VE215 RC5

Erdao Liang, Chongye Yang

UM-SJTU JI

July 13, 2023

Overview

Instantaneous and Average Power

Both $v(t)$ and $i(t)$ here are instantaneous values. (not rms)

$$p(t) = v(t) \cdot i(t)$$

Instantaneous power:

$$\begin{aligned} p(t) &= V_m I_m cos(\omega t + \theta_v) cos(\omega t + \theta_i) \\ &= \frac{1}{2} V_m I_m cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m cos(2\omega t + \theta_v + \theta_i) \end{aligned}$$

Average Power

General definition:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

For sinusoids,

$$P = \frac{1}{2}V_m I_m cos(\theta_v - \theta_i)$$

Represented with phasor $\tilde{V}$ and $\tilde{I}$,

$$P = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^{*})$$

Average Power

- when $\theta_v - \theta_i = 0$, purely resistive load R.

$$P = \frac{1}{2} I_m^2 R = \frac{1}{2} \frac{V_m^2}{R}$$

(the second equality not true when $\theta_v - \theta_i \neq 0$)

- when $|\theta_v - \theta_i| = 90^\circ$, purely reactive load X. It absorbs no average power.
- Generally,

$$\begin{aligned} P &= \frac{1}{2} V_m I_m cos(\theta_v - \theta_i) = \frac{1}{2} \mathrm{Re}(\tilde{V} \tilde{I}^*) = \frac{1}{2} \mathrm{Re}((\tilde{I} Z) \tilde{I}^*) \\ &= \frac{1}{2} \mathrm{Re}(I^2 R + j I^2 X) = \frac{1}{2} I_m^2 R \end{aligned}$$

(Only resistance contributes to the average power absorbance)

Maximum Average Power Transfer

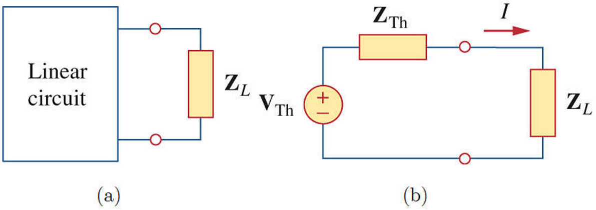


Figure 11.7 Finding the maximum average power transfer (a) circuit with a load, (b) the Thevenin equivalent.

- If there is no restriction on Z_L,

$$R_L = R_{Th} \quad X_L = -X_{Th} \quad P_{max} = \frac{|V_{Th}^2|}{8R_{Th}}$$

- If Z_L is purely resistive,

$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2}$$

Effective or RMS Value

Definition: The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current (similarly for voltage).

$$I_{eff}^2 R = \frac{R}{T}\int_0^T i^2 dt$$

Effective value = RMS (root mean square) value.

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \mathrm{d}t} = I_{eff}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 \mathrm{d}t} = V_{eff}$$

Effective or RMS Value

- Avg power absorbed by a circuit element (generally true):

$$P = I_{rms}^{R} = V_{rms}^{2} \frac{R}{R^2 + X^2}$$

$$P = \frac{1}{2}\mathrm{Re}(\tilde{V}\tilde{I}^*) = \mathrm{Re}(V_{\tilde{rms}} I_{\tilde{rms}}^{*}) = \frac{1}{2} I_m^2 R = I_{rms}^2 R = \frac{1}{2} V_m^2 \mathrm{Re}(\frac{1}{Z^*}) = V_{rms}^2 \frac{R}{R^2 + X^2}$$

- RMS value for a sinusoid sinusoid:

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{V_m}{\sqrt{2}}$$

- Average power absorbed by an element in a sinusoidal circuit:

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Effective or RMS Value

Caution: from now on, unless specified, all values will be assumed to be RMS values.

Complex Power

We define this "complex power" that includes all the information.

$$
\begin{aligned}
\text{Complex Power} = \tilde{S} = \tilde{V_{rms}} \tilde{I_{rms}^{*}} &= |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) \\
&= |S|\angle(\theta_v - \theta_i) \text{ (polar form)} \\
&= P + jQ \text{ (rectangular form)}
\end{aligned}
$$

Value	Name	Meaning	Unit
$\lvert S \rvert$	Apparent power	Magnitude of $\tilde{S}$	VA
$\cos(\theta_v - \theta_i)$	Power factor	Cosine of angle of $\tilde{S}$	/
P	Real power	Real part of $\tilde{S}$	W
Q	Complex power	Imaginary part of $\tilde{S}$	VAR

Complex Power

- Complex power

$$\tilde{S} = \tilde{V_{rms}} \tilde{I^{*}_{rms}} = |I_{rms}||V_{rms}|\angle(\theta_v - \theta_i) = |S|\angle(\theta_v - \theta_i) = P + jQ$$

$$\tilde{S} = |I_{rms}|^2 Z = \frac{|V_{rms}|^2}{Z^*}$$

- Apparent power

$$|S| = |V_{rms}||I_{rms}| = |I_{rms}|^2 |Z| = \sqrt{P^2 + Q^2}$$

- Real power

$$P = \mathrm{Re}(\tilde{S}) = |S| \cos(\theta_v - \theta_i) = |I_{rms}|^2 R$$

- Complex power

$$Q = \mathrm{Im}(\tilde{S}) = |S| \sin(\theta_v - \theta_i) = |I_{rms}|^2 X$$

- Power factor

$$\mathrm{pf} = \frac{P}{|S|} = \cos(\theta_v - \theta_i)$$

Complex Power

Power factor pf $= \frac{P}{|S|} = \cos(\theta_v - \theta_i)$:

