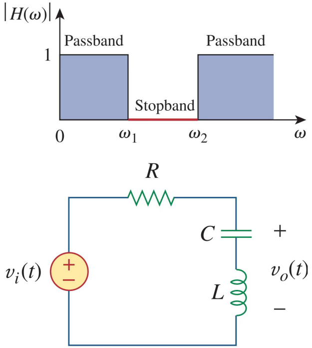
Filters: Passive Filters

Bandpass filter



$$\omega_0 = 1/\sqrt{LC}$$

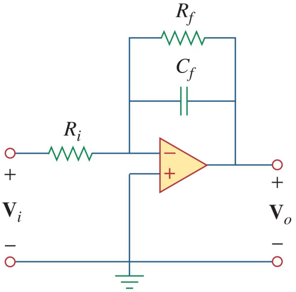
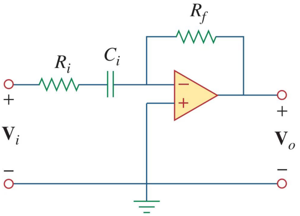
Bandstop filter



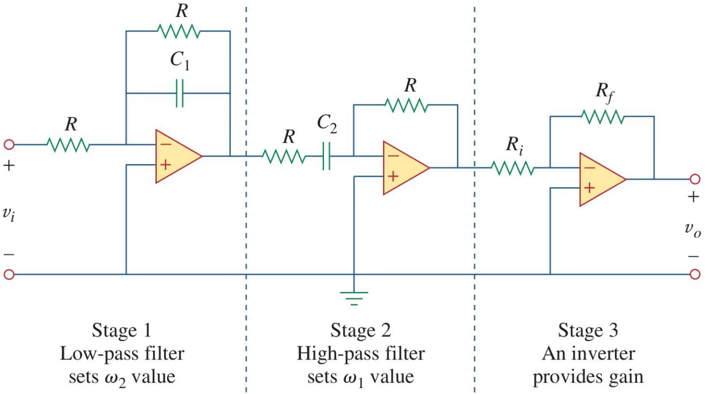
$$\omega_c = 1/\sqrt{LC}$$

Their difference is on where the output is connected across.

Filters: Active filters

<table>
<tr><th>1st-order lowpass filter</th><th>1st-order highpass filter</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$\omega_c = 1/R_f C_f$</td><td>$\omega_c = 1/R_i C_i$</td></tr>
</table>

Filters: Active filters



$$\omega_1 = 1/RC_1, \omega_2 = 1/RC_2$$

References

1. 2023 Summer VE215 slides, Rui Yang
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2022 Fall VE215 Final RC, Zhiyu Zhou, Yuxuan Peng, Yifei Cai

Good luck on your final exam!