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Date:

HW2

1. 1) $XA + A^T = I$

$XA = I - A^T$

$X = A^{-1} - A^T A^{-1}$

2) $X^T C = [2A(X+B)]^T = 1$

$\cancel{X^T C = (X+B)^T 2A^T}$

$X^T C = [2AX + 2AB]^T$

$= (2AX)^T + (2AB)^T$

$= X^T (2A)^T + (2AB)^T$

$X^T C - X^T (2A)^T = (2AB)^T$

$X^T (C - 2A^T) = (2AB)^T$

$X^T = \cancel{(C-2A^T)^{-1}} (2AB)^T (C - 2A^T)^{-1}$

$X = \cancel{((C-2A^T)^{-1} (2AB)^T)^T} ((2AB)^T (C-2A^T)^{-1})^T$

$= \cancel{2(AB)^T ((C-2A^T)^{-1})^T} \cancel{2A} ((C-2A^T)^{-1})^T 2AB$

3) $(Ax-y)^T A = 0$

$(A^T A)$ must be invertible. Since its a scalar,

$\cancel{(Ax-y)^T A A^{-1} = 0}$

$\cancel{(Ax-y)^T = 0}$

$\cancel{Ax - y = 0}$

$\cancel{x = A^{-1} y}$

$A^T (Ax - y) = 0$

$A^T A x - A^T y = 0$

$A^T A x = A^T y$

$x = (A^T A)^{-1} A^T y$

4) $\cancel{(Ax-y)^T A + x^T B = 0}$

$\cancel{(Ax-y)^T + x^T B A^{-1} = 0}$

$\cancel{(Ax)^T - y^T + x^T B A^{-1} = 0}$

$\cancel{x^T A + x^T (BA^{-1}) = y^T}$

$\cancel{x^T (A + BA^{-1}) = y^T}$

$\cancel{x = ((A + BA^{-1})^{-1})^T y}$

$(Ax - y)^T A + x^T B = 0$

$A^T (Ax - y) + B^T x = 0$

$A^T A x + B^T x = A^T y$

$(A^T A + B^T) x = A^T y$

$\therefore x = ((A^T A + B^T)^{-1}) A^T y$

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2 ~~$\nabla f(x) =$~~ Let $\nabla f(x)$ be gradient of $f$ at $x$ where $x \in \mathbb{R}^n$

~~$\nabla f(x)$~~ $v$ be a ~~random~~ unit vector $v \in \mathbb{R}^n$

$\Delta f(x)$ in a direction $v$ is dot product of $\nabla f(x)$ & $v$

$\nabla f(x) \cdot v = |\nabla f(x)||v| \cos\theta$

since $|\nabla f(x)|$ and $|v|$ are constants, the greatest $\Delta f(x)$ achieved is when $\cos\theta = 1$ or when $v$ & $\nabla f(x)$ are in the same direction #QED