- $\theta_v - \theta_i < 0$: leading pf
- $\theta_v - \theta_i > 0$: lagging pf

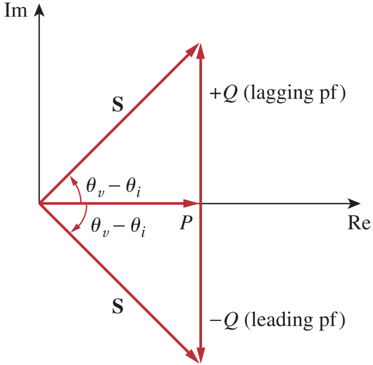
Since $\cos(\theta_v - \theta_i) = \cos(\theta_i - \theta_v)$, the pf value only tells part of the story. Every time you are asked for a power factor, you must declare whether it is leading or lagging.

Complex Power

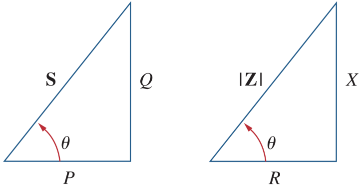
We can use the sign of pf angle or Q to identify the property of the circuit and the loads:

	(1)	(2)	(3)
pf Angle	$\theta_v - \theta_i = 0$	$\theta_v - \theta_i < 0$	$\theta_v - \theta_i > 0$
Sign of Q	$Q = 0$	$Q < 0$	$Q > 0$
Properties	Unity pf I, V in phase $X = 0$ Resistive loads	Leading pf I leads V $X < 0$ Capacitive loads	Lagging pf I lags V $X > 0$ Inductive loads

Complex Power



And observe that the power factor angle is equal to the angle of the impedance of that part of the circuit.



Exercise 1

Question:
A series-connected load draws a current $i(t) = 4cos(100\pi t + 10^{\circ})A$ when the applied voltage is $v(t) = 120cos(100\pi t - 20^{\circ})V$. Find the apparent power and the power factor of the load. Determine the element values that form the series-connected load.

Exercise 1

Solution:
The apparent power is

$$S = V_{\text{rms}} I_{\text{rms}} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$$

The power factor is

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20° - 10°) = 0.866 \qquad \text{(leading)}$$

The pf is leading because the current leads the voltage. The pf may also be obtained from the load impedance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle -20°}{4\angle 10°} = 30\angle -30° = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30°) = 0.866 \qquad \text{(leading)}$$

The load impedance $\mathbf{Z}$ can be modeled by a 25.98-Ω resistor in series with a capacitor with

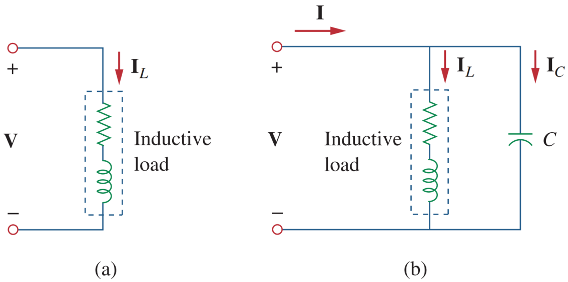
$$X_C = -15 = -\frac{1}{\omega C}$$

or

$$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$$

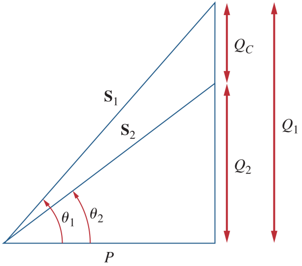
Power Factor Correction

- Goal: increase the pf of a load $\rightarrow$ make it less inductive $\rightarrow$ reduce energy loss
- Solution: add a capacitor in parallel to the load



Power Factor Correction

Goal: increase the pf from $\cos\theta_1$ to $\cos\theta_2$.



Initial:
$P = |S_1|\cos\theta_1$
$Q_1 = |Q_1|\sin\theta_1 = P\tan\theta_1$

The "after" we expect:
$P = |S_2|\cos\theta_2$
$Q_2 = |Q_2|\sin\theta_2 = P\tan\theta_2$

Since $Q_c (= Q_1 - Q_2) = \frac{V_{rms}^2}{X_c}$, then the value of the required capacitance C is

$$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{Q_2 - Q_1}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

Exercise 2

Question:
When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Exercise 2

Question:
When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the pf to 0.95.

Formula: $C = \dfrac{P \tan(\theta_1 - \theta_2)}{\omega \cdot V_{rms}^2}$

Answer: 310.5 μF

References

1. 2023 Summer VE215 slides, Rui Yang
2. Fundamentals of Electric Circuits, 5th e, Sadiku, Matthew
3. 2022 Fall RC5, Yuxuan Peng
4. 2022 Fall RC6, Zhiyu Zhou

Thank you!